



UNIVERSIDADE TÉCNICA DE LISBOA
INSTITUTO SUPERIOR TÉCNICO

Electromagnetic Waves in Artificial Media with Application to Lens Antennas

Mário Gonçalo Mestre Veríssimo Silveirinha
(Licenciado)

Dissertação para obtenção do Grau de Doutor em Engenharia
Electrotécnica e de Computadores

Orientador: Doutor Carlos António Cardoso Fernandes

Presidente: Reitor da Universidade Técnica de Lisboa

Vogais: Doutor Ari Henrik Sihvola
Doutor Ricardo Marqués Sillero
Doutor Henrique José de Almeida e Silva
Doutor Carlos António Cardoso Fernandes
Doutor Carlos Manuel dos Reis Paiva

Abril 2003

Abstract

In recent years there has been an increasing interest in the propagation of electromagnetic waves in artificial materials. Artificial materials consist of a mixture of metallic or dielectric particles in a host medium. It has been shown that artificial materials may significantly improve the performance of antennas, microwave filters and other devices. The characterization and study of these “engineered” materials is thus of increasing relevance.

This thesis comprises a systematic study of electromagnetic waves in artificial materials, and its application to dielectric lens antennas.

We examine the homogenization problem in periodic materials. A new method is proposed to compute the effective parameters of artificial crystals at the long wavelength limit.

The band structure problem in metallic-dielectric crystals is studied. We propose a new hybrid formalism to compute the band structure of periodic media.

The radiation from point sources in metallic crystals is examined. The asymptotic form of the radiated fields in the far field region is derived.

The artificial dielectric concept is put into practice to design artificial material lenses. Several lens configurations with inhomogeneous anisotropic substrate are studied. A new formalism is presented to compute the lens permittivity profile that ensures that the output beam matches a specified target.

Key Words: artificial media, PBG material, homogenization theory, periodic Green functions, radiation in periodic media, lens antennas.

Resumo

Nos últimos anos houve um interesse crescente na propagação de ondas electromagnéticas em meios artificiais. Estes meios consistem numa mistura de partículas metálicas ou dieléctricas embebidas num meio ambiente homogéneo. Este tipo de estruturas pode ter propriedades notáveis que geralmente não são observáveis na natureza. Vários resultados mostram que os materiais artificiais podem melhorar significativamente o desempenho de antenas, filtros para microondas, e outros dispositivos. O estudo e caracterização deste tipo de materiais é pois de crescente importância.

Nesta tese realiza-se um estudo sistemático sobre meios artificiais e investiga-se a sua aplicação em antenas baseadas em lentes.

Primeiro estuda-se o problema de homogeneização em materiais periódicos. É proposto um método para calcular os parâmetros efectivos de cristais artificiais no limite de grandes comprimentos de onda.

Em seguida, estuda-se a estrutura de bandas de cristais dieléctricos ou metálicos. Propõe-se um método híbrido para o cálculo numérico da estrutura de bandas.

A radiação de fontes pontuais elementares é examinada. É deduzida a forma assintótica dos campos radiados na zona distante.

O conceito de dieléctrico artificial é posto em prática para projectar lentes cujo substrato é um material artificial. São examinadas várias configurações de lentes com substrato não uniforme e anisotrópico.

Palavras-chave: meios artificiais, materiais PBG, teoria de homogeneização, funções de Green periódicas, radiação em meios periódicos, antenas com lentes.

To my parents

Acknowledgments

To my supervisor, Professor Carlos António C. Fernandes, for his encouragement, support and guidance.

To *Instituto de Telecomunicações Coimbra-Lisboa*, for the excellent work conditions and for the financial support.

To *Fundação para Ciência e Tecnologia*, for the support under project POSI 34860/99.

To Professor Luís Vieira de Sá, for the support at the University of Coimbra.

To my Parents and family.

Contents

1. INTRODUCTION	1
1.1 Overview	1
1.2 Organization of the Thesis	3
1.3 State of the Art and Main Contributions	4
1.3.1 Homogenization of artificial materials	4
1.3.2 Periodic Green functions	9
1.3.3 Band structure of artificial materials	13
1.3.4 Radiation from point sources	19
1.3.5 Characterization of materials	22
1.3.6 Dielectric lens antennas	23
1.3.7 Non-homogeneous lenses	26
2. HOMOGENIZATION OF ARTIFICIAL MATERIALS	39
2.1 Homogenization of Maxwell-Equations	40
2.1.1 Maxwell-Equations in a composite medium	40
2.1.2 Macroscopic Maxwell-Equations	43
2.1.3 Propagation in the disk-type medium	45
2.2 Effective Permittivity of a Crystal	48
2.2.1 Geometry of dielectric crystals	49
2.2.2 Periodic electric field in a dielectric crystal	49
2.2.3 Effective permittivity dyadic	50
2.2.4 Media with metallic implants	52
2.2.5 Metallic crystals	53
2.2.6 High-volume fraction example	53
2.3 The Effective Permittivity Problem – Integral Equation Based Method	56
2.3.1 Integral representation of the electric potential	57
2.3.2 The Green function	60
2.3.3 Low-volume fraction limit	62
2.4 Computation of the Effective Permittivity of Metallic Crystals	64
2.4.1 Planar inclusions	65
2.4.2 Disk inclusions	67
2.4.3 Numerical results – planar inclusions	69
2.4.4 Wire inclusions	74
2.4.5 Numerical results – wire medium	76
2.4.6 Cylindrical inclusions	78
2.5 Effective Permeability of a Metallic Crystal	80
2.5.1 Periodic induction field	81
2.5.2 Effective permeability dyadic	83

2.5.3	High-volume fraction example	84
2.5.4	Integral equation based formulation	85
2.5.5	Low-volume fraction limit	86
2.6	Computation of the Effective Permeability of Metallic Crystals	87
2.6.1	Integral equation for planar inclusions	87
2.6.2	Planar inclusions	88
2.6.3	Disk inclusions	90
2.6.4	Numerical results – planar inclusions	91
2.6.5	Cylindrical inclusions and Wire inclusions	93
2.7	Metallic Crystals with a Stratified Dielectric Host	94
2.7.1	Geometry of the problem	95
2.7.2	Homogenization of the host medium	96
2.7.3	Effective permittivity	96
2.7.4	Integral equation based formulation	99
2.7.5	The Green function	101
2.8	The Two-Layer Periodic Host Case	103
2.8.1	Geometry of the host medium	103
2.8.2	The homogeneous equation	104
2.8.3	The Floquet mode solutions	105
2.8.4	Amplitude of the spectral potential	107
2.8.5	The metallic crystal	108
2.8.6	A particular case	108
2.8.7	Numerical results – planar inclusions	110
3.	PSEUDO-PERIODIC GREEN FUNCTIONS	119
3.1	Fundamental Solution of the Helmholtz’s Equation	120
3.2	The Layer Green Function	121
3.2.1	The spatial and spectral representations	122
3.3	The Lattice Green Function	123
3.3.1	The spatial and spectral representations	123
3.3.2	The spectral-like representation	125
3.3.3	The mixed-domain representations	128
3.3.4	Hyperbolic tangent	130
3.3.5	Error function	131
3.3.6	Representation for the layer Green function	132
3.3.7	Numerical simulations	133
3.4	The Periodic Green Function	137
3.4.1	Singularities of the lattice Green function	137
3.4.2	The spectral-like representation	138
3.4.3	The mixed-domain representations	139
3.4.4	The spatial representation	140
3.5	Stratified Dielectric Crystal	144
3.5.1	Geometry	144

3.5.2	The electric potential	145
3.5.3	The spectral potential	146
3.5.4	The homogenized medium	148
3.5.5	Some properties of the spectral potential	149
3.5.6	The electric potential in the far field region	152
3.6	The Periodic Green Function – The Stratified Case	154
3.6.1	Definition	154
3.6.2	The layer Green function	156
3.6.3	The lattice Green function	157
3.6.4	The periodic Green function	159
3.7	Integral Operators and Representations of Floquet Waves	163
3.7.1	Single- and double-layer Floquet potentials	164
3.7.2	Representations of scalar potentials	165
3.7.3	Pseudo-periodic vector potentials	166
3.7.4	Stratton-Chu formulas for pseudo-periodic fields	167
3.7.5	Representation of a static pseudo-periodic vector field	168
4.	BAND STRUCTURE OF PERIODIC MEDIA	171
4.1	Two-dimensional Metallic Crystals	173
4.1.1	Geometry	174
4.1.2	Classification of the electromagnetic modes	174
4.1.3	Integral representation of the Floquet waves	176
4.2	Hybrid Method - 2D Metallic Crystals	178
4.2.1	The extended problem	178
4.2.2	The integral-differential eigensystem	180
4.2.3	Band structure of the integral-differential eigensystem	182
4.2.4	Band structure of the metallic crystal	183
4.2.5	Numerical results	185
4.3	Hybrid Method of Higher Order	188
4.3.1	The integral-differential eigensystem	188
4.3.2	Band structure of the integral-differential eigensystem	190
4.3.3	Convergence rate	191
4.4	The Static Limit – 2D Metallic Case	192
4.4.1	H-polarization	192
4.4.2	E-polarization	195
4.5	Hybrid Method - 2D Dielectric Crystals	197
4.5.1	Geometry and formulation	197
4.5.2	The extended problem	199
4.5.3	The integral-differential eigensystem	202
4.5.4	Band structure of the dielectric crystal	203
4.5.5	Numerical results	205
4.6	Three-dimensional Metallic Crystals	209
4.6.1	Geometry	209

4.6.2	Integral representation of the Floquet mode	210
4.6.3	Example: Wire medium	212
4.7	Hybrid Method - 3D Metallic Crystals	213
4.7.1	The extended problem	214
4.7.2	The auxiliary field	215
4.7.3	On the regularity of the auxiliary field	216
4.7.4	Calculation of the auxiliary field	217
4.7.5	The integral-differential eigensystem	219
4.7.6	Discretization of the integral-differential eigensystem	220
4.7.7	The bilinear forms	222
4.7.8	Assembly of the generalized matrix eigensystem	223
4.7.9	Solution of the generalized eigensystem	224
4.7.10	Band structure of the metallic crystal	226
4.8	Disk-Type and Wire Media	226
4.8.1	The disk-type medium	227
4.8.2	The wire medium	235
4.9	The Static Limit - 3D Metallic case	239
4.9.1	Homogenization of the Floquet modes	239
4.9.2	The fundamental bands	241
4.9.3	Numerical examples	243
5.	RADIATION FROM A POINT SOURCE IN A METALLIC CRYSTAL	249
5.1	A Point Charge in a Metallic Crystal	251
5.1.1	The array scanning method	251
5.1.2	The Floquet modes	252
5.1.3	The fundamental band	254
5.1.4	The aperiodic potential in the far field region	256
5.1.5	The electric field in the far field region	260
5.1.6	Physical interpretation of the results	262
5.1.7	Numerical example	263
5.2	A Short Electric Dipole in a Metallic Crystal	264
5.2.1	The aperiodic electric Green dyadic	265
5.2.2	The periodic electric Green dyadic	266
5.2.3	Solution of the aperiodic problem – Slight losses	269
5.2.4	The radiation field and the unphysical modes	270
5.2.5	Solution of the aperiodic problem – Lossless case	271
5.2.6	The radiated power	272
5.3	The Asymptotic Fields	273
5.3.1	An auxiliary result	273
5.3.2	Asymptotic form of the aperiodic Green dyadic	276
5.3.3	Poynting vector	278
5.3.4	Power flow and energy of an electromagnetic Floquet mode	278
5.3.5	Generalized orthogonalization condition	280
5.3.6	The average Poynting vector and the radiation pattern	282
5.3.7	Numerical examples	283

5.4	Radiation from a Line Source in a Three-dimensional Metallic Crystal	290
5.4.1	Geometry and formulation	290
5.4.2	The radiated field	291
5.4.3	The asymptotic electric field	293
5.4.4	Poynting vector	293
5.4.5	Numerical examples	294
6.	CHARACTERIZATION OF MATERIALS	299
6.1	Geometric method for the determination of the dielectric constant	301
6.1.1	Formulation	302
6.1.2	Simplification of the formulation	304
6.1.3	Calculation of the complex permittivity	304
6.1.4	Limitations and refinement of the solution	307
6.1.5	Experimental results	308
6.2	Equivalence between propagation in a rectangular waveguide and propagation in a metallic crystal	310
6.2.1	Waveguide geometry	311
6.2.2	Classification of the electromagnetic waves in the loaded waveguide	312
6.2.3	The equivalent unbounded periodic structure	314
6.2.4	Projection of Floquet solutions into waveguide modes	315
6.2.5	Dispersion characteristic of a periodically loaded waveguide	316
6.2.6	Illumination of the loaded waveguide with the fundamental mode	317
6.3	Characterization of the Disk-Medium	318
7.	GEOMETRICAL OPTICS	323
7.1	From Maxwell's Equations to Geometrical Optics	324
7.1.1	The Debye expansion and Fresnel's differential equation	325
7.1.2	The geometrical optics fields	326
7.1.3	The Poynting vector	327
7.1.4	Consistency of the asymptotic expansion	328
7.1.5	The Fresnel surface of wave normals	329
7.1.6	The Fresnel surface of rays	330
7.1.7	Relations between the two Fresnel surfaces	330
7.2	Integration of the Geometrical Optics Equations	331
7.2.1	Integration of the eikonal	331
7.2.2	Conservation of the energy flux in a ray tube	334
7.2.3	The transfer equation	335
7.3	Geometrical Optics in an Isotropic Medium	336
7.3.1	The transfer equation in an isotropic medium	337
7.3.2	Rotation of the field vectors (Rytov's law)	339
7.3.3	The homogeneous isotropic medium	340
7.4	Radiation from a Point Source	341
7.4.1	Radiation problems and geometrical optics	341
7.4.2	Radiation from a point source in an anisotropic medium	342

7.4.3	The radiated fields over a given surface	343
7.4.4	Radiation from a bounded structure standing in free-space	344
7.5	Reflection and Refraction at an Interface	346
7.5.1	Incidence at an interface	346
7.5.2	Refraction of a bundle of rays	347
7.5.3	Fresnel coefficients for the isotropic case	349
7.5.4	Fresnel coefficients for an anisotropic uniaxial medium	351
7.5.5	Curvatures of the reflected and refracted wave fronts in an isotropic homogeneous medium	353
8.	DIELECTRIC LENS ANTENNAS	359
8.1	Multi-Shell Lenses	361
8.1.1	Shaped beam applications	361
8.1.2	Lens geometry	362
8.1.3	Lens radiation pattern	364
8.1.4	The exit angle	366
8.2	Equations of Synthesis	368
8.2.1	Synthesis formulas	368
8.2.2	The first interface	371
8.2.3	The second interface	371
8.2.4	Geometrical Optics fields	372
8.3	Design of Dielectric Lenses	373
8.3.1	Criterion for maximum power transfer	373
8.3.2	The H-polarization case	375
8.3.3	The general polarization case	376
8.3.4	Computation of the lens index profile	377
8.3.5	Dielectric losses	377
8.4	Radiation Pattern	378
8.4.1	Physical Optics radiation pattern	379
8.4.2	Asymptotic form of the Physical Optics radiation pattern	380
8.4.3	Diffraction effects	384
8.5	Multiple Reflections	385
8.5.1	Application of the SBR method to multi-shell lenses	386
8.5.2	Propagation of the fields along a ray	387
8.6	Numerical Simulations	389
8.6.1	Secant square target	390
8.6.2	Internal reflections – Validation example for the secant square target	395
8.6.3	Internal reflections – Secant square target	399
8.6.4	Flat top target	402
8.6.5	Internal reflections – Validation example for the flat top target	403
8.6.6	Internal reflections – Flat top target	404
8.6.7	Final remarks	405

9. INHOMOGENEOUS LENSES	409
9.1 Radiation in Non-Homogeneous Lenses	412
9.1.1 Lens geometry	412
9.1.2 Electromagnetic fields over the lens surface	414
9.1.3 Lens radiation pattern	415
9.1.4 Lens design	416
9.2 Cylindrical Lenses: Configuration A	417
9.2.1 Lens geometry	418
9.2.2 Propagation inside the lens – The ray initial coordinates	419
9.2.3 Propagation inside the lens – Anisotropic case	420
9.2.4 Propagation inside the lens – Isotropic case	422
9.2.5 Inverse problem – Integral Equation Method	423
9.2.6 Numerical simulations – Feed embedded in the lens	426
9.2.7 Inverse problem – Genetic Algorithm Method	432
9.2.8 Numerical simulations – Feed exterior to the lens	434
9.2.9 Artificial material realization	439
9.3 Cylindrical Lenses: Configuration B	442
9.3.1 Lens geometry	443
9.3.2 Propagation inside the lens – Anisotropic case	444
9.3.3 Propagation inside the lens – Isotropic case	445
9.3.4 Inverse problem	446
9.3.5 Focalization	446
9.3.6 Numerical simulations	449
9.4 Final Remarks	453
10. CONCLUSIONS	457
10.1 Main Contributions	457
10.2 Future Work	460
A. PERIODIC LATTICES	463
A.1 Direct and Reciprocal Lattices	463
A.2 Some Common Lattices	465
A.2.1 Two-dimensional lattices	465
A.2.2 Simple cubic lattice and other orthogonal lattices	466
A.2.3 Face-centered cubic lattice	467
A.2.4 Body-centered cubic lattice	468
A.3 Plane Wave Expansions	468
A.3.1 Scalar fields	468
A.3.2 Vector fields with zero divergence	469
A.4 Pseudo-Periodic Delta Distribution	470

A.5	Poisson Summation Formula	472
A.6	Sub-Spaces of Periodic Lattices	472
B.	MATHEMATICAL RELATIONS	475
B.1	Abel's Integral	475
B.2	Stationary Phase Method	476
B.3	Integral Theorems for Closed Surfaces	478
B.4	Integrals Involving Chebyshev Polynomials and Bessel Functions	479
B.5	Asymptotic formulas	480
B.5.1	Formula A	480
B.5.2	Formula B	480
C.	RADIATION IN AN ANISOTROPIC MEDIUM	483
C.1	Maxwell's Equations in an Anisotropic Homogeneous Medium	483
C.2	Radiation Pattern and Impedance	484
C.2.1	Example: Short dipole	486
D.	GREEN FUNCTIONS	487
D.1	Potential from a 1-D Array of Point Sources	487
D.1.1	3D case	488
D.1.2	2D case	489
D.2	The Auxiliary Function G_1	489
D.3	Depolarization Dyadic for the Unit Cell Region	491
D.4	The Complex Image Method	494
D.5	Spatial Representation of the Periodic Green Function – Stratified Case	497
D.5.1	The sequence of point potentials	497
D.5.2	The difference of potential	498
D.5.3	The spatial representation	502
D.5.4	Depolarization dyadic for the unit cell region	504
E.	METALLIC CRYSTALS: THE STATIC LIMIT	507
E.1	The 2D Metallic case	507
E.1.1	The first order derivatives	507
E.1.2	The second order derivatives: The fundamental band	509
E.1.3	Static periodic fields	510
E.1.4	Effective permittivity	511

F.	IMPLEMENTATION OF THE HYBRID METHOD	513
F.1	Wire Medium	513
F.1.1	The thin-wire approximation	513
F.1.2	The matrix elements	514
F.1.3	Assembly of the eigensystem	515
F.2	The Disk-Type Medium	516
F.2.1	Expansion functions	516
F.2.2	The matrix elements	517
G.	SPHERICAL LENSES	521
G.1.1	Geometry of the lens	522
G.1.2	Trajectory of a generic ray	523
G.1.3	Inverse problem: calculation of the lens profile	523
G.1.4	Numerical simulations	525

Symbol List

a	Lattice constant
a_w	Wire radius
$\mathbf{a}_1, \dots, \mathbf{a}_3$	Primitive vectors
$\mathbf{b}_1, \dots, \mathbf{b}_3$	Reciprocal lattice primitive vectors
A_{cell}	Area of the transversal lattice
\mathcal{B}	Induction field
BZ	Brillouin zone
c	Velocity of light in vacuum
C	Generic constant
D	Domain in the 3D-space
∂D	Boundary of domain D
\mathcal{D}	Dielectric displacement
\mathbf{e}_j	Generic plane wave
\mathbf{E}	Electric field
f	Frequency
f_A	Area fraction
f_V	Volume fraction
g_j	Scalar plane wave
$\overline{\mathbf{G}}_0$	Electric Green dyadic
\mathbf{H}	Magnetic field
$\overline{\mathbf{I}}_d$	Identity dyadic
$\mathbf{I} = (i_1, i_2, \dots)$	Multi-index of integers
$\mathbf{J} = (j_1, j_2, \dots)$	Multi-index of integers
j	$\sqrt{-1}$
\mathbf{J}_c	Density of current over a metallic surface
\mathbf{k}	Wave vector
\mathbf{k}_j	$\mathbf{k}_j = \mathbf{k} + j_1 \mathbf{b}_1 + j_2 \mathbf{b}_2 + \dots$
K_1, K_2	Principal curvatures

$\overline{\mathbf{L}}$	Depolarization dyadic
n	Generic integer
n	Refraction index
\mathbf{m}	Magnetic dipole moment
\mathcal{M}	Magnetization vector
\mathbf{p}	Wave normal or “momentum”
\mathbf{p}	Electric dipole moment
\mathcal{P}	Polarization vector
\mathbf{r}	Observation point
\mathbf{r}'	Source point
\mathbf{r}_1	Lattice point
s	Radiation vector
\mathbf{S}	Poynting vector
$\mathcal{S}^{(n)}$	n -th wave normal surface
$\text{sgn}(\)$	Sign of a real number
\mathcal{T}	Transmissivity
\mathbf{u}	$\mathbf{r} - \mathbf{r}'$
$\hat{\mathbf{u}}_n$	Unit vector along the x_n -direction
U	Radiation intensity
V_{cell}	Volume of the unit cell
V_B	Volume of the Brillouin zone
x_1, \dots, x_3	Cartesian coordinates
Z_0	Impedance in vacuum
β	Free-space wave number
δ	Dirac’s distribution
$\delta_{n,m}$	Kronecker’s delta symbol
ε	Permittivity
ε_h	Permittivity of the host material
ϕ	Electric potential
Φ_0	Free-space Green function
Φ_{p0}	Periodic Green function

$\Phi_{p0,s}$	Periodic Green function for the stratified case
Φ_p	Lattice Green function
φ	Spherical coordinate
γ_f	Exit angle
λ	Wavelength
μ	Permeability
$\hat{\mathbf{v}}$	Outward unit vector
θ	Spherical coordinate
σ_c	Density of electric charge
ω	Angular frequency
Ω	Unit cell
ψ	Wave function
∇	Gradient
∇_s	Surface gradient

