Computers & Operations Research 155 (2023) 106195

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

A deterministic bounding algorithm vs. a hybrid meta-heuristic to deal with a bilevel mixed-integer nonlinear optimization model for electricity dynamic pricing

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ARTICLE INFO

Keywords:
Bi-level mixed-integer nonlinear programming
Hybrid meta-heuristic
Optimal-value-function approach
Electricity retail market
Dynamic tariffs
Demand response

ABSTRACT

In the electricity retail market, the retailer company aims to determine the optimal time-of-use (ToU) prices to maximize profits resulting from buying energy in organized (long-term, day-ahead, balancing) markets and selling it to consumers. Therefore, the retailer should take into account the consumer's demand response actions to minimize the electricity bill in face of time varying prices. In this paper, this problem is formulated as a bilevel mixed-integer nonlinear programming model in which the retailer is the leader and the consumer is the follower. The consumer's problem encompasses the integrated optimization of all home energy resources, considering re-scheduling appliance operation, charging/discharging of electric vehicle and stationary batteries, local micro-generation and (buying and selling) exchanges with the grid. The accurate physical modelling of appliance operation to generate effective load scheduling solutions imposes a high computational burden. Two algorithms are proposed to address this problem: a deterministic bounding algorithm (DBA) using an optimal-value-function approach for bilevel optimization, and a hybrid meta-heuristic using a particle swarm optimization algorithm to tackle the upper-level problem that calls an exact mixed-integer linear programming solver to deal with the lower-level problem. In the framework of the DBA, three different techniques were implemented to deal with the nonlinearities arising from the products of integer and continuous variables (bilinear terms): 1) solving the (non-convex) subproblems of DBA using a mixed-integer nonlinear solver, 2) using the McCormick envelopes to approximate the bilinear terms by linear ones, and 3) expressing the integer variables by binary ones and linearizing the bilinear terms using an exact form. Computational experiments are presented and discussed for real data settings of the problem under study considering a computational budget to compare the different algorithms and techniques employed. The results showed that the DBA with the approximate linearization technique (2) leads to the best solutions.

1. Introduction

The electricity prices in the wholesale market reflect the real-time costs for supplying energy, which are related with generation sources availability and demand patterns. Generation costs increase when plants with higher marginal cost need to operate to satisfy demand. The electricity demand in the residential sector is generally higher in the early morning and early evening (peak hours). In most developed and some developing countries, retail markets for the residential segment have been implemented in the last two decades. To strive for efficient electricity prices, tariff and technological innovation, and enhanced quality of service, retail electricity markets should present low barriers to entry for retailer companies and low barriers to switching for consumers. Retailers procure electricity in wholesale markets and then offer flat or time-differentiated (e.g., peak and off-peak) tariffs to their customers with the prices being valid for long periods (most commonly one-year contract), managing the risk involved. Retailers may profit from dynamic time-of-use (ToU) pricing strategies (i.e., prices varying with possibly higher frequency and magnitude) aimed at incentivizing consumers to reduce consumption during peak (more expensive) hours and enable a "load follows supply" paradigm fostering to increase the share of renewables in the generation mix, thus contributing for a cleaner

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https://doi.org/10.1016/j.cor.2023.106195
Received 28 March 2022; Received in revised form 12 January 2023; Accepted 15 February 2023
Available online 20 February 2023

energy system. With the evolution to smart grids, prices are expected to become more dynamic, being announced with short antecedence (e.g., one day-ahead), with potential benefits derived from inducing changes in consumption patterns for retailers (enabling to maximize profits), consumers (minimizing costs without jeopardizing quality of service), and grid operators (facilitating network management leveraged by demand response capability). Poudineh (2019) assesses the experience of liberalized retail electricity markets, and the ASSET study (European Commission, 2021) evaluates the potential of dynamic retail electricity prices.

Consumers are gaining a more active role in managing their energy resources, becoming *prosumers* (who simultaneously produce and consume energy), being able to optimize exchanges with the grid, load management, local microgeneration and storage assets. Demand response programs leveraged by dynamic retail prices enable to make the most of consumer's flexibility in the integrated utilization of all energy resources. Moglen et al. (2020) present a dynamic programming formulation to schedule demand response events to maximize savings for the retailer by shifting residential consumers' thermostatically-controlled loads from high-priced to lower-priced periods by means of internet-connected thermostats. Chanpiwat et al. (2020) propose a clustered load-profile method to classify residential consumers based on their electricity load profiles to help retailers improving the accuracy and reliability of their demand response programs.

In the residential electricity retail market, the interaction between the retailer and the prosumers has a hierarchical structure in which the retailer first sets electricity prices, with the aim of maximizing profits, and the prosumers then react to those prices by optimizing their energy resources in an integrated manner according to their comfort requirements to minimize the electricity bill. This hierarchical structure can be modeled by a bilevel (BL) optimization problem with the electricity retailer being the *leader* (upper-level decision maker) and the residential prosumers being the *followers* (lower-level decision makers). The optimal design of retail tariffs has been addressed in the literature using BL approaches with different features.

Askeland et al. (2020) formulate a bilevel model of the interaction between the consumers and a distribution system operator. The aim is to design optimal grid tariffs under decentralized decision-making and uncertainty in demand, power prices, and renewable generation to reduce aggregate network peaks thus providing flexibility as a resource to grid management. Grimm et al. (2021) analyze several electricity tariffs with flexible pricing and assess the impact on the prosumer's decisions in what concerns the electricity consumption supplied by the grid, as well as their attractiveness from the retailer's perspective. Other works were developed focusing on the optimal design of retail electricity tariffs. The studies of Mahmoudi et al. (2014) and Gärttner et al. (2018) analyze how time-differentiated electricity prices can incentivize consumers to demand response. The studies in Zugno et al. (2013), Nguyen et al. (2016), Yang et al. (2018) and Besancon et al. (2020) use BL models to compare plain rate tariffs with more flexible tariffs, such as ToU and time-and-level-of-use pricing, considering different incentives for consumers. In these works, demand response models usually rely on simplified formulations, considering only load curtailment and/or load shifting. In addition, for shiftable loads those works consider that a certain amount of energy should be supplied in a certain period disregarding the true operation cycles that can have very different power demand in different operation phases.

The physical modeling of electricity generation and storage in the consumer's problem increases the realism of the BL model, but also makes it computationally harder to solve (Grimm et al., 2021). BL models that include generation and storage, in addition to the accurate representation of load control in the consumer's optimization model, are of increasing importance in the context of smart grids, which should be considered by electricity retailers to design more attractive ToU tariffs (Antunes et al., 2020). Moreover, computationally efficient algorithms should be designed to solve the problems in an acceptable computation time.

In the BL optimization model we present in this paper to assist the retailer establishing ToU tariffs, the prosumer problem (at the lower level) allows for the integrated optimization of all energy resources, by encompassing exchanges with the grid (buying, selling), different types of residential appliances, storage (electric vehicle and static battery) and local microgeneration. According to the type of control, in addition to a base must-serve load, two categories of residential controllable loads are considered: shiftable loads (the operation cycle cannot be interrupted – dishwasher, laundry machine and clothes dryer) and thermostatic loads (controlled by a thermostat device switched by the indoor temperature or the water tank temperature, for the air conditioner system or the electric water heater, respectively). In what concerns storage assets, stationary and electric vehicle batteries are considered. The prosumer's problem requires a rigorous modeling of appliance operation to generate effective load scheduling solutions, respecting their physical operation principles and use patterns in everyday life. Nevertheless, a balance should be sought between the detail level of the optimization models and the computational requirements having in mind generating feasible solutions in a short time.

Our problem is formulated as a BL mixed-integer nonlinear programming (BL-MINLP) problem, with bilinear terms arising from the products of integer variables (the electricity prices) and continuous variables (power the consumer buys from the grid and sell to the grid), as well as a high number of binary variables in the lower-level problem which control appliance operation. The BL-MINLP problem cannot be transformed into a single-level problem using the Karush-Kuhn-Tucker (KKT) conditions because the lower-level problem is non-convex.

Although BL mixed-integer programming has been studied for three decades, both linear and non-linear cases still require more efficient algorithms, either for generic or specific problems. A state-of-the-art algorithm for generic BL mixed-integer linear programming (BL-MILP) problems has been proposed by Lozano and Smith (2017). This algorithm follows similar principles as the (Mitsos, 2010) algorithm, which can also be applied to BL-MINLP problems. These algorithms belong to the category of optimal-value-function approaches for BL problems. Soares et al. (2021) proposed an algorithm based on Mitsos (2010); Lozano and Smith (2017) enhanced to make the most of the particular characteristics of pricing problems in the electricity retail market modelled as BL-MILP, in which the leader's variables do not appear in the follower's constraints, and vice-versa. The study of Liu et al. (2021) adapted the algorithm proposed by Lozano and Smith (2017) to the reserve management problem of an electric vehicle aggregator, by generalizing it to multiple followers.

In the present work, a deterministic bounding algorithm (DBA) using an optimal-value-function approach is proposed for the global optimization of our BL-MINLP problem. The proposed approach is based on the exact deterministic bounding procedure presented in Soares et al. (2021), where the bilinear terms are just due to the product of binary and integer variables, which are easily transformed into equivalent linear functions. However, different techniques are needed to address the more demanding nonlinear terms in the present model. The DBA is an iterative procedure that solves single-level subproblems to compute increasingly tighter lower and upper bounds for the upper-level objective function: (a) MINLP problems consisting of optimizing the upper-level objective function over the set of all constraints of the bilevel problem plus other constraints that are added during the process – the solution to this problem gives an upper bound in each iteration; (b) the lower-level problem for fixed upper-level variables – the solution to this MILP problem gives a lower bound.

We implemented the following techniques in the DBA to solve the successive MINLP problems in (a): 1. Solving the problems using a commercial mixed-integer nonlinear solver; 2. Creating an approximate MILP formulation using the McCormick envelopes (McCormick, 1976) to replace the bilinear terms by linear ones; 3. Expressing the integer variables as binary ones and linearizing the product of binary and continuous variables using an exact form. The MILP problems in 2. and 3. are solved by a MILP solver. For a comparative evaluation of the performance of the DBA, a hybrid meta-heuristic approach was also implemented, using a particle swarm optimization (PSO) algorithm to perform the price search at the upper level, and calling the MILP solver to deal with the lower-level problem for each price setting.

The main contributions of this paper are the following:

- The DBA for global optimization of BL-MINLP problems, which is an extension of the procedure previously developed by the authors for BL-MILP problems.
- Applying different techniques to deal with the nonlinearities arising in the model so that the solutions can be obtained with a MILP solver, and comparing them in a real data setting. The aim is to provide practical answers to the following question: what does provide better results, considering approximate transformations that lead to problems easier to solve, or considering exact problem transformations that are computationally more demanding?
- Comparison of the performance of the DBA and the hybrid meta-heuristic, regarding solution quality and computational effort in real settings considering a limited computational budget to solve each subproblem.

The manuscript is organized as follows. Section 2 presents the BL-MINLP model to optimize ToU prices in the electricity retail market. Section 3 presents two algorithmic approaches to deal with the BL-MINLP problem: the DBA using three different techniques to deal with the nonlinearities and a hybrid PSO-MILP algorithm. Results of a case study are discussed in Section 4. The conclusions are drawn in Section 5.

2. A bilevel mixed-integer nonlinear model to determine ToU prices in the electricity retail market

BL optimization refers to mathematical programming problems which contain another optimization problem in the constraints. In this section, a BL-MINLP model to determine ToU prices in the electricity retail market is presented. The upper-level (UL) problem concerns the aim of the electricity retailer to maximize profit, while the lower-level (LL) problem refers to the consumer's interests to minimize cost.

General nomenclature:

Parameters:

T: number of time intervals resulting from the discretization of the planning horizon $\mathrm{T} = \{1, \cdots, T\}$ (each $t \in \mathrm{T}$ represents the unit of time from $t-1$ to t).

Δt: length of the time interval (unit of time) in hours corresponding to the discretization of the planning horizon (e.g., 1, 5 or 15 min corresponding to $\Delta t = \frac{1}{60}\mathrm{h}, \Delta t = \frac{1}{12}\mathrm{h}, \Delta t = \frac{1}{4}\mathrm{h}$).

I: number of periods of the planning horizon in which different prices apply ($i \in \{1, \cdots, I\}$).

P_i : periods of prices ($P_i = [P_{L_i}, P_{U_i}] \subset \mathrm{T}$), which define the ToU tariff structure (such that $\bigcup_{i=1}^{I} P_i = \mathrm{T}$); $\bar{P}_i = P_{U_i} - P_{L_i} + 1$ is the amplitude of P_i; $P_{L_1} = 1$ and, for $i \in \{2, \cdots, I\}$, $P_{L_i} = P_{U_{i-1}} + 1$).

x^{AVG}: average price for the whole planning horizon T (€/kWh).

$\underline{x}_i \,/\, \bar{x}_i$: minimum/maximum energy price values in each period P_i (€/kWh).

π_t^{market}: energy acquisition prices incurred by the retailer in time t (€/kWh).

c^{sell}: energy remuneration to the consumer by selling energy to the grid (€/kWh).

UL variables:

x_i: energy price (€/kWh) in period P_i.

LL variables appearing in the UL and LL objective functions:

P_t^{G2H}: power from grid to home (G2H) in time t (kW).
P_t^{H2G}: power from home to grid (H2G) in time t (kW).

In the UL problem, the retailer's objective is to maximize profit, i.e. the difference between the revenue with the sale of energy to consumers (first term in the expression (1) below) and the acquisition cost of that amount of energy in the wholesale market (second term in (1)). Constraints at the UL limit the prices to minimum and maximum values (2) and impose an average electricity price for the whole planning horizon (3) to account for competition in the electricity retail market, as proposed by Zugno et al. (2013).

The modeling of the consumer/prosumer's energy resources (load management, exchanges with the grid, electric vehicle and stationary batteries, local photovoltaic generation) incorporates their physical operation principles and control modes that can be implemented in energy management systems. The modeling is based on a comprehensive and modular set of appliance operation MILP models for demand response optimization (Antunes et al., 2022).

At the LL, the consumers' objective is to minimize the net cost, which comprises the costs associated with the energy consumed (equal to the revenue term in the UL objective function – first term in (1) and (4)) minus the revenue with the sale of surplus energy to the grid/market operator (second term in (4)). Constraints at the LL problem (5)–(42) model the integrated management of all energy resources, which involves appliance operation, exchanges with the grid, storage and local microgeneration. The key ideas of each LL constraint group are given below. The mathematical formulation of all the constraints (5)–(42) and their detailed explanation is provided in the Appendix.

Bilevel mixed-integer nonlinear optimization model:

$$\max_{x} F = \sum_{i=1}^{I} \sum_{t \in P_i} \left(x_i P_t^{G2H} \Delta t \right) - \sum_{t=1}^{T} \left(\pi_t^{market} P_t^{G2H} \Delta t \right) \tag{1}$$

s.t.

UL constraints:

$$\underline{x}_i \le x_i \le \bar{x}_i, \quad i = 1, \cdots, I \tag{2}$$

$$\frac{1}{T} \sum_{i=1}^{I} \bar{P}_i x_i \le x^{AVG} \tag{3}$$

$$\min_{P^{G2H}, P^{H2G}} f = \sum_{i=1}^{I} \sum_{t \in P_i} \left(x_i P_t^{G2H} \Delta t \right) - \sum_{t=1}^{T} \left(c^{sell} P_t^{H2G} \Delta t \right) \tag{4}$$

s.t.

LL constraints for:

- Shiftable loads (*Sh*): (5)–(9)
- Electric Water Heater (*EWH*): (10)–(17)
- Air conditioner (*AC*): (18)–(22)
- Stationary Battery (*B*): (23)–(29)
- Electric Vehicle (*EV or V*): (30)–(36)
- Grid to Home/Home to Grid flows (*G2H/H2G*): (37)–(40)
- Power balance: (41)–(42)

In electricity retail markets, prices are always integer values or with a fixed number of decimal places in a given currency per energy unit. In Eurozone countries, electricity retail prices are charged to consumers in €/kWh with four decimal places. Therefore, considering prices as continuous variables is not rigorous and making some type of rounding after the optimization process may lead to underestimate or overestimate the objective function values. Since we are dealing with prices in €/kWh, we have treated the price variables $x_i, i = 1, \cdots, I$, as integer and a scale factor of 10^4 is applied to $\underline{x}_i$, $\bar{x}_i$ and x^{AVG}. The factor 10^{-4} is

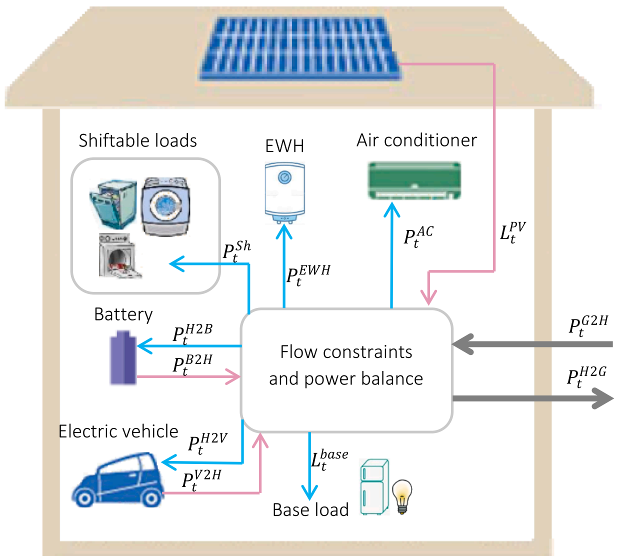

Fig. 1. Representation of the consumer's model.

then applied to the objective function values resulting from solving the models so that values in € are presented.

The UL and LL objective functions in the BL model presented above are nonlinear due to the bilinear terms $x_i P_t^{G2H}$. The LL problem becomes linear after the instantiation of the UL decision variables x.

The set of constraints (5)–(9) models the operation of shiftable loads. Shiftable loads are characterized by a given operation cycle that cannot be interrupted, with a predefined duration, requiring a different power in each operation stage. This load category includes dishwashers, laundry and drying machines, which should operate in time slots defined by the consumer. The decision variables are binary variables s_{jt}^{Sh} that define the time t in which each appliance j starts its operation (i.e., $s_{jt}^{Sh} = 1$ for the starting time t). Constraints (5)–(9) assure that each load j starts only once, its entire operation cycle is properly respected, and it is completed within the comfort time slot specified by the consumer. The constraints simultaneously define the power required by each load j in each time t, i.e., the values of implicit variables P_{jt}^{Sh} that will be used to determine P_t^{G2H} (the power variables entering the objective functions (1) and (4)).

The set of constraints (10)–(17) models the operation of the electric water heater (EWH), which is controlled by a thermostat. The water temperature should be within a minimum and a maximum temperature, considering the water withdrawal and inlet, as well as the losses through the tank envelope (the water reservoir, whose technical characteristics influence the heat losses). The main decision variables are binary variables v_t that define whether the EWH is *on* or *off* in each time t. The constraints compute the power losses and the water temperature in the tank, assuring that a comfort temperature and a maximum allowed temperature are respected. In this model, we also consider constraints to avoid the formation of legionella bacteria, which requires heating the water to a high temperature during a minimum period. The power required by the EWH in each time t, P_t^{EWH}, are implicit variables determined by v_t through the constraints.

The set of constraints (18)–(22) models the operation of the air conditioner (AC) system, which is controlled by a thermostat. The decision variables are binary variables s_t^{AC} that define whether the AC is *on* or *off* in each time t. The constraints determine the indoor temperature considering the control variables s_t^{AC}, the outside temperature and the thermal modeling of the space, assuring that the indoor temperature is within a minimum and a maximum temperature. The constraints also determine the power P_t^{AC} required by the AC in each time t.

The operations of the EV battery (superscript V in the respective notation) and the stationary battery (superscript B) are modeled in a similar way using the set of constraints (23)–(29) and (30)–(36), respectively. The decision variables are binary variables s_t^{H2V}, and s_t^{H2B}, respectively, which define whether the battery is charging or not in each time t. The constraints impose that: both types of batteries should have a charge between minimum and maximum values, considering their charging and discharging efficiencies; the batteries cannot charge and discharge at the same time; the energy available in the batteries cannot be lower than a pre-defined value at the time of departure (V) or the end of planning period (B). The constraints also define the power withdrawn from home to the batteries (P_t^{H2V} and P_t^{H2B}) and the power withdrawn from the batteries to home (P_t^{V2H} and P_t^{B2H}) in each time t. As in the models for the shiftable loads, the EWH and the AC, these (implicit) variables will be used to determine the global power variables that integrate the objective functions - in this case not only P_t^{G2H} but also P_t^{H2G}.

The set of constraints (37)–(40) imposes bounds on P_t^{G2H} (power bought from grid) and P_t^{H2G} (power sold to the grid) and impose that the flow occurs in one direction only (i.e., G2H or H2G) in each time t.

The set of constraints (41)–(42) models the power balance. The total power the home requires from the grid (P_t^{G2H}) and the power sold to the grid (P_t^{H2G}) are balanced by the power required to supply all loads (either base or controllable loads, including the stationary and EV batteries), the power supplied by the PV (determined exogenously) and the charge in batteries.

All the flows of the consumer's model are pictured in Fig. 1.

3. Algorithmic approaches for the optimization of the bilevel mixed-integer nonlinear model

This section is devoted to algorithmic approaches for global optimization of a class of BL-MINLP problems, where neither UL decision variables appear in LL constraints, nor LL decision variables appear in UL constraints. These features are present in our BL model to optimize ToU electricity tariffs as well as in other pricing problems.

The electricity pricing problem we propose in this paper is a BL problem in which the UL variables are the prices and the LL variables are related with the management of energy resources. For simplicity, let us represent the vector of UL decision variables as x and the vector of LL decision variable as y, such that each decision variable vector may include continuous and/or integer variables. Variables x are controlled by the leader (retailer), who wants to optimize his/her objective function $F(x,y)$ – maximize profit (1), over the UL constraints (2)-(3) – and taking into consideration that, for each feasible x, the only feasible y are the ones that optimize the LL problem. The follower (consumer) controls the y variables at the LL aiming to optimize his/her objective function $f(x,y)$ – minimize cost (4), over the LL constraints (5)–(42). The optimistic formulation of the BL problem is assumed, which means that, whenever the LL problem has alternative optimal solutions, the follower chooses the one that most benefits the leader, which may be achieved, for instance, incentivizing the lower-level players with some fringe benefits. To facilitate the algorithm description, the BL model developed in the previous section to determine ToU tariffs is rewritten in the following compact form, where $G(x) \leq 0$ represents the UL constraints

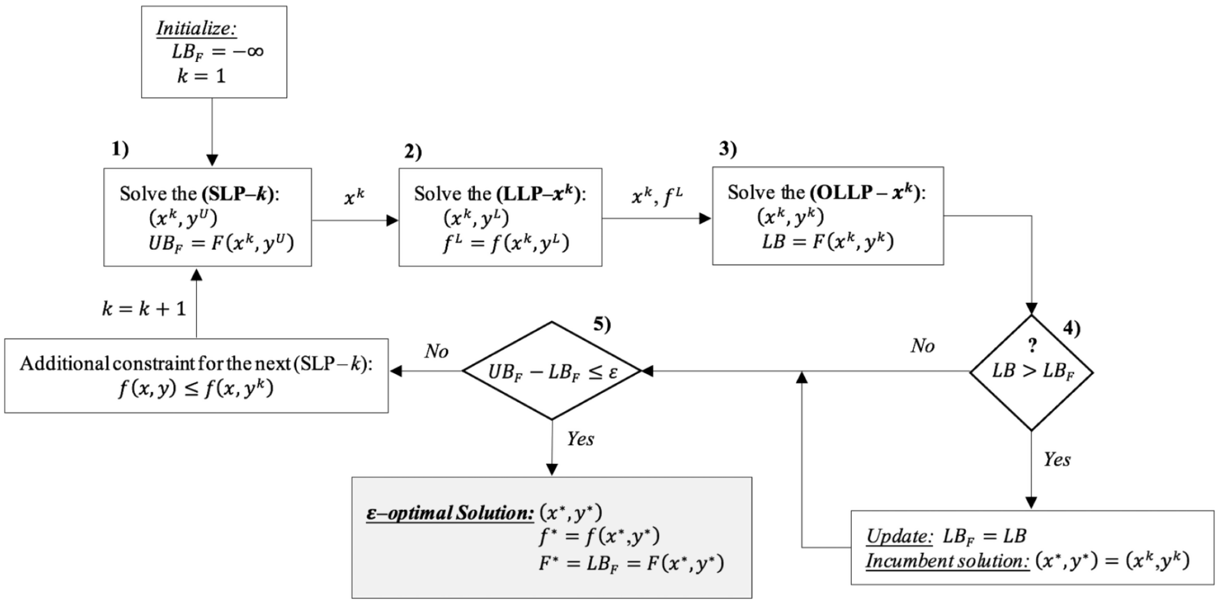

Fig. 2. Flowchart of the DBA.

(2)-(3) and $g(y^*) \leq 0$ represents the LL constraints (5)–(42):

$$\begin{aligned} \max_{x,\ y}\ & F(x,y) \\ s.t.\ & G(x)\leqslant 0 \\ & y\in \arg\min_{y^*}\{f(x,y^*) : g(y^*)\leqslant 0\} \end{aligned} \qquad \text{(BLP)}$$

BL optimization problems are inherently non-convex and, thus, finding a global optimal solution is a challenging task. The difficulties of solving BL problems are further aggravated when the problem has integer variables and nonlinear objective functions and/or constraints. This is the case of our pricing problem, in which the nonlinearities in the UL and LL objective functions arise due to the product between UL and LL decision variables (i.e., prices x multiplied by the power associated with LL variables y controlling the consumer's energy resources). For each instantiation of x, the LL problem becomes a MILP problem, with continuous and binary variables. To cope with these difficulties, we propose two algorithmic approaches: a deterministic bounding algorithm (DBA) considering three different techniques to deal with the nonlinearities; a hybrid algorithm combining a PSO algorithm to address the UL search for x and an external MILP solver (*Cplex*) to solve the LL problem for each x instantiation.

3.1. A deterministic bounding algorithm (DBA)

In this section, a deterministic bounding algorithm devoted to bilevel mixed-integer nonlinear programming is proposed, which is based on the work presented in Soares et al. (2021). The algorithm uses an optimal-value-function approach to obtain increasingly tighter bounds for the UL objective function. The overall strategy is adapted to the features of this class of BL-MINLP (price setting), namely the fact that LL (UL) decision variables do not figure in the UL (LL) constraints.

Let us represent the follower's feasible region as $Y = \{y : g(y) \leq 0\}$, which does not depend on x.

A solution $(x,\ y)$ such that $G(x) \leq 0$, $y \in Y$, is feasible to the BL problem (BLP) if and only if $f(x,y) \leq f(x,\widehat{y})$, for every $\widehat{y} \in Y$ (Lozano and Smith, 2017). Thus, the (BLP) can be rewritten as a single level optimization problem as follows:

$$\begin{aligned} \max_{x,y}\ & F(x,y) \\ s.t.\ & G(x)\leqslant 0 \\ & g(y)\leqslant 0 \\ & f(x,y)\leqslant f(x,\widehat{y}),\quad \forall \widehat{y}\in Y \end{aligned} \qquad \text{(SLP)}$$

If we know all the follower's feasible solutions in Y, then the solution of the problem (SLP) is the optimistic optimal solution (x,y) for the (BLP). However, listing all the follower's feasible solutions is an almost impossible task even in a discrete space; thus, a sub-set $\mathscr{Y} \subset Y$ can be used instead, giving an upper bound for the F^* (optimal F value). A sequence of relaxations of the (SLP) is iteratively solved by adding to $\mathscr{Y}$ a follower's feasible solution $\widehat{y}$ in each iteration (constituting a sample of Y), thus giving successively tighter upper bounds for F^*. The sample $\mathscr{Y}$ is empty in the first iteration of the algorithm in which the so-called *high point relaxation* problem is solved. The LL problem is then solved for the resulting solution x, giving a solution $\widehat{y}$ which is inserted into $\mathscr{Y}$, and the (SLP) is solved again with the new sample $\mathscr{Y}$. This is an iterative deterministic algorithm, whose steps are summarized in the flowchart in Fig. 2 and detailed below.

Step 1) Solve (SLP-k), which is a relaxation of (SLP) with a sample of solutions $\mathscr{Y} = \{y^{(\kappa-1)} \in Y,\ \kappa=2,\ \dots,\ k\}$, where k is the current iteration. In the first iteration ($k=1$), $\mathscr{Y} = \varnothing$.

$$\begin{aligned} \max_{x,y}\ & F(x,y) \\ s.t.\ & G(x)\leqslant 0 \\ & g(y)\leqslant 0 \\ & f(x,y)\leqslant f\left(x,y^{(\kappa-1)}\right),\quad \forall\ 1<\kappa\leqslant k \end{aligned} \qquad (\text{SLP}-k)$$

Let (x^k, y^U) be the optimal solution obtained with (SLP-k). $UB_F = F(x^k, y^U)$ is the current upper bound for the UL objective function.

Step 2) Solve the LL problem for the leader's solution x^k:

$$\min_y f(x^k, y) \quad s.t.\ g(y) \leqslant 0 \qquad (\text{LLP} - x^k)$$

The solution obtained (x^k, y^L) is an optimal follower's reaction for the leader's solution x^k. The f-value is $f^L = f(x^k, y^L)$.

Step 3) Since the optimistic solution to the (BLP) is sought, the optimistic formulation for the LL problem (OLLP-x^k) is solved to obtain the best choice for the leader when alternative optima exist for (LLP-x^k).

$$\begin{aligned} &\max_y F(x^k, y) \\ &s.t.\ g(y) \leqslant 0 \qquad\qquad (\text{OLLP} - x^k) \\ &\quad f(x^k, y) \leqslant f^L + \varepsilon' \end{aligned}$$

However, an ε'-tolerance for the optimal f^L given by (LLP-x^k) is allowed to overcome numerical difficulties arising from floating point operations. Let (x^k, y^k) be the optimal solution to (OLLP-x^k). Assuming $\varepsilon' \to 0$, the F-value here obtained gives a lower bound (LB) for the leader's optimal objective function value $LB = F(x^k, y^k)$.

Step 4) If $LB > LB_F$ (the best lower bound found so far for the UL objective function; initially $LB_F = -\infty$), then LB_F and the incumbent solution (x^*, y^*) are updated: $LB_F = LB$, $(x^*, y^*) = (x^k, y^k)$.

Step 5) The algorithm stops when $UB_F - LB_F \leq \varepsilon$, with ε being a predefined optimality tolerance; the output is an ε-optimal solution (x^*, y^*) with $F^* = LB_F$. Otherwise, $k \leftarrow k+1$ and the algorithm returns to Step 1, including in $\mathscr{Y}$ the last solution y obtained in Step 3.

Implementation note for Step 3:

The MILP solver may be unable to find a feasible solution to the problem (OLLP-x^k) within a reasonable computation time, which would prevent obtaining a LB. To deal with this issue, the problem (OLLP-x^k) is reformulated by introducing an "elastic" variable in the additional constraint, which is penalized in the objective function using a big-M as follows:

$$\begin{aligned} &\max_y \quad [F(x^k, y) - M \times D] \\ &s.t. \quad g(x) \leqslant 0 \qquad\qquad (\text{OLLP} - x^k - \text{D}) \\ &\qquad\ f(x^k, y) \leqslant f^L + \varepsilon' + D \\ &\qquad\ D \geqslant 0 \end{aligned}$$

If the value of D in (OLLP-x^k-D) is different from zero, then the corresponding F-value obtained cannot be accepted as a LB of F because the f-value is not within the ε'-tolerance with respect to f^L. In this case, the solution retained to proceed with the algorithm is the one obtained in (LLP-x^k).

The problems (LLP-x^k) and (OLLP-x^k-D) are MILP problems that can be solved using a state-of-the-art MILP solver. However, the problems (SLP-k) are nonlinear due to the bilinear terms in $F(x, y)$ and in left-hand side of the constraints $f(x, y) \leq f(x, \hat{y})$, thus requiring a MINLP solver or some form of linearizing the problem. Three different techniques to deal with these nonlinearities are proposed: 1) the (SLP-k) are solved using a commercial mixed-integer nonlinear programming solver; 2) an approximate linearization technique using the McCormick envelopes; 3) an exact linearization technique based on the binary expansion of the integer variables. In these techniques, 2) creates an approximate model and 3) reformulates the problem as an equivalent one but it requires a significant number of additional binary variables and constraints.

In the computational experiments to address the (SLP-k) nonlinear problems, in technique 1), we have used *Antigone* (within GAMS[1]) and other nonlinear solvers (within AMPL[2]), such as *Baron*, *Knitro*, and *Gurobi* with the option to tackle bilinear terms. In techniques 2) and 3), the *Cplex*[3] solver was used to deal with the reformulations of (SLP-k) into MILP problems. *Cplex* was also used to solve the (LLP-x^k) and (OLLP-x^k-D) problems in all techniques.

Before presenting the linearization techniques, let us concretize the model (SLP-k) for our problem. For the sake of clarity in writing the model, let us define the auxiliary variable *ConsumerCost*, the expression of which will change for each specific linearization technique:

$$ConsumerCost = \sum_{i=1}^{I} \sum_{t \in P_i} (x_i P_t^{G2H} \Delta t)$$

Constraints (43) are the additional constraints $f(x, y) \leq f(x, y^{(\kappa-1)})$ of (SLP-k), where $y^{(\kappa-1)} = (y_t^{(\kappa-1)G2H}, y_t^{(\kappa-1)H2G})$ are constants equal to the values of (P_t^{G2H}, P_t^{H2G}) obtained in the (OLLP-x^k-D) problem for $x^{(\kappa-1)}$, $\forall 1 < \kappa \leq k$.

$$\begin{aligned} &(\text{SLP-}k): \\ &\max F = ConsumerCost - \sum_{t=1}^{T} (\pi_t^{market} P_t^{G2H} \Delta t) \\ &\text{s.t.} \\ &\quad UL\ constraints\ (2)\text{–}(3) \\ &\quad LL\ constraints\ (5)\text{–}(42) \\ &\quad ConsumerCost - \sum_{t=1}^{T} (c^{sell} P_t^{H2G} \Delta t) \leq \sum_{i=1}^{I} \sum_{t \in P_i} (x_i y_t^{(\kappa-1)G2H} \Delta t) \\ &\qquad - \sum_{t=1}^{T} (c^{sell} y_t^{(\kappa-1)H2G} \Delta t), \forall 1 < \kappa \leq k \end{aligned} \tag{43}$$

Approximate linearization technique:

The problem (SLP-k) includes products between integer and continuous variables in the expression of *ConsumerCost*, which appears in the objective function and in the left-hand side of the additional constraints $f(x, y) \leq f(x, y^{(\kappa-1)})$, $\forall 1 < \kappa \leq k$. The first linearization technique consists of using the McCormick (lower and upper) envelopes.

To approximate the products $x_i P_t^{G2H}$, the auxiliary variables $z_t, \forall t \in \mathrm{T}$, are considered (note that each i depends on t and, therefore, we just need the index t for the additional variables) and the constraints (46)–(49) are imposed to create the approximate version of the (SLP-k) presented below.

The expression of the *ConsumerCost* is replaced by:

$$ConsumerCost\_Al = \sum_{t=1}^{T} (z_t \Delta t)$$

Since the objective function of the problem (Al-SLP-k) is an approximation of the true objective function F, it is denoted by F'.

[1] https://www.gams.com.

[2] https://ampl.com; https://minlp.com/baron-solver.

[3] https://www.ibm.com/analytics/cplex-optimizer.

Approximate linearized problem (Al-SLP-k)*:*

$$\max F' = ConsumerCost\_Al - \sum_{t=1}^{T} \left(\pi_t^{market} P_t^{G2H} \Delta t\right) \tag{44}$$

s.t.

UL constraints (2)–(3)

LL constraints (5)–(42)

$$ConsumerCost\_Al - \sum_{t=1}^{T} \left(c^{sell} P_t^{H2G} \Delta t\right) \le \sum_{i=1}^{I} \sum_{t \in P_i} \left(x_i y_t^{(\kappa-1)G2H} \Delta t\right) - \sum_{t=1}^{T} \left(c^{sell} y_t^{(\kappa-1)H2G} \Delta t\right), \forall 1 < \kappa \le k \tag{45}$$

Additional constraints:

Underestimates:

$$z_t \ge \underline{x}_i P_t^{G2H} \quad i = 1, \cdots, I, \forall t \in P_i \tag{46}$$

$$z_t \ge \bar{x}_i P_t^{G2H} + x_i P_{max}^{G} - \bar{x}_i P_{max}^{G} \quad i = 1, \cdots, I, \forall t \in P_i \tag{47}$$

Overestimates:

$$z_t \le \bar{x}_i P_t^{G2H} \quad i = 1, \cdots, I, \forall t \in P_i \tag{48}$$

$$z_t \le \underline{x}_i P_t^{G2H} x_i + P_{max}^{G} - \underline{x}_i P_{max}^{G} \quad i = 1, \cdots, I, \forall t \in P_i \tag{49}$$

Exact linearized problem (El-SLP-k)*:*

$$\max F = ConsumerCost\_El - \sum_{t=1}^{T} \left(\pi_t^{market} P_t^{G2H} \Delta t\right) \tag{50}$$

s.t.

$$x_i = \underline{x}_i + \sum_{k=0}^{Nb_i} 2^k b_{ik} \quad i = 1, \cdots, I \tag{51}$$

UL constraints (2)–(3)

LL constraints (5)–(42)

$$ConsumerCost\_El - \sum_{t=1}^{T} \left(c^{sell} P_t^{H2G} \Delta t\right) \le \sum_{i=1}^{I} \sum_{t \in P_i} \left(x_i y_t^{(\kappa-1)G2H} \Delta t\right) - \sum_{t=1}^{T} \left(c^{sell} y_t^{(\kappa-1)H2G} \Delta t\right), \forall 1 < \kappa \le k \tag{52}$$

Additional constraints:

$$w_{tk}^{NB} \ge P_t^{G2H} - (1 - b_{ik}) P_{max}^{G} \quad i = 1, \cdots, I, \forall t \in P_i, k = 0, \cdots, Nb_i \tag{53}$$

$$w_{tk}^{NB} \le P_{max}^{G} b_{ik} \quad i = 1, \cdots, I, \forall t \in P_i, k = 0, \cdots, Nb_i \tag{54}$$

$$w_{tk}^{NB} \le P_t^{G2H} \quad i = 1, \cdots, I, \forall t \in P_i, k = 0, \cdots, Nb_i \tag{55}$$

$$b_{ik} \in \{0, 1\} \quad i = 1, \cdots, I, k = 0, \cdots, Nb_i \tag{56}$$

This linearization of the products between the integer variables x_i and the continuous variables P_t^{G2H} by means of the envelopes (46)–(49) enlarges the feasible region of the problem. Therefore, at the first iteration ($k = 1$) of the DBA, the F-value resulting from the (Al-SLP-k) is a true upper bound of the leader's objective function optimal value. However, in the subsequent iterations, the linearization of the additional constraints $f(x, y) \le f\left(x, y^{(\kappa-1)}\right), \forall 1 < \kappa \le k$, creates approximate constraints that may cut feasible solutions of the BL problem. Thus, the solution of (Al-SLP-k) may not give an upper bound for the leader's objective function value. The F-value resulting from the exact resolution of (OLLP-x^k-D) with $D = 0$ represents a true lower bound LB_F.

Accordingly, the DBA stop criterion defined in Step 5 cannot be applied in this case. The algorithm can be stopped when it finds the same solution x in two consecutive iterations because, in these circumstances, no additional different constraints would be introduced in the (Al-SLP-k) in the next iteration and, therefore, the solution would be the same. Practical stopping conditions will be further discussed in the context of the case study in Section 4.

Exact linearization technique:

First, a binary expansion is employed to write each integer variable x_i as a vector of binary variables. Then the nonlinearities due to the products between binary and continuous variables P_t^{G2H} allow for an exact linearization.

Different binary expansion schemes of integer variables exist. A compact reformulation has been adopted, which requires introducing fewer binary variables than an alternative full reformulation that introduces a binary variable for each possible integer value. In the computational experiments performed by Owen and Mehrotra (2002) on MILP problems, the compact reformulation seemed to perform better than the full reformulation.

In the compact reformulation, $x_i = \underline{x}_i + \sum_{k=0}^{Nb_i} 2^k b_{ik}$, where $Nb_i + 1$ is the number of binary variables b_{ik} needed to encode x_i in a binary vector.

Since $x_i P_t^{G2H} = \left(\underline{x}_i + \sum_{k=0}^{Nb_i} 2^k b_{ik}\right) P_t^{G2H} = \underline{x}_i P_t^{G2H} + \sum_{k=0}^{Nb_i} 2^k b_{ik} P_t^{G2H}$, then the nonlinearities now arise from the product of binary variables (b_{ik}) and continuous variables (P_t^{G2H}): $b_{ik} P_t^{G2H}$. Introducing additional auxiliary variables w_{tk}^{NB}, constraints (53)–(56) in the (El-SLP-k) problem are necessary to ensure $w_{tk}^{NB} = b_{ik} P_t^{G2H}$.

The expression of the *ConsumerCost* is replaced by:

$$ConsumerCost\_El = \sum_{i=1}^{I} \sum_{t \in P_i} \left(x_i P_t^{G2H} \Delta t\right) = \sum_{i=1}^{I} \sum_{t \in P_i} \left(\left(\underline{x}_i P_t^{G2H} + \sum_{k=0}^{Nb_i} 2^k w_{tk}^{NB}\right) \Delta t\right)$$

The exact linearized (El-SLP-k) is a MILP problem equivalent to the (SLP-k); therefore, the objective function is F as in the (SLP-k). If (El-SLP-k) is solved until optimality, the resulting F-value is a true upper bound for the leader's objective function value and the stopping condition of the algorithm stated in step 5) ensures an ε-optimal solution to the BL problem.

3.2. A hybrid meta-heuristic

In this section, the hybrid meta-heuristic proposed in Soares et al. (2020) is briefly described, since it will be compared with the DBA. The hybrid meta-heuristic combines a PSO algorithm to address the UL problem (1)–(3) with the *Cplex* solver to solve the LL problem (4)–(42). The steps of the proposed algorithm are summarized in the flowchart in Fig. 3 and are briefly described below. As above, let us represent the UL decision variables as x and the LL decision variables as y.

The hybrid algorithm runs during G iterations with a population of N individuals $x^h = \left(x_1^h, \cdots, x_I^h\right)$, $h = 1, \cdots, N$, each one representing a different price vector. The initial population is randomly generated. The population then evolve according to the PSO principles.

PSO tries to iteratively improve the solutions (designated in PSO as particles) by moving them towards the best directions. The movement of each particle x^h is influenced by its best known position – $x^{h^{best}}$, and the

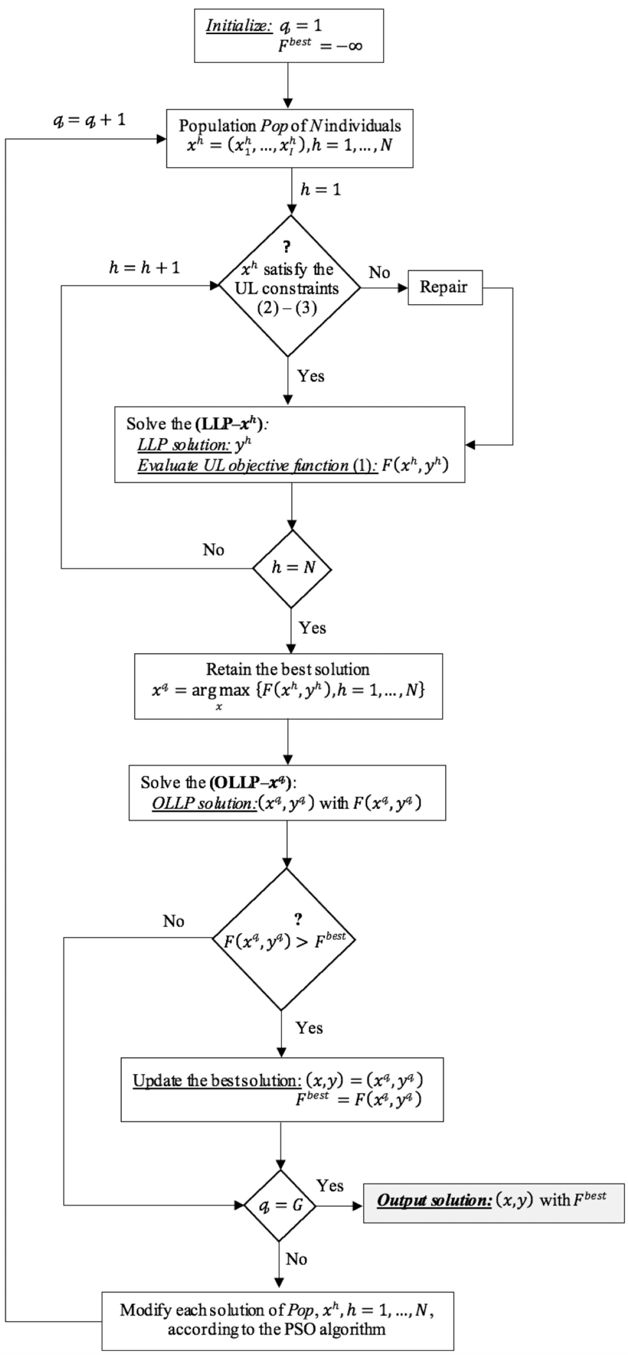

Fig. 3. Flowchart of the hybrid meta-heuristic.

best position of the entire swarm – g^{best}. These positions are updated whenever better positions are found according to the F-value. The particle movements are guided by the velocity $v_i^{h^q}$ and the previous position $x^{h^{q-1}}$, which are determined in each iteration q as follows (Eberhart and Yuhui, 2001):

$$v_i^{h^q} = \eta v_i^{h^{q-1}} + r_1 C_1 \left(x_i^{h^{best}} - x_i^{h^{q-1}}\right) + r_2 C_2 \left(g_i^{best} - x_i^{h^{q-1}}\right), \forall i = 1, \cdots, I$$

$$x^{h^q} = x^{h^{q-1}} + v^{h^q}$$

where η is the inertia weight, C_1 and C_2 are the cognitive and social parameters and r_1 and r_2 are uniform random numbers in the interval [0,1]. For simplicity, the superscript q is omitted in the following, unless it is absolutely necessary.

A *turbulence* operator is applied with a probability p_m aiming to diversify the search, by adjusting x^h as follows:

$x_i^h \leftarrow x_i^h + \zeta$, with the random turbulence
$\zeta \in [-\delta(\bar{x}_i - \underline{x}_i), \delta(\bar{x}_i - \underline{x}_i)], \forall i = 1, \cdots, I$

In each iteration of the algorithm, if the UL constraints (2)–(3) are not satisfied for the price vector x^h, a repair routine is called to fix it. Each solution is repaired using the repairing routine described in (Soares et al., 2020). After repairing x^h, the LL MILP problem (4)–(42), i.e. (LLP-x^k), is solved by *Cplex* for each price vector $x^h, h = 1, \cdots, N$ (imposing a pre-defined maximum computation time), yielding y^h. The optimistic formulation (OLLP-x^k) is then run for the solution with the highest F in the population, and the solution obtained is compared with the current F^{best}. As in the DBA, the (OLLP-x^k-D) can be used instead of (OLLP-x^k) for computational reasons.

If the value of F^{best} has no improvement (i.e., $\frac{F^{best^q} - F^{best^{q-1}}}{F^{best^q}} < \tau$ for a given preset threshold τ) over a predefined number G' of consecutive iterations, the probability p_m of the turbulence operator is increased to promote further exploration.

The output of the algorithm is the solution (x, y) that gives the highest UL objective $F(x, y) \equiv F^{best}$.

4. Case study

The algorithmic approaches proposed in the previous section were applied to the BL-MINLP model to determine optimal ToU electricity prices presented in Section 2, considering a real dataset. First, the problem data and the parameters of the algorithms are described and then the results obtained are analyzed.

A planning horizon of 24 h discretized in units of 15 min ($T = 96$) was considered. The ToU electricity prices charged to the consumers are defined for six tariff periods ($I = 6$).

In the consumer's problem, three shiftable loads (dishwasher, laundry machine and clothes dryer), an electric water heater (EWH), an air conditioner system (AC), and a stationary battery, and an electric vehicle battery (B and EV, respectively) are considered. All the problem data are available at http://dx.https://doi.org/10.17632/j2vr7jgmcz.1, an open-source online data repository hosted at Mendeley Data. These

Table 1
Results obtained in each step of each iteration of the DBA using the *Antigone* solver for the nonlinear (SLP-k) problem.

Iteration (k)	(SLP-k)								(LLP-x^k)		(OLLP-x^k-D)			LB_F
	F	f	Prices (€/kWh)						f	F	D	f	F	
1	14.97220	21.29923	0.0440	0.2776	0.2836	0.0804	0.3240	0.1036	3.10086	−0.75432	0.00000	3.10096	−0.58153	−0.58153
2	5.92096	8.79670	0.0996	0.2692	0.1080	0.2492	0.1548	0.1048	4.89906	1.51819	0.00000	4.89992	1.75825	1.75825
3	5.70537	8.81942	0.0996	0.0936	0.2836	0.2492	0.1704	0.1476	4.71184	1.51570	0.00559	–	–	
4	4.85177	7.71016	0.0996	0.2136	0.2480	0.2100	0.1540	0.0920	6.39974	2.83779	0.03476	–	–	
5	4.23576	7.11667	**0.0996**	**0.1780**	**0.1956**	**0.1892**	**0.2124**	**0.1612**	6.67101	3.55836	0.00000	**6.67111**	**3.77810**	**3.77810**
6	4.23387	7.11667	0.0996	0.1780	0.1956	0.1892	0.2124	0.1612	6.67101	3.55836	0.00000	6.67111	3.77810	

Table 2
Results obtained in each step of each iteration of the DBA using the *Cplex* solver for the approximate MILP problem (Al-SLP-k).

Iteration (k)	(Al-SLP-k)										(LLP-x^k)		(OLLP-x^k-D)			LB_F
	F'	*gap (%)*	F	f	Prices (€/kWh)						f	F	D	f	F	
1	15.10432	0.010	14.96169	21.14540	0.0448	0.2780	0.2836	0.0804	0.3240	0.1012	3.11460	−0.80048	0.01358	–	–	−0.80048
2	5.94174	1.039	5.71773	8.57667	0.0996	0.2644	0.1080	0.2492	0.1560	0.1100	4.93318	1.51060	0.02503	–	–	1.51060
3	5.79389	0.568	5.27279	8.30099	0.0996	0.0952	0.2836	0.2492	0.1676	0.1484	4.76043	1.54359	0.05619	–	–	1.54359
4	4.84418	1.462	4.74587	7.61272	0.0996	0.2136	0.2480	0.2100	0.1540	0.0920	6.39974	2.80015	0.00000	6.39984	3.14620	3.14620
5	4.26890	1.132	4.05070	6.89837	0.0996	0.1780	0.1956	0.1880	0.2140	0.1612	6.67081	3.58477	0.00000	6.67091	3.77776	3.77776
6	4.13852	1.346	4.35113	7.20399	0.0996	0.1928	0.1896	0.1840	0.2048	0.1620	6.81245	3.12686	0.00000	6.81255	3.36443	
7	4.14040	1.295	4.31606	7.16373	0.0996	0.1928	0.1900	0.1948	0.1900	0.1620	6.92999	3.54552	0.00000	6.93009	3.86794	3.86794
8	4.12656	1.182	4.28809	7.13576	0.0996	0.1920	0.1924	0.1924	0.1920	0.1616	6.96974	3.80876	0.00000	6.96984	4.07377	4.07377
9	4.10632	1.680	4.28110	7.14787	**0.0996**	**0.1924**	**0.1924**	**0.1924**	**0.1924**	**0.1608**	6.97310	3.41739	0.00000	**6.97320**	**4.07712**	**4.07712**
10	4.12543	1.221	4.26629	7.11396	0.0996	0.1924	0.1924	0.1924	0.1924	0.1608	6.97310	3.41739	0.00000	6.97320	4.07712	

data define an UL problem with 6 integer variables and 13 constraints. For each instantiation of the UL variables, the LL problem comprises 933 binary variables, 1440 continuous variables and 2232 constraints. The algorithms were run in a computer with an Intel Core i7-7700 K CPU@3.6 GHz and 64 GB RAM.

Since tariffs should be announced with a certain antecedence (e.g., day-ahead), we considered a maximum computation time of 5 min to solve each subproblem. In the DBA, the tolerance parameter for the problem (OLLP-x^k-D) was set to $\varepsilon' = 10^{-4}$ and the tolerance parameter in the stop criterion was set to $\varepsilon = 10^{-3}$.

Firstly, the nonlinear (SLP-k) problems were solved without any transformation (technique 1). Therefore, solvers for mixed-integer nonlinear problems were considered. We made computational experiments with the *Antigone*, *Knitro*, *Baron*, and *Gurobi* (with the option to tackle bilinear terms) solvers. *Antigone* provided the best solution. The other solvers were not able to find competitive final solutions, or even feasible solutions to the (SLP-k) problems within the computational time allowed (5 min for each problem). The results obtained by the DBA using the nonlinear *Antigone* solver for the (SLP-k) problems can be seen in Table 1. The DBA with *Antigone* required 6 iterations and took approximately 90 min, stopping when two consecutive price vectors were obtained. The best solution was found in iteration 5: the prices, in €/kWh, in each period are (0.0996, 0.1780, 0.1956, 0.1892, 0.2124, 0.1612), which result in a consumer's cost of 6.67101€ and a retailer's profit of **3.77810** €.

The single-level problem **(**SLP-k**)** was then reformulated using the approximate linearization (technique 2) resulting in the MILP model (Al-SLP-k) solved by *Cplex*. In this case, the DBA required 10 iterations and took approximately 2h26min, also stopping when two consecutive price vectors were obtained. The results are displayed in Table 2. The best solution was found in iteration 9, with the price vector (0.0996, 0.1924, 0.1924, 0.1924, 0.1924, 0.1608) in €/kWh, the consumer's cost of 6.97320 € and the retailer's profit of **4.07712** €.

The DBA using the MILP model (El-SLP-k) was then run (technique 3), which required 11 iterations and stopped when the F-value of the solution to (El-SLP-k) problem went below the LB_F. This situation can only occur because of the positive MIP gap. The algorithm took approximately 2h47min and the results obtained can be seen in Table 3. The best solution was found in iteration 10, with the price vector (0.0940, 0.1980, 0.2012, 0.1976, 0.2036, 0.1396) in €/kWh, the consumer's cost of 6.87574 € and the retailer's profit of **3.99646** €.

In these tables, the objective function F' that is optimized in problem (Al-SLP-k), the retailer's profit (F-values) and the consumer's cost (f-values) are in €, and the relative MIP gap of the linearized (SLP-k) problems in %. The final solution is in bold. In all techniques, the LL problems were solved using *Cplex* presenting relative MIP gaps in the range 0.5–2.5 % for 5 min runs.

The use of the nonlinear solver resulted in the worst profit for the retailer. Regarding the linearized models, the approximate one resulted in a solution for the retailer ($LB_F = 4.07712$) better than the solution to the exact model ($LB_F = 3.99646$).

The MIP gaps of the solutions to (El-SLP-k) are, in general, higher than the gaps for (Al-SLP-k), since the exact linearized model (El-SLP-k) is computationally more demanding. However, unlike the gap for (Al-SLP-k), the gap for (El-SLP-k) has a valid meaning.

The values of F' and F obtained from the approximate model (Al-SLP-k) are not necessarily upper bounds for the UL objective function and, therefore, the corresponding gap values do not give useful information. In all techniques, the F in (SLP-k) and (El-SLP-k), and the F' in (Al-SLP-k), should not increase along the iterations since new constraints are added to the problems; however, we observe that these values oscillate after a few iterations, which may be explained by the fact that these problems cannot be solved to optimality.

Although the exact linearized model requires a higher computational effort and the gap value increases over iterations, the gap of the solution to the (El-SLP-k) can be used to determine an upper bound for the F-value. The best possible profit for the retailer can thus be calculated as $UB_F = F^{(\text{El-SLP})} \times \left(1 + gap^{(\text{El-SLP})}\right) = 4.12751$€ for the last iteration. No conclusion can be drawn from the gap value of (Al-SLP-k) to derive a valid UB_F.

Note that the DBA did not end using the algorithm stopping condition (cf. step 5: $UB_F - LB_F \le \varepsilon$, with ε being a predefined optimality tolerance) in any technique. When the subproblems cannot be solved to optimality, the DBA may not converge using this stopping condition only, thus not being able to ensure an ε-optimal solution. In addition, if approximate problems are used for the (SLP-k), as in technique 2, there is no actual upper bound UB_F. Therefore, meaningful practical stopping conditions should be considered:

i) Equal price vectors are obtained in consecutive iterations – the next lower-level problem would lead to the same solution of the previous iteration, which means that the constraint to be added to (SLP-$k+1$) would be the same as the one added to (SLP-k).
ii) $UB_F < LB_F$ – this can only happen whenever the (SLP-k) is not solved to optimality due to the computation budget; since (SLP-$k+1$) would be further restricted, then it is not necessary to proceed.
iii) The f values resulting from the (SLP-k) and the corresponding (LLP-x^k) are equal (or within a small tolerance) – this means that if these problems had been solved to optimality, then $UB_F = LB_F$ because the optimal solution to (SLP-k) would be also optimal to (LLP-x^k).

In the computational experiments, the DBA stopped using condition i) in techniques 1 and 2, and condition ii) in technique 3 to address the subproblems (SLP-k).

In the computational experiments with the hybrid approach, the implementation of the PSO considered the parameters $\tau = 0.001$, $\delta =$

Table 3
Results obtained in each step of each iteration of the DBA using the *Cplex* solver for the exact MILP problem (El-SLP-k).

Iteration (k)	(El-SLP-k)									($\text{LLP-}x^k$)		($\text{OLLP-}x^k\text{-}D$)			LB_F
	F	*gap (%)*	f	Prices (€/kWh)						f	F	D	f	F	
1	14.98503	0.013	21.47869	0.0440	0.2776	0.2836	0.0804	0.3240	0.1036	3.11427	−0.79756	0.00000	3.11437	−0.52946	−0.52946
2	5.70597	1.481	8.57774	0.0948	0.2632	0.1080	0.2492	0.1540	0.1248	4.95210	1.57117	0.00000	4.95220	1.82507	1.82507
3	5.53811	2.013	8.60452	0.0948	0.0848	0.2836	0.2492	0.1980	0.1428	4.37919	1.15532	0.00000	4.37929	1.30569	
4	4.79853	1.612	7.66538	0.0940	0.2144	0.2500	0.2172	0.1540	0.0924	6.35011	3.09082	0.00000	6.35021	3.41094	3.41094
5	4.13083	2.230	7.01169	0.0948	0.1804	0.1992	0.1892	0.2360	0.1420	6.54196	3.40570	0.00000	6.54206	3.66575	3.66575
6	4.06962	3.845	6.99026	0.0948	0.2136	0.2124	0.1908	0.2200	0.0984	6.50862	2.71207	0.00000	6.50872	2.99233	
7	3.97043	4.679	6.92576	0.0948	0.2328	0.1976	0.1972	0.1560	0.1428	6.52992	3.07743	0.00000	6.53002	3.44040	
8	3.89758	6.599	6.91425	0.0948	0.1976	0.1952	0.1968	0.2060	0.1428	6.83309	3.40087	0.03262	–	–	
9	3.79929	8.747	6.85940	0.0948	0.2116	0.2092	0.1940	0.1768	0.1428	6.67924	3.15117	0.00000	6.67934	3.48442	
10	4.00069	3.227	6.89089	**0.0940**	**0.1980**	**0.2012**	**0.1976**	**0.2036**	**0.1396**	6.87564	3.20390	0.00000	**6.87574**	**3.99646**	**3.99646**
11	**3.69968**	11.564	6.73702	0.0948	0.1900	0.1868	0.2140	0.2016	0.1428	6.66085	3.35016	0.00000	6.66095	3.60142	

0.2 and $p_m = 0.05$; $G' = 5$ was fixed for the number of consecutive iterations without improvement of F^{best} that increases the probability of the turbulence operator to $p_m = 0.1$. The inertia parameter was set to $\eta = 0.4$ and the learning parameters in both cognitive and social components were $C_1 = C_2 = 2$. Six independent runs of the hybrid approach were performed with $G = 20$ iterations each one, with a population of $N = 10$ individuals. A computation time of 1 min only was given to solve each LL problem using *Cplex*, because N of such problems need to be solved in each iteration. Since a small computation time can result in large MIP gaps for f (i.e., infeasible solutions to the BL problem), no solution was taken as a final result from these quick resolutions: if the value of F^{best} was improved from the resolution of the LL problem for a given x, then its resolution was repeated for a higher computational budget, which was pre-set to 5 min. If the improvement persisted, then the best solution and the corresponding F^{best} value were updated; otherwise, they remained the same for the next iteration. The results of six runs can be seen in Table 4. The best retailer's profit in the hybrid approach was obtained in run 5 with **3.93330** €. The average of the best F-values obtained over the runs is 3.76249 € with a standard deviation of 0.158 €.

The hybrid approach took in average 4 h per run. Giving to the hybrid approach a total computation time similar to that needed by the DBA with the approximate linearized model (about 8800 s), the number of iterations per run was lower: 9 iterations for runs 5 and 6, 11 iterations for runs 2 and 4, and 12 iterations runs 1 and 3. The results obtained in runs 2, 3, 4 and 6 are the same as in Table 4, but in runs 1 and 5 the best results attained were just $F = 3.55055$ and $F = 3.58402$, respectively. Therefore, with this computation budget, the best retailer's profit was $F = 3.80854$ obtained in run 4.

The DBA with the exact and the approximate linearized models revealed better performance than the hybrid approach. Thus, the DBA seems to be a suitable approach to obtain a sound solution in a short time.

In the following, physical results are presented corresponding to the best solution obtained with the DBA using the approximate linearized model (for the electricity prices solution shown in Table 2). Fig. 4 displays the evolution of grid to home and home to grid powers (P_t^{G2H} and P_t^{H2G}, respectively). The electricity prices in €/kWh are also displayed in Fig. 4 to assist in the interpretation of the exchanges with the grid. The evolution of the flows from home to stationary and EV batteries (P_t^{H2B} and P_t^{H2V}, respectively) and batteries to home (P_t^{B2H} and P_t^{V2H}, respectively) is displayed in Fig. 5.

Consumption (P_t^{G2H}) is mainly made in the lower price periods. Consumption in higher price periods is essentially made to charge the EV battery (P_t^{H2V}), due to the period the EV is available for charging/discharging (interval [32,74]). In periods of higher electricity prices, the PV generation (interval [32,72]) and the energy stored in the stationary battery (corresponding to P_t^{B2H}) are used to supplement the supply of loads operating during those periods. The energy stored in the EV battery (corresponding to P_t^{V2H}) is never used to supplement the supply of loads. The possibility of using the energy generated for self-consumption (namely L_t^{PV} and P_t^{B2H}) discourages sales to the grid ($P_t^{H2G} = 0, \forall t \in \mathrm{T}$), since the remuneration of the sale to the grid is low. Experiments with higher remunerations values resulted in positive home to grid power (P_t^{H2G}). The BL model is flexible, enabling to accommodate different regulatory frameworks for tariff design and up-to-date data resulting from the current volatility in electricity markets.

5. Conclusions

In this paper, a comprehensive BL optimization model is presented to address a price setting problem in the electricity retail market. The retailer's goal is to determine the optimal ToU electricity prices to offer to consumers during a planning horizon to maximize profit. The consumer's reaction to minimize cost affects the retailer's profit. The consumer's problem consists of the integrated optimization of all energy resources, namely load management, electric vehicle and stationary battery, local microgeneration and exchanges with the grid. A detailed and accurate modelling of all energy resources makes the model more realistic, but its combinatorial nature poses computational difficulties. The BL model is dealt with two algorithmic approaches: a deterministic bounding algorithm (DBA) using three different techniques to address nonlinearities and a hybrid algorithm in which the UL problem is tackled by a PSO algorithm while the LL problem is solved by an exact MILP solver.

The DBA is an iterative procedure that solves a sequence of single level subproblems to compute successively narrower bounds for the UL objective function. One of these subproblems has bilinear terms in the objective function and in the constraints, arising from products of integer and continuous variables. Three different techniques to deal with these nonlinearities were proposed.

The single-level problems are difficult to solve, namely when a fine-grain time discretization is considered. The exact linearization of the MINLP problems increases the dimension of the problem as it requires the introduction of a significant number of auxiliary binary and continuous variables as well as constraints, thus imposing a higher computational burden. Given the combinatorial nature of the mixed-integer (linear and nonlinear) programming problems, it is not possible to solve them to optimality in an acceptable computation time; then, a realistic computation budget was set for the resolution of each problem. Even if an ε-optimal solution cannot be guaranteed by the DBA because the MIP problems are not solved to optimality, the algorithm is able to compute good quality solutions using meaningful practical stopping conditions.

The DBA using the nonlinear solver gave the worst solution and the exact linearization technique offered a good tight lower/upper bound interval for the retailer's profit. The approximate linearization technique computed the best value for the retailer's profit. The best solution

Table 4
Results obtained in each run of the hybrid approach through 20 iterations for a population of size 10. The best run solution is in bold.

Run	Prices (€/kWh)						F (€)	f (€)	Time (s)
1	0.092	0.1891	0.1909	0.2073	0.1938	0.1619	3.87685	6.77290	14480.320
2	0.0681	0.2139	0.2221	0.2326	0.2223	0.0920	3.49935	6.39538	14493.120
3	0.0985	0.1732	0.1763	0.1793	0.253	0.1609	3.66253	6.54212	13873.410
4	0.0966	0.182	0.2167	0.1942	0.1847	0.1620	3.80854	6.70434	14472.690
5	**0.0979**	**0.1879**	**0.2113**	**0.1906**	**0.1879**	**0.1582**	**3.93330**	**6.82937**	16279.360
6	0.0809	0.1965	0.1972	0.2143	0.1987	0.1581	3.79438	6.69045	16296.040

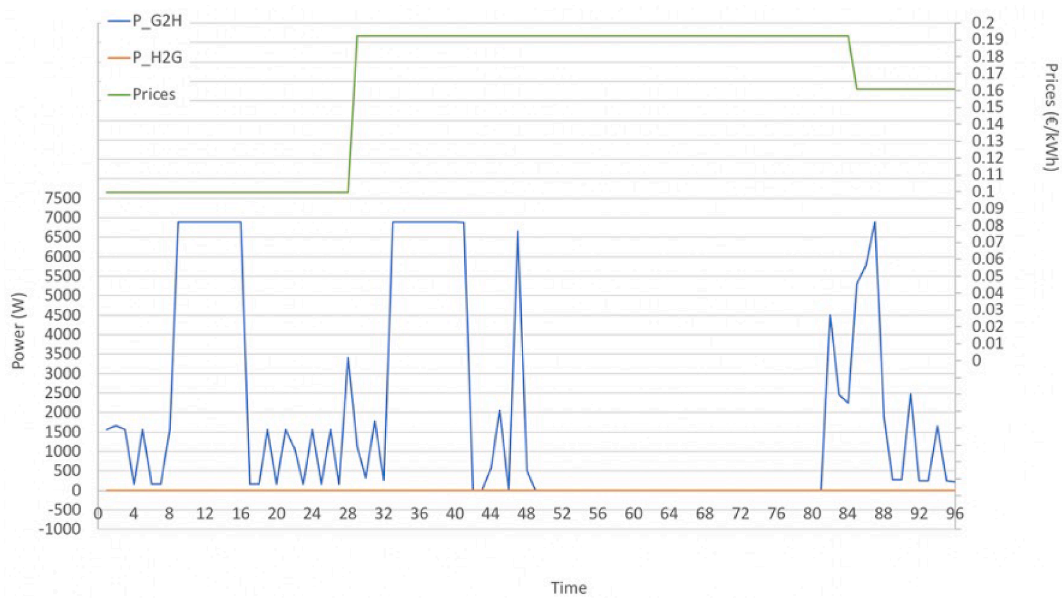

Fig. 4. Grid to home (P_t^{G2H}) and home to grid (P_t^{H2G}) powers.

obtained by the hybrid approach was worse than the ones found by the DBA with the approximate and the exact techniques, even with a higher computation time. The hybrid algorithm required several runs to attain a good quality solution. The DBA can offer sound information with short computation budgets, thus being able to assist the retailer in making sound decisions.

In the future, we aim to investigate more efficient algorithmic approaches able to circumvent the computational difficulties caused by the level of detail of the LL problem, namely using decomposition methods. The use of selected solutions obtained in one of the approaches as a warm start for other approaches will also be pursued. Moreover, we intend to model other features that characterize the consumer's behavior, as considering not only cost but also comfort objective functions, thus assisting the retailer in making decisions accounting for the

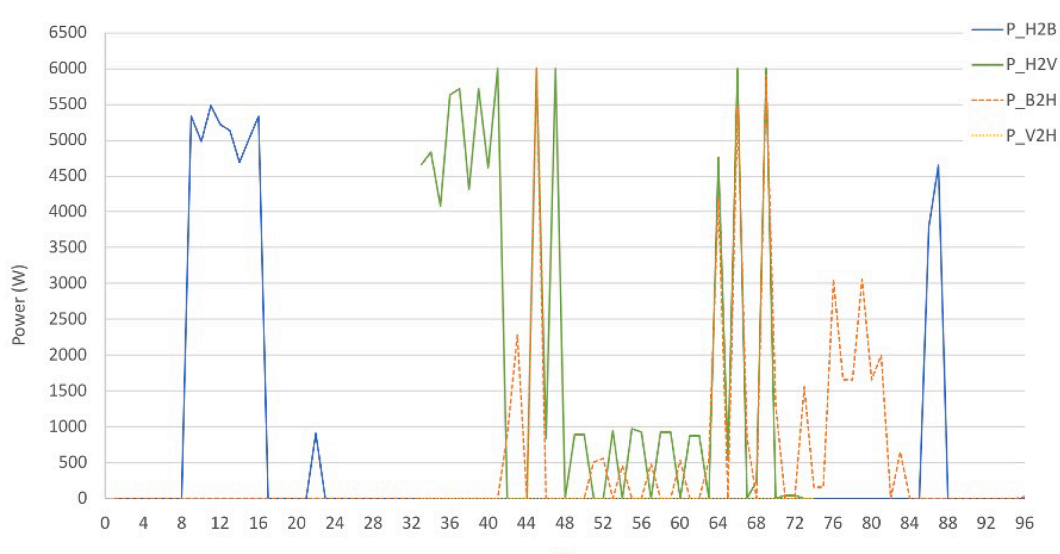

Fig. 5. Home to batteries (P_t^{H2B} and P_t^{H2V}) and batteries to home (P_t^{B2H} and P_t^{V2H}) powers.

exploration of the consumers' economic vs. comfort trade-offs.

CRediT authorship contribution statement

Inês Soares: Investigation, Software. **Maria João Alves:** Investigation, Software. **Carlos Henggeler Antunes:** Investigation, Conceptualization, Data curation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

I have indicated the link to the data in the manuscript.

Acknowledgements

This work was partially supported by projects UIBD/00308/2020 and UIDB/05037/2020 funded by FCT – Portuguese Foundation for Science and Technology. We are indebted to Manuel Amorim for the model implementation in GAMS.

Appendix - Consumer's problem

Nomenclature for shiftable loads:
Parameters:

J: number of shiftable loads ($j \in \{1, \cdots, J\}$).
d_j: duration of load j operation cycle.
f_{jr}: power requested by load j at stage (time) $r = 1, \cdots, d_j$ of its working cycle (kW).
$[T_{1_j}, T_{2_j}] \subset$T: comfort operation time slot allowed by the consumer for load j.

LL variables for shiftable loads:

$$s^{Sh}_{jt} = \begin{cases} 1 & \text{if shiftable appliance } j \text{ begins its operation in time } t \\ 0 & \text{otherwise} \end{cases}, t = T_{1_j}, \cdots, T_{2_j}$$

P^{Sh}_{jt}: power required by load j in time $t \in$ T (kW) (implicit variable determined by s^{Sh}_{jt}).

Constraints for shiftable loads

$$\sum_{t=T_{1_j}}^{T_{2_j}-d_j+1} s^{Sh}_{jt} = 1 \qquad j = 1 \cdots J \tag{5}$$

$$s^{Sh}_{jt} = 0 \qquad j = 1 \cdots J, \quad t = T_{2_j} - d_j + 2 \cdots T_{2_j} \tag{6}$$

$$P^{Sh}_{jt} = \sum_{\substack{r=1 \\ r \le t \wedge r \le t+1-T_{1_j}}}^{d_j} f_{jr} \times s^{Sh}_{j(t-r+1)} \qquad j = 1, \cdots, J, \ t = T_{1_j}, \cdots, T_{2_j} \tag{7}$$

$$P^{Sh}_{jt} = 0 \qquad j = 1, \cdots, J, \ t < T_{1_j} \vee t > T_{2_j} \tag{8}$$

$$s^{Sh}_{jt} \in \{0, 1\} \qquad j = 1, \cdots, J, \ t = T_{1_j}, \cdots, T_{2_j} \tag{9}$$

Constraints (5) and (6) impose that the load operation should start only once and guarantee that the cycle can end within the comfort time slot. Constraints (7) define the power required by each shiftable load j in each time t of its allowed operation slot, according to the starting time determined in (5); r must satisfy $r \le t \wedge r \le t + 1 - T_{1_j}$ to ensure that only existing variables are considered in $s^{Sh}_{j(t-r+1)}$. Constraints (8) force the power to be zero for t outside the allowed comfort time slot.

For the sake of illustration of defining constraints (5)–(8), let us suppose that load j has the comfort time slot $[T_{1_j}, T_{2_j}] = [10, 17]$, with $d_j = 3$ (i.e., 45 min considering $\Delta t = 15$ min) and f_{jr} ($r = 1, \cdots, 3$ – three stages of operation) are given by $f_{j1} = 1.2$ kW, $f_{j2} = 1.5$ kW, $f_{j3} = 0.5$ kW. By (5), the starting time should occur in $[T_{1_j}, (T_{2_j} - d_j + 1)] = [10, 15]$, guaranteeing that the operation ends at most in $t = 17$. For instance, suppose that load j starts the operation at $t = 14$ (i.e., $s^{Sh}_{j14} = 1$ and $s_{jt} = 0, \forall t \ne 14$); thus, it ends at $t = 16$. By (7), $P^{Sh}_{j10} = \sum_{\substack{r=1 \\ r \le 10 \wedge r \le 10+1-10}}^{r=3} f_{jr} s^{Sh}_{j(10-r+1)} = f_{j1} s^{Sh}_{j10} = 0$; similarly, $P^{Sh}_{j11} = P^{Sh}_{j12} = P^{Sh}_{j13} = 0$. For $t = 14$, $P^{Sh}_{j14} = \sum_{\substack{r=1 \\ r \le 14 \wedge r \le 14+1-10}}^{r=3} f_{jr} s^{Sh}_{j(14-r+1)} = f_{j1} s^{Sh}_{j14} + f_{j2} s^{Sh}_{j13} + f_{j3} s^{Sh}_{j12} = f_{j1} s^{Sh}_{j14} = 1.2$. For $t = 15$, the only non-zero term is $f_{j2} s^{Sh}_{j14}$, so $P^{Sh}_{j15} = f_{j2} = 1.5$ and, similarly, $P^{Sh}_{j16} = f_{j3} = 0.5$. For $t = 17$, $P^{Sh}_{j17} = f_{j1} s^{Sh}_{j17} + f_{j2} s^{Sh}_{j16} + f_{j3} s^{Sh}_{j15} = 0$ because $s^{Sh}_{jt} = 0$ for $t = 15$ by (5) and for $t > 15$ by (6).

A first modelling approach for the operation cycle of shiftable loads was proposed by Alves et al. (2016), which requires a higher number of binary variables and constraints. That model and other different models for shiftable loads developed in Antunes et al. (2022) were tested and compared. The computational experiments revealed that model (5)–(9) was the most efficient one among them.

Nomenclature for EWH:
Parameters:

P^R: power of the resistive heating element (kW).
τ_t^{amb}: ambient temperature around the EWH in time t (°C), $t = 1, \cdots, T$.
τ^{net}: inlet water temperature (°C).
m_t: water withdrawal for consumption in time t (kg), $t = 0, \cdots, T$.
M: hot water tank capacity (kg).
A: area of the tank envelope (m^2).
U: heat transfer coefficient of the tank ($kW/m^2.°C$).
c^p: specific heat of the water (J/kg.°C).
τ^{max}: maximum allowed temperature (°C).
τ^{conf}: comfort temperature (°C).
t^{req}: number of time units required to maintain a certain temperature to eliminate the legionella bacteria.
τ^{req}: temperature specified to be kept for t^{req} to eliminate the legionella bacteria (°C).

LL variables for the EWH (for $t \in$ T)*:*

$v_t \in \{0, 1\}$: binary variable defining the off/on control of the heating element in time t (v_0 is a constant).
τ_t: hot water temperature inside the tank in time t (°C), (τ_0 is a constant).
$n_t \in \{0, 1\}$: binary variable equal to 1 in the first t of the period with duration t^{req} in which $\tau_t > \tau^{req}$,$t = 1, \cdots, T - t^{req} + 1$.
P_t^{losses}: power losses through the envelope in time t (kW), (implicit variables determined by τ_t; P_0^{losses} is a constant).
P_t^{EWH}: power required by the EWH in time t (kW), (implicit variables determined by v_t).

Constraints for EWH:

$$P_t^{losses} = AU(\tau_t - \tau_t^{amb}) \qquad t = 1, \cdots, T \qquad (10)$$

$$\tau_{t+1} = \left(\frac{M - m_t}{M}\tau_t + \frac{m_t}{M}\tau^{net}\right) + \frac{v_t P^R - P_t^{losses}}{Mc^p}\Delta t \qquad t = 0, \cdots, T-1 \qquad (11)$$

$$\tau_t \geq \tau^{conf}(1 - v_t) \qquad t = 1, \cdots, T \qquad (12)$$

$$\tau_t \leq \tau^{max} \qquad t = 1, \cdots, T \qquad (13)$$

$$P_t^{EWH} = P^R v_t \qquad (14)$$

$$\sum_{t=1}^{T-t^{req}+1} n_t = 1 \qquad (15)$$

$$\tau_t \geq \sum_{\substack{t'=1 \\ (t' \leq t)}}^{t^{req}} \tau^{req} \times n_{t-t'+1} \qquad t = 1, \cdots, T \qquad (16)$$

$$v_t \in \{0, 1\}, t = 1, \cdots, T; n_t \in \{0, 1\} \qquad t = 1, \cdots, T - t^{req} + 1 \qquad (17)$$

Constraints (10)–(11) compute the power losses through the envelope and the water temperature in the tank. Constraints (12)–(13) enable that the comfort temperature may not be respected when the water is being heated ($v_t = 1$), to account for cases in which the initial temperature $\tau_0 < \tau^{conf}$ or there is a high water withdrawal. These constraints are hard regarding the maximum allowed temperature. Constraints (14) define the power required by the EWH for each t. Constraints (15) – (16) model the sanitary constraints to avoid the formation of legionella bacteria, which requires heating the water to a safety temperature τ^{req} for a specified duration t^{req} over the planning horizon.

Nomenclature for AC:
Parameters:

$\theta^{min}, \theta^{max}$: minimum and maximum allowed indoor temperature during the planning horizon (°C).
θ_t^{ext}: outdoor temperature in time t (°C), $t = 0, \cdots, T$.
P_{AC}^{nom}: nominal power of the AC system (kW).
α, β, γ: coefficients associated with the thermal modeling of the space being conditioned.
$\mathscr{M}$: big positive number.

LL variables for the AC (all for $t = 1, \cdots, T$)*:*

$s_t^{AC} \in \{0, 1\}$: binary variable defining the off/on control of the AC system in time t (s_0^{AC} is a constant).
θ_t^{in}: indoor temperature in time t (°C), (implicit variables determined by s_t^{AC}; θ_0^{in} is a constant).
P_t^{AC}: power required by the AC in time t (kW).

Constraints for AC:

$$\theta_t^{in} = \alpha\theta_{t-1}^{in} + \beta\theta_{t-1}^{ext} + \gamma s_{t-1}^{AC} P_{AC}^{nom} \qquad t = 1, \cdots, T \qquad (18)$$
$$\theta_t^{in} \geq \theta^{min} - \mathscr{M} s_t^{AC} \qquad t = 1, \cdots, T \qquad (19)$$
$$\theta_t^{in} \leq \theta^{max} + \mathscr{M}\left(1 - s_t^{AC}\right) \qquad t = 1, \cdots, T \qquad (20)$$
$$P_t^{AC} = s_t^{AC} P_{AC}^{nom} \qquad t = 1, \cdots, T \qquad (21)$$
$$s_t^{AC} \in \{0, 1\} \qquad t = 1, \cdots, T \qquad (22)$$

Constraints (18) compute the indoor temperature in time t as a function of the indoor temperature, the outdoor temperature and the AC status in time $t-1$. Constraints (19)–(20) model the AC to operate in an "on/off" mode by ensuring that the AC is "on" when the indoor temperature is below the minimum temperature (forcing $s_t^{AC} = 1$) and is "off" when the indoor temperature is above the maximum allowed temperature (forcing $s_t^{AC} = 0$), respectively. Constraints (21) define the power required by the AC for all t.

Nomenclature for the static battery (B) and the EV battery (V):
Parameters for B:

$\eta_{ch}^B, \eta_{dch}^B$: charging and discharging efficiency of the battery.
E_{min}^B, E_{max}^B: minimum and maximum allowed battery charge.
E_0^B: initial battery charge (kWh).
$P_{max}^{Bch}/P_{max}^{Bdch}$: maximum charge/discharge power allowed for the battery (kW).

Parameters for EV:

t_a, t_d: time of arrival and departure of the EV.
$\eta_{ch}^V, \eta_{dch}^V$: charging and discharging efficiency of the EV battery.
E_{min}^V, E_{max}^V: minimum and maximum allowed EV battery charge (kWh).
$E_{t_a}^V$: initial EV battery charge, at the time of arrival t_a (kWh).
E_{req}^V : EV battery charge requested at the time of departure t_d (kWh).
$P_{max}^{Vch}, P_{max}^{Vdch}$: maximum charge and discharge power allowed for the EV battery (kW).

LL variables for B (all for $t = 1, \cdots, T$):

$s_t^{H2B} \in \{0, 1\}$: binary variables that are 1 (0) when the battery B is (is not) charging in time t.
$s_t^{B2H} \in \{0, 1\}$: binary variables that are 1 (0) when the battery B is (is not) discharging in time t.
E_t^B: energy in the battery in time t (kWh), (E_0^B is a constant).
P_t^{B2H}: power withdrawn from the battery to home (B2H) in time t (battery discharge).
P_t^{H2B}: power withdrawn from the home to the battery (H2B) in time t (battery charge).

LL variables for EV (all for $t = t_a + 1, \cdots, t_d$):

$s_t^{H2V} \in \{0, 1\}$: binary variables that are 1 (0) when the EV battery is (is not) charging in time t.
$s_t^{V2H} \in \{0, 1\}$: binary variables that are 1 (0) when the EV battery is (is not) discharging in time t.
E_t^V: energy in the EV battery in time t ($E_{t_a}^V$ is a constant).
P_t^{H2V}: power withdrawn from the home to the EV battery (H2V) in time t (EV battery charge).
P_t^{V2H}: power withdrawn from the EV battery to home (V2H) in time t (EV battery discharge).

Constraints for B:

$$E_t^B = E_{t-1}^B + \left(\eta_{ch}^B P_t^{H2B} \Delta t\right) - \left(P_t^{B2H} \Delta t / \eta_{dch}^B\right) \qquad t = 1, \cdots, T \qquad (23)$$
$$E_{min}^B \leq E_t^B \leq E_{max}^B \qquad t = 1, \cdots, T \qquad (24)$$
$$0 \leq P_t^{H2B} \leq P_{max}^{Bch} s_t^{H2B} \qquad t = 1, \cdots, T \qquad (25)$$
$$0 \leq P_t^{B2H} \leq P_{max}^{Bdch} s_t^{B2H} \qquad t = 1, \cdots, T \qquad (26)$$
$$s_t^{H2B} + s_t^{B2H} \leq 1 \qquad t = 1, \cdots, T \qquad (27)$$
$$E_T^B \geq E_0^B \qquad (28)$$
$$s_t^{H2B}, s_t^{B2H} \in \{0, 1\} \qquad t = 1, \cdots, T \qquad (29)$$

Constraints for EV:

$$E_t^V = E_{t-1}^V + \left(\eta_{ch}^V P_t^{H2V}\Delta t\right) - \left(P_t^{V2H}\Delta t/\eta_{dch}^V\right) \qquad t = t_a + 1, \cdots, t_d \qquad (30)$$
$$E_{min}^V \le E_t^V \le E_{max}^V \qquad t = t_a + 1, \cdots, t_d \qquad (31)$$
$$0 \le P_t^{H2V} \le P_{max}^{Vch} s_t^{H2V} \qquad t = t_a + 1, \cdots, t_d \qquad (32)$$
$$0 \le P_t^{V2H} \le P_{max}^{Vdch} s_t^{V2H} \qquad t = t_a + 1, \cdots, t_d \qquad (33)$$
$$s_t^{H2V} + s_t^{V2H} \le 1 \qquad t = t_a + 1, \cdots, t_d \qquad (34)$$
$$E_{t_d}^V \ge E_{req}^V \qquad (35)$$
$$s_t^{H2V}, s_t^{V2H} \in \{0, 1\} \qquad t = 1, \cdots, T \qquad (36)$$

Constraints (23) and (30) compute the energy in the batteries considering the charging/discharging events. Constraints (24) and (31) impose a minimum and maximum allowed energy in the batteries. Constraints (25)–(26) and (32)–(33) define the power exchanges as non-negative and limit to a maximum allowed power for charging (when $s_t^{H2B} = 1$ and $s_t^{H2V} = 1$, respectively) and discharging (when $s_t^{B2H} = 1$ and $s_t^{V2H} = 1$, respectively) modes. Constraints (27) and (34) impose that the battery cannot charge and discharge at the same time. Constraints (28) define that at the end of the planning period the energy available in the stationary battery cannot be lower than at the beginning of the planning period. Constraints (35) specify a minimum charge in the EV battery at the time of departure (t_d).

Nomenclature for G2H and H2G flows:
Parameters:

P_{max}^G: maximum power allowed for exchanges with the grid.

LL variables for exchanges with the grid (all for $t = 1, \cdots, T$)*:*
$s_t^{G2H} \in \{0, 1\}$: binary variables that are 1 (0) when the energy is (is not) flowing from grid to home in time t
$s_t^{H2G} \in \{0, 1\}$: binary variables that are 1 (0) when the energy is (is not) flowing from home to grid in time t

Constraints for G2H/H2G:

$$0 \le P_t^{G2H} \le P_{max}^G s_t^{G2H} \qquad t = 1, \cdots, T \qquad (37)$$
$$0 \le P_t^{H2G} \le P_{max}^G s_t^{H2G} \qquad t = 1, \cdots, T \qquad (38)$$
$$s_t^{G2H} + s_t^{H2G} \le 1 \qquad t = 1, \cdots, T \qquad (39)$$
$$s_t^{G2H}, s_t^{H2G} \in \{0, 1\} \qquad t = 1, \cdots, T \qquad (40)$$

Nomenclature for Power balance:
Parameters (all for $t = 1, \cdots, T$)*:*

L_t^{Base}: power of non-controllable base load in time t (kW), corresponding to appliances that are not deemed for control (e.g., lighting, fridge, oven, etc.).
L_t^{PV}: local PV power in time t (kW).

Constraints for Power balance:

$$P_t^{G2H} - P_t^{H2G} + L_t^{PV} = L_t^{Base} + \sum_{j=1}^{J} P_{j,t}^{Sh} + P_t^{EWH} + P_t^{AC} + \left(P_t^{H2B} - P_t^{B2H}\right), \qquad t \le t_a \lor t > t_d \qquad (41)$$
$$P_t^{G2H} - P_t^{H2G} + L_t^{PV} = L_t^{Base} + \sum_{j=1}^{J} P_{j,t}^{Sh} + P_t^{EWH} + P_t^{AC} + \left(P_t^{H2B} - P_t^{B2H}\right) + \left(P_t^{H2V} - P_t^{V2H}\right), \qquad t = t_a + 1, \cdots, t_d \qquad (42)$$

The set of constraints (41)–(42) models the power balance, and they only differ in the time slot allowed for the EV battery operation. The total power required from the grid P_t^{G2H} plus the local generation L_t^{PV} and the charge in batteries, P_t^{B2H} and P_t^{V2H}, must be equal to the power required to supply all loads (base load, controllable loads, stationary and EV battery charging) plus the power sold to the grid P_t^{H2G}.

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