Department of Economics

Business Cycle Synchronization and International Risk Sharing

Pedro André Cerqueira

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, May 2007

EUROPEAN UNIVERSITY INSTITUTE
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DEDICATION

This thesis is dedicated to my parents, wife and sister, who offered me unconditional love and support throughout the course of this thesis.

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Part I

Introduction

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Since the seminal work of Arthur Burns and Wesley Mitchell in their 1946 book much of the subsequent economic literature has been devoted to the study of fluctuations in economic activity in the mid term or in other words at what is now called the business cycle frequency. Originally the term business cycle referred to combination of periods of economic expansion and decline, however the concept has been enlarged to include fluctuations of growth around the trend, that is, combinations of periods when growth is higher than the long term growth with periods when it is lower. These last are called deviation cycles as opposed to the previous ones, studied by Burns and Mitchell, that are called classical cycles. Contrary what the name suggests they are not regular in timing and duration - on the contrary, booms and recessions occur at irregular intervals and last for varying lengths of times.

We can split the literature into two branches. On one side theorists have proposed different causes of the business cycles occurrence and, using general equilibrium, have predicted agents behavior during the fluctuations. In the end they try to match the results from the models to ones from data.

On the other side, more determinedly empirical work has tried to measure and date the business cycle accurately as well as describing the behavior of agents, again, checking whether that behavior is accord with the predictions of theoretical models.

From the intersection of these literatures several puzzles have arisen, as theoretical models have found great difficulty in replicating some of the facts measured. Among others, and more related to business cycles we can point to the Feldstein-Horioka puzzle, the consumption correlation puzzle, the portfolio home bias, the trade home bias puzzle and several price anomalies. As the focus of the current work is, as the title indicates, business cycle synchronization issues and the consumption correlation puzzle and because we will, also examine the price anomaly of the behavior of the terms of trade through the cycle we will just give a brief explanation of these two puzzles/anomalies, leaving to the following chapters a more complete characterization of each.

The consumption correlation puzzle arises from the fact that if we assume that international financial markets are fully integrated then consumption should not depend too much on nation-specific shocks to output and therefore cross country correlations of consumption growth or of the deviations from trend should be quite high. This relation continues to hold true even if the capital markets are segmented but agents in different countries would be able to trade bonds. However when we look at the data we usually find values that are much lower than the ones predicted by theoretical models, and even cross-country correlations of consumption lower than the cross-country correlations of output.

As for the price anomaly, in theoretical models when a country suffers a negative shock

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and output declines then the prices of goods produced relative to foreign ones should increase (and vice-versa, after a boom the relative prices should decrease), therefore the correlation between output and terms of trade, defined as $\frac{\text{Price of Exports}}{\text{Price of Imports}}$, should be negative. However, when we look at the data, while it is true that for a significant number of countries that correlation has the correct sign, for most it is not as strongly negative as the models would predict, and furthermore for some countries, including the US, that correlation is positive. This fact has been corrected in some models when they allow for a very low cross-country elasticity of substitution, however this comes at the expense of a counterfactual lower volatility of the terms of trade when compared with that of output.

The work presented in the following chapters has as its objective to characterize business cycle synchronization across countries, to measure how much consumption risk sharing is done and in what way across countries, as well as, to propose a different channel to match the correlations measured in data and in a theoretical model. Therefore, as the first two issues are mainly empirical and the last is theoretical, we divided the work into two parts.

The first of these two parts is composed of two chapters. The first chapter is called "How Pervasive is the World Business Cycle?". Here we tried to establish whether business-cycle synchronization across countries is due to a single world business cycle, to a diversity of regional business cycles or a combination of both, using an approximate factor model to estimate the common components and the loadings. Contrary to previous papers that examine the same issue, the methodology we use does not impose any prior hypothesis about regional composition. If distinctive regional business cycles exist, the regional composition would be estimated from the data used and the countries will form a region in *substrictus sensus*[1]. The results indicate that there are four common components, but none specific to a geographical or cultural region. However we found that one of those common factors was more associated with high and one to middle income countries and therefore we can consider that those two country groups form the only regions in *substrictus sensus.* As for the low income countries they seem detached from any factor, as most of the cyclical behavior is idiosyncratic. Afterwards we looked closely at the dependence structure from the estimated common factors and the degree of commovement between countries. From that analysis we found some sets of countries that have a similar factor dependence and a high degree of commovement . We will call those groups regions in the *latus sensus*[2]*.* In the high income countries group we found two groups that we labeled the French-Japanese (as it

[1]From latin, meaning in narrow sense.
[2]From latin, meaning in broad sense.

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includes France, some other European countries neighbors of France and Japan) and German-American (as it includes Germany, some neighbors of Germany and the USA) groups. In the middle income countries *substrictus sensus* region we only found a set of groups in South East Asia that has a similar dependence structure but do not commove (the idiosyncratic components are quite high), and therefore we did not classify this group of countries as a *latus sensus* region.

In the second chapter, called "Consumption smoothing at business cycle frequency", we measure the importance of the consumption puzzle for different countries and, at the same time, disentangle the strength of the several sources of consumption smoothing through the cycle in a panel of countries. For this purpose a factor model is applied to data from 23 OECD countries. This approach allows us to measure the strength of the different channels of consumption smoothing for the different countries rather than estimating an aggregate value for all. In the aggregate we found similar results for the different smoothing channels as in the existing literature; however, the individual estimates show that those aggregations hide considerable differences across countries. For instance, the smoothing through international portfolio diversification that at an aggregate level is not different from zero, ranges from -17,4% to 16,4% and the smoothing through the credit market that is around 39% at aggregate level ranges from 2,5% to 82,9%.

Having individual estimators for each country we could check to which indicators can they be related. Concerning the portfolio diversification channel we found some puzzling results: the more a country is synchronous with others, and therefore has less to gain from smoothing the impact of national shocks on consumption, the more it smooths and the more open a country is, and so it has more access to foreign goods, the less it smooths through this channel. Moreover the portfolio diversification channel has no correlation with any of the financial indicators. As for the smoothing through the credit market the higher is bank concentration and the lower are interest margins the more the consumers smooth through this channel.

The last part of the thesis is a chapter entitled "International Real Business Cycles and R&D" where we look at the impact of *variety expansion* on the moments of an international real business cycle model.

The literature that tries to match the moments of the data with those from theoretical models, even in the recent vintage of models in which the number of varieties is endogenous, did not give much attention to the fact that consumers may smooth their utility by varying the diversity of varieties that they consume rather than the quantity and that the number

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of varieties might change, not only due to shocks in the total factor productivity of goods production, but also due to the variability of the creativity process of the R&D sector responsible for the introduction of new varieties in the economy.

Incorporating this idea into a formal model might enable us to account for the "consumption correlation puzzle" that has plagued the empirical confirmation of RBC models. Suppose, for example, that in one country agents smooth consumption by diversifying over a bigger array of varieties whilst in another agents smooth consumption in a traditional way. Because of the fact that consumer price indices and hence deflated consumption volume cannot accurately register changes in the composition of the consumption basket, it is entirely possible that consumption will not appear to be highly correlated in these two countries. In other words, the model might help explain the consumption correlation puzzle.

In addition, an increase in the number of varieties produced in one country could lead to a reduction of units produced of each (even if it could increase the sum of the total) and this would make them more valuable relative to the varieties produced by the other country. In the end this can lead to a measured economic expansion and an appreciation of the terms of trade reproducing the "price anomaly" found for some countries, as for instance for the US.

In the formal model we incorporated a R&D sector responsible for the introduction of new varieties with the objective of checking whether shocks in the creativity process alone or in combination with the traditional shocks in the total factor productivity of final goods could confirm these conjectures.

The results showed that shocks in R&D sector alone are not able to replicate most of the (unconditional) moments of the data and exhibit the same problems as the shocks in the productivity of the final goods sector, although with different magnitudes. However, even if the impact of shocks on different variables lead to similar moments the fact is that the relative response of the variables is not the same in both cases and for certain variables is even the opposite. Therefore, when we allowed both shocks to coexist we found that we could replicate most of the moments from the data for reasonable values of the standard parameters together with a strong correlation between the shocks in R&D and the ones in the productivity in the final sector and with negative contamination across countries in the stochastic process of the R&D shocks.

In the model the reason for matching the relative sizes of the cross–country correlations of output and consumption is that the utility-smoothing through the cycle is done through variation in the quantity consumed of each good when the shocks are to the productivity of the production sector and through the diversity of goods when the shocks hit the R&D

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sector. Due to the different strategies, even if utility is smoothed, *measured* consumption diverges, leading to a smaller cross country correlation of consumption than of output (the consumption correlation puzzle) as we've conjectured.

As for the correlation between output and the terms of trade they were around zero, and the volatility in the terms of trade was higher than in output. Even if we did not got the positive correlation that we have thought , the fact is that the volatility found was such that the value found in the model was not statistically different for most of the values found for this correlation in the data (either positive or negative), therefore together with the size of the volatility of the TOT relative to the one of output found in the moments from the model we can claim that the results were better than the ones we expected, replicating the price anomaly.

Summarizing we can say that the work presented in the next chapters sheds some new insights in the business cycle literature, both at the empirical and theoretical level.

At the empirical level we showed how the world can be grouped in terms of business cycle synchronization departing from the *a prioristc* assumption of continental divisions or from the traditional North-South split. Moreover, contrary to previous works it shows that the groups in the developed world are not due to specific cycles but due to different sensibility to different factors that affect the worldwide economy. As for the North-South split, it shows that the non-developing world cannot be put all together in the same region.

A second empirical contributions is how consumption risk-sharing across countries is done giving us a more detailed picture of each country than the aggregate estimations from previous papers could provide. This more detailed picture shows that the developed countries use the different mechanisms with different magnitudes and that those differences can be related to population size, degree of world integration, openness and to two financial indicators: net interest margin and degree of concentration of the financial system. Moreover, some of the relations found are puzzling and they might deserve further research.

At the theoretical level we were able to replicate the consumption correlation puzzle and the price anomaly without having to resort to rigidities and distortions in the economies such as trade costs, tradables vs. non-tradables or financial ones. Instead, we offer the explanation that those puzzles in the data could be the result of creativity shocks combined with traditional productivity shocks. This result justifies further research at empirical level in order to estimate shocks at business cycle frequency to the creativity process in order to check if the estimated stochastic processes are equal to the ones necessary to replicate both puzzles as the model shows.

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Part II

Empirical Chapters

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CHAPTER 1

HOW PERVASIVE IS THE WORLD BUSINESS CYCLE?

1.1 Introduction

This chapter looks at the question of whether there is a world business cycle and/or a number of regional business cycles. This question has, already, been the subject of several papers and has been under renewed attention. The main reasons of this new interest are the creation of the EMU and the slowdown at the beginning of the 21st century.

The creation of the EMU led to an interest in whether the business cycles of European countries were or not synchronized, and therefore, whether a single monetary policy could fit all. The studies done concluded in favour of the existence of a core and a periphery. However, they raised the question whether national business cycle synchronization is the reflex of a truly European Business Cycle, or just the reflex of a World Business Cycle. This kind of question was the subject of Artis and Zhang(1997) and Artis(2003).

The slowdown verified in 2000/2001 was initially expected to be constrained to the US economy, but it spread to most of the developed economies, indicating that those economies are linked. This raised the question of whether the synchronization observed was a reflex of a world business cycle or just a coincidence and if the links between economies have or not increased. To answer this questions several papers studied the synchronization of the G7 business cycles: Gregory and Head(1997), Andreano and Savio(2002), Helbling and Bayoumi(2003), Bordo and Helbling(2003). Other studies were more ambitious and tried to include more countries, Lumsdaine and Prasad(2003) included all OECD countries, Kose et al.(2003) used data from 1960 to 1990 for 60 countries and Mansour(2003) used GDP data for 113 countries from 1961 to 1989. However, the studies that try to cluster the countries into regions do not try to estimate the world and the regional cycles and their importance to the national cycle. The ones that try to estimate the world business cycle and its importance for each country either do not take into account regional cycles, or if they do, as Kose et al.(ibid.) and Mansour(ibid.), assume the identity of the regions rather than estimating them from the data.

This chapter tries to estimate the world and regional cycles (if they exist) as well as the importance of each for each country studied and, at the same time, to cluster the countries into regions. In order to perform the study I use an approximate factor model, applied to

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data of 58 countries from 1970 to 2001, for which the asymptotic distribution theory was developed by Bai(2003). To the best of my knowledge this particular method has not been used before to study this question. Other studies have used factor models, but of a different kind: Helbling and Bayoumi (ibid.) and Mansour(ibid.) used a dynamic general factor model (DGFM) proposed by Forni et al.(2002), Kose et al.(ibid.) used a Bayesian method described in Otrok and Whiteman(1998), Lumsdaine and Prasad(ibid.) used a time-varying weighted method.

The main advantages of the factor model used in this paper are that the estimators have an asymptotic distribution, rendering it possible to perform inference over them (which is not possible in the DGFM or in Lumsdaine and Prasad's approach), and we do not have to assume regional composition beforehand.

The structure of the chapter is the following: the next section describes the econometric model used; the following shows the empirical results and the final one concludes.

1.2 Econometric Methodology

To address the problem of the existence of a world business cycle and/or regional business cycles I used the cyclical component of real GDP of each country. These cyclical components were constructed from the difference of two HP filters as discussed in Artis et al. (2003).

Afterwards I applied to these GDP cycles an approximate factor model for which the inferential theory was developed by Bai(2003). The model is represented by:

$$\begin{aligned} X_{it} &= \lambda_i' F_t + e_{it} = C_{it} + e_{it} \qquad (1.1) \\ i &= 1, 2, 3, ..., N \\ t &= 1, 2, 3, ..., T \end{aligned}$$

where X_{it} represents the value of the i^{th} national GDP business cycle at time t, λ_i is the loading vector $r \times 1$, F_t is a $r \times 1$ vector representing the value of r factors at time t and e_{it} is the idiosyncratic component of the series at time t.

In this framework and under the assumption A described in Bai (ibid.), this factor model allows F_t to be dynamic such that $A(L)F_t = \epsilon_t$. However the relationship between X_{it} and F_t is static[1].

The first problem to solve is to decide the number of factors that should be included in the model. Bai and Ng(2002) constructed three information criteria (ICs) that can be used consistently to estimate the number of common factors.

[1] A more general factor model where the relationship between X_{it} and F_t can be dynamic is the DGFM described by Forni et al.(2002).

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However, if there is too much cross-heteroscedasticity those IC estimates will identify too many common factors when used in finite samples. The Montecarlo studies[2] conducted showed that in this case if the series are not standardized those ICs almost always select the maximum number of factors that we test for. If we standardize the data the IC that gets closer to the true number is the IC2 (even if occasionally it overestimates them), as the IC1 and IC3 always select a number bigger than the true one. Therefore when analyzing the results we should be aware of the fact that even using the IC2 criteria we may be overestimating the number of factors.

Then, to check for the importance of each factor we can perform the variance decomposition of the indices given by:

$$\begin{aligned} var(Ind^c) &= var\left(\sum_{r=1}^{R} \lambda_{cr}.F_r + e_c\right) \Leftrightarrow \qquad (1.2) \\ &\Leftrightarrow 1 = \frac{var(\lambda_{c1}.F_1)}{var(Ind^c)} + \frac{var(\lambda_{c2}.F_2)}{var(Ind^c)} + ... + \frac{var(\lambda_{cR}.F_R)}{var(Ind^c)} + \frac{var(e_c)}{var(Ind^c)} \\ \text{where} &: Ind^c,\ F_r \text{ and } e_c \text{ are } t \times 1 \text{ vectors} \\ & \quad \text{and } \lambda_{cr} \text{ is the loading of factor } r \text{ associated with the country } c \end{aligned}$$

The last equality results from the estimation procedure of the approximate factor model. The method imposes that $(F'F)/T = I_r$, (F is a $t \times r$ matrix and I_r is an identity matrix of order r) the factors are orthogonal, and so the variance of the sum is equal to the sum of the variances.

Moreover, the factors are standardized, their variance is equal to one, so the importance of each factor is given by:

$$\frac{var(\lambda_{cr}.F_r)}{var(Ind^c)} = \frac{(\lambda_{cr})^2}{var(Ind^c)} \quad ; \quad r = 1, 2, 3, ..., R \qquad (1.3)$$

If there is a regional business cycle, how should we decide for its existence and the regional composition? As said before regions are not assumed before the estimation of the common components. Their existence has to be inferred from the data. The method used is to compare the importance of a factor for each country and/or a subsets of countries using equation (1.3). If the ratio estimates that the factor is only important for a limited number of countries then we are in the presence of a regional factor, and the set of countries is said to be the region *in substrictus sensus*. If the factor is important for a large subset of countries then the component is a global one.

[2] See appendix 1.D.

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A second definition of region will be in *latus sensus*. In this case, independently of whether we find or not a regional factor, there can be groups of countries with a similar dependence structure from the common components. In this case these sets of countries would form regions in *latus sensus.* To check for the existence of this kind of region we will perform a cluster analysis to check how similar are the countries' dependence structures and how they group.

1.3 Empirical work

This section will present the estimated results. It is divided into two subsections. The first describes the data used and the following shows the results obtained.

1.3.1 Data used

The data used were those for Gross Domestic Product (GDP) calculated from the expenditure approach of 58 countries at constant prices (base year: 1990). All the series were collected from the WTI 2004 World Bank Database, selecting those for which we could get a complete data set from 1970 to 2001. A complete list of the countries invloved is given in appendix 1.A.

The use of annual data rather than data at a higher frequency was dictated by the availability of the series for as many countries and as long time span as possible. Quarterly data are not available for most countries since 1970. Had we chosen a shorter interval either the number of countries that could be used would be too small or the time span would be too short.

Finally, to estimate the business cycle indices of each series I chose to retain the fluctuations from two to eight years. This interval follows the one that Baxter and King (1999) adopt, between 6 and 32 quarters.

1.3.2 Results

This subsection presents the results and it is divided into five subsections. The first one estimates the number of factors to be included in the model, the next presents the estimated factors and the following the relative importance of each for each country. The fourth tries to identify regions in *substrictus sensus.* The last point clusters the countries together to check if there are regions in *latus sensus* or not.

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1.3.2.1 Estimating the number of common factors

When we analyze the estimated business cycle indices of each country[3] we realize that there are some regularities common to most countries: a trough around 1973, coinciding with the first oil crisis, another around 1981, and a third trough at the beginning of the nineties. These coincidences make us suspect that the countries are linked by a world business cycle. However, some of those coincidences are not common to all countries nor are the variations registered on a equal basis. Therefore we could assume as a starting hypothesis that, beyond a common factor and idiosyncratic shocks, some countries are subject to some regional components. But to be certain of our suspicions we should try to find the regional cycles and the regions themselves.

To apply the model of equation (1.1) we need to know how many factors should be included in our analysis. Using a statistical criterion is in this context better than assuming how many factors to test based on our beliefs of how do the world and/or regional cycles should look like, as that would lead to some prior beliefs about regions and region composition.

However, before selecting the number of common components we have to select the maximum number of factors that we allow for ($Rmax$). In time series a rule such as $8 * int[(T/100)^{1/4}]$ considered in Schwert(1989) is sometimes used to set $Rmax$. However for panel data no such rule is available. Bai and Ng(ibid.) considered $8*int[(\min(T,N)/100)^{1/4}]$ and in their Montecarlo simulations for samples of a size similar to ours they used $Rmax = 8$.

So is 8 a parsimonious value? Kose et al.(ibid.) in their study have divided the world into 7 regions (the six continents, splitting Asia into developed and non-developed); therefore, they estimated a total of eight common components. Mansour(2003) selected 8 geographical regions (the 6 continents plus the Mediterranean and Maghreb area) and 12 trade agreements. However in many of those trade agreements most countries are of either one or two geographical groupings, and if there is a specific component to that trade agreement they will be the same as the ones found in the geographical regions. Therefore using this division we should select around 9 common components. Taking into account these two studies we used 9 as $Rmax$.The estimated results for the number of common components are in table 1.1.

We argued before that if there is too much cross-heteroscedasticity in the idiosyncratic components the ICs applied to non-standardized data almost always select the $Rmax$. As regarding the standardized data, even if the IC2 sometimes overestimate the number of

[3]See appendix 1.B.

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Table 1.1: Number of common components

	Standardized Data	Non Standardized Data
IC1	$9(Rmax)$	$9(Rmax)$
IC2	4	$9(Rmax)$
IC3	$9(Rmax)$	$9(Rmax)$

common factors, this criterion is the one closer to it[4]. From the previous table it seems that we should select 4 as the number of common components to use.

This number should be put into perspective regarding previous studies:

- In the Kose at al.(2003) study only the World component and the North American factor seemed important in explaining commovements. In this case two would be the number of common components;
- In Mansour's study he found that 2 regions (Africa and Asia) were quite heterogeneous as three regions had a quite high value of commovement (Europe and especially the EC; Oceania and South America) as the others had a degree of commovement not much different from the world average. Therefore, even if he only decomposed the possible regional component for the EC, from the previous results the number of common components to be estimated should be around 4, one for each of the three regions with high commovement and one for the whole sample: the world component;
- Helbling and Bayoumi(2003) have estimated 2 world common components for the G7 countries;
- Baxter and Kouparitsas(2004) when searching for indicators to explain commovement showed that the only characteristic that was robust to several specifications was whether a country is a developed one or not.

From those studies if we select 4 as the number of common components it seems it would be in the range that previous studies would suggest (somewhere between 2 and 4).

Taking into account this comparison with other studies and the fact that the IC2 occasionally overestimates the number of common components we should consider that we might be including too many (not too few) factors. Anyway, if that happens, it should be

[4]In fact we found evidence of cross-heteroscedasticity when estimating the model without standardizing the model, see appendix 1.C. Furthermore, the results of table 1.1 by selecting $Rmax$ with non-standardized data give some support for standardizing it.

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apparent from the importance of each factor in the variation of the business cycle of the countries; that is, if the last factor is only important to one country, the importance to the others being only marginal, then that factor is capturing a country business cycle and not a common component. At this point the model can be restricted[5]. Therefore, we proceed with the study using 4 as the number of common components to be estimated.

Looking at the ratio of eigenvalues we can assess how far the first n common components explain the common variation of the series:

Table 1.2: Ratio of Eigenvalues

Eigenvalue	1	2	3	4	5
Ratio	0.2391	0.1184	0.1142	0.0842	0.0683
Eigenvalue	6	7	8	9	10
Ratio	0.0537	0.0473	0.0458	0.0373	0.0322

From table 1.2 we see that the first eigenvector can explain 23.91% of the common variation of the series. Cumulatively, the first four factors explain up to 55.5% of the common variation These values are smaller than those estimated by Helbling and Bayoumi(2003). Those authors, using the methods of Forni et al.(2002), found that the first factor would explain 60% of the variation and the first three factors would explain about 90%. Those results are for GDP data of the G7 countries. However, when they add other series their values decrease to 44% and 71% respectively. But, we can expect that the more countries/series are used, the smaller will be the estimated ratios as we add more sources of idiosyncratic variation. If we do the same exercise for the OECD countries, the first four factors would explain almost 70% of the common variation.

1.3.2.2 The estimated common components

The first question is what do the first four common components look like. Figure 1.1 depicts the estimated factor associated with the biggest eigenvalue. The dotted lines are the 95% confidence level band for the estimated value. At this point we should stress that assumptions A-H described in Bai(2003) are needed to compute the confidence interval of the

[5]At this point we should note that we do not need to reestimate the model, as the estimated factors are orthogonal, the estimated loadings for the first n factors are equal if we use n or $n+m$ factors . However the same is not true to the estimated variance-covariance matrixes and, hence,for the confidence intervals. Restricting the model would lead to wider confidence intervals.

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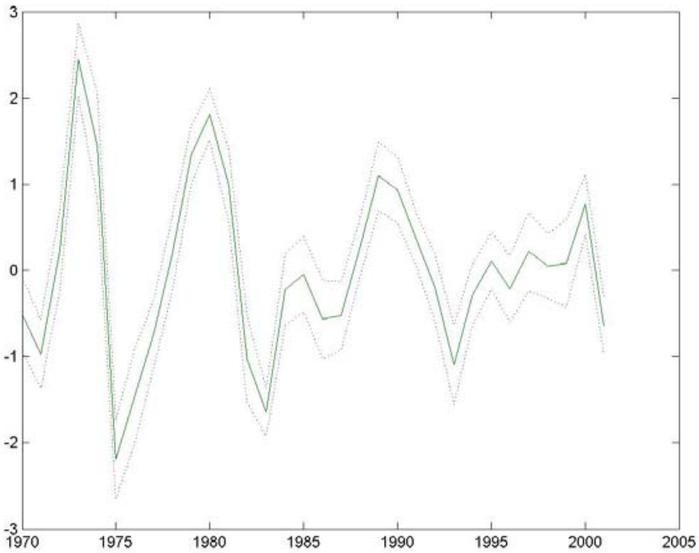
factors when $N, T \rightarrow \infty$ (this is the weaker condition imposed). Of importance is assumption H, it imposes that the residuals of equation(1.1) are homoscedastic and not serial correlated. However we found for some countries traces of autocorrelation, therefore the asymptotic theory is only valid when $\sqrt{N}/T \rightarrow 0$[6].

Figure 1.1: The First Factor



From table 1.2 we realized that the first factor explains almost 24% of the variation of the countries' indices. From figure 1.1[7] we realize that this first common component captures most of the stylized facts of the world business cycle described in the literature. It peaks[8] in 1973, 1980, 1989 and 2000, reaches a trough in 1975, 1983, 1993 and also reflects the slowdown at the beginning of the new millennium.

[6]From those assumptions is clear that cross-section heteroscedasticity and cross-section correlation are still allowed, thus the model continues to be an approximate factor model.

In this sample I performed a white test to detect heteroscedasticity . The test did not reject the null of homoscedasticiy at 5% significance level for any country.

As for autocorrelation, doing a LM test for AR, 8 out of 58 countries rejected no autocorrelation of 1st order and 28 out of 58 of 2nd order, both at 5 % significance level.

In this case the asymptotic distribution of the factors is only valid when $\sqrt{N}/T \rightarrow 0$. In our sample $N = 26 \rightarrow \sqrt{N} \approx 5$ which is much smaller than $T = 32$. The use of the asymptotic distribution is still plausible, as long as assumptions A-G of Bai(idem) are satisfied.

[7]The method is not able to estimate the factors themselves, but a rotation of them (HF_t).Therefore in this study I assumed that the loadings of the factors should be positive in the majority of the G7 countries, and multiplied the estimated eigenvector by -1 every time that the estimated loadings would not obey to that assumption.

[8]We will say that the index reaches a peak in a given year if it is both positive and is the bigger value between troughs. A index reaches a trough if it is negative and if it is the smaller value between peaks.

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Kose et al. (2003) using annual data from 1961 to 1990 for 60 countries found, after 1970, peaks in 1973, 1976 and 1988, troughs in 1975 and 1982 and a slowdown in the end of the sample.

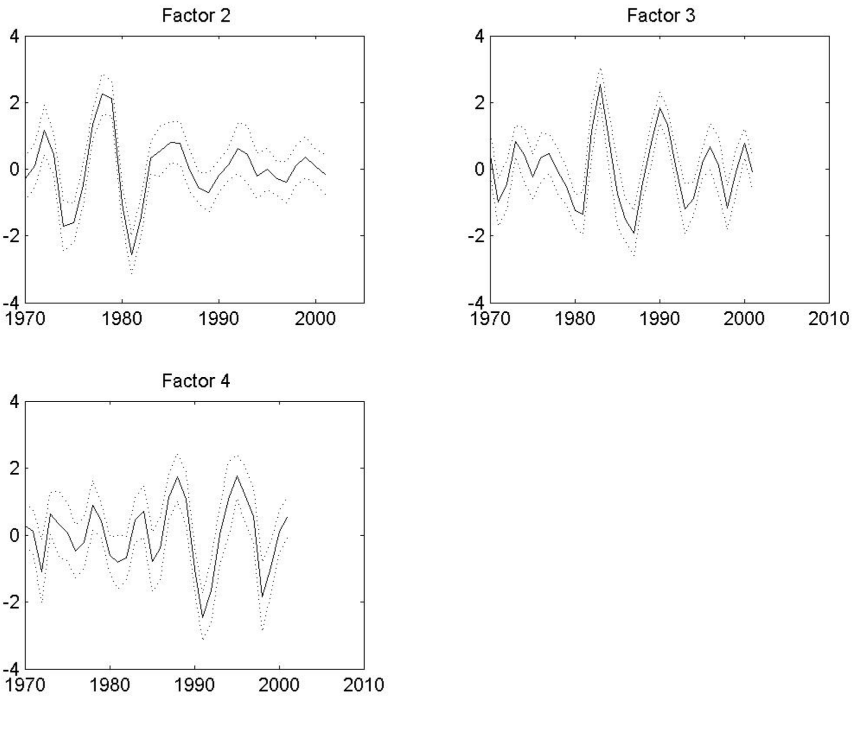
Gregory et al. (1997) studying the G7 countries with quarterly data from 1970:1 to 1993:4 found recessions in 1975, 1982 and a slowdown in the early nineties, upturns in the early and late seventies and late eighties. They also found during the mid-eighties a long period where the cycle was below the zero level.

Helbling and Bayoumi (2003) when estimating the G7 weighted gap using GDP quarterly data from 1973 to 2000 found troughs in 1974, 1982, 1986 and 1992 and peaks in 1978/1979, 1985, 1990 and 2000. Their sample end also depicts a slowdown, even if it is much smaller than the one depicted in figure 1.1.

It seems that this first factor is in line with estimations found in the literature, however this does not mean that it is the word business cycle. For that we have to know how important this factor is in the business cycle of each country. Does it affect all countries (being it a global component) or just some?

In figure 1.2 we can see the evolution of the remaining three factors.

Figure 1.2: The 2nd, 3rd and 4th Factors



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As the first factor, the second common component also captures the effects of the first and second oil crisis (it reaches troughs in 1974/75 and 1981). After that it seems to fluctuate around the zero value depicting smaller peaks in 1986 and 1992 and a trough in 1989. It seems that is just capturing the effects of the two oil crises.

The third factor peaks in 1983, 1990, 1996, 2000 and reaches troughs in 1981, 1987, 1993 and 1998. It seems that this factor is just capturing the fluctuations of the eighties and the nineties.

The fourth factor fluctuates around zero until the late eighties when it reaches a peak in 1988 and other in 1995, with troughs in 1991 and 1998. It seems that is just capturing the fluctuations in the world economy due to other causes than to the oil crises.

On the whole this three factors seem to split the fluctuations of the 70's, 80's and 90's between them. This might indicate that these fluctuations in different decades affected different countries differently, being global components, or that were specific to some set of countries and thus regional cycles. However, without analyzing the impact of each factor in the different countries we are not able to distinguish between these two alternatives.

1.3.2.3 The importance of the factors for each country

The first question is how much the first four factors explain the volatility of the national business cycles. Figure 1.3 shows the sum of the importance of the first four factors for each country. We can see that in Europe it is only for Iceland, Norway, Sweden and Turkey that the first four factors explain less than 50% of the GDP cycle.

In America the countries for which the four factors are not able to explain at least half of the volatility of the GDP cycle are Bolivia, Colombia, Dominican Rep. and Ecuador. In Asia that benchmark is not achieved by Hong Kong, India and Pakistan and in Africa just Ghana, South Africa and Zimbabwe have more than 50% of the GDP cycle volatility explained by the first four factors. From this first analysis we can say that Africa seems to be the continent more asynchronous with the rest of the world and where most of the variations are due to idiosyncratic shocks.

However for a more rigorous assessment we need to analyze the loadings component by component as well as their significance. Tables 1.3 to 1.6 report the loadings of the common components for each country as well as the 95% confidence bands and the importance of each factor for the GDP business cycle variance (VD, variance decomposition) following equations (1.2) and (1.3)[9].

[9] Remember that by equation (1.3) and because we standardized the GDPs business cycle, the square of the loadings is also the importance of the factor in the countries' business cycle variance.

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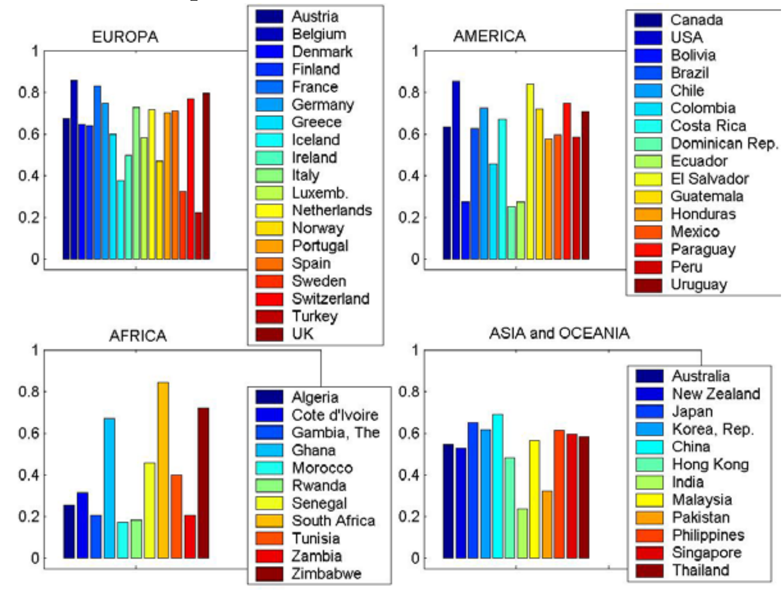
Table 1.3: Factor loadings and Importance for Europe

	1st Factor Load	1st Factor VD	2nd Factor Load	2nd Factor VD	3rd Factor Load	3rd Factor VD	4th Factor Load	4th Factor VD
AUT	0.55*** (0.37;0.74)	0,31	0.23** (0.05;0.41)	0.05	0.35*** (0.17;0.54)	0.12	-0.41*** (-0.55;-0.27)	0.17
BEL	0.80*** (0.72;0.89)	0.64	-0.28*** (-0.40;-0.16)	0.08	0.27*** (0.19;0.34)	0.07	-0.19*** (-0.26;-0.11)	0.03
DEN	0.55*** (0.35;0.73)	0.29	0.57*** (0.38;0.76)	0.32	-0.07 (-0.32;0.18)	0.00	0.09 (-0.07;0.25)	0.00
FIN	0.55*** (0.28;0.82)	0.30	-0.45*** (-0.69;-0.20)	0.20	0.34*** (0.16;0.51)	0.11	0.05 (-0.23;0.33)	0.00
FRA	0.72*** (0.60;0.84)	0.52	0.13** (0.03;0.23)	0.02	0.51*** (0.37;0.65)	0.26	-0.05 (-0.19;0.08)	0.00
GER	0.70*** (0.55;0.85)	0.49	0.32*** (0.17;0.46)	0.10	0.12 (-0.05;0.30)	0.02	-0.35** (-0.62;-0.07)	0.12
GRE	0.56*** (0.37;0.75)	0.31	0.48*** (0.16;0.79)	0.23	-0.03 (-0.14;0.08)	0.00	-0.20*** (-0.36;-0.04)	0.04
ICE	0.50*** (0.27;0.73)	0.25	-0.13 (-0.43;0.18)	0.02	-0.31** (-0.60;-0.03)	0.10	-0.01 (-0.22;0.20)	0.00
IRE	0.51*** (0.18;0.83)	0.26	0.09 (-0.11;0.28)	0.01	0.37*** (0.12;0.61)	0.13	-0.29*** (-0.42;-0.15)	0.09
ITA	0.80*** (0.66;0.93)	0.63	0.24** (-0.46;-0.024)	0.06	0.11 (-0.03;0.26)	0.01	0.03 (-0.12;0.18)	0.00
LUX.	0.72*** (0.49;0.95)	0.52	0.11 (-0.17;0.39)	0.01	0.11 (-0.13;0.35)	0.01	-0.13 (-0.32;0.06)	0.02
NTH	0.68*** (0.48;0.88)	0.46	0.25** (0.01;0.50)	0.07	-0.01 (-0.20;0.18)	0.00	-0.41*** (-0.56;-0.26)	0.17
NOR	0.04 (-0.27;0.35)	0.00	0.10 (-0.13;0.33)	0.01	-0.64*** (-0.97;-0.32)	0.41	-0.17 (-0.41;0-07)	0.03
POR	0.72*** (0.44;1.00)	0.52	-0.09 (-0.30;0.11)	0.01	0.33*** (0.09;0.57)	0.11	-0.21** (-0.40;-0.11)	0.04
SPA	0.61*** (0.46;0.76)	0.38	-0.13 (-0.34;0.08)	0.02	0.51*** (0.33;0.69)	0.26	-0.18 (-0.45;-0.08)	0.03
SWE	0.37 (-0.11;0.86)	0.14	-0.34* (-0.72;0.05)	0.11	0.23 (-0.10;0.55)	0.05	0.09 (-0.07;0.24)	0.01
SWI	0.81*** (0.56;1.01	0.65	-0.20*** (-0.35;-0.06)	0.04	0.01 (-0.11;0.12)	0.00	-0.23*** (-0.39;-0.72)	0.05
TUR	-0.28** (-0.50;-0.06)	0.08	0.34*** (0.20;0.48)	0.12	-0.06 (-0.24;0.12)	0.00	-0.13 (-0.41;0.15)	0.02
UK	0.63*** (0.51;0.75)	0.40	0.34*** (0.19;0.48)	0.11	0.16* (-0.01;0.33)	0.03	0.49*** (0.25;0.73)	0.24

Numbers between parenthesis are the 95% confidence level interval; VD - variance decompostion
* Significantly different from zero at 10% level
** Significantly different from zero at 5% level
*** Significantly different from zero at 1% level

Figure 1.3: Cumulative importance of the first four factors in the GDP cycles.



As we can see from table 1.3 the first factor has a very significant influence in most countries in Europe, the exceptions being Norway and Sweden for which the first factor's influence is not significantly different from zero even at 10% significance level. Also Turkey has a weaker dependence from the first factor than the other European countries. In fact the loading is only significant at the 5% significance level, which contrasts with the other countries for which the loading is significant even at 1%.

The influence of the second factor is less all encompassing than the first, however it is significant at 1% for Belgium, Denmark, Finland, Germany, Greece, Switzerland, Turkey and UK and at 5% for Austria, France, Italy and Netherlands. However, for these countries,, it's importance in the national business cycle variances is quite small, being over 10% only to Denmark, Finland, Greece, Turkey and UK and almost 10% for Germany.

The third common component is significant for Austria, Belgium, Finland, France, Ireland, Norway, Portugal and Spain. It is, also, significant at 5% level for Iceland. With the exception of Belgium and Iceland, the third factor accounts for more than 10% of the national business cycle variance of these countries (for Iceland it accounts for almost 10%).

The fourth factor, despite being significant for some countries, only accounts for more than 10% of the variance decomposition for Austria, Germany, Netherlands and UK.

For North and South America (see table 1.4), as for Europe, the first factor is significant

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Table 1.4: Factor loadings and Importance for North and South America

	1st Factor		2nd Factor		3rd Factor		4th Factor	
	Load	VD	Load	VD	Load	VD	Load	VD
CAN	0.68*** (0.36;0.99)	0.46	-0.05 (-0.18;0.08)	0.00	-0.22* (-0.45;0.01)	0.05	0.32*** (0.10;0.54)	0.10
USA	0.75*** (0.63;0.87)	0.56	0.46*** (0.36;0.56)	0.22	-0.16*** (-0.26;-0.06)	0.02	0.17* (-0.00;0.34)	0.03
BOL	0.28 (-0.06;0.62)	0.08	0.13 (-0.25;0.51)	0.02	0.11 (-0.21;0.42)	0.01	-0.40*** (-0.59;-0.21)	0.16
BRA	0.46*** (0.31;0.61)	0.21	-0.30*** (-0.43;-0.17)	0.09	-0.50*** (-0.75;-0.24)	0.25	0.25** (0.05;0.44)	0.06
CHI	0.64*** (0.49;0.79)	0.41	-0.09 (-0.27;0.10)	0.01	-0.53*** (-0.71;-0.34)	0.28	-0.08 (-0.25;0.08)	0.01
COL	0.58*** (0.41;0.75)	0.34	-0.14* (-0.31;0.02)	0.02	-0.20** (-0.40;-0.01)	0.04	0.20* (-0.02;0.42)	0.04
CRI	0.57*** (0.42;0.07)	0.31	0.32*** (0.18;0.46)	0.10	-0.48*** (-0.68;-0.28)	0.23	0.11 (-0.09;0.31)	0.01
DOM	0.19 (-0.07;0.44)	0.04	-0.30*** (-0.53;-0.07)	0.09	-0.05 (-0.40;0.31	0.00	0.34 (-0.19;0.88)	0.12
ECU	0.17** (0.01;0.34)	0.03	-0.45*** (-0.70;-0.19)	0.20	-0.16 (-0.42;0.10	0.03	-0.10 (-0.36;0.17)	0.01
ESV	0.20** (0.03;0.36)	0.04	0.80*** (0.55;1.05)	0.64	-0.19*** (-0.30;-0.08)	0.04	0.32*** (0.21;0.44)	0.10
GUA	0.80*** (0.66;0.95)	0.64	-0.12 (-0.41;0.17)	0.01	-0.21* (-0.43;0.02)	0.04	-0.04 (-0.17;0.10)	0.00
HND	0.53*** (0.35;0.70)	0.28	0.42** (0.10;0.74))	0.18	-0.28*** (-0.45;-0.10)	0.08	0.16 (-0.05;0.37)	0.03
MEX	0.32*** (0.17;0.47)	0.10	-0.44*** (-0.52;-0.37)	0.20	-0.23* (-0.48;0.02)	0.05	-0.48*** (-0.62;-0.33)	0.23
PRG	0.53*** (0.30;0.77)	0.29	-0.61*** (-0.085;-0.36)	0.37	-0.26** (-0.49;-0.03)	0.07	-0.07 (-0.26;0.12)	0.01
PER	-0.03 (-0.22;0.17)	0.00	-0.46*** (-0.63;-0.30)	0.22	-0.53*** (-0.87;-0.19)	0.28	0.26*** (0.08;0.44)	0.07
URG	0.15* (-0.00;0.30)	0.02	-0.45*** (-0.69;-0.21)	0.20	-0.67*** (-0.75;-0.59)	0.45	-0.10 (-0.28;0.07)	0.01

Numbers between parenthesis are the 95% confidence level interval

* Significantly different from zero at 10% level

** Significantly different from zero at 5% level

*** Significantly different from zero at 1% level

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at least at the 5% level for most countries, the exceptions being Uruguay where it is significant only at the 10% level and Bolivia, Dominican Rep., and Peru for which it is not significant.

The second common component is not significant for Canada, Bolivia, Chile and Guatemala and for Colombia it is significant only at 10% level.

The third factor beyond being significant at 1% level for the USA (however it's importance in the variance decomposition is quite low) it affects the majority of the Latin American countries with quite high values for the variance decomposition as the fourth factor affects only a small number of countries (Canada, Bolivia, Brazil, El Salvador, Mexico and Peru).

Table 1.5: Factor loadings and Importance for Asia and Oceania

	1st Factor		2nd Factor		3rd Factor		4th Factor	
	Load	VD	Load	VD	Load	VD	Load	VD
AUS	0.54*** (0.30;0.77)	0.29	-0.07 (-0.26;0.12)	0.01	-0.37*** (-0.60;-0.14)	0.14	0.31* (-0.01;0.63)	0.10
NZL	-0.02 (-0.30;0.27)	0.00	-0.40** (-0.73;-0.7)	0.16	0.22* (-0.03;0.46)	0.05	0.55*** (0.32;0.79)	0.31
JAP	0.69*** (0.55;0.84)	0.48	0.17* (-0.02;0.36)	0.03	0.33*** (0.23;0.43)	0.11	-0.12 (-0.32;0.08)	0.01
KOR	0.09 (-0.09;0.27)	0.01	0.38*** (0.23;0.53)	0.14	0.37** (0.07;0.67)	0.14	0.55*** (0.17;0.94)	0.31
CHN	0.16* (0.02;0.30)	0.03	0.12 (-0.12;0.36)	0.02	-0.53*** (-0.67;-0.39)	0.28	0.59*** (0.45;0.72)	0.35
HKG	0.48*** (0.27;0.70)	0.23	-0.01 (-0.42;0.40)	0.00	-0.04 (-0.26;0.18)	0.00	0.48*** (0.21;0.75)	0.23
IND	-0.26 (-0.62;0.11)	0.07	0.03 (-0.34;0.39)	0.00	0.35** (0.02;0.69)	0.13	0.19 (-0.04;0.42)	0.04
MAL	0.34*** (0.18;0.50)	0.11	-0.09 (-0.26;0.08)	0.01	0.56*** (0.36;0.75	0.31	0.34** (0.04;0.64)	0.12
PAK	0.00 (-0.15;0.50)	0.00	-0.48*** (-0.70;-0.26)	0.23	0.21 (-0.05;0.46)	0.04	0.19 (-0.09;0.47)	0.04
PHI	0.09 (-0.06;024)	0.01	-0.29* (-0.61;0.04)	0.08	0.68*** (0.47;0.88)	0.46	0.22** (0.02;0.42)	0.05
SIN	0.41*** (0.29;0.52)	0.16	-0.16 (-0.38;0.06)	0.03	0.53*** (0.26;0.80)	0.28	0.33*** (0.15;0.51)	0.11
THA	0.23** (0.02;0.45)	0.06	0.03 (-0.21;0.26)	0.00	0.47*** (0.18;0.77)	0.22	0.53** (0.10;0.97)	0.29

Numbers between parenthesis are the 95% confidence level interval
* Significantly different from zero at 10% level
** Significantly different from zero at 5% level
*** Significantly different from zero at 1% level

The picture of Asia and Oceania (table 1.5) is a bit different: the first factor affects those

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countries that we can classify as developed (with the exceptions of New Zealand and Korea), Malaysia and Thailand. The second factor affects only New Zealand, Korea and Pakistan. The third one seems to encompass all the region with the exception of Pakistan and Hong Kong, as well as New Zealand for which it is only significant at 10%. Also the fourth factor encompasses a number of countries in a significant way.

Table 1.6: Factor loadings and Importance for Africa

	1st Factor		2nd Factor		3rd Factor		4th Factor	
	Load	VD	Load	VD	Load	VD	Load	VD
ALG	0.26*** (0.09;0.44)	0.07	0.29*** (0.10;0.48)	0.08	0.28* (-0.03;0.59)	0.08	-0.12 (-0.24;-0.00)	0.02
CIV	0.57 (-0.18;0.29)	0.00	0.54*** (0.17;0.91)	0.29	-0.02 (-0.27;0.22)	0.00	-0.11 (-0.35;0.14)	0.01
GAM	-0.38** (-0.72;-0.05)	0.15	0.00 (-0.25;0.26))	0.00	0.23 (-0.08;0.55)	0.05	0.01 (-0.27;0.28)	0.00
GHA	0.69*** (0.50;0.87)	0.47	-0.02 (-0.32,0.28)	0.00	-0.41*** (-0.50;-0.33)	0.17	0.08 (-0.13;0.29)	0.01
MOR	-0.14 (-0.40;0.11)	0.02	0.19* (-0.03;0.42)	0.04	0.06 (-0.21;0.33)	0.00	-0.33*** (-0.50;-0.15)	0.11
RWA	0.03 (-0.12;0.18)	0.00	0.08 (-0.07;0.23)	0.01	0.04 (-0.17;0.25)	0.00	-0.41** (-0.78;-04)	0.17
SEN	-0.54*** (-0.73;-0.35)	0.30	0.11 (-0.12;0.34)	0.01	0.35*** (0.18;0.52)	0.12	-0.11 (-0.40;0.18)	0.01
SAF	0.15 (-0.05;0.35)	0.02	-0.89*** (-1.01;-0.76)	0.79	0.06 (-0.13;0.24)	0.00	0.07 (-0.02;0.16)	0.01
TUN	0.07 (-0.08;0.21)	0.00	0.10 (-0.20;0.41)	0.01	-0.39*** (-0.60;-0.19)	0.15	-0.47*** (-0.76;-0.17)	0.21
ZAM	-0.08 (-0.36;0.20)	0.01	-43* (-0.66;-0.20)	0.18	-0.04 (-0.25;0.17)	0.00	-0.09 (-0.41;0.23)	0.01
ZIM	0.11 (-0.04;0.25)	0.01	-0.65*** (-0.79;-051)	0.42	0.24*** (0.01;0.37)	0.06	-0.46*** (-0.64;-0.28)	0.21

Numbers between parenthesis are the 95% confidence level interval
* Significantly different from zero at 10% level
** Significantly different from zero at 5% level
*** Significantly different from zero at 1% level

In Africa (table 1.6) the picture is quite mixed, as no factor seems to be more important for the region than any other. All of them are significant for more or less the same number of countries (4/5). The variance decompositions reveal, also, the same picture.

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1.3.2.4 Trying to identify regions in *substrictus sensus*

At this point, having identified the common components, as well as which one is more important for each country we can try to check if there is some factor associated with a set of countries defining a region in *substrictus sensus.*

From our previous definitions this kind of region would be characterized from the existence of a common component which would be important just to a specific set of countries. So the straightforward method would be to check which countries the factors affect significantly. However this would be a too simple consideration, because even if the common component is a regional one it may affect countries outside that region due to economic linkages. Therefore the regional characteristics of a factor should be evident from the fact that similar countries are the ones more affected by it and not the only ones affected by it.

To see which setting of groups are more affected by each component, the model is adjusted to restrict to zero those loadings (and variance decompositions) that were not significantly different from zero at the 5% significance level. Then I estimated the variance decomposition average for a number of country sets grouped by economic, cultural or geographical characteristics and compared them to the importance of the factor to the whole sample (world average). The regional characteristic of a factor should came from the fact that the set of countries that compose the region should be more affected by that common component than the sample as a whole.

From table 1.7 we can see that the first factor is more important than the world average for a number of groupings, however those groups have a common characteristic: most of its members are high-income countries. In fact from the economic division we can see that, the smaller the income the smaller is the importance of this factor.

Therefore, even if the importance for many non-developed countries is not zero, the first factor seems to be associated primarily with developed economies Two apparent exceptions should be taken into consideration: developed Asia and Oceania. In both cases lower values than the world average are due to the small values of two countries: South Korea and New Zealand.

As for the second factor the analysis is more difficult. First, it seems that geographically it is more important for Latin America, Sub-Saharian Africa and for North America (this is due to USA). In cultural terms it seems to be more important to Spanish speaking countries (a result that anyway is due to the fact that most countries are from South America) and to Commonwealth countries . In terms of economic division it seems to affect more developing

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Table 1.7: Comparision of variance decomposition averages of possible regions

	1st Factor	2nd Factor	3rd Factor	4th Factor	Idioss. comp.
World	0,2238	0,1053	0,0988	0,0704	0,5017
Geographical Divisions					
Europe	0,3688+	0,0695	0,0834	0,0500	0,4283
America (North and South):	0,2270+	0,1554+	0,1080+	0,0449	0,4647
North America	0,5085+	0,1073+	0,0122	0,0509	0,3211
South America	0,1867	0,1623+	0,1217+	0,0441	0,4852
Africa:	0,0893	0,1606+	0,0455	0,0650	0,6400
North African Countries	0,0232	0,0279	0,0511	0,1131+	0,7847
Sub-Saharian Countries	0,1141	0,2103+	0,0434	0,0470	0,5852
Asia:	0,1070	0,0374	0,1781+	0,1394+	0,5381
Developed Asia*	0,2187	0,0356	0,0963	0,1619+	0,4875
Developing Asia*	0,0326	0,0386	0,2327+	0,1245+	0,5716
Oceania	0,1441	0,0803	0,0691	0,1524+	0,5541
Developed Asia*+Oceania	0,1938	0,0505	0,0872	0,1587+	0,5098
Cultural Divisions					
Anglo-Saxon countries***	0,3259+	0,0812	0,0494	0,1218+	0,4217
France, former colonies and Belgium(4*)	0,1914	0,0587	0,0757	0,0676	0,6066
German speaking countries(5*)	0,4773+	0,0481	0,031	0,1274+	0,3162
Spanish speaking countries(6*)	0,1994	0,1514+	0,1309+	0,0412	0,4771
Commonwealth countries plus HK(7*)	0,1756	0,1228+	0,062	0,0827+	0,5569
Economic Divisions					
High Income countries**	0,3503+	0,0663	0,0820	0,0771+	0,4243
Developing countries:	0,1249	0,1161+	0,1161+	0,0671	0,5758
Low Income countries**	0,1014	0,1254+	0,0526	0,0418	0,6788
Lower-middle income countries**	0,1218	0,116+	0,1166+	0,0851+	0,5605
Upper-middle income countries**	0,1683	0,1024	0,2102+	0,0571	0,462
Severely or moderately indebted**	0,0952	0,1565+	0,1226+	0,055	0,5707
Less indebted or not classified**	0,2914+	0,0784	0,0863	0,0785+	0,4654

* According the high-not high income division used in the economic groupings

** According to the world bank classification

*** Australia, Canada, Ireland, New Zealand, USA and UK

(4*) France, Belgium, Senegal, Morocco, Algeria, Tunisia, Cote d'Ivoire and Rwanda

(5*) Germany, Austria Switzerland and Netherlands

(6*) Spain plus South America without Brazil

(7*) England, Australia, Canada, New Zealand, Gambia, Ghana, India, Malaysia, Pakistan, Zambia, South Africa and Hong Kong

\+ Above world value

and severely or moderately indebted countries than the others. But in those groups the weight of Sub-Saharian Africa and Latin America drives the result.

From those results is hard to find a common characteristic. On one side it seems that is affecting low or lower-middle income. countries, but how do we explain afterwards the importance for USA or the aggregate of Commonwealth countries? So we will not associate it directly with a certain economic, cultural or regional division. It seems more a factor that has affected USA and contaminated countries in South America, Africa, in the Pacific region(New Zealand and Korea) and in Europe (as for instance Denmark, Sweden, UK and Germany).

The third factor seems to be affecting mostly middle income countries mainly in Asia and Latin America. Furthermore if we perform the factor estimation only for OECD countries this third factor is not identified, which did not happen with the remaining estimated common components. This fact leads us to associate this factor with middle-income economies.

As for the fourth factor if we look to geographical divisions it seems to be associated with Asia and Oceania. However also the North African countries and from the cultural groupings the Anglo-Saxon and the German speaking countries are also more affected than the world average. Therefore, like the second common component, we will not associate it with a certain group of countries but we will consider it as a factor that bridges different countries across the world.

From the previous tables we may say that the first factor is more important to developed countries than for the others; anyway it encompasses all the world. Our conclusion is that it is a world component that has its origin in the developed countries and spreads toward the developing world. In this sense we can define the high-income countries as a region in *substrictus sensus.*

The third common component seems to be almost specific to middle income countries (even if it is also transmitted to developed countries of Asia). Therefore it will define the middle income countries as our second region in *substrictus sensus.*

Finally, the other two common components seem to, at least, form a bridge across countries that in geographical, cultural or economical terms would belong to different groups. In this sense they are global or supra-regional components not defining any set of countries as a region.

In conclusion, we can define two regions in *substrictus sensus*: the high income and the middle income countries as they seem to have a specific common component. A third group would be the lower-income countries due to the absence of a specific factor. In fact, for them it seems that they are the ones more detached from the global commovement as it is for this group that the idiosyncratic component has the higher importance. Therefore they seem not only outside the global world cycle but also undetached from any region.

1.3.2.5 Clustering analysis

From the previous section the only set of countries that could be called regions *sub-strictus sensus* would be the broad economic division between developed and middle-income countries. However, this does not mean that countries in those areas have similar business cycles as their structural dependence from the different common components might differ. This section tries, by clustering the countries[10], to see if we can group countries according to that structure, thus defining regions *in latus sensus.*

Hard clustering :
The first technique used was the hard cluster that groups together the countries for which the distance between their characteristics is smaller.

In the first method the distance between any two countries is measured only by the loading distance:

$$d_{c1,c2} = \sqrt{\sum_{i=1}^{r} \left(\left(\lambda_i^{c1}\right) - \left(\lambda_i^{c2}\right) \right)^2} \tag{1.4}$$

where: $\left(\lambda_i^{cj}\right)$ is the loading of factor i to country c_j, j=1,2.

By not considering the idiosyncratic component we just verify which countries are more similar in their structure of dependence relative to the common factors. However this measure does not take into consideration if the dependence rate from common factors is high or not. Therefore it just measures how similar is their dependence from the common factors, not commovement.

To take into account commovement we have to consider the importance of the idiosyncratic component.

$$d_{c1,c2} = \sqrt{\sum_{i=1}^{r} \left(\left(\lambda_i^{c1}\right) - \left(\lambda_i^{c2}\right) \right)^2 + (s^{c1})^2 + (s^{c2})^2} \tag{1.5}$$

[10] The clustering techniques were applied to a restricted model, where the loadings that were not significantly different from zero at least at 5% level were set to zero.See appendix 1.E for a short decription of the method.

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where: $\left(\lambda_i^{cj}\right)$ is the loading of factor i to country c_j ; $(s^{cj})^2$ is the importance of the idiosyncratic component of country c_j to it's business cycle and $j = 1, 2$.

In this way even if the structure dependence from common factors is similar but weak, and therefore the countries business cycle is mainly influenced by the idiosyncratic components, they will not appear in the same group but apart. In this case, countries that cluster together do not only have a similar structure of dependence but also commove. We can say that those business cycles are more synchronized.

We implement these ideas first for the OECD countries and then for the sample as a whole.

From figure 1.4 and making the divisions when a group of more than two clustered countries joins with another group we can identify three central groups. The first composed by Austria, France, Japan, Spain, Portugal and Ireland. The second has Belgium, Italy, Switzerland, Luxembourg and Netherlands and the third Denmark, Greece, Germany and USA. These three groups cluster together before any other remaining country. They seem to form a core.

As for the remaining countries, five have a distance to these three groups that is not large: Australia, Iceland, Canada, UK and Finland. The remaining ones seem to have a structure of dependence from the common factors more dissimilar to the three central core groups. These countries are Mexico, Sweden, Turkey, Norway, South Korea and New Zealand,

Looking at the figure 1.5 the picture has some differences. A first group is composed by France, Japan, Spain, Portugal, Belgium, Italy, Switzerland Netherlands and Finland and the second is composed by Denmark, Greece, Germany, USA and UK. As we can realize, in terms of commovement the first group cluster together the first two groups of the previous analysis as the second one is the third group of the previous analysis.

Therefore it seems that the OECD can be characterized by a core, itself divided into two groups (or regions in *latus sensus).* The first can be called the French-Japanese group as it clusters France, Japan, Spain, Portugal and probably Belgium, Italy, Switzerland Netherlands and Finland. The second can be called the USA-German one as it includes both countries, Denmark, Greece and probably the UK. The remaining countries are a periphery to these two groups. Being that those that are further away in terms of comovement are also less similar in terms of structure: Sweden, Turkey, Norway, South Korea and New Zealand.

The next two figures (1.6 and 1.7) show the cluster tree for all countries. From figure 1.6 we get a rather complex image. However we can see that the sub-group IIC is composed by Austria, France, Japan, Spain, Portugal and Ireland and IID includes the USA, Germany,

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Figure 1.4: Hard Cluster for the OECD with the loadings and without the idiosyncratic component

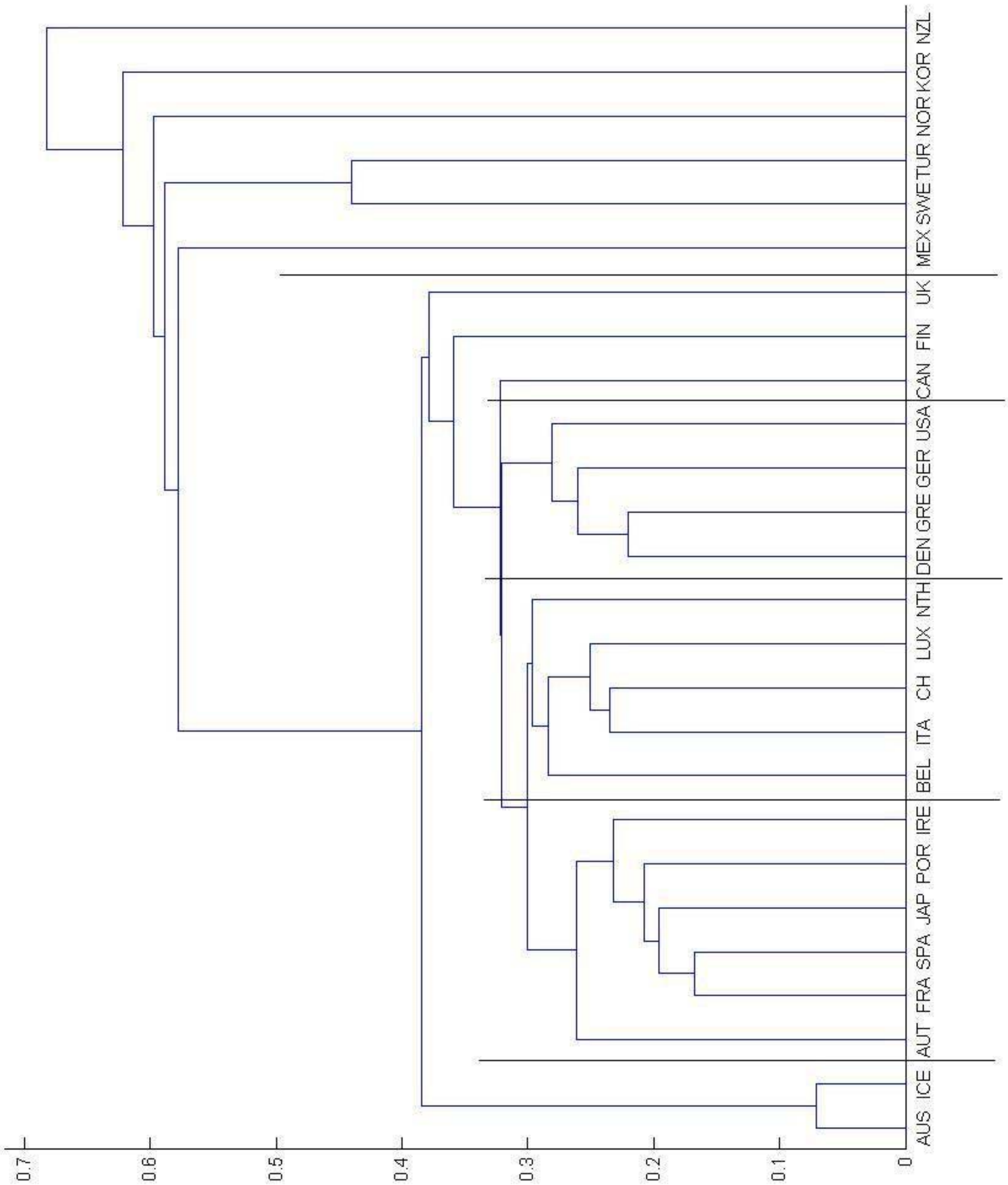

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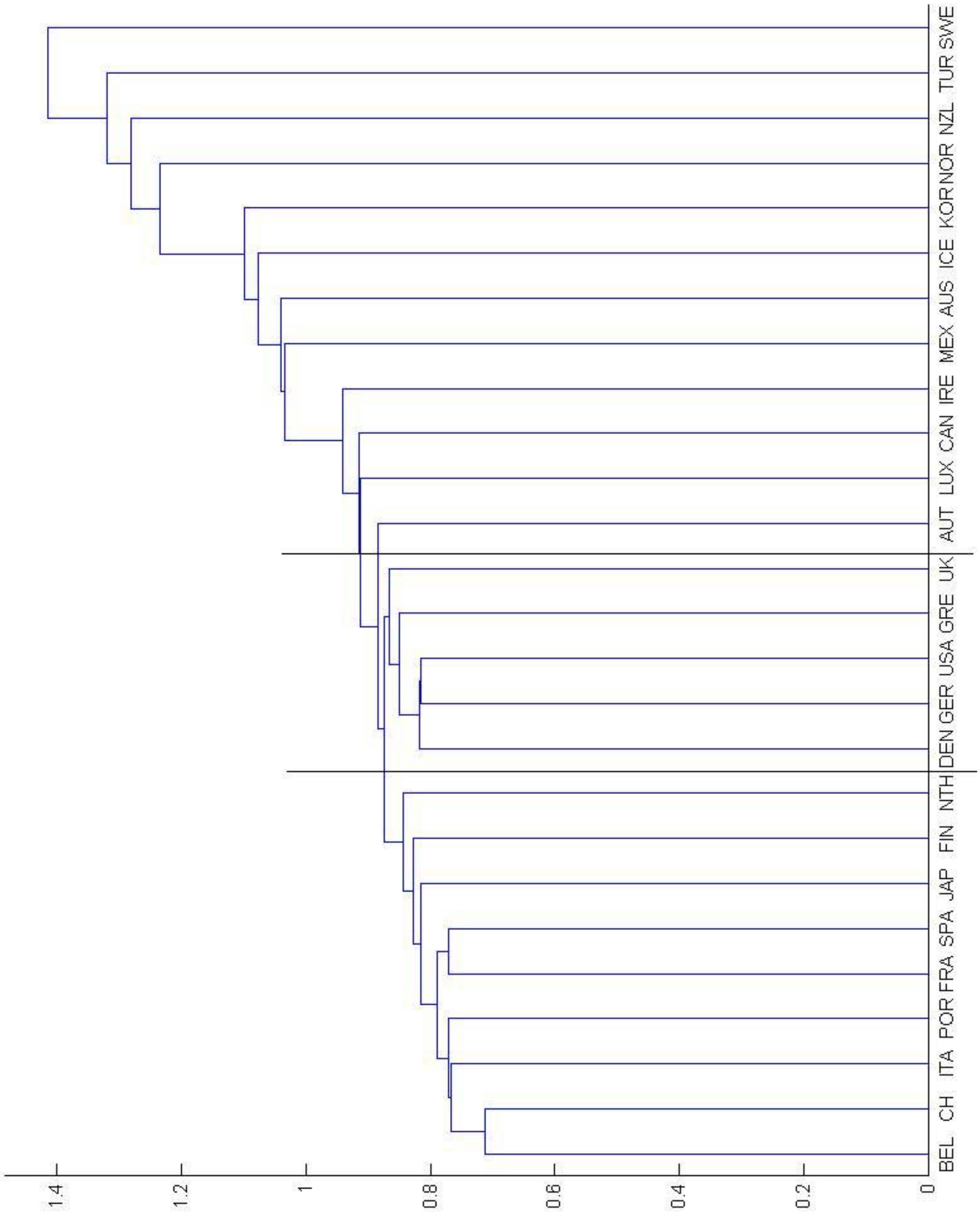
Figure 1.5: Hard Cluster for the OECD with the loadings and the idiosyncratic component



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Greece and Denmark plus Honduras and Costa Rica. Therefore the French-Japanese and the American-German groups are also identified as before.

As before the French-Japanese group does not cluster immediately with the American-German one. First it clusters together with an aggregate composed by the groups IIA[11], IIB[12] , Belgium and Netherlands. Finally a number of countries cluster subsequently, being these Algeria, Canada, Hong-Kong, Finland and UK as well as an African grouping: Cote d'Ivoire, Turkey, Gambia and Senegal. The remaining countries appears in group I or III being the first those with weak links with the common components and the latter with more dissimilar structure.

From the commovement analysis, figure 1.7, there are only two groups surviving: the French-Japanese composed by France, Japan, Spain, Portugal, Belgium, Switzerland, Italy, Finland, Guatemala and Netherlands. Therefore in terms of commovement the French-Japanese identified in terms of structural similarity loses Austria and Ireland but aggregates the previous group IIB (without Luxembourg) and the two intermediate countries (Belgium and Netherlands). The second group is the American-German as defined before plus the United Kingdom.

Therefore it seems that those two groups form a center (where only three developing countries seem to commove with: Guatemala, Costa Rica and Honduras). On the other side, of the twenty countries that commove less with this center we only have four OECD countries: Sweden, Norway, New Zealand and Turkey.

Fuzzy clustering :
However the last approach has the limitation that allocates a country to a certain group, and sometimes the country position may be halfway through two centers, in that case the method allocates it arbitrary to one of the groups. In order to correct that shortcoming this subsection presents the results for fuzzy clustering, where countries are not allocated certainly to a group but will be given a probability measure of belonging to the several groups.

The study was done without the idiosyncratic component, hence, it is just checking for similar dependence on common factors.

The first problem is to decide for the number of clusters. One indicator is the normalized Dunn-coefficient that measures how much is the fuzzy clustering close to a hard cluster. However this measure has the backdrop that its limit is one when the number of clusters

[11]Composed by Australia, Iceland, Colombia, Chile and Ghana.
[12]Composed by Guatemala, Luxembourg, Italy and Switzerland

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Figure 1.6: Hard Cluster for the world sample with the loadings and without the idiosyncratic component

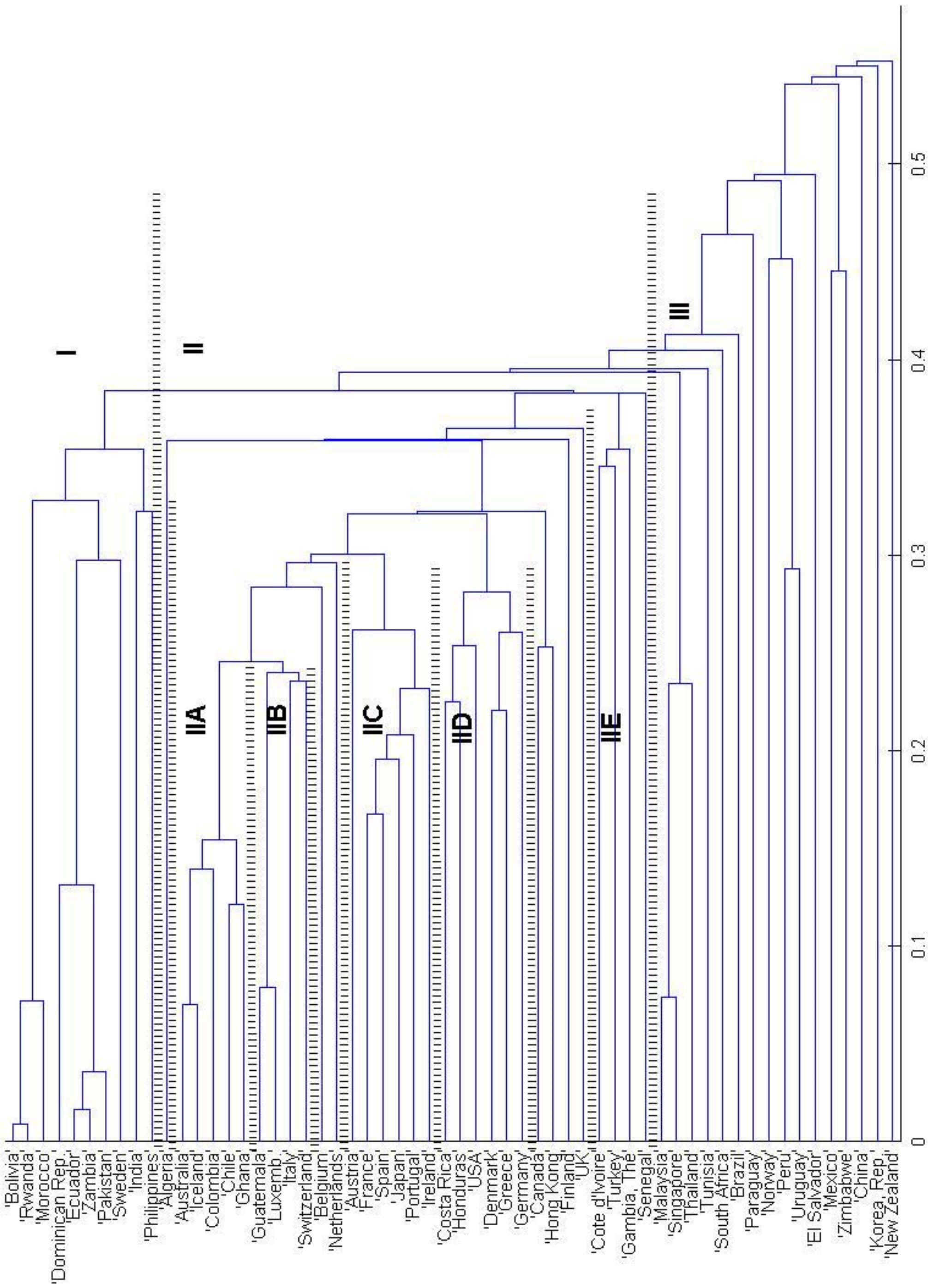

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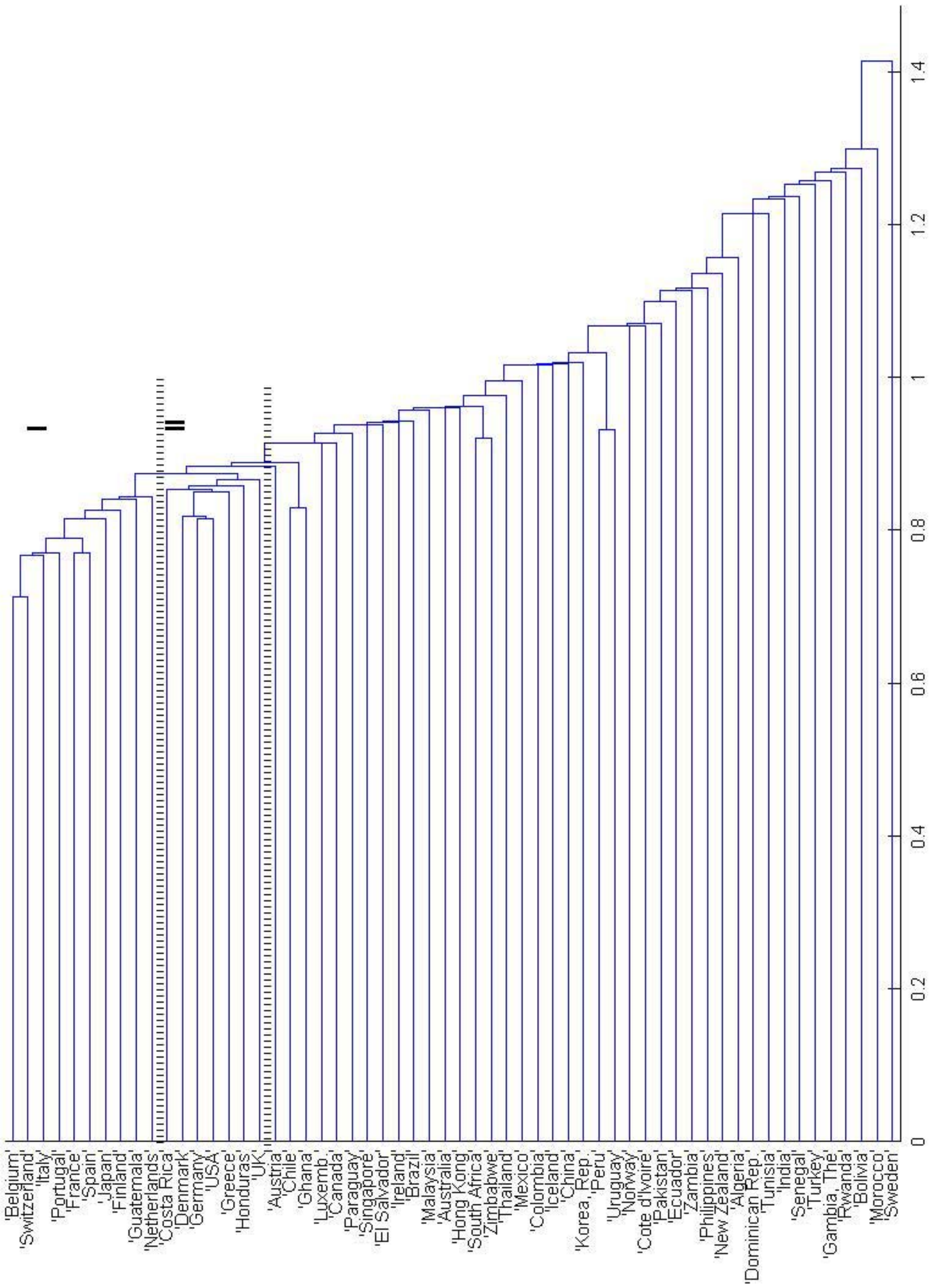
Figure 1.7: Hard Cluster for the whole sample with the loadings and the idiosyncratic component



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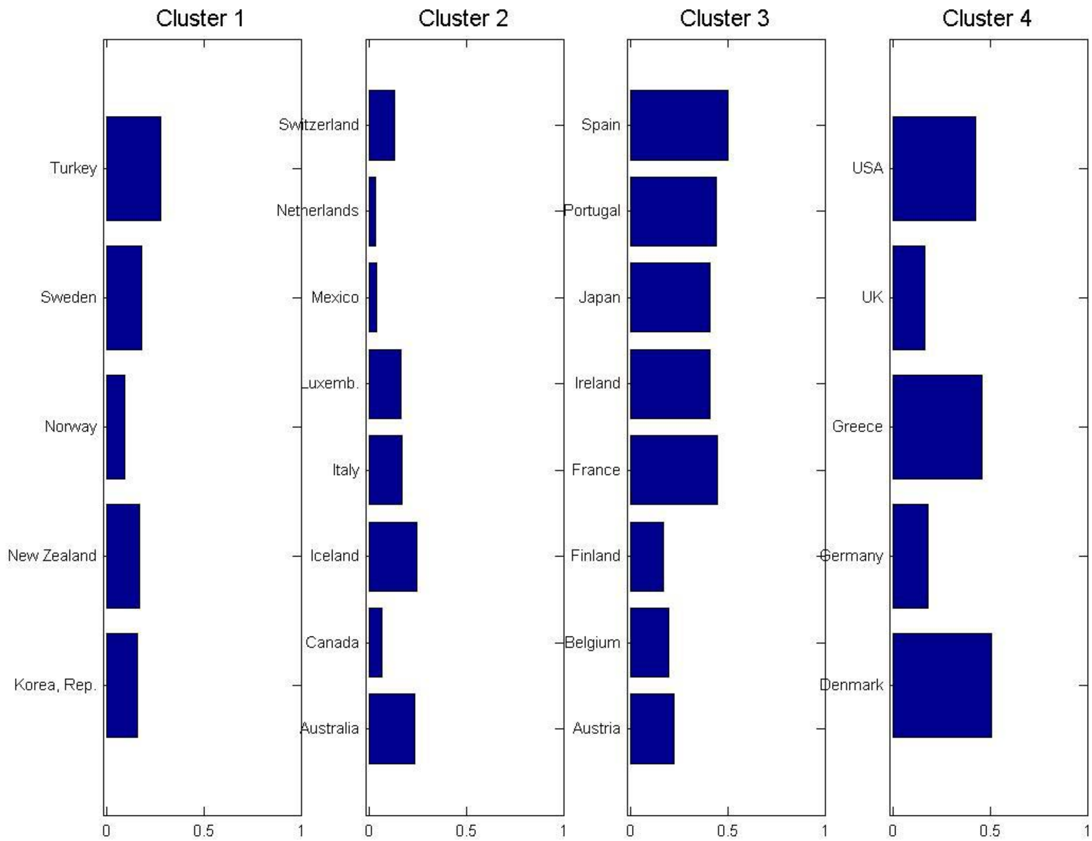
aproach the number of countries. A second criterion would be to select the number of clusters that would give the maximum value for the cluster silhouettes average.

Using this criterion for the OECD countries it selected 4 as the number of clusters. The next figure shows the cluster composition and the country silhouette (the complete table is in appendix 1.F - table F1).

Figure 1.8: Fuzzy clustering and country silhouette



From figure 1.8 we can see that the French-Japanese group appears in Cluster 3. In this case France, Japan, Spain, Portugal and Ireland seem to have a high probability of belonging to this group (high silhouettes and in table F1 higher probabilities), as Finland, Belgium and Austria have somewhat a lower probability of being part of this group.

It is in Cluster 4 that we find the German-American group with USA, Greece and Denmark having a quite high probability of being part of this cluster as UK and Germany

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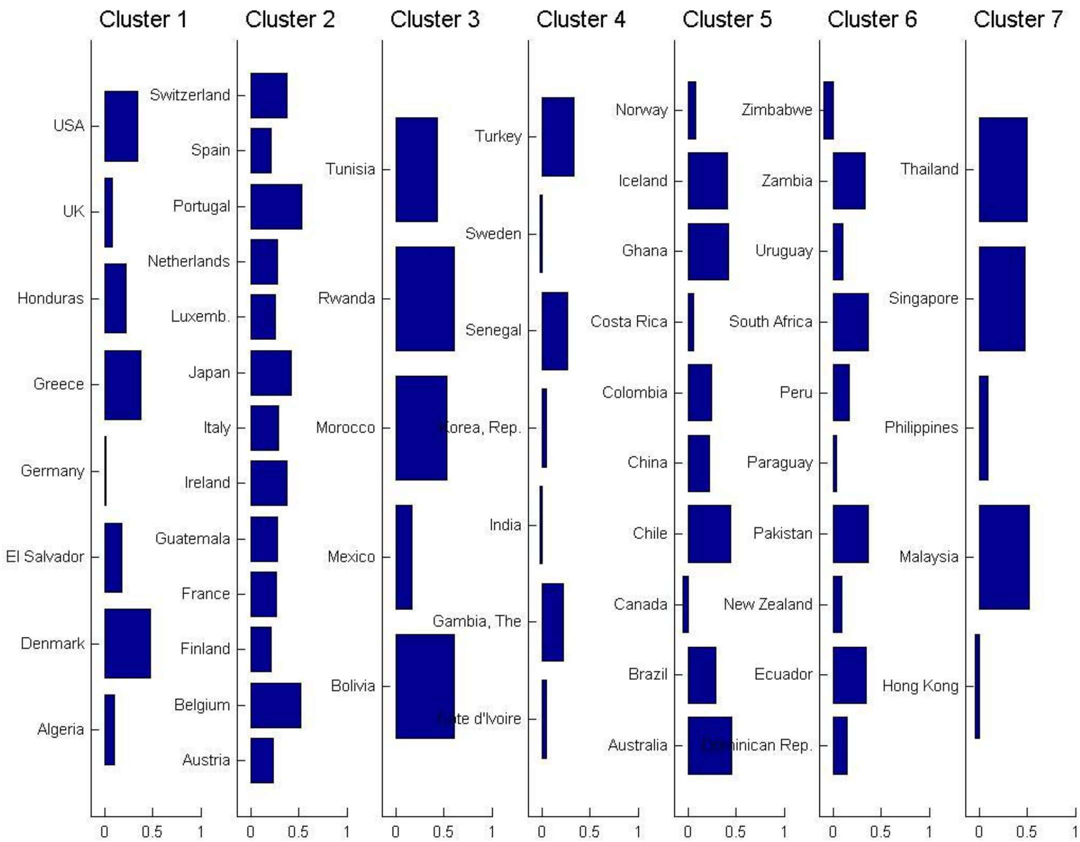
have a lower degree of association. In fact from table F1 in appendix 1.F this two countries also have high probabilities of belonging to other clusters, which leads to the idea that the UK and Germany are the connectors between the USA and Europe.

Finally in group 1 we find the countries that have lower dependence from the common factors as in group 2 we find the remaining countries. We should say that the degree of association of these two clusters is somewhat lower than the other two

For the world sample the method selected 7 clusters. Figure 1.9 shows the cluster composition and the country silhouette (the complete table is in appendix 1.F - table F2)

Figure 1.9: Fuzzy clustering and country silhouette



The first observation is that there is a clear division between developed and developing countries. The first are concentrated mostly in cluster 1 and 2 as the latter are hardly present

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in those clusters (just Algeria, El Salvador and Honduras in the first cluster and Guatemala in the second) and compose the other 5 clusters.

The second is that cluster one is the German-American group and the second is the French-Japanese group that includes some more countries relative to the previous figure (Guatemala, Italy, Luxembourg, Netherlands and Switzerland). Also here Germany and the UK have a lower association with cluster 1 than the USA. As before, if we look to table F2 in appendix 1.F we can realize that these two countries also have a higher probability of belonging to cluster 2. Once more, we can reach the same conclusion as before: Germany and UK bridge the cycles between the USA and the French-Japanese cluster (mainly Europeans countries).

Aside cluster 7 that seems a Southeast Asiatic group (Hong Kong, Malaysia, Philippines, Singapore and Thailand) the remaining do not seem to have any geographical or cultural common characteristic

1.4 Conclusion

As a main conclusion, if we want to characterize the world business cycle we should consider the existence of more than one component, even if the first estimated factor is the more all encompassing of them all and can be used to summarize the world fluctuations. This conclusion has some similarities with those of Bayoumi and Helbling(2003). In their study, using a different approach, they also found that the common variations across the G7 countries were due to two components. They interpreted the first component as reflecting common global shocks, and the second reflecting country shocks that were transmitted to other countries. In the present study I do not go as far as to give a formal interpretation of the four components, however the first component has many similarities with the world business cycles estimated in other papers.

A second conclusion is that most of the developing countries are very weakly linked to the different components of the world business cycle. This results are in line with those found in Kose et al.(2003) and Mansour(2003). Also those studies showed that most of the developing countries' cycles are idiosyncratic.

The third and main conclusion is about regional composition. Even if the first common component has a statistically significant impact on most countries in the world, its importance is higher in high income countries. In this sense those countries can be defined as a region in *substrictus sensus.* The third common component affects mostly middle income countries and therefore we can group those countries as a second *substrictus sensus* region. The other common components bridge different groups, and there is no further evidence for

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distinctive geographical or cultural regions. Therefore this study confirmed the research of Artis(2003) that found if there existed a European region also Japan and USA would be part of it (rendering it non-European) and of Baxter and Kouparitsas(2004) that found that the only indicator that explained commovement was whether a country is developed or not.

Inside those *substrictus sensus* regions we can find further subdivisions regarding the dependence structure from the different factors. In the high income countries we found evidence of two groups that we label the French-Japanese and the German-American groups. These two groups are evident if we speak about similar structure or commovement and therefore they form regions in *latus sensus.* Furthermore these two groups seem to form a world core in terms of commovement as the remaining countries seem to be peripheral, being that the developing countries are the ones that commove less with this center. Also, Germany and the UK although part of the USA-German group seem to be the countries that link these two groups.

In the middle income countries we found a South East Asiatic group that has a similar dependence structure but do not commove. This leads us to consider that those countries may be starting to form a group but their integration is still not strong enough to classify them as a region in *latus sensus.*

Finally the limitations of the work should be pointed out: in the first place we should note that the statistical inference was done using the asymptotic distribution. That raises the question whether this distribution is close enough to the finite sample distribution to be reliable. To try to answer that question a small Monte-Carlo is presented in appendix 1.G, where the finite distribution of the common component and the asymptotic distribution are compared.

Second, the factor model is static, and does not capture potential dynamics in the transmission process from the world business cycle to each country.

Third, it assumes that the importance of the factors is constant through time. Therefore, to try to include some variation across time on the loadings as Stock and Watson(1999) suggest could help into getting a clear idea of the world synchronization and how it has evolved.

Finally, the model does not try to explain what the factors are. Previous studies using factors were unable to find a single variable that could explain the factors' variation and therefore the identification of what it is that the factors are depicting is still a open question.

These two last issues, the variability of loadings and an attempt to identify what the common components are describing are left for future work

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Appendices

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Appendix 1.A List of countries

Table A1: List of countries

1	(ALG)	Algeria	21	(GHA)	Ghana	41	(PER)	Peru
2	(AUS)	Australia*	22	(GRE)	Greece*	42	(PHI)	Philippines
3	(AUT)	Austria*	23	(GUA)	Guatemala	43	(POR)	Portugal*
4	(BEL)	Belgium*	24	(HND)	Honduras	44	(RWA)	Rwanda
5	(BOL)	Bolivia	25	(HKG)	Hong Kong	45	(SEN)	Senegal
6	(BRA)	Brazil	26	(ICE)	Iceland*	46	(SIN)	Singapore
7	(CAN)	Canada*	27	(IND)	India	47	(SAF)	South Africa
8	(CHI)	Chile	28	(IRE)	Ireland*	48	(SPN)	Spain*
9	(CHN)	China	29	(ITA)	Italy*	49	(SWE)	Sweden*
10	(COL)	Colombia	30	(JAP)	Japan*	50	(SWI/CH)	Switzerland*
11	(CRI)	Costa Rica	31	(KOR)	Korea (South)*	51	(THA)	Thailand
12	(CIV)	Cote d'Ivoire	32	(LUX)	Luxembourg*	52	(TUN)	Tunisia
13	(DEN)	Denmark*	33	(MAL)	Malaysia	53	(TUR)	Turkey*
14	(DOM)	Dominican Rep.	34	(MEX)	Mexico*	54	(UK)	Un. Kingdom*
15	(ECU)	Ecuador	35	(MOR)	Morocco	55	(USA)	USA.*
16	(ESV)	El Salvador	36	(NTH)	Netherlands*	56	(URU)	Uruguay
17	(FIN)	Finland*	37	(NZL)	New Zealand*	57	(ZAM)	Zambia
18	(FRA)	France*	38	(NOR)	Norway*	58	(ZIM)	Zimbabwe
19	(GAM)	Gambia, The	39	(PAK)	Pakistan			
20	(GER)	Germany*	40	(PAR)	Paraguay			

* - OECD countries.

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Appendix 1.B Country cycles

Figure B1: 1st set

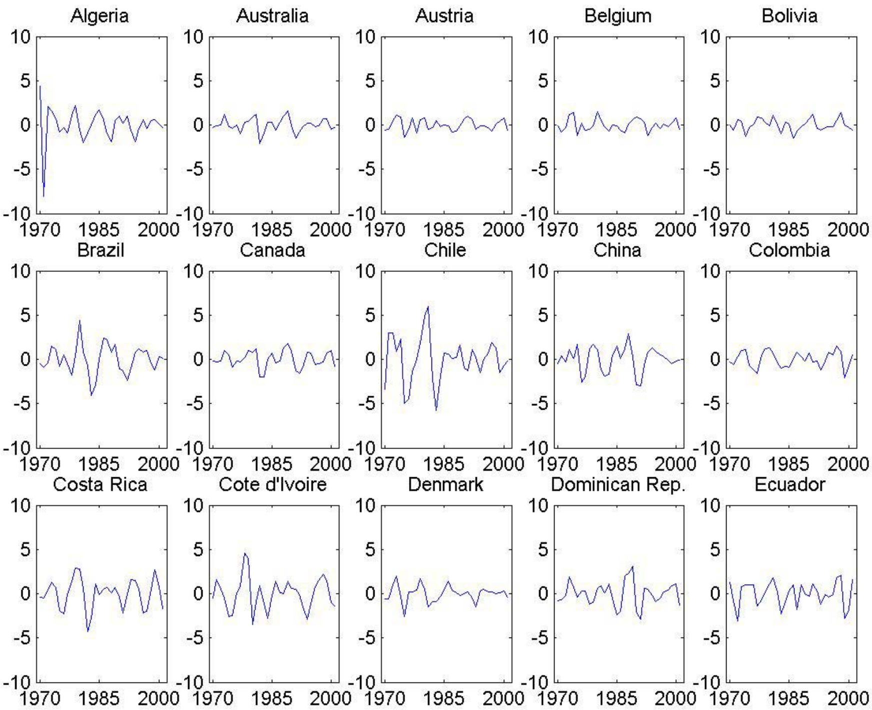

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Figure B2: 2nd set

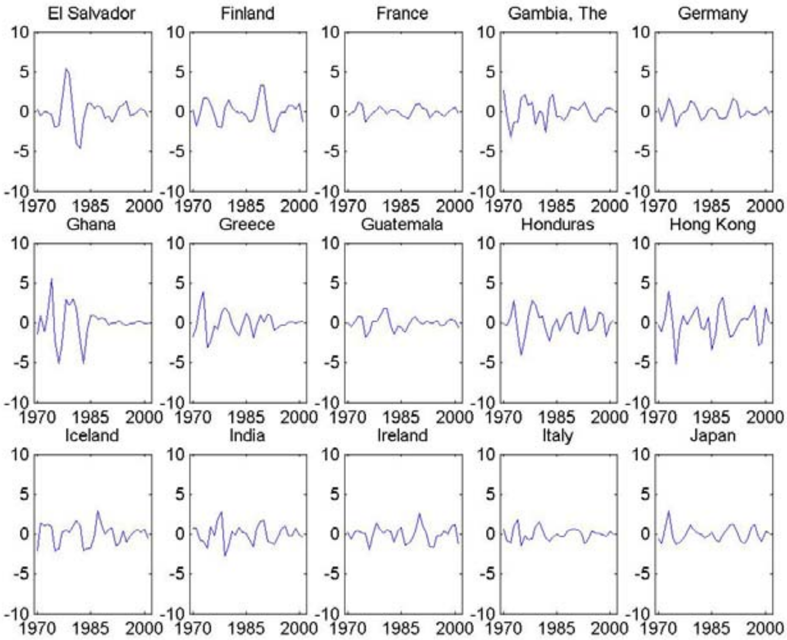


Figure B3: 3rd set

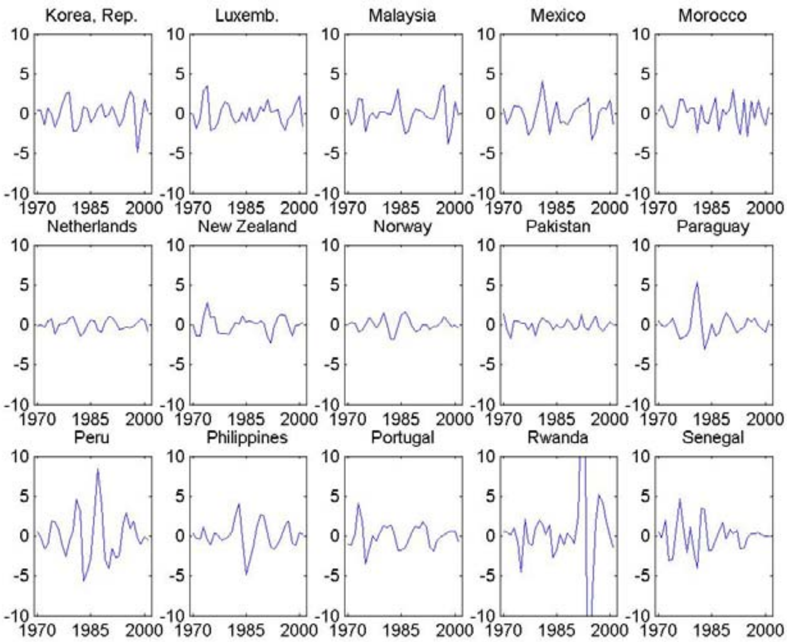
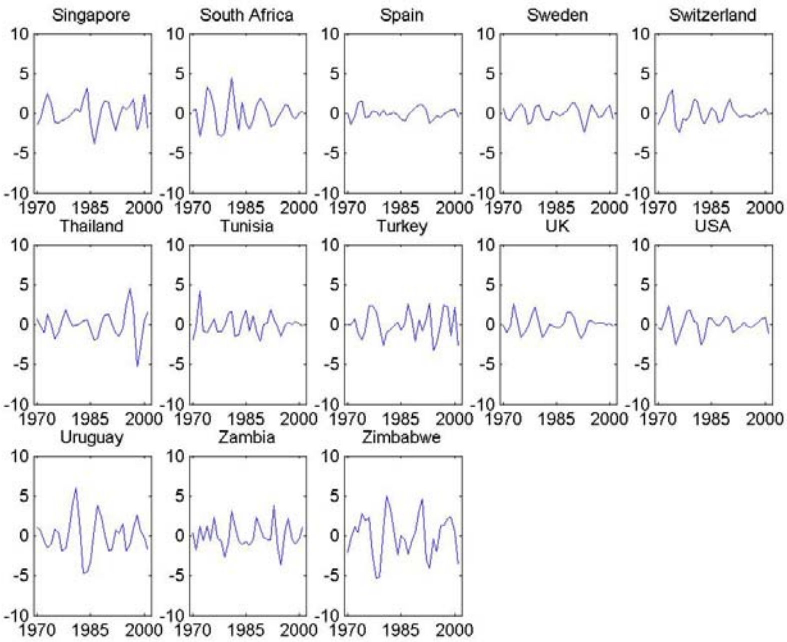


Figure B4: 4th set



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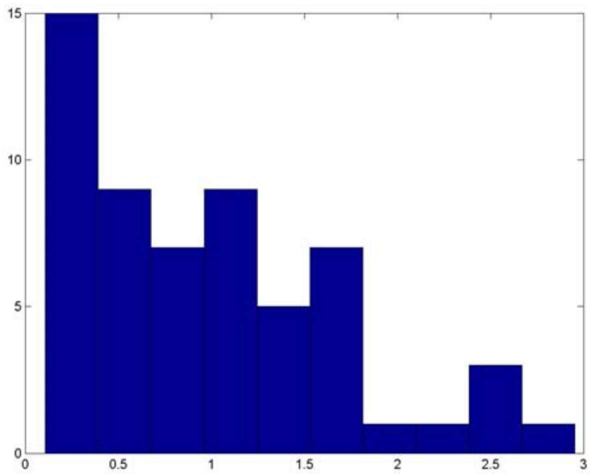
Appendix 1.C Are the idiosyncratic components cross-heteroscedastic?

The next graphics show the variance of the estimated residuals for the model without standardizing the data.

The world sample depict quite high cross-section heteroscedasticity Therefore this gives us some support for standardizing the data as we can see in the next appendix whenever there is cross-heteroscedasticity the method always overestimate the number of factors whenever we do not standardize the data.

Figure C1: Estimated variance of the residuals



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Appendix 1.D Monte Carlo simulation for detecting the number of common factors

This simulation tries to see how many factors do the IC indicate for the factor model:

$$\begin{aligned} X_{it} &= \lambda_i' F_t + e_{it} = C_{it} + e_{it} \\ i &= 1, 2, 3, ..., N \text{ and } t = 1, 2, 3, ..., T \end{aligned}$$

in which the number of factors was fixed at 2, 3 and 4. The factors are AR(1)processes described by:

$$\begin{aligned} F_t &= 0.5 * F_{t-1} + \epsilon_t \\ &\quad \epsilon_t \text{ are i.i.d.; randomly generated with variance equal to 1} \end{aligned}$$

The loadings λ_i' were generated randomly.

As for the idiosyncratic component e_{it} I considered two cases:

→ Cross-heteroscedastic: $\frac{e_i' e_i}{T} = 0.1$ for $\frac{1}{4}$ of the series;
$\frac{e_i' e_i}{T} = 1$ for $\frac{1}{2}$ of the series
$\frac{e_i' e_i}{T} = 10$ for $\frac{1}{4}$ of the series

→ Cross-homoscedastic: $\frac{e_i' e_i}{T} = 1$ for all the N series

Being $T = 32$ and N equal to 26 or 59.

In both cases they are non-serial and non-cross correlated and homoscedastic on the time dimension.

The number of replications was 5000, and were done for standardized (Std) and non-Standardized (non-Std) data.

From table D1 we realize that if the data is cross heteroscedastic the IC with non-standardized data select a number of factors higher than the true one. With standardized data the IC select a smaller number of factors than with non-standardized data but in any case higher than the true number. From the three IC the one that gets a number closer to the true number is the IC2.

In the case the data is cross-homoscedastic (see table D2) the IC1 and IC2 with non-standardized data get values very close to their true number (in any case IC2 is more precise). If we standardized the data both IC (1 and 2) would select a number bigger than the true one.

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Table D1: Cross-Heteroscedastic data

		2 factors		3 factors		4 factors	
		Std	non-Std	Std	non-Std	Std	non-Std
IC1	N=26	5.47 (1.177)	6 (0.0)	5.94 (0.346)	6 (0.0)	5.99 (0.133)	6 (0.0)
	N=59	2.1 (0.401)	5.76 (0.788)	3.70 (1.01)	5.87 (0.473)	5.15 (0.88)	5.91 (0.34)
IC2	N=26	2.65 (1.294)	6 (0.031)	4.55 (0.327)	5.99 (0.114)	5.50 (0.807)	5.97 (0.23)
	N=59	2 (0.045)	3.152 (1.492)	3.06 (0.281)	4.16 (1.24)	4.20 (0.525)	5.01 (0.88)
IC3	N=26	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.0)
	N=59	5.95 (0.34)	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.0)

Table D2: Cross-Homoscedastic data

		2 factors		3 factors		4 factors	
		Std	non-Std	Std	non-Std	Std	non-Std
IC1	N=26	4.11 (1.56)	2 (0.063)	4.92 (0.111)	3.02 (0.125)	5.4 (0.781)	4.05 (0.231)
	N=59	2.08 (0.331)	2 (0.0)	3.29 (0.061)	3 (0.0)	4.33 (0.572)	4 (0.0)
IC2	N=26	2.28 (0.712)	2 (0.0)	3.54 (0.779)	3 (0.0)	4.44 (0.644)	4 (0.032)
	N=59	2 (0.063)	2 (0.0)	3.04 (0.219)	3 (0.0)	4.09 (0.320)	4 (0.0)
IC3	N=26	6 (0.0)	6 (0.017)	6 (0.0)	6 (0.0)	6 (0.0)	6 (0.032)
	N=59	5.6 (0.949)	2.02 (0.318)	5.84 (0.499)	3.21 (0.572)	5.92 (0.313)	4.348 (0.628)

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Appendix 1.E Short description of the clustering method

1.E.1 Hard cluster trees

The hard-cluster trees are built by applying the following steps:

1. Compute the following distances for all pair of countries:

1A Without the idiosyncratic component	$d_{c1,c2} = \sqrt{\sum_{i=1}^{r}\left(\left(\lambda_i^{c1}\right)^2 - \left(\lambda_i^{c2}\right)^2\right)^2}$
1B With the idiosyncratic component	$d_{c1,c2} = \sqrt{\sum_{i=1}^{r}\left(\left(\lambda_i^{c1}\right)^2 - \left(\lambda_i^{c2}\right)^2\right)^2 + (s^{c1})^2 + (s^{c2})^2}$

where $\left(\lambda_i^{cj}\right)^2$ is the importance of factor i to country cj;, $j = 1;2$ and $(s^{cj})^2$ is the idiosyncratic component of country cj national cycle, $j = 1;2$.

2. Find the minimum value,

If the minimum value is between two original countries go to 3A.

If the minimum value is between a clustered group and other country or clustered group go to 3B

3A Delete those two countries from the sample and add an artificial country where the loadings are equal to the average of the clustered countries.

3B Delete the clustered groups and add an artificial country where the loadings are equal to the average of all clustered original countries.

4 Go to step one.

1.E.2 Fuzzy cluster

For the fuzzy clustering we minimize the following objective function using the algorithm described in Kaufman and Rousseeuw (1990):

$$\sum_{v=1}^{K} \frac{\sum_{i,j=1}^{N} u_{iv}^2 u_{jv}^2 d(i,j)}{2\sum_{j=1}^{N} u_{jv}^2}$$

where K is the number of clusters and N is the total number of countries; $d(i;j)$ is the distance between country i and j defined in 1A of the hard cluster tree algorithm and u_{iv} stands for the membership of country i in cluster v;

The normalized Dunn coefficient is given by:

$$NDC = \frac{DC - (1/K)}{1 - (1/K)} \text{ where } DC = \sum_{i=1}^{N}\sum_{v=1}^{K} u_{iv}^2/n$$

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If NDC = 0 complete fuzziness, if NDC = 1 perfect hard-clustering

The silhouette value for each country is a measure of how similar that country is to countries in its own cluster against. countries in other clusters and ranges from -1 to +1. It is defined as:

$$S_i = \frac{\min[b_i - a_i]}{\max[b_i; a_i]}$$

where a_i is the average distance of country i to all other countries in its own cluster and b_i is the average distance of country i to all other countries belonging to other cluster.

If silhouette value is close to 1, it means that sample is well-clustered and it was assigned to a very appropriate cluster. If silhouette value is about zero, it means that the sample could be assign to another closest cluster as well, and the sample lies equally far away from both clusters. If silhouette value is close to -1, it means that sample is misclassified and is merely somewhere in between the clusters.

The overall average silhouette width for the entire plot is simply the average of the S_i for all countries in the whole dataset.

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Appendix 1.F Fuzzy cluster tables

Table F1: Fuzzy Cluster Table for the OECD countries

Countries	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Country Silhouette
Australia	0,173	**0,4402**	0,1326	0,2541	0,237
Austria	0,0908	0,1924	**0,4761**	0,2408	0,2242
Belgium	0,0537	0,3579	**0,5017**	0,0867	0,196
Canada	0,1295	**0,4384**	0,2197	0,2124	0,0655
Denmark	0,0557	0,0666	0,0656	**0,812**	0,5084
Finland	0,1164	0,348	**0,4317**	0,104	0,1717
France	0,0547	0,1459	**0,6769**	0,1225	0,4451
Germany	0,0693	0,2065	0,195	**0,5292**	0,1795
Greece	0,0393	0,0633	0,0634	**0,834**	0,4564
Iceland	0,1778	**0,4517**	0,1294	0,2411	0,2465
Ireland	0,0558	0,1512	**0,7**	0,093	0,4078
Italy	0,0399	**0,7417**	0,1516	0,0668	0,1676
Japan	0,0202	0,102	**0,834**	0,0438	0,4065
Korea	**0,5246**	0,1419	0,1312	0,2022	0,1597
Luxembourg	0,0254	**0,7895**	0,107	0,0781	0,1627
Mexico	0,2013	**0,3611**	0,2811	0,1565	0,0391
Netherlands	0,0762	**0,4273**	0,2929	0,2036	0,0317
New Zealand	**0,469**	0,216	0,1732	0,1418	0,1702
Norway	**0,4886**	0,1976	0,1214	0,1923	0,0955
Portugal	0,0159	0,0815	**0,8642**	0,0384	0,4394
Spain	0,0462	0,1191	**0,7604**	0,0743	0,5029
Sweden	**0,8563**	0,0527	0,0444	0,0466	0,178
Switzerland	0,047	**0,6426**	0,2169	0,0935	0,1302
Turkey	**0,6506**	0,1026	0,1013	0,1454	0,2764
UK	0,1929	0,2383	0,1944	**0,3745**	0,1617
USA	0,0494	0,1034	0,0736	**0,7736**	0,4243
Cluster Silhouette	0,176	0,135	0,3492	0,3461	
Normalized Dunn coefficient = 0,2910					

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Table F2: Fuzzy Cluster Table for the whole sample

Countries	Clusters						
Countries	1	2	3	4	5	6	7
Algeria	**0,3**	0,0991	0,1714	0,1955	0,1117	0,0557	0,0666
Australia	0,0109	0,0063	0,0052	0,0058	**0,9641**	0,0045	0,0031
Austria	0,1682	**0,3484**	0,1336	0,0944	0,0928	0,0633	0,0992
Belgium	0,067	**0,5913**	0,0599	0,0525	0,0883	0,0626	0,0785
Bolivia	0,0094	0,0097	**0,9177**	0,0302	0,0099	0,0164	0,0068
Brazil	0,0954	0,0889	0,0837	0,1106	**0,4123**	0,1409	0,0684
Canada	0,1667	0,2091	0,0607	0,0907	**0,2428**	0,072	0,1579
Chile	0,0949	0,0553	0,045	0,0475	**0,6875**	0,0409	0,0289
China	0,1357	0,0868	0,1044	0,189	**0,2296**	0,1449	0,1097
Colombia	0,0743	0,0596	0,0287	0,0324	**0,7585**	0,0246	0,0218
Costa Rica	0,3253	0,0717	0,0678	0,0761	**0,3704**	0,0458	0,0429
Cote d'Ivoire	0,2152	0,0803	0,168	**0,2696**	0,0974	0,0731	0,0965
Denmark	**0,7332**	0,059	0,0384	0,0458	0,0618	0,0216	0,0402
Dominican Rep.	0,017	0,0232	0,0719	0,1205	0,0278	**0,7134**	0,0263
Ecuador	0,001	0,0016	0,0037	0,0046	0,0018	**0,9856**	0,0017
El Salvador	**0,3081**	0,0937	0,1077	0,1643	0,1381	0,0732	0,1148
Finland	0,0689	**0,3219**	0,0899	0,0937	0,1005	0,1542	0,1708
France	0,109	**0,4612**	0,0588	0,064	0,0705	0,0486	0,1879
Gambia, The	0,0537	0,0468	0,2062	**0,4114**	0,057	0,1579	0,0669
Germany	**0,3584**	0,2208	0,095	0,0729	0,1453	0,0474	0,0601
Ghana	0,0745	0,0472	0,0299	0,0316	**0,7683**	0,027	0,0214
Greece	**0,6555**	0,0887	0,0566	0,0529	0,0789	0,0268	0,0405
Guatemala	0,153	**0,4072**	0,0519	0,0551	0,2166	0,0468	0,0695
Honduras	**0,711**	0,0433	0,0363	0,0428	0,1192	0,0214	0,0261
Hong Kong	0,1402	0,1351	0,0696	0,133	0,1875	0,0961	**0,2386**
Iceland	0,0138	0,0083	0,0069	0,008	**0,9531**	0,0059	0,004
India	0,0625	0,0878	0,1696	**0,317**	0,0545	0,1268	0,1818
Ireland	0,0858	**0,5169**	0,1021	0,0754	0,0679	0,0596	0,0923
Italy	0,0996	**0,3875**	0,0645	0,0662	0,2217	0,0771	0,0834
Japan	0,0456	**0,7741**	0,0252	0,0276	0,0378	0,0225	0,0673
Korea, Rep.	0,1528	0,0859	0,1045	**0,2503**	0,1152	0,1011	0,1902
Luxembourg	0,1491	**0,405**	0,0518	0,0562	0,2254	0,0459	0,0665
Malaysia	0,0122	0,0228	0,0108	0,0176	0,0102	0,0128	**0,9137**

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Table F3: Fuzzy Cluster Table for the whole sample - continuation

Countries	Clusters						
	1	2	3	4	5	6	7
Mexico	0,0727	0,1508	**0,2424**	0,1193	0,1211	0,2254	0,0682
Morocco	0,0014	0,0015	**0,9857**	0,0061	0,0015	0,0027	0,0011
Netherlands	0,1533	**0,3529**	0,1188	0,0768	0,171	0,0667	0,0605
New Zealand	0,0685	0,0817	0,0952	0,1919	0,1047	**0,2915**	0,1665
Norway	0,1156	0,0694	0,1745	0,211	**0,2174**	0,1513	0,0609
Pakistan	0,003	0,0045	0,0098	0,012	0,0051	**0,9607**	0,0049
Paraguay	0,0808	0,1411	0,1085	0,1082	0,2359	**0,2428**	0,0826
Peru	0,0767	0,0738	0,1241	0,1737	0,1672	**0,3073**	0,0771
Philippines	0,0764	0,1253	0,1379	0,1743	0,0622	0,1205	**0,3033**
Portugal	0,041	**0,8157**	0,0275	0,024	0,0333	0,0204	0,0381
Rwanda	0,0113	0,0117	**0,9026**	0,0351	0,0118	0,0194	0,0081
Senegal	0,0727	0,075	0,2017	**0,2806**	0,0683	0,175	0,1266
Singapore	0,0135	0,0272	0,0108	0,0172	0,0112	0,0128	**0,9075**
South Africa	0,0499	0,0802	0,1132	0,1159	0,0841	**0,4755**	0,0813
Spain	0,0766	**0,4764**	0,0565	0,0625	0,0593	0,0509	0,2178
Sweden	0,0058	0,0052	0,0253	**0,9364**	0,0067	0,0144	0,0063
Switzerland	0,1036	**0,4338**	0,0771	0,0633	0,1867	0,0706	0,065
Thailand	0,0409	0,0557	0,0358	0,0677	0,0368	0,0486	**0,7145**
Tunisia	0,0874	0,0659	**0,4044**	0,1603	0,1195	0,1183	0,0442
Turkey	0,0936	0,0577	0,2025	**0,4008**	0,0707	0,0943	0,0803
UK	**0,2733**	0,1369	0,0669	0,1079	0,1712	0,0648	0,179
USA	**0,6713**	0,0756	0,0374	0,0414	0,1119	0,0245	0,0379
Uruguay	0,0835	0,0771	0,1556	0,1653	0,1847	**0,2672**	0,0667
Zambia	0,0009	0,0013	0,0032	0,0041	0,0015	**0,9874**	0,0015
Zimbabwe	0,0603	0,1104	0,2113	0,1342	0,0825	**0,3155**	0,0859
Cluster Silhouette	0,2206	0,3259	0,4663	0,123	0,2561	0,1839	0,3075
Normalized Dunn coefficient = 0,2887							

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Appendix 1.G Monte Carlo simulation to compare the finite with the asymptotic distribution

This simulation tries to see if the asymptotic distribution used in the estimation and inference of the model can approximate the finite distribution.

The model used is:

$$\begin{aligned} X_{it} &= \lambda_i' F_t + e_{it} = C_{it} + e_{it} \\ i &= 1, 2, 3, ..., N \text{ and } t = 1, 2, 3, ..., T \end{aligned}$$

in which the number of factors was fixed at 4. The factors are AR(1)processes described by:

$$\begin{aligned} F_t &= 0.5 * F_{t-1} + \epsilon_t \\ & \epsilon_t \text{ are i.i.d.; randomly generated with variance equal to 0.75} \end{aligned}$$

The loadings λ_i' were generated randomly.

The idiosyncratic component e_{it} were cross-homoscedastic: $\frac{e_i' e_i}{T} = 1$ for all the N series Being $T = 32$ and N equal to 59

As the number of factors is four and have equal variance, it makes it difficult to know which factor estimated (and respective loadings) corresponds to which true factor (and respective true loadings). Therefore the study was only made for the asymptotic distribution of the common factor, as is described in Bai(2003). He showed that:

$$\begin{aligned} c_{it} &= \frac{\widehat{C}_{it} - C_{it}^0}{\sqrt{\frac{1}{N} V_{it} - \frac{1}{T} W_{it}}} \xrightarrow{d} N(0,1), \qquad i = 1, 2, 3, ..., N \text{ and } t = 1, 2, ..., T \\ \text{where} &: V_{it} \text{ and } W_{it} \text{ are described in Bai(ibidem)} \\ & \widehat{C}_{it} \text{ is the estimated common component } (\sum_{i=1}^{4} \widehat{\lambda}_{ri} \widehat{F}_{rt}) \\ & C_{it}^0 \text{ is the true common component } (\sum_{i=1}^{4} \lambda_{ri} F_{rt} \end{aligned}$$

In the following graphic the histograms depict the distribution of c_{it} using the estimated $\widehat{V}_{it}$ and $\widehat{W}_{it}$, and the plot of the N(0,1) density function.

As we can see for this sample, the estimated distribution has fatter tails than the standard Gaussian distribution. Even if they are not that far from each other, rendering the use of the asymptotic distribution useful for inference, seems that a better distribution for a finite sample would be an improvement for this kind of studies. As final note, it does not seem that standardizing or not the data modifies the finite distribution.

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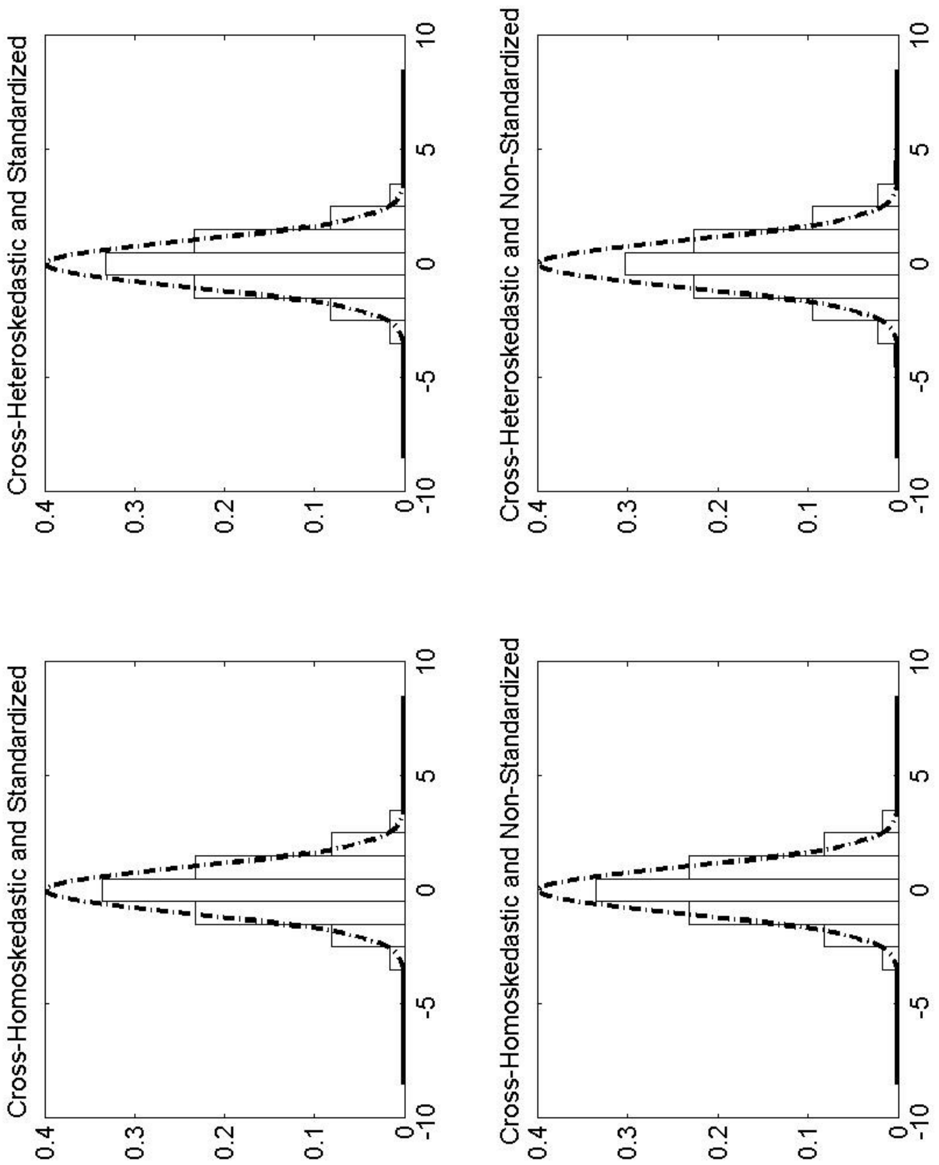
Figure G1: Histograms



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CHAPTER 2

CONSUMPTION SMOOTHING AT BUSINESS CYCLE FREQUENCY

2.1 Introduction

A substantial number of papers have been written on the symmetry of business cycles across countries[1]. While those papers concentrate on the symmetry of fluctuations in output or in general economic indicators, the agent's utility is usually depicted as a function of consumption and leisure. Therefore, if the agent can smooth her utility over the cycle by intertemporal substitution and/or international risk sharing, the degree of symmetry of the output business cycle is of little importance, as consumption should respond only to world-wide output variations. To accomplish this, there are several ways in which the agent is able to risk share the country specific shocks to output.

The first one is through portfolio diversification. As Obstfeld and Rogoff (1996) show, using a modified version of Lucas (1982) model, agents would optimally hold a diversified portfolio of financial assets in which the importance of each country's asset would be proportional to that country's economic wealth. In this framework the consumption level would only vary according to world output fluctuations and not to country specific ones. However, French and Poterba (1991), Lane (2000) and Kraay et al. (2000) show that countries do not optimally diversify their portfolios, having most of their wealth invested in home assets[2]. The main arguments given to explain this behavior are financial transaction costs [3]; the existence of non-tradable goods[4] and the regulation of financial markets[5].

[1] Some recent studies on business cycle syncronization at an European level are Artis et al. (1999 and 2003), Wynne and Koo (2000), Altissimo et al. (2000) and Forni and Reichlin (2001).At a world level we have Gregory et al. (1997), Andreano and Savio (2002), Helbling and Bayoumi (2003), Bordo and Helbling (2003), Lumsdaine and Prasad (2003), Mansour (2003), Kose et all. (2003) and Cerqueira (2005).

[2] In the literature this is known as the home bias puzzle.

[3] Cole and Obstfeld (1991) estimated that the welfare gains from portfolio diversification are very small, therefore small impediments can avoid the agents to diversiy their portfolios. However, Van Wincoop (1994) and Tesar and Werner (1995) dismissed this argument. The first by showing that the gains from portfolio diversification can be much higher than those estimated by Cole and Obstfeld; the second by noting that the turnover rate on equities is higher for non-residents than for residents.

[4] Therefore consumption of this kind of goods would not be risk-sharable.

[5] In every year from 1966 to 1994 the proportion of countries with international capital market restricitons was around 75%, Lewis (1996) .

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The portfolio diversification channel is an ex-ante insurance mechanism, as it will work automatically in the presence of asymmetric shocks. We will call it the portfolio diversification or asset market channel.

Even if agents do not optimally diversify their portfolio, they can still engage in risk sharing behavior by using the international credit markets (by lending and borrowing) adjusting their consumption level. In fact, in general equilibrium models with imperfect financial markets, where only an international bond can be traded and in the absence of other costs/rigidities, the cross country consumption correlations are higher than output correlations, indicating that there is some amount of risk sharing (see Baxter and Crucini (1995) or Kollman (1996)). Even if we introduce some modifications to the base model like imperfect substitutability between home and foreign goods it is only for very low levels of the elasticity of substitution that the cross-correlation of consumption is lower than the one for output, see Corsetti et al. (2004). In the case of trade costs (as for example in Ravn and Mazenga(2004)[6]) or the existence of non-tradables (for a recent paper where tradability is endogenous see Ghironi and Melitz (2004)), the consumption cross-correlation level drops but continues to be, in most cases, higher than the output cross-correlations. There are some exceptions as in the models presented by Ubide (1999) and Olivero (2004), the first by introducing shocks on the firms mark-up and government expenditures, the second by introducing an oligopolist banking system.

The use of the credit markets is an ex-post insurance mechanism, as it is put in practice after the revelation of asymmetric shocks. Moreover in practical terms it is difficult to disentangle this risk-sharing mechanism from inter-temporal smoothing through savings, therefore in the empirical part of the paper we will call it the credit market or savings channel.

Notwithstanding the prediction of the majority of theoretical models, the observed cross-consumption correlation across countries is inferior to the output cross-correlation[7] giving rise to the consumption correlation puzzle[8]; moreover, a number of empirical studies that have tried to measure the amount of risk sharing [9] have found that the amount of risk sharing

[6]Even if Rogoff and Obstfeld(2000) argue that trade costs solve the consumption correlation puzzle. See also the criticism from Engel(2000).

[7]See Backus et al. (1995) or Hess and Shin (1997).

[8]We should refer that, adding to the previous exceptions, also the presence of taste shocks can reduce the gap between the cross-correlations observed and the ones given by the theoretical models as it is shown by Stockman and Tesar (1995), even in the presence of complete asset markets.

[9]See Lewis (1996), Asdruballi et all. (1996), Sorenson and Yosha (1998a, 1998b, 2000), Arreaza et all. (1998); Kalemli-Ozcan et all. (1999), Crucini (1999), Melitz and Zummer (1999), Del Negro (2000), Artis and Hoffman (2003) and Marinheiro (2004).

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is relatively small, far from perfect and that the bulk of smoothing is done through the credit markets/saving channel. Even if those articles have shed some light on how risk sharing is achieved, the fact is that, when trying to measure the different channels they are only able to do aggregate estimations (the EMU area, the US States, etc.). Once the number of countries/regions is reduced the methods lose power making inference difficult.

In this chapter I purpose to use a factor model[10] to perform this study using a Bayesian method purposed by Otrok and Whiteman (1999) to calculate the distributions of the estimated parameters. The use of factor methods has the advantage that panel information can be used in order to get more accurate estimators while it can retrieve estimators for each individual/country in the sample. In this way we will be able to estimate the different channels of risk sharing and consumption smoothing for each country.

The next section derives a simple theoretical framework of how international risk sharing is achieved and how can it be related to the empirical methods used. The following section describes the empirical method, the data and the results. The final section concludes the chapter.

2.2 Risk sharing and international consumption smoothing: a simple theoretical model

The model presented is a "*demonstration model*" to illustrate how international risk sharing works and how can it be related to an empirical factor model. This section is divided into four subsections. The first presents a model where markets are complete and so risk sharing is complete too; the following considers that not all agents have access to the financial markets and so the risk sharing measured at country level is incomplete. The next section considers the case when only an international bond is traded and compares the results with those of the previous models. The last section considers the previous models in the presence of taste shocks, as this kind of shocks can account for the consumption correlation puzzle as is described by Stockman and Tesar (1995).

2.2.1 Complete markets and full risk sharing

Consider an endowment economy with n different countries, where all produce the same internationally tradable homogeneous good, having in equilibrium the same GDP per capita

[10] The first to use a factor model to study this issue was Del Negro (2000), however the estimation methods used did not allow him to disentagle the parameters for individual regions.

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$(\overline{Y}^i)$. At each point the world economy is hit by a shock (ϵ_t^w) which causes world output (Y_t^w) to deviate from it's steady state $(\overline{Y}^w)$:

$$Y_t^w = \overline{Y}^w(1+\epsilon_t^w) = \overline{Y}^w + \overline{Y}^w.\epsilon_t^w \tag{2.1}$$

Each country i is subject to this worldwide shock with different magnitudes that can vary through time (β_t^i) plus an asymmetric shock (ϵ_t^i):

$$Y_t^i = \overline{Y}^i(1+\beta_t^i.\epsilon_t^w + \epsilon_t^i) = \overline{Y}^i + \overline{Y}^i.\beta_t^i.\epsilon_t^w + \overline{Y}^i.\epsilon_t^i \tag{2.2}$$

such that $cov(\epsilon_t^w;\epsilon_t^i) = 0$. Aggregating the n economies we have that:

$$\begin{aligned} Y_t^w &= \sum_{i=1}^n Y_t^i = \sum_{i=1}^n \left(\overline{Y}^i + \overline{Y}^i.\beta_t^i.\epsilon_t^w + \overline{Y}^i.\epsilon_t^i\right) = \\ &= \sum_{i=1}^n \overline{Y}^i + \epsilon_t^w \sum_{i=1}^n \beta_t^i.\overline{Y}^i + \sum_{i=1}^n \overline{Y}^i.\epsilon_t^i \end{aligned} \tag{2.3}$$

Equating equations (2.3) and (2.1) :

$$\begin{cases} \sum_{i=1}^n \overline{Y}^i = \overline{Y}^w \\ \sum_{i=1}^n \beta_t^i.\overline{Y}^i = \overline{Y}^w \\ \sum_{i=1}^n .\overline{Y}^i.\epsilon_t^i = 0 \end{cases} \tag{2.4}$$

which implies that:

$$\frac{\sum_{i=1}^n \beta_t^i.\overline{Y}^i}{\sum_{i=1}^n \overline{Y}^i} = 1 \tag{2.5}$$

in words, the weighted average impact of the world shock is 1.

At each point in time, consumers in each economy will try to maximize their lifetime utility (small letters denote *per capita* values):

$$\max_{c^i} U(c^i) = \max_{c_\tau^i} E_t \left(\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u(c_\tau^i)\right) \tag{2.6}$$

$$s.t: \sum_{i=1}^n P_i.y_\tau^i = \sum_{i=1}^n P_i.c_\tau^i$$

where: c_τ^i *per capita* consumption in country i

y_τ^i *per capita* output in country i and

P_i is the i^{th} country population and

E_t denotes the expectations at t. (2.7)

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If we consider that there are complete asset markets, the optimal solution for each consumer would be equal to the social planner's problem that would maximize the sum of individual utilities:

$$\max_{c^i} \sum_{i=1}^{n} U(C^i) = \max_{c^i_\tau} \sum_{i=1}^{n} w_i.E_t \left(\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u(c^i_\tau) \right) \tag{2.8}$$

$$s.t: \sum_{i=1}^{n} P_i.y^i_\tau = \sum_{i=1}^{n} P_i.c^i_\tau$$

$$w_i = \frac{P_i}{P_w}$$

where: P_w is the world population.

It can be shown that in this economy the agents consumption would depend only on the worldwide deviations from the equilibrium, and not on the asymmetric shocks. The first order conditions at each point in time of the problem depicted by (2.8) are:

$$\begin{cases} \frac{P_i}{P_w}\ u'(c^i_t) = P_i\lambda_t \Leftrightarrow\ u'(c^i_t) = P_w\lambda_t \\ \sum_{i=1}^{n} P_i.y^i_t = \sum_{i=1}^{n} P_i.c^i_t \end{cases} \tag{2.9}$$

The marginal utilities are all equalized, therefore $c^i_t = c^j_t$,implying[11]:

$$\begin{aligned} \sum_{i=1}^{n} P_i.y^i_t &= P_w.c^{i,*}_t \Leftrightarrow \\ &\Leftrightarrow\ c^{i,*}_t = \frac{Y^w_t}{P_w} \Leftrightarrow \\ &\Leftrightarrow\ P_i.c^{i,*}_t = P_i.\frac{Y^w_t}{P_w} \Leftrightarrow \\ &\Leftrightarrow\ C^{i,*}_t = w_i.Y^w_t \end{aligned} \tag{2.10}$$

and therefore:

$$C^{i,*}_t = w_i(\overline{Y}^w + \overline{Y}^w \epsilon^w_t) = \frac{P_i}{P_w}.(\overline{Y}^w + \overline{Y}^w.\epsilon^w_t) = \overline{Y}^i + \frac{P_i}{P_w}.\overline{Y}^w \epsilon^w_t \tag{2.11}$$

As GDP *per capita* in the steady state is equal across countries, then:

$$\frac{P_i}{P_w}.\overline{Y}^w = P_i.\overline{y}^w = \overline{Y}^i \tag{2.12}$$

[11] From now on the $*$ denotes the consumption level of the representative agent ($c^{i,*}_t$), the aggregate consumption country level ($C^{i,*}_t$), or the relative deviation from the steady state ($\Delta C^{i,*}_t$) when there is full risk sharing.

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Therefore equation (2.11) can be written as:

$$C_t^{i,*} = \overline{Y}^i + \overline{Y}^i.\epsilon_t^w \Leftrightarrow \Delta C_t^{i,*} = \epsilon_t^w \tag{2.13}$$

So, in this framework, the asymmetric shocks would be perfectly smoothed and consumption would respond to the worldwide average effect of the world shock and not to the specific impact on each country.

2.2.2 Limited participation in the financial markets

If we consider that in each country part of the population is excluded from the financial markets[12], living in financial autarky, the problem would have to be split into two parts: the optimization problem for the agents with access to financial markets (denoted by F) and for those who are living in financial autarky (denoted by FA) .

In this case we can define the world shock as the deviation of aggregate world income for agents with access to financial markets from its equilibrium:

$$Y_t^{w,F} = \overline{Y}^{w,F}(1+\epsilon_t^w) = \overline{Y}^{w,F} + \overline{Y}^{w,F}.\epsilon_t^w \tag{2.14}$$

Therefore in each economy, output evolves as:

$$Y_t^i = Y_t^{i,F} + Y_t^{i,FA} = \overline{Y}^{i,F}(1+\beta_t^i.\epsilon_t^w + \epsilon_t^i) + \overline{Y}^{i,FA}(1+\beta_t^i.\epsilon_t^w + \epsilon_t^i) \tag{2.15}$$

Linking equation (2.15) and (2.14) we have that:

$$\begin{cases} \sum_{i=1}^n \overline{Y}^{i,F} = \overline{Y}^{w,F} \\ \sum_{i=1}^n \overline{Y}^{i,F}\beta_t^i.\epsilon_t^w = \overline{Y}^{w,F}.\epsilon_t^w \Leftrightarrow \frac{\sum_{i=1}^n \overline{Y}^{i,F}\beta_t^i}{\overline{Y}^{w,F}} = 1 \\ \sum_{i=1}^n \overline{Y}^{i,F}.\epsilon_t^i = 0 \end{cases} \tag{2.16}$$

The world economy, as a whole, evolves as:

$$\begin{aligned} Y_t^w &= Y_t^{w,F} + Y_t^{w,FA} = \sum_{i=1}^n \left(Y_t^{i,F} + Y_t^{i,FA}\right) = \\ &= \sum_{i=1}^n \left[\overline{Y}^{i,F}(1+\beta_t^i.\epsilon_t^w + \epsilon_t^i) + \overline{Y}^{i,FA}(1+\beta_t^i.\epsilon_t^w + \epsilon_t^i)\right] = \\ &= \overline{Y}^w + \left(\overline{Y}^{w,F} + \sum_{i=1}^n \overline{Y}^{i,FA}\beta_t^i\right).\epsilon_t^w + \sum_{i=1}^n \overline{Y}^{i,FA}\epsilon_t^i \end{aligned}$$

[12] Both internal and internationally. If they were able to access internal financial markets they could smooth consumption with the agents which have access to international markets. The latter would borrow more than they would need in order to lend to the other ones.

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Note that if the share of people living in financial autarky is zero we would be back with our initial problem depicted by equations (2.1) to (2.8).

Those who live in financial autarky would consume:

$$c_t^{i,FA} = \overline{y}^i + \overline{y}^i.\beta_t^i.\epsilon_t^w + \overline{y}^i.\epsilon_t^i \tag{2.17}$$

As for the financially integrated agents the problem can be set as a social planner maximization problem:

$$\max_{c_\tau^i} \sum_{i=1}^{n} s_F^i.E_t\left(\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u(c_\tau^{F,i})\right) \tag{2.18}$$

$$s.t: \sum_{i=1}^{n} P_i.(1-s_{FA}^i).y_\tau^i = \sum_{i=1}^{n} P_i.(1-s_{FA}^i).c_\tau^{F,i}$$

where s_{FA}^i is the share of people living in financial autarky

and $c_\tau^{F,i}$ is the per capita consumption of the financially integrated people

maximizing the previous problem we would get that $c_t^{F,i}$ would be equal in all countries therefore:

$$\begin{aligned} c_t^{F,i} &= \frac{1}{\sum_{j=1}^{n}(P_j.s_F^j)}.\sum_{j=1}^{n}\left(P_j.y_t^j.s_F^j\right) = \\ &= \frac{1}{\sum_{j=1}^{n}(P_j.s_F^j)}.\sum_{j=1}^{n}\left(P_j.\overline{y}^j.(1+\beta_t^j.\epsilon_t^w+\epsilon_t^i).s_F^j\right) = \\ &= \frac{1}{\sum_{j=1}^{n}(P_j^F)}.\sum_{j=1}^{n}\left(\overline{Y}^{j,F}.(1+\beta_t^j.\epsilon_t^w+\epsilon_t^i)\right) \end{aligned} \tag{2.19}$$

The overall consumption in the economy would be:

$$\begin{aligned} C_t^i &= C_t^{i,FA} + C_t^{i,F} = \\ &= \left(\overline{Y}^i + \overline{Y}^i.\beta_t^i.\epsilon_t^w + \overline{Y}^i\epsilon_t^{i,FA}\right).s_{FA}^i + \frac{P_i^F}{\sum_{j=1}^{n}(P_j^F)}.\sum_{j=1}^{n}\left(\overline{Y}^{j,F}.(1+\beta_t^j.\epsilon_t^w+\epsilon_t^i)\right) \end{aligned} \tag{2.20}$$

from equation (2.16) $\sum_{j=1}^{n}\left(\overline{Y}^{j,F}.\varepsilon^i\right) = 0$ and $\sum_{i=1}^{n}\overline{Y}^{i,F}\beta_t^i.\epsilon_t^w = \overline{Y}^{w,F}.\epsilon_t^w$, therefore:

$$C_t^i = \left(\overline{Y}^{i,FA} + \overline{Y}^{i,FA}.\beta_t^i.\epsilon_t^w + \overline{Y}^{i,FA}\epsilon_t^{i,FA}\right) + \frac{P_i^F}{P^{w,F}}.\overline{Y}^{w,F}.(1+\epsilon_t^w)$$

As GDP per capita in steady state is equal $\left(\frac{\overline{Y}^{w,F}}{P^{w,F}} = \frac{\overline{Y}^{i,F}}{P_i^F}\right)$, then:

$$C_t^i = \left(\overline{Y}^{i,FA} + \overline{Y}^{i,FA}.\beta_t^i.\epsilon_t^w + \overline{Y}^{i,FA}\epsilon_t^{i,FA}\right) + \overline{Y}^{i,F}.(\epsilon_t^w. + 1) =$$
$$= \overline{Y}^i + \overline{Y}^i\left(\left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w + s_{FA}^i.\epsilon_t^i\right)$$

Therefore the evolution of each economy can be characterized as:

$$\begin{aligned} Y_t^i &= \overline{Y}^i(1 + \beta_t^i.\epsilon_t^w + \epsilon_t^i) \\ C_t^i &= \overline{Y}^i + \overline{Y}^i\left(\left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w + s_{FA}^i.\epsilon_t^i\right) \end{aligned} \tag{2.21}$$

re-labeling $\overline{Y}^i = \overline{C}^i$[13] in the second expression and dividing each by the equilibrium level we get[14]:

$$\begin{aligned} \Delta Y_t^i &= \beta_t^i.\epsilon_t^w + .\epsilon_t^i \\ \Delta C_t^i &= \left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w + s_{FA}^i.\epsilon_t^i \end{aligned} \tag{2.22}$$

The non-smoothed ratio would be: $\frac{\Delta C_t^i - \Delta C^{i,*}}{\Delta Y_t^i - \Delta Y^{w*}}$ where $\Delta C^{i,*}$ and ΔY^{w*} are, respectively, the deviation of consumption from the equilibrium for country i and the world output deviation from equilibrium if risk sharing was complete, which from the model in section 2.2.1 would be $\Delta C_t^* = \Delta Y_t^{w,*} = \epsilon_t^w$:

$$\begin{aligned} \frac{\Delta C_t^i - \Delta C_t^*}{\Delta Y_t^i - \Delta Y_t^w} &= \frac{\left[\left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w + s_{FA}^i.\epsilon_t^i\right] - \left[\epsilon_t^w\right]}{\left[\beta_t^i.\epsilon_t^w + \epsilon_t^i\right] - \left[\epsilon_t^w\right]} = \\ &= \frac{s_{FA}^i.(\beta_t^i - 1).\epsilon_t^w + s_{FA}^i.\epsilon_t^i}{(\beta_t^i - 1).\epsilon_t^w + \epsilon_t^i} = s_{FA}^i.\frac{(\beta_t^i - 1).\epsilon_t^w + \epsilon_t^i}{(\beta_t^i - 1).\epsilon_t^w + \epsilon_t^i} = s_{FA}^i \end{aligned} \tag{2.23}$$

We obtain the same result if we build the previous ratio due to the asymmetric shock:

$$\frac{[s_{FA}^i.\epsilon_t^i] - [0]}{[\epsilon_t^i] - [0]} = s_{FA}^i \tag{2.24}$$

or due to the asymmetric impact of the world shock:

$$\frac{\left[\left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w\right] - \left[\epsilon_t^w\right]}{\left[\beta_t^i.\epsilon_t^w\right] - \left[\epsilon_t^w\right]} = \frac{\left(s_{FA}^i.(\beta_t^i) + s_F^i\right).\epsilon_t^w - (s_{FA}^i + s_F^i).\epsilon_t^w}{\beta_t^i.\epsilon_t^w - \epsilon_t^w} = \frac{s_{FA}^i.(\beta_t^i - 1)}{(\beta_t^i - 1)} = s_{FA}^i \tag{2.25}$$

[13]As we are working with an endowment economy, in the steady state total consumption in country i is equal to its own total production.

[14]Note that, if the percentage of people with access to finantial markets is equal across countries, the consumption equation would be:

$$\Delta C_t^i = \epsilon_t^w + \epsilon_t^i.s_{FA}$$

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2.2.3 Incomplete financial markets

The smoothing performance of the consumption behaviour depicted in the previous sections was done through international diversification of portfolios (in the national accounts this smoothing would be measured by comparing GDP with GNI). Even if complete markets do not exist, and there was only trade in an internationally bond, in an endowment economy, the same result holds as it is showed by Crucini(1999). In this case this smoothing would be measured by comparing DNI with Consumption. The problem for a given agent at time t would be:

$$\max_{c_\tau^i, b_\tau^i} E_t \left[\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u(c_\tau^i) \right] \tag{2.26}$$

$$\text{s.t.:} \quad y_\tau^i + b_\tau^i \geq c_\tau^i + (1 + r_{\tau-1}).b_{\tau-1}^i$$

and the market clearing equations:

$$\begin{aligned} \sum_{i=1}^{n} P^i.c_\tau^i &= \sum_{i=1}^{n} P^w.y_\tau^i \\ \sum_{i=1}^{n} P^i.b_\tau^i &= 0 \end{aligned} \tag{2.27}$$

The first order conditions at each point t are :

$$\begin{cases} u'(c_t^i) = \lambda_t \\ \lambda_t = \delta.E_t \left[\lambda_{t+1}.(1 + r_t) \right] \end{cases} \tag{2.28}$$

... the Euler equation is ...

$$u'(c_t^i) = \delta.E_t \left[u'(c_{t+1}^i).(1 + r_t) \right]$$

Considering that the intertemporal discount factor (δ) and the interest rate (r) are equal for all agents independently of the country, then all of them will choose the same level of c_t^i. The first market clearing condition can be written as

$$c_t^{i,*} \sum_{i=1}^{n} P^i. = Y_t^w \Leftrightarrow c_t^{i,*} P^w = Y_t^w \Leftrightarrow c_t^{i,*} = \frac{Y_t^w}{P^w} \tag{2.29}$$

Calculating each country aggregate consumption :

$$C_t^{i,*} = P^i.\frac{Y_t^w}{P^w} = P^i.\frac{\overline{Y}^w + \overline{Y}^w \epsilon_t^w}{P^w} = \overline{Y}^i + \frac{P_i}{P_w}.\overline{Y}^w \epsilon_t^w$$

which is the same condition as the one depicted in equation (2.11).

So, if we restrict the financial market participation and redefine the world shock accordingly, as we did in section 2.2.2, we will obtain the same results as the ones of that subsection.

2.2.4 Impact of taste shocks

Finally assume that in the complete markets model we insert additive taste shocks φ_τ^i :

$$\begin{aligned} \max_{c^i} U(c^i) &= \max_{c^i_\tau} E_t \left(\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u\left(c^i_\tau \left(1 + \frac{\varphi^i_\tau}{c^i_\tau} \right) \right) \right) \\ s.t &: \sum_{i=1}^{n} P_i.y^i_\tau = \sum_{i=1}^{n} P_i.c^i_\tau \end{aligned} \tag{2.30}$$

The first order conditions at time t are:

$$\begin{cases} u'(c^i_t + \varphi^i_t) = \lambda_t \\ \sum_{i=1}^n P_i.y^i_t = \sum_{i=1}^n P_i.c^i_t \end{cases} \tag{2.31}$$

So c^i_t+ φ^i_t is equalized across countries (re-label as ct_t) , so $c^i_t = ct_t - \varphi^i_t$, so from the clearing market equation:

$$\begin{aligned} \sum_{i=1}^{n} P_i.y^i_t &= \sum_{i=1}^{n} P_i.\left(ct_t - \varphi^i_t\right) \Leftrightarrow \\ &\Leftrightarrow \sum_{i=1}^{n} P_i.y^i_t = ct_t \sum_{i=1}^{n} P_i - \sum_{i=1}^{n} P_i.\varphi^i_t = \\ &\Leftrightarrow Y^w = ct_t.P_w - \sum_{i=1}^{n} P_i.\varphi^i_t \Leftrightarrow ct_t = y^w + \frac{1}{P_w} \sum_{i=1}^{n} P_i.\varphi^i_t \end{aligned} \tag{2.32}$$

which implies:

$$\begin{aligned} C^i_t &= P_i.y^w + \frac{P_i}{P_w} \sum_{i=1}^{n} P_i.\varphi^i_t - P_i.\varphi^i_t = \\ &= P_i\left(\overline{y}^w + \overline{y}^w.\epsilon^w_t\right) + \frac{P_i}{P_w} \sum_{i=1}^{n} P_i.\varphi^i_t - P_i.\varphi^i_t = \\ &= \overline{Y}^i + \overline{Y}^i.\epsilon^w_t + P_i.\left(\overline{\varphi}^w_t - \varphi^i_t\right) \\ &\Leftrightarrow \Delta C^i_t = \epsilon^w_t + P_i.\frac{\left(\overline{\varphi}^w_t - \varphi^i_t\right)}{\overline{C}^i} \end{aligned} \tag{2.33}$$

where $\overline{\varphi}^i_t$ represents the worldwide average taste shock.

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If we consider that part of the population is living in financial autarky the problem for those with access to the financial markets may be written as:

$$\max_{c_t^{F,i}} U(c_t^{F,i}) = \max_{c_\tau^{F,i}} E_t \left(\sum_{\tau=t}^{+\infty} \delta^{\tau-t}.u\left(c_\tau^{F,i} \left(1+\frac{\varphi_\tau^i}{c_\tau^{F,i}} \right) \right) \right) \tag{2.34}$$
$$s.t : \sum_{i=1}^{n} P_i^F.y_\tau^i = \sum_{i=1}^{n} P_i^F.c_\tau^{F,i}$$

Solving the problem gives that:

$$C_t^{F,i} = \overline{Y}^i s_F + \overline{Y}^i.s_F.\epsilon_t^w + \frac{\overline{\varphi}^{w.F} - \varphi^{F,i}}{\overline{Y}^i}.P_i^F \tag{2.35}$$

where $\overline{\varphi}^{w,F}$ is the world weighted average of the taste shock of the people with access to financial markets and equal to $\frac{\sum_{i=1}^n P_i^F.\varphi^{F,i}}{\sum_{i=1}^n P_i^F}$.

The people that live in financial autarky will consume their income:

$$C_t^{FA,i} = \overline{Y}^i s_{FA} + \overline{Y}^i.\beta_i.\epsilon_t^w.s_{FA} + \overline{Y}^i.\epsilon_t^i.s_{FA} \tag{2.36}$$

Aggregating terms gives:

$$C_t^i = \overline{Y}^i + \overline{Y}^i.(s_F + \beta_i.s_{FA}).\epsilon_t^w + \overline{Y}^i.\epsilon_t^i.s_{FA} + \left(\overline{\varphi}^{w.F} - \varphi^{F,i} \right).P_i^F \tag{2.37}$$

consumption and output deviation are:

$$\begin{cases} \Delta Y_t^i = \beta_t^i.\epsilon_t^w + \epsilon_t^i \\ \Delta C_t^{,i} = (s_F + \beta_i.s_{FA}).\epsilon_t^w + \epsilon_t^i.s_{FA} + \frac{\overline{\varphi}^{w,F} - \varphi^{F,i}}{\overline{C}^i}.P_i \end{cases} \tag{2.38}$$

Note that if we compute the share of non-smoothed consumption, applying directly the formula of equation (2.23) we would get:

$$\frac{\Delta C_t^i - \Delta C_t^*}{\Delta Y_t^i - \Delta Y_t^w} = \frac{\left[(s_F + \beta_i.s_{FA}).\epsilon_t^w + \epsilon_t^i.s_{FA} + \frac{\overline{\varphi}^{w,F} - \varphi^{F,i}}{\overline{C}^i}.P_i \right] - \left[\epsilon_t^w + P_i.\frac{\left(\overline{\varphi}_t^w - \varphi_t^i \right)}{\overline{C}^i} \right]}{\left[\beta_t^i.\epsilon_t^w + .\epsilon_t^i \right] - \left[\epsilon_t^w \right]} =$$
$$= s_{FA+} \frac{\left[\frac{\overline{\varphi}^{w,F} - \overline{\varphi}_t^w}{\overline{C}^i}.P_i \right] + \left[\frac{\left(\varphi_t^{i,F} - \varphi_t^i \right)}{\overline{C}^i} \right]}{\left[\left(\beta_t^i - 1 \right).\epsilon_t^w + .\epsilon_t^i \right]}$$

Therefore the taste shocks might produce a bias if the taste shocks of people with access to the financial markets are different from those living in financial autarky ($\varphi_t^{i,F} \neq \varphi_t^i$) or if

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the world weighted average of the taste shock differs from the world weighted average of the taste shock of the people with access to financial markets:

$$\overline{\varphi}^{w,F} \neq \overline{\varphi}_t^w \Leftrightarrow \frac{\sum_{i=1}^n P_i^F.\varphi^{F,i}}{\sum_{i=1}^n P_i^F} \neq \frac{\sum_{i=1}^n P_i.\varphi_t^i}{P_W}$$

So to calculate the non-smoothed ratio of consumption we have to use the national asymmetric shock or the differentiated impact of the world shock as in equations (2.24) or (2.25).

The models presented in this section are simple but their main purpose is to show how can the level of international risk sharing be estimated. More complex models, with a production function and capital as in Baxter and Crucini (1995) or Kollman(1996), with trade costs as in Ravn and Mazenga(2004) or endogenous tradability as in Ghironi and Melitz(2004) do not yield a closed form solution. However from their simulations we can see that those extensions do not solve the consumption correlation puzzle.

2.3 The empirical study

In this section we will show how the consumption smoothing implied by the previous model can be estimated and present the results. The next subsection will present the link between the theoretical model and the empirical methodology. The next subsection will describe the data and the following presents the empirical results.

2.3.1 The methodology

The model in equation (2.38), has a natural empirical counterpart in a factor model:

$$\Delta x_{j,t}^i = \lambda_j^{i,w}.F_t^w + \lambda_j^i.F_t^i + \varepsilon_{j,t}^i \tag{2.39}$$

where F_t^w and F_t^i would be, respectively, the global factors that would affect all the series of all countries (the world shocks) and the country i factor affecting the series of each country (the asymmetric country shocks), $\lambda_j^{i,w}$ is the loading of the world factor on series j of country i, λ_j^i is the loading of the country i factor on series j and $\varepsilon_{i,t}^j$ would represent the idiosyncratic components of each series[15].

[15] In case of the consumption they can represent the taste shocks depicted in equation (2.38) that would alter the consumption level but are unrelated to shocks in total production.

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Connecting the model of equation (2.38) with the factor equation (2.39) we have for output:

$$\begin{aligned} \text{World component:} \quad & \beta_t^i.\epsilon_t^w = \lambda_y^{i,w}.F_t^w \\ \text{National Component:} \quad & \epsilon_t^i = \lambda_y^i.F_t^i \\ \text{Idiosyncratic component} \quad : \quad & [\varnothing] = \varepsilon_{y,t}^i \end{aligned} \tag{2.40}$$

and for consumption:

$$\begin{aligned} \text{World component:} \quad & \left(s_{FA}^i.\beta_t^i + s_F^i\right).\epsilon_t^w = \lambda_c^{i,w}.F_t^w \\ \text{National Component:} \quad & s_{FA}.\epsilon_t^i = \lambda_c^i.F_t^i \\ \text{Idiosyncratic component} \quad : \quad & \frac{\overline{\varphi}^{w,F} - \varphi^{F,i}}{\overline{C}^i}.P_i = \varepsilon_{c,t}^i \end{aligned} \tag{2.41}$$

in the end, the consumption smoothing parameter from the theoretical model is $1 - s_{FA}$. As we saw in the theoretical model this value can be obtained in several ways; however, from the estimation method we can recover it from the comparison of the national components:

$$b = 1 - s_{FA} = 1 - \frac{[s_{FA}^i.\epsilon_t^i]}{[\epsilon_t^i]} = 1 - \frac{\lambda_c^i F_t^i}{\lambda_y^i F_t^i} = 1 - \frac{\lambda_c^i}{\lambda_y^i} \tag{2.42}$$

To estimate the different channels of consumption smoothing we will use the same channels as described by Sorensen and Yosha (1998b). Therefore the smoothing from GDP to Gross National Income(GNI) will be considered as the one that is achieved due to international portfolio diversification or the asset markets channel. The smoothing from GNI to National Income (NI) measures the smoothing due to the variation of capital depreciation. The third layer is from NI to Disposable National Income (DNI), this will take into consideration international transfers from countries that are experiencing booms to ones that are experiencing recessions (this layer is more likely to exist among regions that have a joint budget like the states of US). Finally the last channel, due to the credit market/saving channel, is measured from DNI to Private Consumption (C).

Therefore the estimated factor model will be:

$$\begin{cases} \Delta GDP_t^i = \lambda_{GDP}^{i,w}.F_t^w + \lambda_{GDP}^i.F_t^i + \varepsilon_{GDP,t}^i \\ \Delta GNI_t^i = \lambda_{GNI}^{i,w}.F_t^w + \lambda_{GNI}^i.F_t^i + \varepsilon_{GNI,t}^i \\ \Delta NI_t^i = \lambda_{NI}^{i,w}.F_t^w + \lambda_{NI}^i.F_t^i + \varepsilon_{NI,t}^i \\ \Delta DNI_t^i = \lambda_{DNI}^{i,w}.F_t^w + \lambda_{DNI}^i.F_t^i + \varepsilon_{DNI,t}^i \\ \Delta C_t^i = \lambda_C^{i,w}.F_t^w + \lambda_C^i.F_t^i + \varepsilon_{C,t}^i \end{cases} \tag{2.43}$$

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where:

$$\begin{bmatrix} A(L)F_t^w = \epsilon_t^w \text{ and } \epsilon_t^w \sim N(0,\sigma^w) \\ A(L)F_t^i = \epsilon_t^i \text{ and } \epsilon_t^i \sim N(0,\sigma^j) \\ A(L)\varepsilon_{j,t}^i = u_{j,t}^i \text{ and } u_{j,t}^i \sim N(0,\sigma_j^i) \end{bmatrix} \text{and} \begin{bmatrix} E(\epsilon_t^w.\epsilon_t^i) = 0 \\ E(\epsilon_t^{i_1}.\epsilon_t^{i_2}) = 0 \\ E(\epsilon_t^w.u_{j,t}^i) = 0 \\ E(\epsilon_t^i.u_{j,t}^i) = 0 \\ E(u_{j_1,t}^{i_1}.u_{j_2,t}^{i_2}) = 0 \end{bmatrix} \text{for } j_1 \neq j_2 \text{ and } i_1 \neq i_2$$

with:

$$i = 1, 2, 3, ..., N \text{ (countries)}$$
$$t = 1, 2, 3, ..., T \text{ (time frame)}$$
$$j = GDP, GNI, NI, DNI, C$$

The model is patterned as a dynamic factor model as in Stock and Watson (1999), the dynamics enter through the fact that the factors are AR processes as the dependence of the series on the factor is static[16].

At this point we should note that the scale of the loadings and the factors cannot be estimated independently, therefore, we opted for normalizing the variance of the world and country factors to a constant.

As for the idiosyncratic components, one interpretation is that they are errors of measurement. However for that to be true, they would have to be independent across series. As GNI is derived from GDP , NI from GNI and DNI from NI, if we have an error measurement in one series it will contaminate all the series that are derived from it and mixed in the national component. Moreover the international transfers used to build the GNI, the NI and the DNI have to be consistent across countries. Errors of measurement in one country will also contaminate the other countries series. It is probable that most errors of measurement are mixed in the world and national components. The only way to be able to capture those errors would be to have data from two independent sources of the same aggregate which is not, in most cases, available. As for consumption they can be considered as a mix of errors of measurement and, as we saw before, taste shocks.

We should, also, note that if we estimate the model of equation (2.43) the national component will only be captured if it is not perfectly risk shared by portfolio diversification. If there is perfect risk sharing, the national shock to GDP will be captured in the idiosyncratic component and the model won't estimate any national component. Moreover, we can also

[16]More general dynamic factor models(GDFM) where the relationship between the factors and the series is dynamic can be found in Forni et al. (2002).

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assume that national shocks to GDP are composed by several parts where some are completely risk-shared and will be captured in the idiosyncratic component and the others in the national component.

Also, for the other aggregates (GNI, NI, DNI) there might exist some shocks specific to those series that are completely smoothed at the following level; therefore, those components will be captured in the idiosyncratic component.

In order to take into account these aspects the estimated consumption smoothing at each layer should not only take into account the estimated national components but also the estimated idiosyncratic ones. Therefore the formulas used should be:

via international portfolio diversification (layer GDP/GNI):

$$\widehat{b}_f = 1 - \frac{cov(\lambda^i_{GDP}.F^i_t + \varepsilon^i_{GDP,t}\ ,\ \lambda^i_{GNI}.F^i_t)}{var(\lambda^i_{GDP}.F^i_t + \varepsilon^i_{GDP,t})} = \tag{2.44}$$

as the idiosyncratic component and the factors are orthogonal, the covariance and the variance can be written as:

$$= 1 - \frac{cov(\lambda^i_{GDP}.F^i_t\ ,\ \lambda^i_{GNI}, F^i_t) + cov(\varepsilon^i_{GDP,t}\ ,\ \lambda^i_{GNI}.F^i_t)}{var(\lambda^i_{GDP}.F^i_t) + var(\varepsilon^i_{GDP,t})} = 1 - \frac{\lambda^i_{GDP}.\lambda^i_{GNI}.var(F^i_t)}{\left(\lambda^i_{GDP}\right)^2 var(.F^i_t) + var(\varepsilon^i_{GDP,t})}$$

via depreciation of capital (layer GNI/NI):

$$\widehat{b}_d = 1 - \frac{\lambda^i_{GNI}.\lambda^i_{NI}.var(F^i_t)}{\left(\lambda^i_{GNI}\right)^2 .var(F^i_t) + var(\varepsilon^i_{GNI,t})} \tag{2.45}$$

via international transfers (layer NI/DNI):

$$\widehat{b}_t = 1 - \frac{\lambda^i_{NI}.\lambda^i_{DNI}.var(F^i_t)}{\left(\lambda^i_{NI}\right)^2 .var(F^i_t) + var(\varepsilon^i_{NI,t})} \tag{2.46}$$

via savings (layer DNI/C):

$$\widehat{b}_s = 1 - \frac{\lambda^i_{DNI}.\lambda^i_{C}.var(F^i_t)}{\left(\lambda^i_{DNI}\right)^2 .var(F^i_t) + var(\varepsilon^i_{DNI,t})} \tag{2.47}$$

and total smoothing (layer GDP/C):

$$\widehat{b}_{total} = 1 - \frac{\lambda^i_{GDP}.\lambda^i_{C}.var(F^i_t)}{\left(\lambda^i_{GDP}\right)^2 .var(.F^i_t) + var(\varepsilon^i_{GDP,t})} \tag{2.48}$$

Note that if there is no idiosyncratic component on the GDP the above equation is:

$$\widehat{b}_{total} = 1 - \frac{\lambda^i_{GDP}.\lambda^i_{C}.var(F^i_t)}{\left(\lambda^i_{GDP}\right)^2 .var(F^i_t)} = 1 - \frac{\lambda^i_{GDP}.\lambda^i_{C}}{\left(\lambda^i_{GDP}\right)^2} = 1 - \frac{\lambda^i_{C}}{\lambda^i_{GDP}}$$

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which is exactly the same as equation (2.42).

To estimate this model and making inference we opted to use Otrok and Whiteman's(1998) approach[17]. This approach allow us to compute the ratio at each iteration recovering, in the end, the distribution of the smoothing parameter. However this approach has strong assumptions, as for instance the innovations not being cross-correlated and we cannot guarantee that the idiosyncratic components of consumption are not cross-correlated in the presence of taste shocks. From equation (2.41) the idiosyncratic component of the consumption deviation of country i is:

$$\left(\frac{\sum_{j=1}^{n}\left(P_j^F.\varphi^{F,j}\right)}{\sum_{j=1}^{n}P_j^F}-\varphi^{F,i}\right).\frac{P_i}{\overline{C}^i}.$$

Therefore if we assume that $\varphi^{F,j}$ are i.i.d. with variance equal to σ_φ^2 the variance of this term is:

$$\left(\frac{\sum_{j=1}^{n}\left(P_j^F\right)^2\sigma_\varphi^2}{\left(\sum_{j=1}^{n}P_j^F\right)^2}+\sigma_\varphi^2\right).\left(\frac{P_i}{\overline{C}^i}\right)^2$$

as the covariance of it between two countries is:

$$\frac{\sigma_\varphi^2}{\overline{c}^i.\overline{c}^j}\left[\frac{\sum_{s=1}^{n}\left(P_s\right)^2}{\left(P_w\right)^2}-\frac{P_i}{P_w}-\frac{P_j}{P_w}\right]$$

If we consider that all countries are equally populated we would have for the variance:

$$\left(\frac{n.\sigma_\varphi^2}{n^2}+\sigma_\varphi^2\right).\left(\frac{P_i}{\overline{C}^i}\right)^2=\left(\frac{\left(n+n^2\right).\sigma_\varphi^2}{n^2}\right).\left(\frac{P_i}{\overline{C}^i}\right)^2=\frac{\left(1+n\right).\sigma_\varphi^2}{n}.\left(\frac{P_i}{\overline{C}^i}\right)^2$$

and for the covariance:

$$-\frac{\sigma_\varphi^2}{\overline{c}^i.\overline{c}^j}.\frac{1}{n}$$

So as $n\rightarrow\infty$ the covariance tends to zero and the variance tends to $\sigma_\varphi^2.\left(\frac{P_i}{\overline{C}^i}\right)^2$, therefore the correlation tends to zero. So, at least, these idiosyncratic components are asymptotically not cross correlated.

Because Otrok and Whiteman's approach can be used to compute the smoothing ratios distributions while it is recovering the parameters distributions and at least asymptotically the idiosyncratic terms of the consumption deviations are not cross correlated we opted to use this one[18].

[17] An alternative approach would be the one described by Bai (2003). Bai's method is valid for a larger array of models; however, inference is only valid when $N,T\rightarrow+\infty$ (or in more stringent cases when $N,T\rightarrow+\infty$ when $\sqrt{N}/T\rightarrow 0$). However the estimation of the smoothing parameters is rather complex, as they are ratios involving several estimators, and with the Bai approach we can only have the asymptotic distribution of the parameters in the model and not for the smoothing ratios.

[18] See appendix 2.A for a short description of the method.

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2.3.2 The data used

To estimate the equations of the model implied by equation (2.43) we used annual data from 1970 to 2001 for a sample of 23 countries[19] of OECD. The data collected were those of GDP, GNI, NI, DNI and private consumption at current prices taken from the OECD Main Indicators 2003. Then we calculated the *per capita* values at constant prices using the population and consumption price index taken from the same source.[20].

Afterwards we de-trended the data in order to take the deviation cycles. We used a band pass filter as is described in Artis et al. (2003) and retain the fluctuations between 2 and 8 years. This interval follows Baxter and King (1999), as they used the interval between 6 and 32 months (in year terms 1.5 and 8).

We should note that the same study can be done with different cycle intervals. This might allow us to see how the consumption smoothing differs when we use different time horizons.

2.3.3 Results

From the method used we can get as a by-product a measure of the world business cycle and its importance relative to the national one in the different series. This analysis allows us to compare the estimated world business cycle with those from other studies and see if they coincide. We can, also, check if the relative importance of the national component versus the world one diminishes when we move from GDP to Consumption. This will give us a first idea of how much smoothing is done through the different channels. These points will be the subject of the first part. The second part will report the estimation of the smoothing parameters for the different countries in the sample, and a third subsection will relate the differences found to different economic and financial indicators.

[19]The countires used were: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany , Greece, Ireland, Italy, Japan, South Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States .
The data for Germany was corrected for the break in 1990 by the OECD Secretariat.

[20]It should be noted that we did not transform the data by accounting the purchasing power parity. We can think that some risk sharing can be done through relative price movements. This transformation and comparision with the present results are left for future work.

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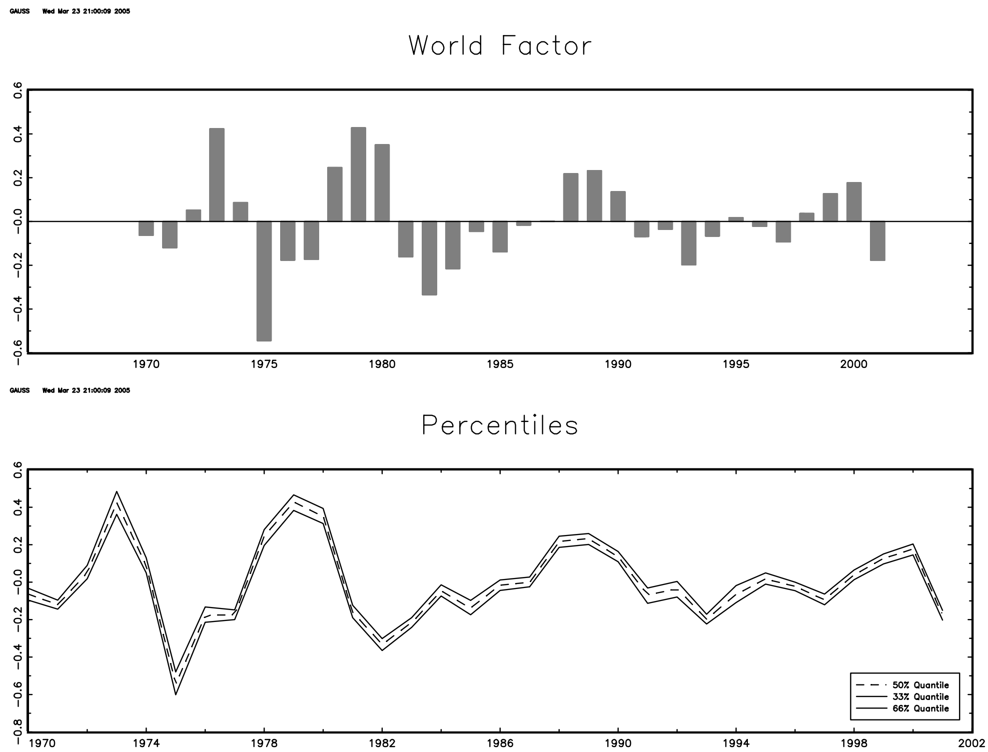
2.3.3.1 Business cycle symmetry

Figure 2.1 displays the median of the estimated world component[21], as well as the 33^{rd} and 66^{th} percentile[22] of the estimations.

Figure 2.1: World business cycle



From this figure we can see that the estimation shows troughs on 1975, 1982/85, 1991/1993 and a recession in 2001. It depicts peaks in 1973, 1979, 1989, 1995 and 2000. There is also a period covering all the eighties were the world economy was in a downturn cycle.

These estimates are similar to other estimates using different methods. Gregory et al. (1997) studying the G7 countries with quarterly data from 1970:1 to 1993:4, found recessions in 1975, 1982 and a slowdown in the early nineties, upturns in the early and late seventies and late eighties. They also found during the mid-eighties a long period where the cycle deviation was negative. Helbling and Bayoumi (2003) when estimating the G7 weighted gap

[21] As we said before, the size and signal of the loadings and of the factors are not estimable independently of each other. As for the size we fixed the variance of the factors as explained before. As for the signal we considered that the world component would have a positive loading for the US GDP, and the national components a positive loading for the respective country's GDP.

[22] This percentile choice follows Otrok and Whiteman(1998).

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using GDP quarterly data from 1973 to 2000 found troughs in 1974, 1982, 1986 and 1992 and peaks in 1978/1979, 1985, 1990 and 2000. Their sample end also depicts a slowdown, even if it is much smaller than the one illustrated in our figure. Cerqueira(2005) using national indexes constructed from GDP, GFCF and consumption from 1970 to 2000 found that the first estimated common component[23] peaks in 1973, 1980, 1990 and 2000 and has troughs in 1975, 1983/6, 1993 and also reflects the slowdown at the beginning of the new millennium. Kose et al.(2003) using the same method for GDP, investment and consumption from 1960 to 1990 found similar results. The biggest difference was the relative size of the recession after the first oil crisis in respect to the one after the second oil crisis. They found the crisis in the eighties was stronger than the one in the seventies, which was in contrast to most papers in the literature.

Table 2.1 shows the importance of the world factor for the different series.

If we compare the median value across series, we can see that when we move from GDP to GNI, and from NI to DNI the sensitivity of the series to the world component, on average, increases, indicating that those channels of international risk sharing (through portfolio diversification in the first case and international transfers in the second) seem to be working. However, when moving from DNI to Consumption, the majority of the countries experience a decrease in the sensitivity to the world factor (the exceptions are Canada, Ireland, Mexico and Norway but for the last two the dependence of the world component is very weak for any series). Even if this replicates the consumption correlation puzzle, the fact is that it does not mean that there is no smoothing through the savings channel, as that sensitivity reduction can be due to the existence of errors of measurement on the consumption or taste shocks (either would reduce the sensitivity of consumption but not of DNI to the world component).

On the other hand, if we check for which countries the GDP commoves more with the world cycle, those are Belgium, Germany, Ireland, France, UK and US (with more than 50%) Austria, Japan, Netherlands and Switzerland (between 50 and 40%) and Greece, Italy and Spain (between 30% and 40%).

The importance of the national component on the variance of each series can be seen in table 2.2. As we saw in the model, if perfect risk sharing existed, through portfolio diversification, the importance of the national component in GNI should be very small. In

[23] It should be recalled that Helbling and Bayoumi(2003) and Cerqueira(2005) found that the world business cycle was composed by two orthogonal factors. However, as the variance decompositions estimated by Cerqueira showed the second component was not as all as encompassing as the first, in fact it was only important for US, Germany and some german neighbours. The approach used in this paper allows for one common component, the second one, if exists, will be mingled in the national components. However, as the second component is not as global as the first we can think that the countries to which some of the cycle deviations are caused by it can smooth those deviations with the countries that are little or not affected by it.

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Table 2.1: Importance of the World factor in the series variance, 1970-2001

	GDP			GNI			NI			DNI			Consumption		
	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3
Aus	.094	.113	.137	.094	.112	.132	.086	.105	.126	.089	.108	.13	.003	.008	.019
Aut	.379	.436	.477	.379	.434	.475	.393	.449	.488	.416	.471	.511	.024	.033	.041
Bel	.559	.586	.619	.551	.578	.613	.633	.662	.693	.645	.675	.707	.29	.333	.378
Can	.171	.208	.254	.17	.207	.254	.185	.224	.273	.185	.224	.273	.261	.293	.342
Dnk	.154	.184	.207	.181	.214	.241	.204	.239	.268	.204	.239	.269	.034	.047	.061
Fin	.133	.154	.18	.14	.16	.186	.143	.163	.187	.141	.162	.185	.145	.159	.176
Fra	.602	.64	.672	.648	.688	.717	.661	.7	.731	.676	.717	.748	.301	.332	.356
Ger	.502	.567	.605	.527	.589	.627	.537	.599	.639	.687	.751	.785	.252	.296	.332
Gre	.298	.335	.359	.286	.324	.348	.27	.307	.33	.279	.315	.337	.241	.275	.297
Ire	.473	.509	.533	.479	.51	.535	.418	.447	.471	.469	.503	.526	.593	.614	.633
Ita	.361	.397	.433	.43	.466	.5	.506	.544	.58	.507	.543	.578	.188	.215	.249
Jap	.391	.433	.466	.383	.426	.459	.399	.446	.48	.4	.448	.481	.327	.364	.39
Kor	.029	.035	.041	.038	.044	.05	.028	.034	.039	.03	.036	.041	.006	.01	.015
Mex	.049	.063	.079	.058	.072	.089	.049	.061	.076	.052	.064	.079	.062	.077	.092
Nth	.466	.498	.523	.539	.572	.595	.525	.56	.581	.513	.545	.568	.431	.476	.51
Nzl	.109	.128	.152	.12	.139	.163	.123	.143	.167	.122	.142	.166	.098	.116	.14
Nor	.017	.023	.03	.008	.012	.017	.001	.003	.005	.001	.003	.006	.005	.009	.015
Por	.152	.18	.213	.174	.202	.235	.154	.183	.216	.239	.275	.316	.069	.087	.106
Spa	.333	.373	.416	.337	.377	.419	.379	.421	.463	.389	.43	.472	.165	.196	.237
Swe	.052	.068	.085	.079	.098	.118	.078	.097	.117	.076	.095	.115	.027	.033	.041
Swi	.378	.42	.46	.4	.437	.479	.418	.45	.487	.424	.455	.491	.374	.431	.478
UK	.494	.538	.57	.457	.502	.534	.448	.492	.523	.45	.495	.523	.46	.492	.519
US	.485	.528	.555	.511	.552	.577	.509	.549	.574	.514	.553	.578	.399	.443	.476

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Table 2.2: Importance of the National factor in the series variance, 1970-2001

	GDP			GNI			NI			DNI			Consumption		
	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3
Aus	.835	.859	.878	.853	.874	.892	.873	.894	.913	.869	.891	.910	.687	.702	.710
Aut	.510	.550	.607	.519	.559	.615	.507	.546	.602	.482	.521	.577	.415	.431	.449
Bel	.302	.333	.362	.338	.374	.401	.302	.333	.362	.286	.317	.348	.168	.218	.279
Can	.735	.781	.818	.743	.790	.828	.724	.774	.813	.725	.775	.814	.379	.423	.455
Dnk	.759	.782	.812	.755	.782	.815	.728	.757	.793	.721	.751	.786	.660	.676	.690
Fin	.806	.832	.853	.810	.836	.857	.812	.836	.856	.813	.837	.858	.661	.680	.694
Fra	.303	.335	.374	.277	.306	.345	.265	.295	.335	.246	.277	.318	.434	.465	.501
Ger	.379	.418	.483	.368	.406	.469	.356	.395	.458	.181	.214	.276	.384	.425	.472
Gre	.632	.656	.693	.650	.674	.712	.669	.692	.729	.657	.680	.715	.434	.456	.490
Ire	.425	.450	.485	.457	.481	.512	.519	.542	.572	.433	.457	.492	.086	.097	.114
Ita	.539	.576	.612	.490	.524	.560	.412	.449	.486	.412	.447	.483	.464	.503	.534
Jap	.520	.553	.598	.529	.562	.608	.514	.548	.594	.512	.546	.593	.383	.408	.446
Kor	.945	.951	.958	.945	.952	.958	.954	.960	.966	.954	.959	.965	.677	.686	.692
Mex	.880	.896	.910	.897	.914	.928	.922	.938	.950	.920	.935	.948	.790	.804	.819
Nth	.292	.315	.347	.395	.417	.450	.412	.434	.468	.424	.446	.478	.131	.153	.190
Nzl	.734	.758	.779	.821	.845	.864	.828	.852	.873	.828	.852	.872	.302	.323	.340
Nor	.941	.948	.953	.971	.976	.980	.993	.996	.998	.993	.996	.997	.153	.155	.157
Por	.774	.807	.836	.758	.791	.820	.776	.809	.838	.657	.697	.733	.431	.470	.493
Spa	.557	.600	.641	.571	.614	.655	.534	.576	.618	.524	.567	.608	.550	.595	.628
Swe	.877	.894	.909	.875	.895	.914	.882	.902	.921	.884	.904	.922	.380	.388	.396
Swi	.413	.453	.495	.502	.543	.580	.511	.548	.580	.506	.543	.574	.220	.259	.306
UK	.400	.431	.476	.461	.493	.538	.472	.503	.548	.466	.494	.539	.347	.375	.409
US	.417	.444	.487	.419	.444	.485	.423	.448	.488	.416	.442	.481	.388	.422	.469

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fact we can see that the values are close to the ones for the GDP, reflecting that here is hardly any consumption smoothing of the national shocks due to portfolio diversification. The second channel (the savings mechanism) reflects a different story. When comparing the importance of the national component for DNI and Consumption we can see that in most countries (the exceptions are France, Germany, Italy and Spain[24]) consumption is less affected by this component than is DNI. This reduction is, however, not equal among all the countries. There are substantial reductions in some, as for instance New Zealand, Norway and Ireland, whilst for others it is very small, as in the case of the US (where the 33rd to 66th percentile interval of the variance decomposition of the national component in DNI and Consumption overlap).

The question that arises from the two previous tables is how can the consumption be less dependent on the world cycle than GDP (giving rise to the consumption correlation puzzle), but at the same time be also less dependent from the national component (indicating that there is some risk-sharing and consumption smoothing across countries)? The answer is that, for consumption the idiosyncratic component of the series is much more important than for the other series. This may reflect errors of measurement or taste shocks. That is what we can observe in table 2.3.

We can see that for all series, except consumption, the idiosyncratic component is very small (except for the GDP of Netherlands, N. Zealand and Switzerland). For consumption the figure is quite different, ranging from 11.9% for Mexico to 83.5% for Norway.

These last tables allow us to analyze the existence or not of a consumption correlation puzzle. We have seen before that consumption is subject to more idiosyncratic shocks (that can come from taste shocks or measurement errors) than GDP. The question is to know what are the relative sizes if we compare only the national and world component. Figure 2.2 presents the relationship of the relative size of the national component to the sum of the common components (world plus national) for GDP and consumption.

If in fact consumption is less dependent on the national shocks than on the world shocks, then we should find the markers above the 45^0 degree line. On the contrary, if there was a clear consumption correlation puzzle we would find most of the markers bellow the 45^0 line. From the figure we can see that the number of markers bellow and above the 45^0 line is more or less equal and most are very close to that line. This indicates that once we account for the idiosyncratic components of the series the consumption correlation across countries is

[24] Only for Italy and Spain do the 33rd to 66th percentile interval of the importance of this component for either series overlap each other.

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Table 2.3: Importance of the idiosyncratic component in the series variance, 1970-2001

	GDP			GNI			NI			DNI			Consumption		
	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3	1/3	Med.	2/3
Aus	.027	.028	.029	.014	.014	.015	.000	.001	.001	.001	.001	.001	.282	.289	.295
Aut	.012	.013	.014	.004	.005	.006	.004	.004	.005	.006	.007	.008	.520	.534	.548
Bel	.076	.081	.084	.045	.048	.051	.002	.003	.005	.004	.005	.007	.403	.437	.453
Can	.010	.011	.012	.002	.003	.003	.001	.002	.002	.001	.001	.002	.279	.285	.290
Dnk	.032	.034	.036	.003	.004	.005	.003	.003	.004	.009	.010	.011	.268	.276	.285
Fin	.013	.014	.015	.003	.003	.004	.001	.001	.001	.001	.001	.002	.160	.162	.163
Fra	.023	.024	.026	.004	.005	.006	.003	.003	.005	.004	.005	.006	.199	.208	.220
Ger	.014	.015	.016	.002	.003	.004	.003	.004	.005	.032	.035	.039	.277	.283	.289
Gre	.008	.009	.009	.001	.001	.002	.001	.001	.002	.005	.005	.006	.267	.268	.270
Ire	.039	.041	.043	.006	.008	.010	.009	.010	.012	.038	.041	.043	.274	.285	.295
Ita	.024	.028	.032	.008	.010	.012	.005	.006	.008	.007	.009	.012	.272	.281	.291
Jap	.009	.017	.019	.008	.014	.016	.002	.004	.009	.002	.004	.009	.220	.230	.235
Kor	.012	.013	.015	.003	.004	.005	.005	.006	.007	.004	.005	.006	.293	.302	.313
Mex	.041	.042	.042	.013	.014	.014	.001	.001	.001	.001	.001	.001	.118	.119	.120
Nth	.180	.184	.188	.009	.010	.011	.005	.006	.007	.007	.008	.009	.357	.368	.379
Nzl	.110	.114	.117	.014	.016	.018	.003	.004	.005	.004	.005	.006	.549	.557	.564
Nor	.028	.029	.031	.011	.012	.013	.001	.001	.001	.001	.001	.001	.830	.835	.839
Por	.012	.013	.014	.005	.006	.007	.007	.008	.009	.025	.028	.030	.403	.440	.489
Spa	.025	.027	.028	.008	.009	.010	.001	.002	.003	.002	.002	.003	.206	.211	.218
Swe	.038	.038	.039	.007	.008	.008	.001	.001	.001	.001	.001	.001	.577	.579	.580
Swi	.123	.125	.127	.018	.018	.019	.001	.001	.002	.001	.002	.003	.297	.308	.317
UK	.029	.030	.031	.003	.004	.005	.003	.004	.005	.009	.010	.011	.128	.131	.134
US	.026	.027	.028	.002	.003	.004	.001	.002	.003	.003	.004	.005	.131	.134	.136

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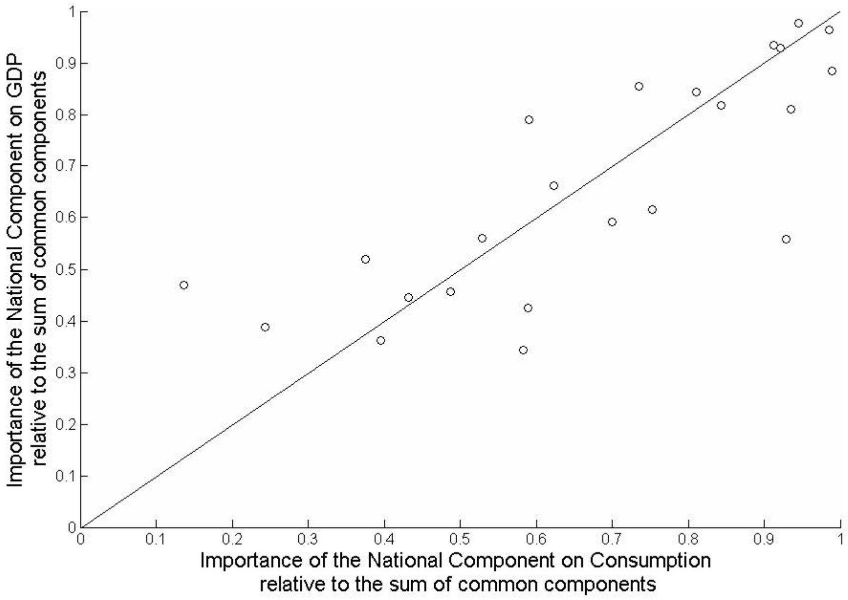
Figure 2.2: Relative size of the national component to the sum of world and national component



similar to that for GDP.

2.3.3.2 Consumption smoothing channels

From the previous section, it seems that countries engage in partial consumption smoothing through the savings channel, while the smoothing by portfolio diversification is very small or even non-existent. To look into more detail at the importance of each channel, table 2.4 show the estimated values[25] for the different smoothing channels using the formulas in equations (2.44) to (2.48).

As we can see from table 2.4, the amount of consumption smoothing achieved by international portfolio diversification ($\widehat{\beta}_f$) is small for most countries. The countries for which this channel is more important are Belgium (17.4%), France (8.8%), Italy (5.4%) and US (5.2%). However for a number of countries, this channel, has an unsmoothing effect: Netherlands (-16.4%), Sweden (-10.4%) , Portugal (-8.8%), Denmark (-6.0%), Finland (-5.5%) and Norway (-4.9%). From these lists, it seems that the countries that smooth more through this channel are big countries (with the exception of Belgium) or countries where the GDP

[25] In Appendix 2.B we can see the 5^{th}, 10^{th}, 20^{th}, 25^{th}, 33^{th}, 50^{th} (the median), 66^{th}, 75^{th}, 80^{th}, 90^{th} and 95^{th} percentiles.

Table 2.4: Importance of Consumption Smoothing channels, 1970-2001

	β_f			β_d			β_t			β_s			β_{total}		
	.33	Med.	.66	.33	Med.	.66	.33	Med.	.66	.33	Med.	.66	.33	Med.	.66
Aus	-.010	.006	.023	-.059	-.049	-.040	-.005	-.002	.001	.397	.424	.450	.357	.387	.417
Aut	-.012	.000	.012	-.055	-.044	-.033	.023	.033	.045	.170	.228	.284	.145	.205	.262
Bel	.133	.174	.213	.022	.054	.085	-.033	-.019	-.006	.309	.406	.507	.375	.462	.550
Can	-.008	.001	.010	-.085	-.080	-.074	-.003	.001	.005	.527	.558	.589	.487	.521	.555
Dnk	-.077	-.060	-.043	-.148	-.139	-.130	-.001	.009	.019	-.017	.025	.068	-.237	-.185	-.131
Fin	-.069	-.055	-.040	-.149	-.141	-.134	-.011	-.007	-.003	.524	.540	.555	.420	.439	.459
Fra	.064	.088	.111	-.096	-.082	-.067	.001	.016	.031	.241	.295	.353	.263	.307	.351
Ger	.004	.025	.046	-.098	-.085	-.071	.218	.268	.308	.159	.228	.297	.241	.288	.336
Gre	.012	.021	.031	-.042	-.038	-.034	-.015	-.008	-.001	.542	.566	.590	.529	.553	.578
Ire	.011	.033	.054	-.062	-.047	-.030	-.022	.000	.021	.341	.396	.452	.297	.357	.416
Ita	.035	.054	.072	-.115	-.097	-.079	-.013	.000	.012	.124	.177	.231	.055	.110	.166
Jap	-.029	-.011	.014	-.149	-.108	-.053	.001	.006	.012	.269	.303	.336	.169	.211	.250
Kor	-.056	-.045	-.034	-.073	-.064	-.055	.014	.022	.030	.104	.145	.186	.011	.056	.101
Mex	-.016	.000	.016	-.117	-.107	-.097	.010	.013	.016	.164	.191	.217	.072	.103	.134
Nth	-.237	-.164	-.094	-.138	-.123	-.109	-.019	-.005	.009	.488	.544	.597	.320	.386	.453
Nzl	-.042	-.017	.008	-.091	-.078	-.065	-.014	-.006	.003	.584	.613	.641	.529	.562	.595
Nor	-.073	-.049	-.027	-.197	-.185	-.174	-.031	-.027	-.023	.793	.829	.865	.731	.778	.825
Por	-.101	-.087	-.073	-.027	-.015	-.003	.106	.127	.146	.580	.635	.688	.565	.621	.676
Spa	-.041	-.021	-.003	-.081	-.067	-.054	-.007	-.002	.003	.200	.252	.304	.109	.165	.224
Swe	-.123	-.104	-.084	-.113	-.104	-.095	-.006	-.002	.001	.409	.444	.479	.270	.314	.359
Swi	-.060	-.028	.007	-.022	-.006	.010	.016	.020	.024	.551	.600	.652	.536	.586	.642
UK	-.054	-.033	-.012	-.100	-.090	-.080	-.006	.007	.020	.192	.230	.267	.072	.118	.163
US	.037	.052	.066	-.134	-.127	-.120	-.001	.006	.013	.273	.327	.396	.221	.279	.353

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dependence on the world cycle is higher, but it does not include all of them (in that group Italy is the one for which the GDP is less related with the world component, 39.7%). The countries that unsmooth through this channel are small and/or with a lower dependence from the world component (the exception is Netherlands which is also the biggest one with 15,92 million inhabitants in 2000).

As for the second channel (variation of capital depreciation - $\widehat{\beta}_d$), it has an unsmoothing effect on all countries with the exception of Belgium (being Switzerland the smallest with -0.6% and Norway the biggest with -18.5%). This is in line with previous studies (see referenced papers in footnote 9).

International transfers are not important for most countries. The fact that international transfers aren't important for most countries is not surprising as this channel would depict the transfers made across states through international aid to catastrophes, automatic stabilizers through common budgets if they existed and private sector remittances[26]. In this sample, the only common budget arrangement is the one that encompasses the EU countries, but it is not built to provide automatic stabilization, therefore, even for most of the EU countries the effects are small. In the countries studied the exceptions are Germany with 26.8% (the biggest net contributor to the EU budget and an immigrant receiver) and Portugal with 12.7%. (one of the cohesion countries and a country that is at the same time an immigrant receiver - from the former African colonies - and an emigrant provider - mostly to France, Germany, USA and South America).

The bulk of the smoothing is done through the final channel, savings and credit markets ($\widehat{\beta}_s$)[27]. But the importance of it is not the same in every country. For some it smooths more than 50% of the cycle deviations - Norway (82.9%), Portugal (63.5%), New Zealand (61.3%), Switzerland (60%), Greece (56.6%) , Canada (55.8%), Netherlands (54.4%), and Finland (54%). For others it account for less than 20% - Denmark (2.5%), Italy (17.7%) South Korea(14.5%) and Mexico (19.1%).

Finally, comparing total smoothing from GDP to private consumption, the groups of countries that smooth more and less mimics the groups formed from the importance of the credit market channel as this one was the more important and heterogeneous[28].

[26] Account code D75 of the 1993 SNA, see http://unstats.un.org/unsd/sna1993/toctop2.asp.
Previous studies about this issue have disregarded the existence of these private remmitances at this level. A more accurate look to these transfers might be a issue of future research.

[27] Moreover if we add to this, the fact that the importance of the national component for the consumption and GDP in respect to the sum of the common components (national plus world) is similar - see figure (2.2) - we can suspect that most of the smoothing is due to intertemporal smoothing and not by using the international credit markets.

[28] The total smoothing parameter does not have to be equal to the sum of the other parametres as at each level the idiosyncratic component of the series is considered in the smoothing process.

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2.3.3.3 Possible relations with economic and financial indicators

At this point we can raise the question as to why the values are different across countries[29]. One obvious justification would be the importance of the world component in the countries GDP cycle. We could think that the more a country commoves with the world cycle the less it has to gain by entering into smoothing mechanisms. Figures 2.3 and 2.4 show the relation between the importance of the world component for the GDP cycles and the smoothing parameters through asset markets ($\widehat{\beta}_f$) and credit markets ($\widehat{\beta}_s$).

From the first graph we can see that there is a positive relationship between the importance of the World cycle on the national GDPs and the total smoothing through the capital markets. The R^2 of the linear regression is 0.2318 and the parameters are significant at 5% level.

The second figure gives the impression that there is an inverse relationship, but the R^2 is only 0.0603 and the estimated parameters are insignificant at 5% level.

So, the idea that countries that commove less with the world cycle would smooth more is not confirmed, and if any relationship exists is through the asset markets but positive (the ones that commove more also smooth more through this channel).

A second idea that we could take from the previous tables, would be that there could be a relation between country size and consumption smoothing as most of the countries that have higher smoothing values through the savings/credit markets are small ones.

Figures 2.5 and 2.6 depict the relationship between population size and the consumption smoothing channels.

Although the regression lines seem to depict some relationships the R^2 are 0,079 and 0,12 and the parameters are insignificant at 5% level. However if we take out the US, the first relationship (asset markets channel vs population) continues to be unimportant, but the second (savings/credit market channel vs population) has a R^2 of 0,25 and the parameter is significantly negative at 5% level.

Another relationship that we can consider is the degree of openness[30]. We could think that countries that have a higher degree of openness are more integrated and would have easier access to the capital markets and/or international credit markets[31].

[29] The analysis presented are simple correlations between the median values founded and several indicators. They should be read as indicators of the differences found and directions that deserve further research.

[30] The openess indicator was calculated by averaging the ratio of (exports+imports)/GDP from 1970 to 2000.

[31] Below we relate the values found with indices measuring the financial integration. However, the best

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Figure 2.3: Relation between the importance of the World component on the GDP cycle and the consumption smoothing through the asset markets

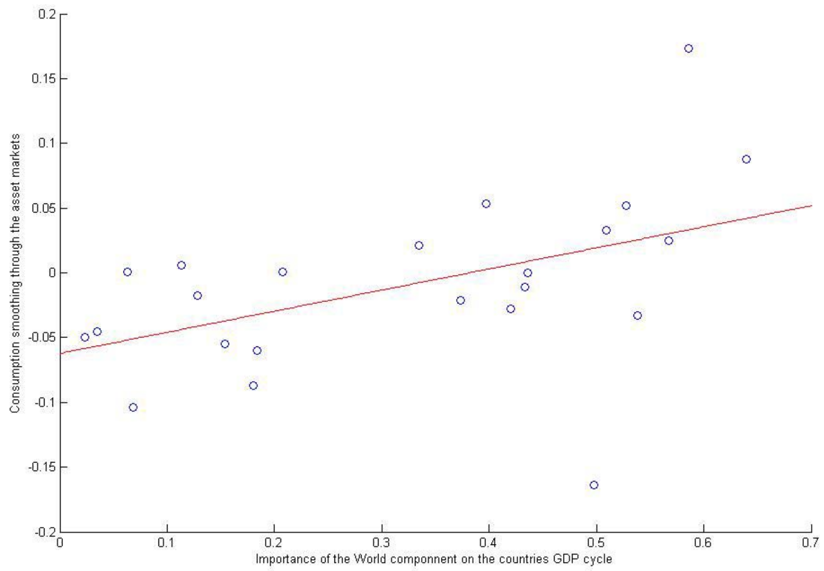

Figure 2.4: Relation between the importance of the World component on the GDP cycle and the consumption smoothing through the savings and credit markets

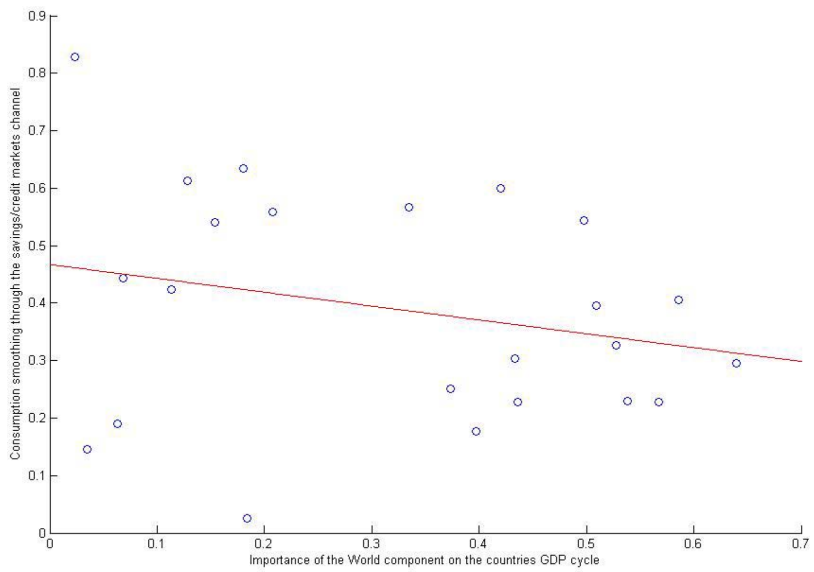

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Figure 2.5: Relation between the population size and consumption smoothing through the asset markets

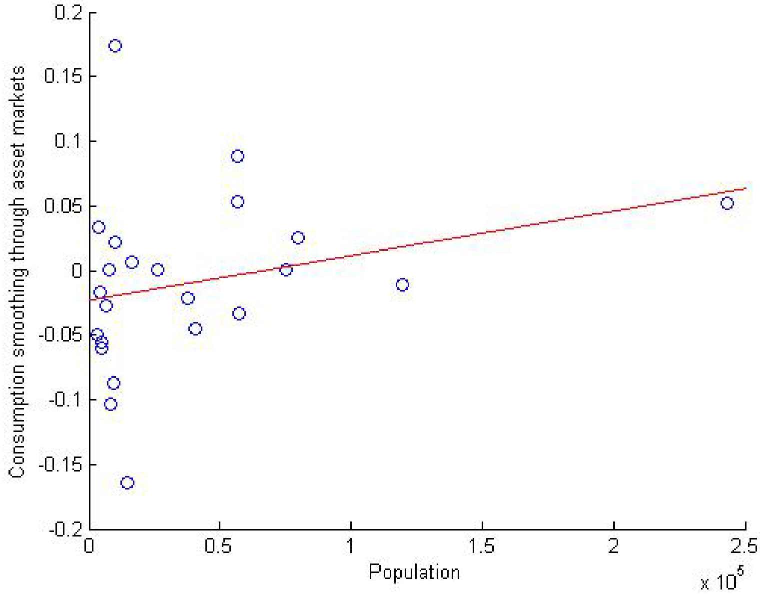
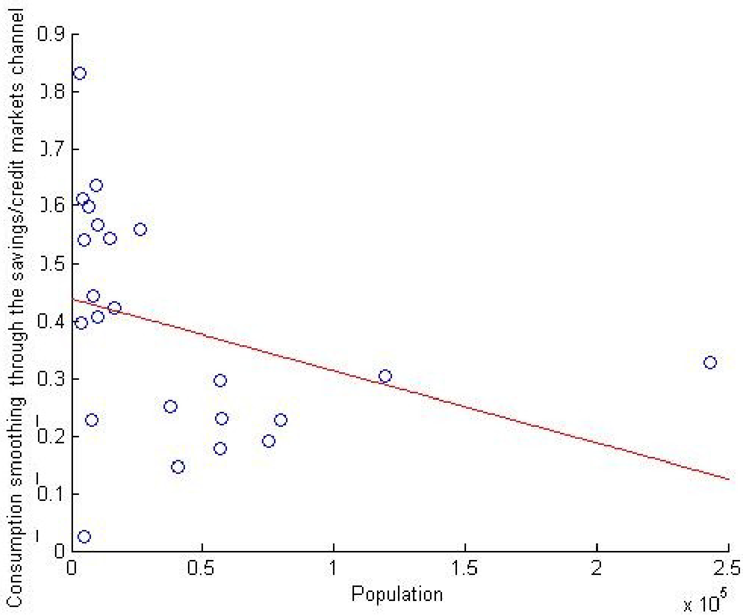


Figure 2.6: Relation between the population size and consumption smoothing through the savings and credit markets



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Figures 2.7 and 2.8 depict the relationship between openness and the consumption smoothing channels.

From the first graph we can see that there is no relationship between the openness level and the smoothing through the capital markets. The R^2 of the linear regression is 0.0011 and the regression parameter is zero. However, if we do the same regression without the three countries that have a higher level of openness (Belgium: 1.11 , Ireland: 0.99 and Netherlands: 0.88), there is a negative relationship between openness and the smoothing channel. The R^2 is 0.2848 and the parameter is negative at 5% significance level.

The second figure gives the impression that there is a positive relationship, but the R^2 is only 0.0612 and the estimated parameters are insignificant at 5% level. If we do the same regression without the same countries as before, the R^2 increases to 0.092 but the estimated parameters continue to be insignificant.

Finally, and because the two main channels of consumption smoothing discussed in papers are through portfolio diversification and savings, we tried to relate the values found with some indicators of the financial structure of the countries studied. The indicators used were taken from the database on Financial Development and Structure revised in October 2003, for a description of this database see Beck and al.(1999), the dataset on Bank Concentration & Competition and the data collected from Lane and Millesi-Ferretti (2006)[32]. However, from the whole batch of indicators few seemed to be significant.

As concerning the portfolio diversification channel only the 'Public bond market capitalization to GDP' and the 'Average Portfolio Equity Net / GDP' indicators had significant correlations. The first with R^2 of 0.204 and the second with a R^2 of 0.1934, however both results were overmost due to the values of Belgium and if we perform both regressions without Belgium the relation is insignificant at 5% (see figures 2.9 and 2.10 - dotted line represents the regression without Belgium).

For the savings/credit market channel seems that the only significant indicators are : Net interest margin ($R^2 = 0.2259$) and Concentration ($R^2 = 0.2124$) (see figures 2.11 and 2.12[33]).

From the first graph it seems that the more efficient are the commercial banks into

known to us is the Coordinated Portfolio Investment Survey (CPIS) conducted by the IMF but it is only available on an annual basis since 2001 onwards and therefore does not cover the period studied. Instead we used several indicators of financial openness and others as "Foreign ownership", "Fraction of foreign entry applications denied", "Economic freedom" that can be seen as proxies of international financial integration.

[32] The indicator list used from each database and the weblink are transcribed in appendix 2.C.

[33] The indicators of entry denials showed to be significant with a negative correlation. However most of the countries had a zero value for this index and the correlation coeficents were basically due to two or three countries.

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Figure 2.7: Relation between openness and consumption smoothing through the asset markets

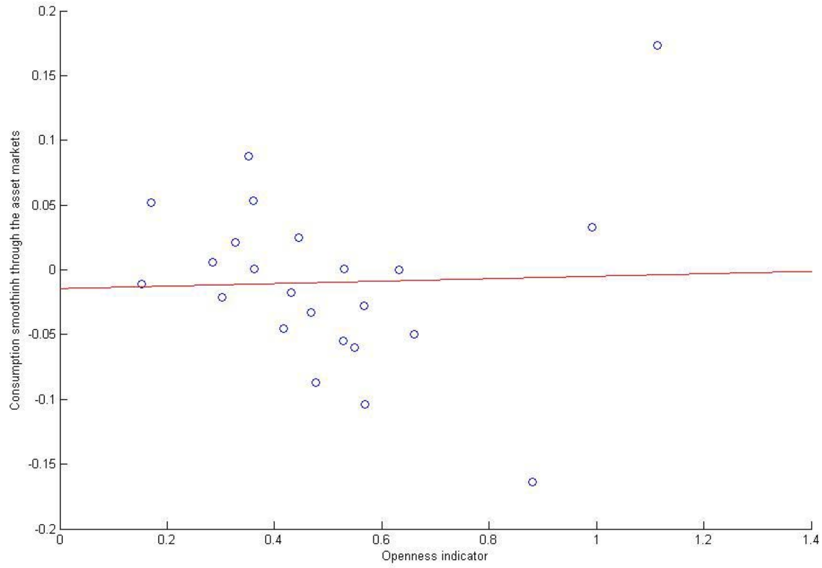

Figure 2.8: Relation between openness and the consumption smoothing through the savings and credit markets

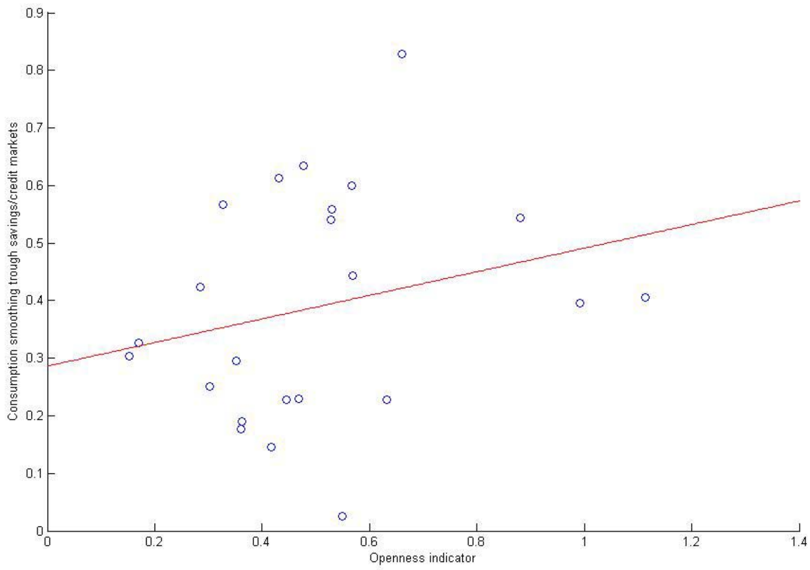

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Figure 2.9: Relation between Public bond market capitalization to GDP and consumption smoothing through the asset markets

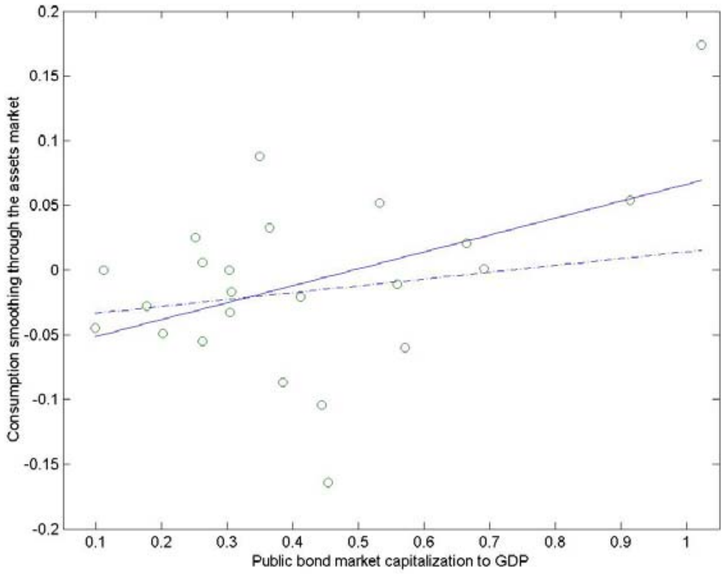

Figure 2.10: Relation between Average Portfolio Equity Net / GDP and consumption smoothing through the asset markets

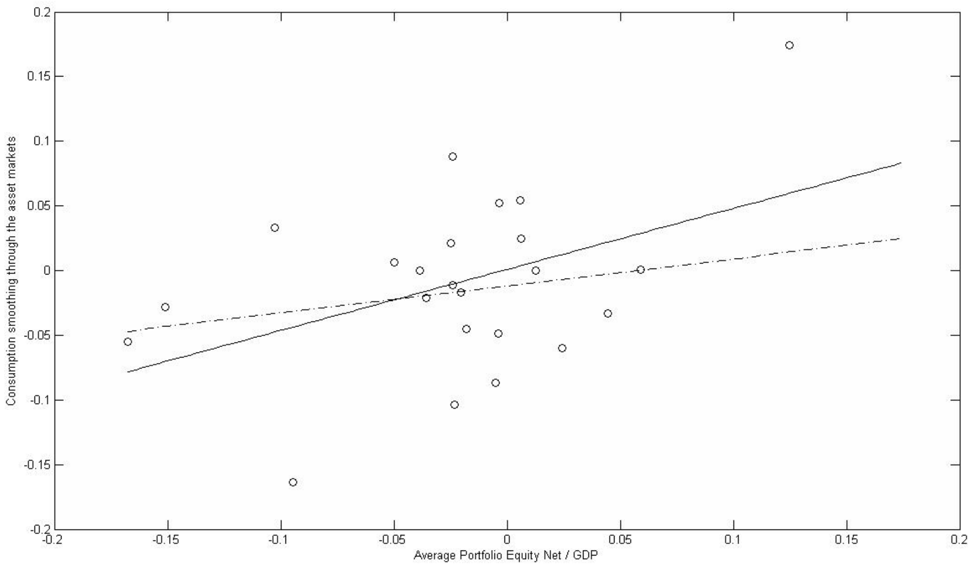

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Figure 2.11: Relation between Net Interest Margin and consumption smoothing through the savings/credit market

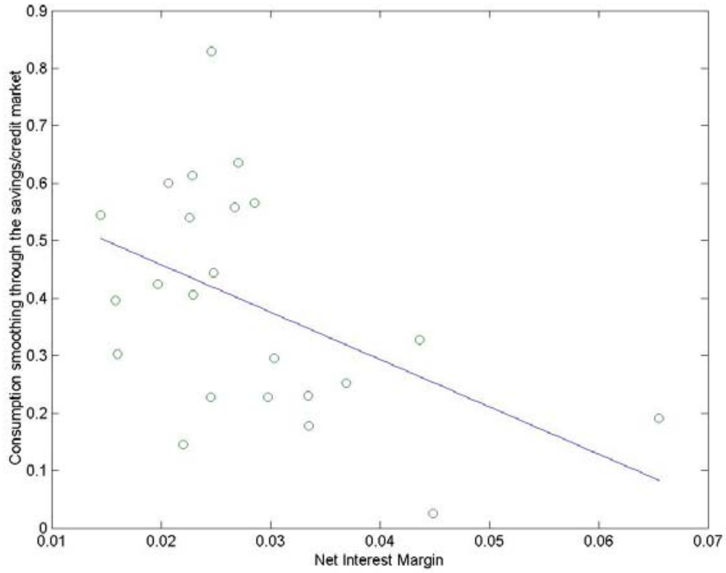
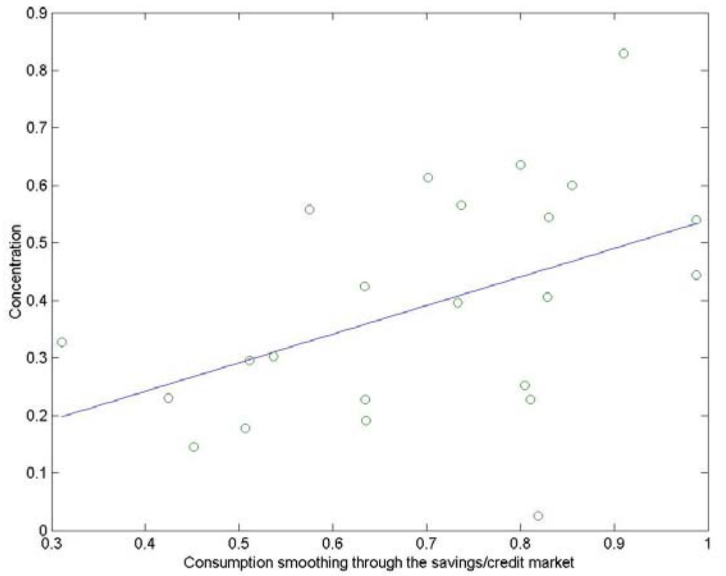


Figure 2.12: Relation between Concentration and consumption smoothing through the savings/credit market



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channeling funds from savers to investors the more able are agents to use the savings/credit market to smooth the consumption through the business cycle.

On the other side the relationship between concentration and consumption smoothing is positive. The concentration indicator might have two interpretations: highly concentrated commercial banking sector might result in lack of competitive pressure to attract savings and channel them efficiently to investors or a highly fragmented market might be evidence for undercapitalized banks. From the relationship depicted by the graph it seems that a higher level of concentration actually improves the ability of agents to smooth consumption, therefore it appears that fears of a highly concentrated market leading to a reduction in efficiency may be misplaced.

Finally, we should refer that the indexes that could be related to financial integration[34] do not show any correlation with any of the consumption smoothing channels. Even if Imbs(2005) found that regions across countries do not have a consumption correlation puzzle which, in the author opinion, would be due to the fact that those regions would be more financial integrated than the countries among themselves, in a subsequent study, the same author - Imbs(2006) - shows that more financial integrated countries have, simultaneously, a higher degree of consumption correlation and business cycle synchronization and therefore do not show any signs of increased consumption smoothing, which is in line with the results obtained

2.4 Conclusion

In conclusion, we can say that the results found are in the aggregate not different from those existing in the literature. Asset markets are marginally important (if we aggregate the data the world value is zero), capital depreciation contributes to unsmooth consumption (-8% at world level), international transfers are not important (0% for the world) and the bulk of consumption is done through the savings/credit market mechanism (at the world level this accounts for 39%).

However these aggregations hide some differences among countries. Smoothing through the asset markets channel ranges from -17.4% to 16.4%, as the international transfers (with the exception of Germany and Portugal) are zero for most countries. The savings channel ranges from 2.5% to 82.9%.

When we tried to see to which indicators we could relate to those differences, we found

[34]Such as "Fraction of foreign entry applications denied", "Activity restrictions" ; "Banking freedom", "State ownership", "Foreign ownership" or "Economic freedom", or financial openness as the ones that measure the weight of assets and liabilities to GDP, being it in equities, FDI, financial derivatives, debt or total assets.

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that the asset markets channel is positively related with the importance of the world component in the country cycles and negatively (when we take out Belgium, Netherlands and Ireland) related to the degree of openness. Both results can be considered unexpected. First, we should think that the less important is the world component in the national cycle, the more the country has to gain from engaging in risk sharing through portfolio diversification, but we found the inverse relationship. As for the second result we might think that the more open is a country, the more integrated it is and therefore it would have easier access to international asset markets, however this hypothesis is not only not confirmed but, if anything, we found the inverse relationship.

When we did the same analysis for the saving channel we only found a negative relationship with the population size (when we do not consider US) indicating that smaller countries smooth more through this channel. This can be seen as expected, as smaller countries have a smaller impact on international credit markets and can lend/borrow to/from more agents.

On the other hand, when we relate the results to financial indicators we did not find any indicator that would be related with to the asset market channel. This might indicate that the reasons for the diference through this channel are insensitive to financial market structure or that the indicators used do not capture the relevant diferences to explain the heterogenous values found for this channel. As for the saving mechanism, we found that the indicator of efficiency (net interest margin) and of market structure (concentration) were related to this channel. More efficient banks (lower net interest margin) and a more concentrated market structure would lead to a more efficient consumption smoothing through the savings/credit channel.

However the previous relations should be read as indications of what is causing the heterogeneity found across countries on the smoothing mechanisms and which directions deserve further research.

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Appendix 2.A Empirical methodology

The estimation of model depicted by equation (2.43) is accomplished through a MonteCarlo Markov Chain applied to dynamic factor models, as is described by Otrok and Whiteman(1998) and Kose et al. (2003).

If we consider a specification of a Gaussian probability density for the data $\{y_t\}$ conditional on a set of latent parameters $\{\varphi\}$ and a set of latent variables $\{f_t\}$, call this density function $g_y(Y|\varphi, F)$. In addition there is a Gaussian probability density function $g_f(F)$ for the F itself. Given a prior distribution for φ, $h(\varphi)$, the joint posterior distribution for the parameters and the latent variables (the factors) is: $h(\varphi, F|Y) = g_y(Y|\varphi, F) * g_f(F) * h(\varphi)$.

Otrok and Whiteman(ibid.) showed that, although $h(\varphi, F|Y)$ is hard to derive, under a conjugate prior for φ we can get $h(\varphi|F, Y)$ and $h(F|\varphi, Y)$.

So starting with a guess for F^0 (in the support of the posterior distribution) we generate a random drawing for φ^1 from $h(\varphi|F^0, Y)$. Then get a random drawing F^1 from $h(\varphi|F^1, Y)$

The sample produced is a realization of a Markov chain whose invariant distribution is the joint posterior $h(\varphi, F|Y)$.

In this paper we have an intermediate factor (the national factor) therefore the estimation was done in the following steps:

1. Get a guess for world factor (F_W^0) and country factors (F_C^0) in the support of their distributions. The priors used were $N(0, \sigma_i^2)$. For the world factor the variance was fixed to the average of the variances of all series, for the country factors it was fixed to the average of the variances of the countries series. Remember that the size of the loadings and the factors cannot be estimated independently By fixing the variance of the factors we also fix the size of the loadings.

2. Draw a random drawing φ^1 from $h(\varphi|F_W^0, F_C^0, Y)$

3. Draw a random drawing F_W^1 from $h(F_W|\varphi^1, F_C^0, Y)$

4. Draw a random drawing F_C^1 from $h(F_C|\varphi^1, F_W^1, Y)$

5. Go to step 2.

This iteration was done 100000 times, and then we eliminate the first 30000 iterations. The distributions were taken from the last 70000 iterations. We tried with smaller chain lengths, but from 10000 (eliminating the first 5000) onward the results were similar.

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As in Kose et al.(ibid.) the prior used on the loadings was a $N(0,1)$ .The length of the idiosyncratic and factor autoregressive polynomials was 3 (I tried with larger polynomial orders with final similar results). The prior on the autoregressive polynomials parameters was $N(0,\Sigma)$ with $\Sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$ .As in Otrok and Whiteman(ibidem) the prior on the innovation variances in the observable equation is an Inverted Gamma(6,0.001).

For more details see the above cited papers.

Appendix 2.B Smoothing parameters distributions estimated percentiles

In the next pages we can see the percentiles of the smoothing parameters distributions. In those tables the symmetric interval around the median where the parameter keeps the signal is in bold.

Table B1: bf - Asset market channel

	Percentile										
	1/20	1/10	1/5	1/4	1/3	1/2	2/3	3/4	4/5	9/10	19/20
Aus	-0.062	-0.045	-0.027	-0.020	-0.010	**0.006**	0.023	0.032	0.039	0.056	0.070
Aut	-0.049	-0.037	-0.024	-0.019	-0.012	**0.000**	0.012	0.019	0.024	0.038	0.049
Bel	**0.007**	**0.047**	**0.092**	**0.109**	**0.133**	**0.174**	**0.213**	**0.235**	**0.250**	**0.288**	**0.319**
Can	-0.040	-0.029	-0.018	-0.014	-0.008	**0.001**	0.010	0.016	0.020	0.031	0.041
Dnk	-0.123	**-0.109**	**-0.093**	**-0.087**	**-0.077**	**-0.060**	**-0.043**	**-0.032**	**-0.025**	**-0.005**	0.012
Fin	-0.111	**-0.098**	**-0.083**	**-0.078**	**-0.069**	**-0.055**	**-0.040**	**-0.032**	**-0.027**	**-0.012**	0.001
Fra	**0.003**	**0.020**	**0.042**	**0.051**	**0.064**	**0.088**	**0.111**	**0.123**	**0.132**	**0.154**	**0.172**
Ger	-0.047	-0.032	-0.014	-0.007	**0.004**	**0.025**	**0.046**	0.059	0.068	0.091	0.110
Gre	-0.015	-0.007	**0.003**	**0.006**	**0.012**	**0.021**	**0.031**	**0.037**	**0.040**	0.050	0.059
Ire	-0.055	-0.035	-0.010	-0.002	**0.011**	**0.033**	**0.054**	0.066	0.074	0.096	0.114
Ita	-0.020	-0.002	**0.017**	**0.025**	**0.035**	**0.054**	**0.072**	**0.082**	**0.090**	0.109	0.126
Jap	-0.141	-0.096	-0.050	-0.040	-0.029	**-0.011**	0.014	0.031	0.043	0.073	0.101
Kor	**-0.086**	**-0.077**	**-0.066**	**-0.062**	**-0.056**	**-0.045**	**-0.034**	**-0.029**	**-0.025**	**-0.013**	**-0.004**
Mex	-0.063	-0.049	-0.031	-0.025	-0.016	**0.000**	0.016	0.025	0.031	0.047	0.060
Nth	-0.470	-0.397	**-0.312**	**-0.281**	**-0.237**	**-0.164**	**-0.094**	**-0.058**	**-0.033**	0.030	0.080
Nzl	-0.110	-0.090	-0.066	-0.056	-0.042	**-0.017**	0.008	0.023	0.033	0.060	0.082
Nor	-0.143	-0.123	**-0.097**	**-0.087**	**-0.073**	**-0.049**	**-0.027**	**-0.014**	**-0.006**	0.015	0.031
Por	**-0.144**	**-0.131**	**-0.115**	**-0.110**	**-0.101**	**-0.087**	**-0.073**	**-0.065**	**-0.059**	**-0.044**	**-0.031**
Spa	-0.104	-0.083	-0.060	-0.052	**-0.041**	**-0.021**	**-0.003**	0.008	0.015	0.036	0.055
Swe	**-0.178**	**-0.161**	**-0.142**	**-0.134**	**-0.123**	**-0.104**	**-0.084**	**-0.073**	**-0.065**	**-0.044**	**-0.027**
Swi	-0.144	-0.120	-0.089	-0.077	-0.060	**-0.028**	0.007	0.028	0.044	0.091	0.141
UK	-0.115	-0.096	-0.074	-0.066	**-0.054**	**-0.033**	**-0.012**	0.000	0.008	0.031	0.051
US	**0.008**	**0.006**	**0.022**	**0.028**	**0.037**	**0.052**	**0.066**	**0.075**	**0.080**	**0.094**	**0.105**

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Table B2: bd – Capital depreciation

	Percentile										
	1/20	1/10	1/5	1/4	1/3	1/2	2/3	3/4	4/5	9/10	19/20
Aus	**-0.086**	**-0.077**	**-0.068**	**-0.064**	**-0.059**	**-0.049**	**-0.040**	**-0.035**	**-0.031**	**-0.021**	**-0.012**
Aut	**-0.092**	**-0.079**	**-0.066**	**-0.061**	**-0.055**	**-0.044**	**-0.033**	**-0.028**	**-0.023**	**-0.013**	**-0.004**
Bel	-0.134	-0.071	-0.015	**0.001**	**0.022**	**0.054**	**0.085**	**0.102**	0.115	0.147	0.175
Can	**-0.104**	**-0.098**	**-0.091**	**-0.089**	**-0.085**	**-0.080**	**-0.074**	**-0.070**	**-0.068**	**-0.061**	**-0.053**
Dnk	**-0.174**	**-0.166**	**-0.157**	**-0.153**	**-0.148**	**-0.139**	**-0.130**	**-0.125**	**-0.122**	**-0.112**	**-0.104**
Fin	**-0.173**	**-0.166**	**-0.157**	**-0.154**	**-0.149**	**-0.141**	**-0.134**	**-0.129**	**-0.126**	**-0.119**	**-0.113**
Fra	**-0.134**	**-0.122**	**-0.109**	**-0.104**	**-0.096**	**-0.082**	**-0.067**	**-0.058**	**-0.052**	**-0.035**	**-0.022**
Ger	**-0.135**	**-0.124**	**-0.110**	**-0.105**	**-0.098**	**-0.085**	**-0.071**	**-0.064**	**-0.058**	**-0.043**	**-0.031**
Gre	**-0.052**	**-0.049**	**-0.045**	**-0.044**	**-0.042**	**-0.038**	**-0.034**	**-0.032**	**-0.031**	**-0.026**	**-0.023**
Ire	-0.107	-0.093	**-0.077**	**-0.071**	**-0.062**	**-0.047**	**-0.030**	**-0.021**	**-0.015**	0.002	0.017
Ita	**-0.175**	**-0.156**	**-0.134**	**-0.126**	**-0.115**	**-0.097**	**-0.079**	**-0.070**	**-0.063**	**-0.045**	**-0.030**
Jap	-0.203	-0.189	-0.170	**-0.162**	**-0.149**	**-0.108**	**-0.053**	**-0.033**	**-0.022**	0.002	0.019
Kor	**-0.099**	**-0.091**	**-0.082**	**-0.078**	**-0.073**	**-0.064**	**-0.055**	**-0.050**	**-0.047**	**-0.038**	**-0.030**
Mex	**-0.146**	**-0.137**	**-0.126**	**-0.123**	**-0.117**	**-0.107**	**-0.097**	**-0.092**	**-0.088**	**-0.078**	**-0.069**
Nth	**-0.182**	**-0.168**	**-0.152**	**-0.146**	**-0.138**	**-0.123**	**-0.109**	**-0.100**	**-0.094**	**-0.078**	**-0.065**
Nzl	**-0.131**	**-0.118**	**-0.104**	**-0.098**	**-0.091**	**-0.078**	**-0.065**	**-0.058**	**-0.053**	**-0.040**	**-0.030**
Nor	**-0.239**	**-0.225**	**-0.210**	**-0.205**	**-0.197**	**-0.185**	**-0.174**	**-0.168**	**-0.163**	**-0.152**	**-0.143**
Por	-0.062	-0.051	-0.039	-0.034	**-0.027**	**-0.015**	**-0.003**	0.004	0.008	0.022	0.033
Spa	**-0.137**	**-0.117**	**-0.097**	**-0.090**	**-0.081**	**-0.067**	**-0.054**	**-0.047**	**-0.043**	**-0.030**	**-0.020**
Swe	**-0.139**	**-0.131**	**-0.121**	**-0.118**	**-0.113**	**-0.104**	**-0.095**	**-0.090**	**-0.086**	**-0.077**	**-0.069**
Swi	-0.074	-0.057	-0.038	-0.031	-0.022	**-0.006**	0.010	0.019	0.025	0.041	0.054
UK	**-0.129**	**-0.120**	**-0.109**	**-0.105**	**-0.100**	**-0.090**	**-0.080**	**-0.075**	**-0.071**	**-0.061**	**-0.052**
US	**-0.156**	**-0.149**	**-0.141**	**-0.138**	**-0.134**	**-0.127**	**-0.120**	**-0.116**	**-0.113**	**-0.106**	**-0.100**

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Table B3: bt – International transfers

	Percentile										
	1/20	1/10	1/5	1/4	1/3	1/2	2/3	3/4	4/5	9/10	19/20
Aus	-0.013	-0.010	-0.007	-0.006	-0.005	**-0.002**	0.001	0.002	0.003	0.007	0.010
Aut	-0.010	0.000	**0.012**	**0.017**	**0.023**	**0.033**	**0.045**	**0.051**	**0.056**	0.068	0.079
Bel	-0.070	-0.059	-0.046	-0.040	**-0.033**	**-0.019**	**-0.006**	0.002	0.008	0.024	0.039
Can	-0.016	-0.012	-0.007	-0.006	-0.003	**0.001**	0.005	0.007	0.009	0.015	0.020
Dnk	-0.028	-0.020	-0.010	-0.006	-0.001	**0.009**	0.019	0.026	0.030	0.042	0.052
Fin	-0.023	-0.019	**-0.015**	**-0.013**	**-0.011**	**-0.007**	**-0.003**	**-0.001**	0.001	0.005	0.008
Fra	-0.037	-0.026	-0.012	-0.007	**0.001**	**0.016**	**0.031**	0.039	0.045	0.060	0.072
Ger	**0.053**	**0.098**	**0.160**	**0.184**	**0.218**	**0.268**	**0.308**	**0.329**	**0.343**	**0.380**	**0.411**
Gre	-0.034	-0.028	-0.021	-0.018	**-0.015**	**-0.008**	**-0.001**	0.003	0.005	0.012	0.018
Ire	-0.083	-0.064	-0.042	-0.034	-0.022	**0.000**	0.021	0.033	0.041	0.062	0.081
Ita	-0.052	-0.039	-0.025	-0.020	-0.013	**0.000**	0.012	0.020	0.025	0.041	0.055
Jap	-0.058	-0.026	-0.007	-0.003	**0.001**	**0.006**	**0.012**	0.017	0.022	0.049	0.075
Kor	-0.010	-0.002	**0.006**	**0.009**	**0.014**	**0.022**	**0.030**	**0.035**	**0.038**	0.047	0.055
Mex	**0.001**	**0.004**	**0.007**	**0.008**	**0.010**	**0.013**	**0.016**	**0.018**	**0.019**	**0.022**	**0.025**
Nth	-0.060	-0.047	-0.032	-0.027	-0.019	**-0.005**	0.009	0.017	0.023	0.039	0.052
Nzl	-0.039	-0.031	-0.022	-0.019	-0.014	**-0.006**	0.003	0.008	0.012	0.022	0.032
Nor	**-0.043**	**-0.039**	**-0.035**	**-0.034**	**-0.031**	**-0.027**	**-0.023**	**-0.021**	**-0.019**	**-0.014**	**-0.010**
Por	**0.047**	**0.065**	**0.087**	**0.095**	**0.106**	**0.127**	**0.146**	**0.158**	**0.166**	**0.187**	**0.206**
Spa	-0.024	-0.019	-0.013	-0.011	-0.007	**-0.002**	0.003	0.006	0.008	0.014	0.020
Swe	-0.016	-0.013	-0.009	-0.008	-0.006	**-0.002**	0.001	0.003	0.004	0.008	0.011
Swi	**0.002**	**0.007**	**0.012**	**0.013**	**0.016**	**0.020**	**0.024**	**0.026**	**0.028**	**0.032**	**0.036**
UK	-0.047	-0.034	-0.019	-0.014	-0.006	**0.007**	0.020	0.026	0.031	0.044	0.055
US	-0.021	-0.015	-0.007	-0.005	-0.001	**0.006**	0.013	0.017	0.020	0.027	0.034

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Table B4: bs– Savings/Credit markets channel

	Percentile										
	1/20	1/10	1/5	1/4	1/3	1/2	2/3	3/4	4/5	9/10	19/20
Aus	**0.322**	**0.345**	**0.372**	**0.383**	**0.397**	**0.424**	**0.450**	**0.466**	**0.477**	**0.506**	**0.531**
Aut	-0.018	**0.043**	**0.112**	**0.135**	**0.170**	**0.228**	**0.284**	**0.315**	**0.337**	**0.392**	0.437
Bel	**0.039**	**0.121**	**0.219**	**0.256**	**0.309**	**0.406**	**0.507**	**0.565**	**0.605**	**0.716**	**0.812**
Can	**0.434**	**0.463**	**0.497**	**0.509**	**0.527**	**0.558**	**0.589**	**0.607**	**0.620**	**0.654**	**0.687**
Dnk	-0.133	-0.098	-0.056	-0.040	-0.017	**0.025**	0.068	0.094	0.112	0.161	0.204
Fin	**0.478**	**0.492**	**0.509**	**0.515**	**0.524**	**0.540**	**0.555**	**0.564**	**0.570**	**0.586**	**0.601**
Fra	**0.104**	**0.144**	**0.193**	**0.212**	**0.241**	**0.295**	**0.353**	**0.388**	**0.411**	**0.468**	**0.508**
Ger	-0.037	**0.023**	**0.094**	**0.120**	**0.159**	**0.228**	**0.297**	**0.334**	**0.358**	**0.417**	0.463
Gre	**0.470**	**0.493**	**0.519**	**0.528**	**0.542**	**0.566**	**0.590**	**0.603**	**0.612**	**0.636**	**0.656**
Ire	**0.178**	**0.228**	**0.287**	**0.309**	**0.341**	**0.396**	**0.452**	**0.483**	**0.504**	**0.561**	**0.612**
Ita	-0.029	**0.017**	**0.072**	**0.093**	**0.124**	**0.177**	**0.231**	**0.262**	**0.284**	**0.346**	0.399
Jap	**0.164**	**0.196**	**0.234**	**0.249**	**0.269**	**0.303**	**0.336**	**0.355**	**0.367**	**0.400**	**0.426**
Kor	-0.013	**0.022**	**0.065**	**0.081**	**0.104**	**0.145**	**0.186**	**0.210**	**0.226**	**0.268**	0.303
Mex	**0.089**	**0.113**	**0.140**	**0.150**	**0.164**	**0.191**	**0.217**	**0.232**	**0.242**	**0.270**	**0.294**
Nth	**0.318**	**0.371**	**0.432**	**0.455**	**0.488**	**0.544**	**0.597**	**0.627**	**0.647**	**0.697**	**0.740**
Nzl	**0.505**	**0.529**	**0.557**	**0.568**	**0.584**	**0.613**	**0.641**	**0.657**	**0.668**	**0.699**	**0.726**
Nor	**0.690**	**0.721**	**0.758**	**0.773**	**0.793**	**0.829**	**0.865**	**0.886**	**0.901**	**0.939**	**0.971**
Por	**0.397**	**0.457**	**0.523**	**0.547**	**0.580**	**0.635**	**0.688**	**0.718**	**0.738**	**0.790**	**0.834**
Spa	**0.061**	**0.102**	**0.152**	**0.172**	**0.200**	**0.252**	**0.304**	**0.334**	**0.355**	**0.412**	**0.462**
Swe	**0.310**	**0.339**	**0.375**	**0.389**	**0.409**	**0.444**	**0.479**	**0.499**	**0.513**	**0.551**	**0.583**
Swi	**0.437**	**0.468**	**0.509**	**0.526**	**0.551**	**0.600**	**0.652**	**0.684**	**0.706**	**0.765**	**0.813**
UK	**0.076**	**0.113**	**0.155**	**0.170**	**0.192**	**0.230**	**0.267**	**0.288**	**0.302**	**0.341**	**0.374**
US	**0.164**	**0.193**	**0.231**	**0.248**	**0.273**	**0.327**	**0.396**	**0.440**	**0.471**	**0.546**	**0.599**

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Appendix 2.C List of financial indicators used

From the database available on:

http://web.worldbank.org/WBSITE/EXTERNAL/EXTDEC/EXTRESEARCH/...

...EXTPROGRAMS/EXTFINRES/...

0,,contentMDK:20352338~menuPK:806638~pagePK:64168182~piPK:64168060~theSitePK:478060,00.html

Central Bank Assets to total financial assets
Deposit Money Bank Assets to total financial assets
Other Financial Institutions Assets to total financial assets
Deposit money bank vs. central bank assets
Liquid liabilities to GDP
Central Bank Assets to GDP
Deposit Money Bank Assets to GDP
Other Financial Institutions Assets to GDP
Private credit by deposit money banks to GDP
Private credit by deposit money banks and other financial institutions to GDP
Bank deposits
Financial system deposits
Concentration
Overhead Costs
Net Interest Margin
Life insurance penetration
Non-life insurance penetration
Stock market capitalization to GDP
Stock market total value traded to GDP
Stockmarket turnover ratio
Private bond market capitalization to GDP
Public bond market capitalization to GDP

From the database available on:

http://www.worldbank.org/research/interest/confs/042003/data.htm

Fraction of entry applications denied
Fraction of domestic entry applications denied
Fraction of foreign entry applications denied
Activity restrictions
Banking freedom

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State ownership

Foreign ownership

Economic freedom

KKZ index (A composite of six governance indicators (1998 data): voice and accountability, political stability, government effectiveness, regulatory quality, rule of law. And corruption. Higher values correspond to better governance)

Private credit

Total value traded

From Lane and Milesi-Ferretti(2006) database available on:
http://www.tcd.ie/iis/pages/planedata.php

Average Portfolio equity ssets + liabilities / GDP

Average Portfolio equity net / GDP

Average FDI assets + liabilities / GDP

Average FDI net / GDP

Average Debt equity assets + liabilities / GDP

Average Debt equity net / GDP

Average derivatives assets + liabilities / GDP

Average derivatives net / GDP

Average Total assets liabilities / GDP

Average Net external position

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Part III

Theoretical Chapter

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CHAPTER 3

INTERNATIONAL REAL BUSINESS CYCLES AND R&D

3.1 Introduction

Since the seminal paper of Backus et al. (1992) that extended the closed economy real business cycle (RBC) framework[1] to open economies this kind of model has been the workhorse for analyzing business cycle fluctuations and commovements across countries. With time, these models have been extended to cover a number of alternatives, away from the original one. Among many, one of the assumptions of the original model that has been relaxed was the typology of consumption goods produced. In the first models there was only one homogeneous good produced by all countries. This original assumption was a simplification, some of the implications of which were well understood, as for instance the impossibility of the existence of exports and imports by the same country at the same time. It is, therefore, not surprising that people working in international trade theory were among the first to consider seriously the existence of different varieties as it allows for intrasectoral trade among countries.

The works of Krugman(1979) and Helpman (1981) using Dixit-Stiglitz (1977) preferences and Lancaster(1980) with the concept of "ideal variety" introduced the idea to trade theory that the production of each country is composed of a set of different varieties. These models, contrary to previous work in the field, allowed for intrasectoral trade across countries and furthermore provided a framework to explain why fast growing countries do not have a declining trend in their terms of trade(TOT) as previous models predicted. Krugman(1988) argues that: "*Fast growing countries expand their share of world markets, not by reducing the relative prices of their goods, but by expanding the range of goods that they produce as their economies growth. What we measure are not ... fixed set of goods but instead aggregates whose definitions change over time*". In this way, fast growing countries expand the set of goods that they export, the relative prices of the individual goods do not change and therefore the aggregate index of the terms of trade (TOT) is kept constant.

In the international business cycle literature we face a similar puzzle. The moments taken from the data show that for some countries the TOT are weakly correlated with output or that the correlation is even positive. This fact is difficult to replicate in theoretical models

[1] This framework was itself introduced in the literature by Kydland and Prescott (1982).

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(where this correlation is, usually, negative) and furthermore the relative volatility of the TOT to output is in data higher than in the theoretical models. This puzzle, which is usually called the price anomaly, is in fact similar to the absence of declining TOT for fast growing countries and therefore we might assume that a similar mechanism (that the fluctuations are not only over the quantities but also over the varieties produced) might be working at the business cycle frequency.

A second puzzle that fluctuations in produced varieties might solve is the consumption correlation puzzle, i.e., the puzzle that measured cross country correlations of output are usually higher than those of consumption which is difficult to replicate in theoretical models. These fluctuations might solve this puzzle due to two mechanisms: first, if the fluctuations in varieties shuts down international risk sharing through relative prices and might, even, work in the opposite direction then it reduces the cross country correlation of consumption. Second, measured consumption is done over the quantity of goods consumed, but utility is driven from a mixture of quantity and diversity. Therefore, if produced varieties change we might observe a low cross country correlation of consumption because different countries might smooth utility by varying the quantity-diversity mix in different ways.

When we look to the business cycle literature we realize that the idea that each country produces a set of symmetric varieties has, already, a long history. The first models to incorporate this idea assumed that goods from different countries were perceived as being of different varieties (although each country produces a homogenous good). Examples of this framework are Backus,Kehoe and Kydland (1994), Stockman and Tesar(1995) and Zimmerman (1997). Afterwards, the models have extended the previous one by considering that each country produces a set of different varieties, the support for them being static; a recent example is the model of Alessandria and Choi(2004)[2]. More recently these kinds of model have endogeneized the number of varieties, such as in the model of Head(2002) with different varieties of intermediate tradable inputs, where the number of firms is related to a fixed cost or in the models of Ghironi and Melitz(2005)[3] and Corsetti et al. (2005)[4] which have introduced a non-static set of varieties in the model by indexing the entry cost to wages and a specific productivity factor of innovation.

Even if these models have fluctuations of varieties over time, the simulations done fail to

[2] Even if in this case they introduce a endogenous tradability mechanism, making that the number of goods available at a certain point in time in each market varies.

[3] Extending the framework of Melitz(2003) where besides the entry of varieties mechanism he also considers heterogeneous firms and a export cost making the tradability of each good endogenous to the model.

[4] This paper model is used to illustrate the sign of the reaction of the variables to changes in the factor productivity of goods or in the entry mechanism but do not presents a dynamic simulation to study the moments.

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replicate the moments from the data, and therefore the mechanism explained above is not strong enough to solve consistently both puzzles. We think that this failure is due to the fact that the simulations studied in most papers only have shocks in the productivity of the goods sector or in the case of Ghironi and Melitz(ibid.), which have a specific productivity component for the entry mechanism, they only consider simulations where the entry factor productivity is linked to the productivity in the goods sector, and therefore the shocks affect simultaneously both sectors. Therefore, in those simulations, the only source of fluctuations in the number of firms/varieties in a given period is due to the shock in the productivity of the goods sector and not an independent shock in the innovation/entry mechanism.

We argue that if we consider shocks in the total factor productivity of the final goods sector there is no reason not to consider shocks in the innovation process not perfectly correlated with the previous ones. In fact, when we look at variables that proxy the number of new varieties introduced in the market each year ("registered trademarks"), the observed volatility in data is higher than the one that we obtain in the models when we consider, only, shocks in the productivity of the goods sector. Therefore, we will consider that the number of varieties introduced in the market at each point is at same time a function of the shocks hitting the production sector and of "creativity" shocks that increase/decrease the productivity of those who are working in the R&D sector responsible for the creation of new varieties.

With this in mind, this paper will introduce into an otherwise standard flexible price, two countries IRBC model with a flexible labour market a R&D sector[5] responsible for the introduction of new symmetric varieties, as is done in the endogenous growth literature in the case of horizontal specialization[6]. However, the exit strategy is going to be modelled as a fixed probability that the variety ceases to exist. The introduction of an explicit R&D sector will allow us to check how the behavior of the economies differs when the shocks are in the consumption goods sector, in the R&D sector, or when they hit the economy simultaneously in both sectors. Although the introduction of the R&D sector will, in the end, be equivalent to the formulation of the entry cost used in previous models[7], the formulation used splits off the creation process from the production process which might be useful for future refinements of the model such as the inclusion of foreign direct investment or outsourcing.

[5] In the models that index the entry cost to wages, like Corsetti et al. (ibid.) or Ghironi and Melitz(ibid.), the indexing is a proxy for an unmodelled R&D sector.

[6] See Romer(1987 and 1990). We could also think that if we considered the R&D sector with vertical innovations as purposed by Segerstorm et al (1990) we would get similar results.

[7] Therefore the model studied in this paper can be considered an extension of Corsetti et al. (ibidem) model, where we look to the shocks dynamics at business cycle frequency, as well as, the sensability to different labour elasticities and home-bias, which are not adressed in that paper.

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From the results we will see that the impact of shocks in the two sectors have the opposite effects on most variables, but alone they do not replicate the consumption correlation puzzle or the price anomaly. However, when we allow for the existence of both kinds of shocks simultaneously we verify that, in a number of cases, the model is able to solve the two anomalies described above and increases the volatility of new varieties introduced in the market at each period, matching the moments found in the data.

The chapter is structured along four sections. The first will describe the model and derive the FOCs. The following will give the results for the non–stochastic steady state. The next checks the dynamics of the model and the final one concludes and sets up the path for future research.

3.2 The model

We will consider that each country has two sectors and is populated by n agents. The first sector will produce consumption goods; however, these goods are not homogeneous, but will be of different varieties set in a continuum . The second will produce new varieties, expanding therefore the consumption goods support. The home country at time t produces the varieties indexed from 0 to $v_{H,t}$ and the foreign country will produce those in the interval 0 to $v_{F,t}$. At each time the agents will decide how much to consume of each variety and how to allocate their time between leisure and labour in either sector. We can see the economic environment in figure 3.1.

All contracts in the economy are expressed in nominal terms and prices are flexible. Due to exogenous shocks the consumption basket changes over time, affecting the price level and the terms of trade. So we have to consider the existence of money, but other than as an unit of account money does not play any role so we'll resort to a cashless economy (as in Woodford(2003)) and will express a worldwide demand for money as in the quantity theory of money: $P.Y = M^S.k$, just to anchor prices. Otherwise, as we see later, even if we obtain a determinate solution for relative prices, the nominal ones would be undetermined.

3.2.1 The Household´s problem

The representative household will want to maximize its utility function[8] at the present moment (set to zero):

$$E_0 \sum_{t=0}^{+\infty} \beta^t.U(C_{H,t}; H_{H,t}) \tag{3.1}$$

[8]The problem will be stated for the home (H) country. Exactly analagous conditions will apply to the foreign (F) country.

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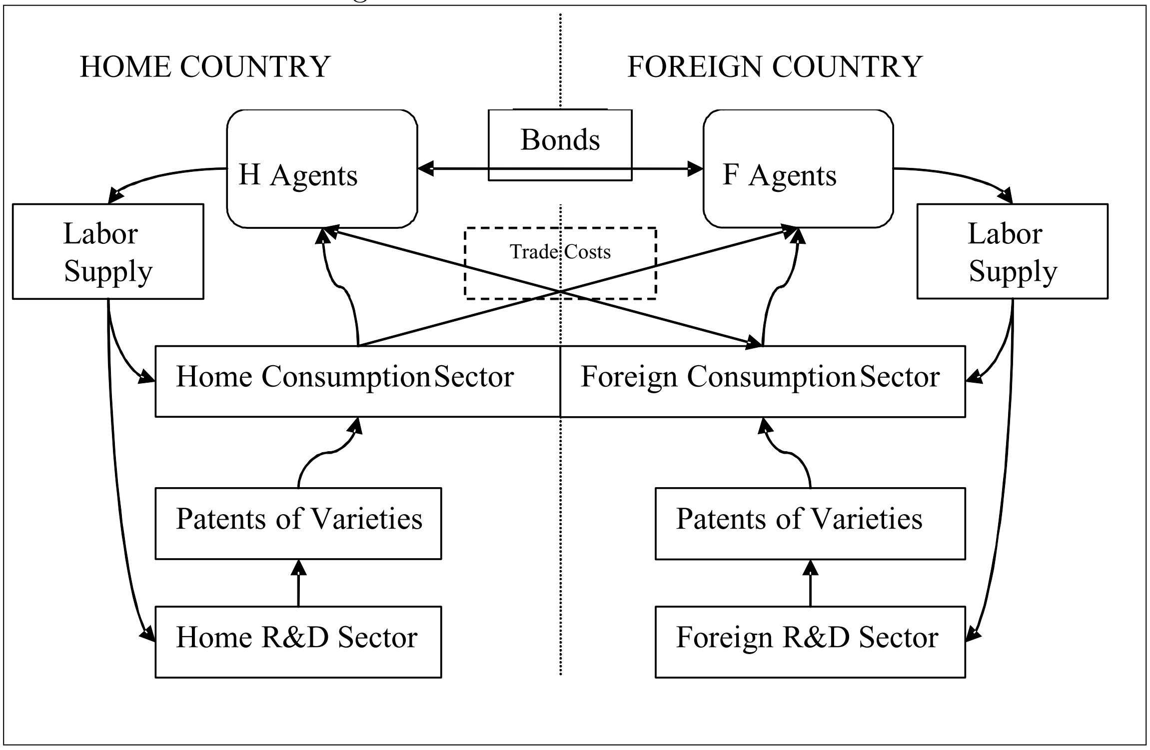
Figure 3.1: A sketch of the economies



where β is the intertemporal discount factor;

$C_{H,t}$ is the home aggregate consumption index of an individual agent given by:

$$C_{H,t} = \left(\int_0^{v_{H,t}} \gamma_H^H . \left(c_{H,t}^H(s) \right)^{\frac{\theta-1}{\theta}} ds + \int_0^{v_{F,t}} \gamma_H^F . \left((1-T) c_{H,t}^F(s) \right)^{\frac{\theta-1}{\theta}} ds \right)^{\frac{\theta}{\theta-1}} \tag{3.2}$$

$c_{H,t}^H(s)$ is the *per capita* consumption of variety s produced in the home country and consumed by home agents;

$c_{H,t}^F(s)$ is the *per capita* consumption of variety s produced in the foreign country and bought by home agents;

γ_H^H and γ_H^F are parameters to model potential bias towards home goods such that $\gamma_H^H = (0.5 + bias)^{\frac{1}{\theta}}$ and $\gamma_H^F = (0.5 - bias)^{\frac{1}{\theta}}$.

T is the trade cost, modelled as an iceberg cost. Agents buy the quantity $c_{H,t}^F$ but part *melts* on the way and therefore they only are able to consume $(1-T)c_{H,t}^F$;

$H_{H,t}$ is the amount of labor supplied and;

θ is the elasticity of substitution across varieties

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The budget constraint is:

$$\begin{aligned} w_t^H.H_t^H + b_t^H + \pi_t^H \geq & \int_0^{v_{H,t}} p_{H,t}(s).c_{H,t}^H(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c_{H,t}^F(s)ds + \\ & + b_{t-1}^H.(1+i_{t-1}) + \psi(b_t^H - b_{SS}^H)^2 \end{aligned} \quad (3.3)$$

where:

w_t^H is the nominal wage in the home country;

b_t^H is the total value of international bonds sold at t by the representative home household (b_{SS}^H being the steady state value which is zero);

π_t^H are the total profits earned at time t by the representative household (i.e., the *per capita* profits). This is equal to the sum of the profits from the consumption goods sector and the R&D sector of the home economy. We will assume that there is no foreign ownership.

$p_{H,t}$ is the price of home goods.

$p_{F,t}$ is the price of foreign goods.

i_t is the interest rate at time t.

ψ is the portfolio adjustment costs away from the steady state; the presence of those is needed to enforce stability in the system as is shown by Schmitt-Grohe and Uribe (2003).

Moreover we are considering that the nominal exchange rate is fixed and equal to 1[9].

The time constraint is:

$$TT_H = L_{H,t} + H_{H,t} = L_{H,t} + \int_0^{v_{H,t}} h_{H,t}(s)ds + h_{H,t}^{RD} \quad (3.4)$$

TT_H is the total time available to the home agent;

$H_{H,t}$ is the total amount of labor that is split between :

the producers of the different $v_{H,t}$ varieties: $\int_0^{v_{H,t}} h_{H,t}(s)ds$;

and the amount of labor provided to the R&D sector: $h_{H,t}^{RD}$.

and $L_{H,t}$ is leisure.

3.2.1.1 The consumption price index and the demand equations

The consumption price index, following Obstfeld and Rogoff (1996), will be given by the expenditure minimization problem to buy $C_{H,t}$[10].

The home consumption price index, denoted by $P_{H,t}$, is:

$$P_{H,t} = \left(\int_0^{v_{H,t}} \left(\gamma_H^H\right)^{\theta} . \left(p_{H,t}(s)\right)^{1-\theta} ds + \int_0^{v_{F,t}} \left(\gamma_H^F\right)^{\theta} . \left(\frac{p_{F,t}(s)}{(1-T)}\right)^{1-\theta} ds \right)^{\frac{1}{1-\theta}} \quad (3.5)$$

[9] See appendix 3.D
[10] See appendix 3.A

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and the demand equations of the home agent are:

$$c^H_{H,t}(s) = \left(\frac{p_{H,t}(s).}{P_{H,t}.\gamma^H_H.}\right)^{-\theta} C_{H,t} \quad (3.6)$$

$$c^F_{H,t}(s) = \left(\frac{p_{F,t}(s)}{P_{H,t}.\gamma^F_H.(1-T)}\right)^{-\theta} \frac{C_{H,t}.}{(1-T).} \quad (3.7)$$

Similar equations hold for the foreign representative household.

3.2.1.2 The intertemporal problem

The intertemporal problem is[11]:

$$\max E_0 \sum_{t=0}^{+\infty} \beta^t.U(C_{H,t}; L_{H,t}) \quad (3.8)$$

subject to:

$$\begin{aligned} w^H_t.H^H_t + b^H_t + \pi^H_t &\geq \int_0^{v_{H,t}} p_{H,t}(s).c^H_{H,t}(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c^F_{H,t}(s)ds + \\ &+b^H_{t-1}.(1+i_{t-1}) + \psi(b^H_t - b^H_{SS})^2 \\ TT_H &= L_{H,t} + H_{H,t} = L_{H,t} + \int_0^{v_{H,t}} h_{H,t}(s)ds + h^{RD}_{H,t} \end{aligned}$$

The first order conditions (FOC$_S$) of this problem, see appendix 3.B[12], are:

$$\begin{aligned} \frac{\partial U}{\partial C_{H,t}} &= \lambda^H_{BC,t}.P_{H,t} \quad (3.9) \\ \frac{\partial U}{\partial L_{H,t}} &= \lambda^H_{L,t} \\ \lambda^H_{BC,t}.w^H_t &= \lambda^H_{L,t} \\ \lambda^H_{BC,t} &= \beta E_t\left[\lambda^H_{BC,t+1}.(1+i_t)\right] - 2\psi(b^H_t - b^H_{SS}) \end{aligned}$$

plus the restrictions.

Replacing $\lambda^H_{BC,t}$ in the fourth equation by the value from the first equation we get the Euler equation:

$$\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}} = \beta E_t\left[\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}.(1+i_t)\right] - 2\psi(b^H_t - b^H_{SS}) \quad (3.10)$$

[11] Because $L_{H,t} = TT_H - H_{H,t}$, we replace the condition into the utility function making consumption and leisure the dependent variables.

[12] As total time available (TT_H) is fixed the problem was solved considering that $TT_H = 1$.

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3.2.2 Firms' Problem

In both sectors firms will use labor as the only input factor.

3.2.2.1 The consumption goods sector

The home(foreign) country will produce a set of symmetric s varieties on the support vector $[0; v_{H,t}]$ ($[0, v_{F,t}]$).

Total production of goods of variety s in the home country is at time t :

$$\overline{Y}_t^H(s) = A_t^H.\overline{H}_t^H(s) \tag{3.11}$$

where $\overline{H}_t^H(s)$ is the total labour used to produce variety s at time t and A_t^H is the productivity parameter subject to shocks, specified later.

Therefore total profits are:

$$\overline{\Pi}_t^H(s) = p_t^H(s).\overline{Y}_t^H(s) - w_t^H.\overline{H}_t^H(s) - pv_t^H(s) \tag{3.12}$$

where $p_t^H(s)$ is the price of home produced variety s at time t; w_t^H is the nominal value of home wages at time t and $pv_t^H(s)$ is the price of the right to produce the variety at time t paid to the home R&D sector[13].

At each point the R&D sector will charge the profit maximizing price for the royalty, therefore consumption goods companies will at each point in time maximize the operating profit:

$$\overline{\Pi o}_t^H(s) = p_t^H(s).\overline{Y}_t^H(s) - w_t^H.\overline{H}_t^H(s) \tag{3.13}$$

Rearranging equation (3.11) we get $\overline{H}_t^H(s) = \frac{\overline{Y}_t^H(s)}{A_t^H}$.

Moreover, total production of variety s has to be equal to the total consumption in both countries:

$$\overline{Y}_t^H(s) = \overline{c}_{H,t}^H(s) + \overline{c}_{F,t}^H(s) \tag{3.14}$$

where $\overline{c}_{H,t}^H(s)$ (the total home consumption of home produced s variety) is equal to $n_H.c_{H,t}^H(s)$ and $\overline{c}_{F,t}^H(s)$ (the total foreign consumption of home produced s variety) is equal to $n_F.c_{F,t}^H(s)$.

So, operating profits can be written as:

$$\begin{aligned} \overline{\Pi o}_t^H(s) &= p_t^H(s).\left(n_H.c_{H,t}^H(s) + n_F.c_{F,t}^H(s)\right) - w_t^H.\frac{\overline{Y}_t^H(s)}{A_t^H} = \\ &= p_t^H(s).\left(n_H.c_{H,t}^H(s) + n_F.c_{F,t}^H(s)\right) - w_t^H.\frac{n_H.c_{H,t}^H(s) + n_F.c_{F,t}^H(s)}{A_t^H} \end{aligned} \tag{3.15}$$

[13] This price is as a royalty paid at all points in time by the production sector to the R&D sector which owns the patents. Moreover, we will assume that these rights to produce are not tradable internationally.

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Per capita operating profits are:

$$\pi o_t^H(s) = \left(p_t^H(s) - \frac{w_t^H}{A_t^H}\right) \cdot \left(c_{H,t}^H(s) + \frac{n_F}{n_H} . c_{F,t}^H(s)\right) \tag{3.16}$$

Replacing the consumption level of variety s by their demand functions:

$$\begin{aligned} \pi o_t^H(s) &= \left(p_t^H(s) - \frac{w_t^H}{A_t^H}\right) \cdot \left(\begin{array}{c} \left(\frac{p_{H,t}(s).}{P_{H,t}..\gamma_H^H.}\right)^{-\theta} C_{H,t} + \\ +\frac{n_F}{n_H}.\left(\frac{p_{H,t}(s)}{P_{F,t}.\gamma_F^H.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).} \end{array}\right) = \\ &= \left(p_t^H(s)\right)^{1-\theta} . \left(\left(\frac{1}{P_{H,t}.\gamma_H^H.}\right)^{-\theta} C_{H,t} + \frac{n_F}{n_H}.\left(\frac{1}{P_{F,t}.\gamma_F^H.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).}\right) - \\ &\quad - \left(p_t^H(s)\right)^{-\theta} . \frac{w_t^H}{A_t^H} . \left(\begin{array}{c} \left(\frac{1}{P_{H,t}..\gamma_H^H.}\right)^{-\theta} C_{H,t} + \\ +\frac{n_F}{n_H}.\left(\frac{1}{P_{F,t}.\gamma_F^H.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).} \end{array}\right) \end{aligned} \tag{3.17}$$

The firm will choose $p_t^H(s)$ in order to maximize the operating profits, considering that the company is small enough that it would not influence the aggregate consumption price indexes ($P_{H,t}$ and $P_{F,t}$) and the consumption bundles ($C_{H,t}$ and $C_{F,t}$) and simplifying the above expression by defining $\Omega_1 = \left(\frac{1}{P_{H,t}..\gamma_H^H.}\right)^{-\theta} C_{H,t} + \frac{n_F}{n_H}.\left(\frac{1}{P_{F,t}.\gamma_F^H.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).}$, we get:

$$\begin{aligned} \frac{\partial \pi o_t^H(s)}{\partial p_t^H(s)} &= 0 \Leftrightarrow \\ &\Leftrightarrow \left((1-\theta) . \left(p_t^H(s)\right)^{-\theta} + \theta . \left(p_t^H(s)\right)^{-1-\theta} . \frac{w_t^H}{A_t^H}\right) . \Omega_1 = 0 \Longleftrightarrow \\ &\Leftrightarrow \left((1-\theta) + \theta . \left(p_t^H(s)\right)^{-1} . \frac{w_t^H}{A_t^H}\right) . \left(p_t^H(s)\right)^{-\theta} . \Omega_1 = 0 \Longleftrightarrow \end{aligned} \tag{3.18}$$

$$\Leftrightarrow \begin{array}{c} \left(p_t^H(s)\right)^{-\theta} . \Omega_1 = 0, \\ \vee \\ \left((1-\theta) + \theta . \left(p_t^H(s)\right)^{-1} . \frac{w_t^H}{A_t^H}\right) = 0 \end{array} \Leftrightarrow \tag{3.19}$$

As Ω_1 cannot be zero because consumption is strictly positive and and as $\theta > 0$ and the prices will be finite $\left(p_t^H(s)\right)^{-\theta} \neq 0$, the product of both terms cannot be zero.Therefore the first condition of equation (3.19) is impossible, from the second we get:

$$\left(p_t^H(s)\right) = \frac{\theta}{\theta - 1} . \frac{w_t^H}{A_t^H} \tag{3.20}$$

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The total profits will be zero, as they will use the operating profits to pay the royalty, which will be given by:

$$\begin{aligned} \overline{\Pi}_t^H(s) &= \frac{\theta}{\theta-1}.\frac{w_t^H}{A_t^H}.\overline{Y}_t^H(s) - w_t^H.\overline{H}_t^H(s) - pv_t^H(s) = 0 \Leftrightarrow \\ \Leftrightarrow &\ \frac{\theta}{\theta-1}.\frac{w_t^H}{A_t^H}.A_t^H\overline{H}_t^H(s) - w_t^H.\overline{H}_t^H(s) - pv_t^H(s) = 0 \Leftrightarrow \\ \Leftrightarrow &\ \frac{\theta}{\theta-1}.w_t^H.\overline{H}_t^H(s) - w_t^H.\overline{H}_t^H(s) - pv_t^H(s) = 0 \Leftrightarrow \left(\frac{\theta}{\theta-1} - 1\right) w_t^H.\overline{H}_t^H(s) = pv_t^H(s) \Leftrightarrow \\ \Leftrightarrow &\ \frac{1}{\theta-1} w_t^H.\overline{H}_t^H(s) = pv_t^H(s) \Leftrightarrow \frac{1}{\theta-1} w_t^H.n_H.h_t^H(s) = pv_t^H(s) \Leftrightarrow \\ \Leftrightarrow &\ \frac{pv_t^H(s)}{n_H} = \frac{1}{\theta-1} w_t^H.h_t^H(s) \end{aligned} \tag{3.21}$$

3.2.2.2 The R&D sector

We will consider that the R&D sector is populated by a large number of small companies that act competitively in the market; each of them takes market prices (wages, royalty, etc.) as fixed. The aggregate production of R&D at time t is:

$$\overline{Y}_t^{H,RD} = A_t^{H,RD}.\overline{H}_t^{H,RD} \tag{3.22}$$

where $A_t^{H,RD}$ is the productivity parameter subject to stochastic shocks.

The new varieties developed at time t are only available to the consumer goods sector on the following period. The evolution of varieties is:

$$v_{H,t+1} = (1-\varphi^V)v_{H,t} + \overline{Y}_t^{H,RD} \tag{3.23}$$

where φ^V is the number of varieties that disappear in each period.

Total Profits of the R&D sector are:

$$\overline{\Pi}_{H,t}^{RD} = \int_0^{v_{H,t}} pv_t^H(s)ds - w_t^H.\overline{H}_t^{H,RD} \tag{3.24}$$

At this point the R&D producers have to decide the number of varieties to produce. The value is given by the arbitrage R&D condition (see derivation in appendix 3.C):

$$E_t\left[\sum_{\tau=t+1}^{+\infty} \beta^{\tau-t}.\frac{\partial U}{\partial C_{H,\tau}}.\frac{pv_\tau^H}{P_{H,\tau}}.(1-\varphi^V)^{\tau-t-1}\right] = \frac{w_t^H}{A_t^{H,RD}.P_{H,t}}.\frac{\partial U}{\partial C_{H,t}} \tag{3.25}$$

Or in words, that the present real cost of developing a new variety in terms of utility variation (rhs) is equal to the actualized future real value of the variety in terms of future utility variation (lhs).

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This is just an entry condition, and is therefore only valid when the shocks to the economy are small. If the shocks were too big, such that the optimal level of varieties in t would be inferior to $(1-\varphi^V)\upsilon_{H,t-1}$ in this formulation there is no mechanism to store varieties for later use[14]. This formulation, despite being restrictive, is the same has been used in previous models, whether in trade models as in Melitz(2003), in IRBC models as in Ghironi and Melitz(2005) or in endogenous growth theory, see for instance Aghion and Hewitt(1999).

So, in the remainder of the study we will consider that the shocks hitting the economies are small enough not to violate the lower bound of existing varieties at each point in time: $(1-\varphi^V)\upsilon_{H,t-1}$.

3.2.2.3 Specification of productivity shocks.

We will assume that the productivity parameters are subject to exogenous shocks, such that:

$$\log(A_t) = B * \log(A_{t-1}) + \varepsilon_t \tag{3.26}$$

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\varepsilon_t \sim N(0, \sigma^{H,RD}) \ , \ E(\varepsilon_t, \varepsilon_{t-j}) = 0 \text{ for } j \neq 0$$

3.2.2.4 Aggregating country total production

At this point we have to know how total production is calculated. The problem is that we have more than one sector in each country. The nominal value of the production of the final goods sector is:

$$A_t^H(s) * \overline{H}_t^H(s) * \upsilon_t^H * p_H(s) \tag{3.27}$$

For the R&D sector the value of production is the number of varieties produced in that period times the value of each variety patent. As we saw in equation (3.25) the cost of producing a patent is equal to the future expected returns of it, therefore if anyone wants to sell the patent that he owns in that period the price asked is exactly the cost of inventing a new one. Therefore the total value of the production of the R&D sector in terms of the

[14] In fact, mathematically the only way to achieve a variety level below $(1-\varphi^V)\upsilon_{H,t-1}$ would be to allocate a negative amount of labour to the R&D sector, violating the non-negativity condition of this variable.

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consumption bundle $C_{H,t}$ is:

$$\frac{w_t^H}{A_t^{H,RD}.P_{H,t}} * A_t^{H,RD} * \overline{H}_t^{H.RD}$$

which in nominal terms is:

$$w_t^H * \overline{H}_t^{H.RD}$$

Therefore the nominal production level is given by:

$$GDP_{H,t}^{Nom.} = A_t^H(s) * \overline{H}_t^H(s) * v_t^H * p_H(s) + w_t^H * .\overline{H}_t^{H,RD} \tag{3.28}$$

where the first parcel is the value of the consumption goods produced (that pays salaries and the yearly return on existent patents) and the second is the value of the new varieties patents produced.

3.2.2.5 The measured CPI, GDP and Consumption

As to the CPI, the institutions responsible for collecting statistics to measure it keep constant the shares of expenditures over varieties and readjust it only at discrete points in time due to permanent changes of consumption reallocation of expenditures and not due to transitory ones; hence they are not measuring $P_{H,t}$.

In our framework, in the steady state home agents consume $v_{H,SS}.c_{H,SS}^H$ home goods and $v_{F,SS}.(1-T).c_{H.ss}^F$ of foreign goods. So these are going to be the weights given to home and foreign prices.

Total expenditure is:

$$v_{H,SS}.c_{H,ss}^H.p_{H,t} + v_{F,SS}.c_{H,ss}^F.p_{F,t}$$

It can be transformed into:

$$v_{H,SS}.c_{H,ss}^H.p_{H,t} + v_{F,SS}.(1-T).c_{H,ss}^F.\frac{p_{F,t}}{1-T}$$

So the price of home goods, as perceived by the home agent is $p_{H,ss}$ and for the foreign good is $\frac{p_{F,ss}}{(1-T)}$.

Therefore the price index at any point in t is[15]:

$$CPI_{H,t} = \frac{v_{H,SS}.c_{H,ss}^H.p_{H,t} + v_{F,SS}.(1-T).c_{H,ss}^F.\frac{p_{F,t}}{1-T}}{v_{H,SS}.c_{H,ss}^H + v_{F,SS}.(1-T).c_{H,ss}^F}$$

[15] This is a fixed-base Laspeyres index. For details of different ways of calculating the CPI and on the imputed bias see Feenstra and Shapiro(2001).

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The nominal consumption (expenditure) is:

$$C_{H,t}^{Nom} = P_{H,t}.C_{H,t}$$

which, deflated by the CPI, gives:

$$C_{H,t}^{C} = \frac{P_{H,t}.C_{H,t}}{CPI_{H,t}} \tag{3.29}$$

As for the GDP to obtain the real value we deflate the nominal one defined by equation (3.28) by the PPI[16], which is:

$$PPI_{H,t} = \frac{p_{H,t}(s)}{p_{H,t}^{SS}(s)} = \text{(by normalizing the steady state price to 1)} = p_{H,t}(s)$$

So the real GDP in terms of home consumed goods is:

$$GDP_{H,t}^{\text{Real}} = \frac{A_t^H(s) * \overline{H}_t^H(s) * v_t^H * p_H(s) + w_t^H * \overline{H}_t^{H.RD}}{p_{H,t}(s)} \tag{3.30}$$

Therefore when we calibrate/parameterize the model it is these definitions of real GDP and consumption that we will compare with the data.

3.2.3 Market clearing conditions

Finally we have to state the market clearing conditions.

For the consumption good markets we have that total consumption is equal to total production, for both countries:

$$\begin{cases} n_H * c_{H,t}^H(s) + n_F * c_{F,t}^H(s) = \overline{Y}_t^H(s) \\ n_H * c_{H,t}^F(s) + n_F * c_{F,t}^F(s) = \overline{Y}_t^F(s) \end{cases} \Leftrightarrow \tag{3.31}$$

$$\Leftrightarrow \begin{cases} c_{H,t}^H(s) + \frac{n_F}{n_H} * c_{F,t}^H(s) = \frac{A_t^H.\overline{H}_t^H(s)}{n_H} \\ \frac{n_H}{n_F} * c_{H,t}^F(s) + c_{F,t}^F(s) = \frac{A_t^F.\overline{H}_t^F(s)}{n_F} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} c_{H,t}^H(s) + \frac{n_F}{n_H} * c_{F,t}^H(s) = A_t^H.h_t^H(s) \\ \frac{n_H}{n_F} * c_{H,t}^F(s) + c_{F,t}^F(s) = A_t^F.h_t^F(s) \end{cases} \Leftrightarrow$$

No net bonds:

$$n_H * b_t^H + n_F * b_t^F = 0 \tag{3.32}$$

[16] We saw before that varieties have an economic value, however in real data that price is in most cases non-observable (most of the patents or trademarks are not traded). The construction of the PPI is usally based on the prices of physical products.

Finally, if we use the budget constraint:

$$w_t^H.H_t^H + \pi_t^{H,RD} + b_t^H = P_{H,t}.C_{H,t} + b_{t-1}^H.(1+i_{t-1}) + \psi(b_t^H - b_{SS}^H)^2 \tag{3.33}$$

considering that $w_t^H.H_t^H + \pi_t^{H,RD} = \int_0^{v_{H,t}} (p_{H,t}(s)).Y_t^H(s)ds$ and $P_{H,t}.C_{H,t} = \int_0^{v_{H,t}} p_{H,t}(s).c_H^H(s)ds$ $+ \int_0^{v_{F,t}} p_{F,t}(s).c_H^F(s)ds$ and subtracting from both sides own country goods consumption $(\int_0^{v_{H,t}} p_{H,t}(s).c_H^H(s)ds)$ and ignoring the financial asset (anyway in steady state $b_t^H = 0$), we get:

$$\begin{aligned} & \int_0^{v_{H,t}} (p_{H,t}(s)).Y_t^H(s)ds - \int_0^{v_{H,t}} p_{H,t}(s).c_H^H(s)ds \\ = & \\ & \int_0^{v_{H,t}} p_{H,t}(s).c_H^H(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c_H^F(s)ds - \int_0^{v_{H,t}} p_{H,t}(s).c_H^H(s)ds \end{aligned} \tag{3.34}$$

which is equal to:

$$\int_0^{v_{H,t}} (p_{H,t}(s)).\left(Y_t^H(s) - c_H^H(s)\right) ds = \int_0^{v_{F,t}} p_{F,t}(s).c_H^F(s)ds \tag{3.35}$$

On the left hand side we have the *per capita* production of the consumption good sector that is exported, on the right hand side we have *per capita* imports, therefore the trade balance for the home country is:

$$n_H \left[\int_0^{v_{H,t}} (p_{H,t}(s)).\left(Y_t^H(s) - c_H^H(s)\right) ds - \int_0^{v_{F,t}} p_{F,t}(s).c_H^F(s)ds \right] = TradeB_{H,t} \tag{3.36}$$

Note that if the trade balance is different from zero, the sum of both trade balances is not equal to zero due to the portfolio allocation costs: replacing equation (3.36) in the budget constraint we get for the home country:

$$TradeB_{H,t} = n_H \left[b_{t-1}^H.(1+i_{t-1}) - b_t^H + \psi(b_t^H - b_{SS}^H)^2 \right] \tag{3.37}$$

and for the foreign country:

$$TradeB_{F,t} = n_F \left[b_{t-1}^F.(1+i_{t-1}) - b_t^F + \psi(b_t^F - b_{SS}^F)^2 \right] \tag{3.38}$$

adding the previous equations:

$$\begin{aligned} & TradeB_{H,t} + TradeB_{F,t} \\ = & \\ & n_H \left[b_{t-1}^H.(1+i_{t-1}) - b_t^H + \psi(b_t^H - b_{SS}^H)^2 \right] + n_F \left[b_{t-1}^F.(1+i_{t-1}) - b_t^F + \psi(b_t^F - b_{SS}^F)^2 \right] \end{aligned} \tag{3.39}$$

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using the no net bonds conditions at any point in time and considering that $b^*_{SS} = 0$:

$$TradeB_{H,t} + TradeB_{F,t} = \psi\left[n_H.(b_t^H)^2 + n_F(-\frac{n_H}{n_F}b_t^H)^2\right] = \tag{3.40}$$
$$\psi\left[n_H.(b_t^H)^2\left(1+\frac{n_H}{n_F}\right)\right] = \psi\left[(n_H.b_t^H)^2\left(\frac{1}{n_H}+\frac{1}{n_F}\right)\right]$$

3.2.4 The First Order Conditions

The first order conditions are given by the price indexes[17]:

$$P_{H,t} = \left(\int_0^{v_{H,t}} \left(\gamma_H^H\right)^\theta . (p_{H,t}(s))^{1-\theta} ds + \int_0^{v_{F,t}} \left(\gamma_H^F\right)^\theta . \left(\frac{p_{F,t}(s)}{(1-T)}\right)^{1-\theta} ds\right)^{\frac{1}{1-\theta}} \tag{3.41}$$

$$P_{F,t} = \left(\int_0^{v_{F,t}} \left(\gamma_F^F\right)^\theta . (p_{F,t}(s))^{1-\theta} ds + \int_0^{v_{H,t}} \left(\gamma_F^H\right)^\theta . \left(\frac{p_{H,t}(s)}{(1-T)}\right)^{1-\theta} ds\right)^{\frac{1}{1-\theta}} \tag{3.42}$$

the demand equations[18]:

$$c_{H,t}^H(s) = \left(\frac{p_{H,t}(s)}{P_{H,t}.\gamma_H^H}\right)^{-\theta} C_{H,t} \tag{3.43}$$

$$c_{H,t}^F(s) = \left(\frac{p_{F,t}(s)}{P_{H,t}.\gamma_H^F.(1-T)}\right)^{-\theta} \frac{C_{H,t}.}{(1-T).} \tag{3.44}$$

$$c_{F,t}^F(s) = \left(\frac{p_{F,t}(s)}{P_{F,t}.\gamma_F^F}\right)^{-\theta} C_{F,t} \tag{3.45}$$

$$c_{F,t}^H(s) = \left(\frac{p_{H,t}(s)}{P_{F,t}.\gamma_F^H.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).} \tag{3.46}$$

the FOC from the consumption intertemporal problem[19]:

$$\frac{\partial U}{\partial C_{H,t}} = \lambda_{BC,t}^H.P_{H,t} \tag{3.47}$$

$$\frac{\partial U}{\partial L_{H,t}} = \lambda_{L,t}^H \tag{3.48}$$

$$\lambda_{BC,t}^H.w_t^H = \lambda_{L,t}^H \tag{3.49}$$

[17] For the home country it is given by equation (3.5)
[18] For the home country it is given by equations (3.6) and (3.7)
[19] For the home country this is given by equation (3.9)

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$$\lambda^H_{BC,t} = \beta E_t \left[\lambda^H_{BC,t+1}.(1+i_t)\right] - 2\psi(b^H_t - b^H_{SS}) \tag{3.50}$$

$$\frac{\partial U}{\partial C_{F,t}} = \lambda^F_{BC,t}.P_{F,t} \tag{3.51}$$

$$\frac{\partial U}{\partial L_{F,t}} = \lambda^F_{L,t} \tag{3.52}$$

$$\lambda^F_{BC,t}.w^F_t = \lambda^F_{L,t} \tag{3.53}$$

$$\lambda^F_{BC,t} = \beta E_t \left[\lambda^F_{BC,t+1}.(1+i_t)\right] - 2\psi(b^F_t - b^F_{SS}) \tag{3.54}$$

the price conditions[20]:

$$\left(p^H_t(s)\right) = \frac{\theta}{\theta-1}.\frac{w^H_t}{A^H_t} \tag{3.55}$$

$$\frac{pv^H_t(s)}{n_H} = \frac{1}{\theta-1}w^H_t.h^H_t(s) \tag{3.56}$$

$$\left(p^F_t(s)\right) = \frac{\theta}{\theta-1}.\frac{w^F_t}{A^F_t} \tag{3.57}$$

$$\frac{pv^F_t(s)}{n_F} = \frac{1}{\theta-1}w^F_t.h^F_t(s) \tag{3.58}$$

the R&D profits and arbitrage condition[21]:

$$\overline{\Pi}^{RD}_{H,t} = \int_0^{v_{H,t}} pv^H_t(s)ds - w^H_t.\overline{H}^{H,RD}_t \tag{3.59}$$

$$\overline{\Pi}^{RD}_{F,t} = \int_0^{v_{F,t}} pv^F_t(s)ds - w^F_t.\overline{H}^{F,RD}_t \tag{3.60}$$

$$E_t\left[\sum_{\tau=t+1}^{+\infty}\beta^{\tau-t}\frac{\frac{\partial U}{\partial C_{H,\tau}}}{P_{H,\tau}}pv^H_\tau.(1-\varphi^V)^{\tau-t-1}\right] = \frac{w^H_t}{A^{H,RD}_t}.\frac{\frac{\partial U}{\partial C_{H,t}}}{P_{H,t}} \tag{3.61}$$

$$E_t\left[\sum_{\tau=t+1}^{+\infty}\beta^{\tau-t}\frac{\frac{\partial U}{\partial C_{F,\tau}}}{P_{F,\tau}}pv^F_\tau.(1-\varphi^V)^{\tau-t-1}\right] = \frac{w^F_t}{A^{F,RD}_t}.\frac{\frac{\partial U}{\partial C_{F,t}}}{P_{F,t}} \tag{3.62}$$

the budget and time constraint[22]:

$$\begin{aligned} w^H_t.H^H_t + b^H_t + \pi^H_t &= \int_0^{v_{H,t}} p_{H,t}(s).c^H_{H,t}(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c^F_{H,t}(s)ds + \\ &\quad + b^H_{t-1}.(1+i_{t-1}) + \psi(b^H_t - b^H_{SS})^2 \end{aligned} \tag{3.63}$$

[20] For the home country this is given by equations (3.20) and (3.21).

[21] For the home country this is given by equations (3.24) and (3.25).

[22] For the home country this is given by equations (3.3) and (3.4).

$$w_t^F.H_t^F + b_t^F + \pi_t^F = \int_0^{v_{F,t}} p_{F,t}(s).c_{F,t}^F(s)ds + \int_0^{v_{H,t}} p_{H,t}(s).c_{F,t}^H(s)ds + \quad (3.64)$$
$$+b_{t-1}^F.(1+i_{t-1}) + \psi(b_t^F - b_{SS}^F)^2$$

$$1 = L_{H,t} + H_{H,t} = L_{H,t} + \int_0^{v_{H,t}} h_{H,t}(s)ds + h_{H,t}^{RD} \quad (3.65)$$

$$1 = L_{F,t} + H_{F,t} = L_{F,t} + \int_0^{v_{F,t}} h_{F,t}(s)ds + h_{F,t}^{RD} \quad (3.66)$$

the consumption goods market clearing conditions[23]:

$$c_{H,t}^H(s) + \frac{n_F}{n_H} * c_{F,t}^H(s) = A_t^H.h_t^H(s) \quad (3.67)$$

$$\frac{n_H}{n_F} * c_{H,t}^F(s) + c_{F,t}^F(s) = A_t^F.h_t^F(s) \quad (3.68)$$

the no net bonds condition[24]:

$$n_H * b_t^H + n_F * b_t^F = 0 \quad (3.69)$$

and the varieties evolution[25]:

$$v_{H,t+1} = (1-\varphi^V)v_{H,t} + \overline{Y}_t^{H,RD} \quad (3.70)$$

$$v_{F,t+1} = (1-\varphi^V)v_{F,t} + \overline{Y}_t^{F,RD} \quad (3.71)$$

3.2.5 Equilibrium

An equilibrium of this economy at time t is a collection of allocations for home consumers $c_{H,t}^H(s)$, $c_{H,t}^F(s)$, $h_{H,t}(s)$ $h_{H,t}^{RD}$ and $b_{H,t}$ for $s \in [0, v_t^H]$; allocations for foreign consumers $c_{F,t}^F(s)$, $c_{F,t}^H(s)$, $h_{F,t}(s)$ $h_{F,t}^{RD}$ for $s \in [0, v_t^F]$ and $b_{F,t}$; allocations and prices for home final goods producers $[n_H.c_{H,t}^H(s) + n_F.c_{F,t}^H(s)]$ and $p_{H,t}(s)$ for $s \in [0, v_t^H]$; allocations and prices for foreign goods producers $[n_F.c_{F,t}^F(s) + n_H.c_{H,t}^F(s)]$ and $p_{F,t}(s)$ for $s \in [0, v_t^F]$; allocations for the number of varieties in the home country $v_{H,t}$ and in the foreign country $v_{F,t}$; allocations for production of new varieties at home, $v_{H,t+1} - \varphi^V v_{H,t}$ and in the foreign country $v_{F,t+1} - \varphi^V v_{F,t}$; the price of varieties in both countries, $pv_{H,t}$ and $pv_{F,t}$.and the nominal wages $w_{H,t}$ and $w_{F,t}$ such that: (i) the consumer allocations solve the consumer's problem; (ii) the consumption good producers allocations solve their profit maximization problems; (iii) the production and price of varieties solve the R&D producers profit maximization problems and (iv) the market clearing conditions hold.

[23]For the home country this is given by equations (3.31)

[24]For the home country is given by equations (3.32)

[25]For the home country is given by equations (3.23)

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3.3 The Non-Stochastic Steady State

The above system is non-linear and it does not have a closed form solution. Therefore we make use of numerical methods to study the dynamic properties of the model. Even if it might not be necessary to derive the non-stochastic steady state (NSSS), it is a good starting point as an initial guess; moreover, if we want to approximate the model with a first or second order linearization, that solution is the usual point around which we linearize.

As we will see while deriving the NSSS, it is impossible to derive this explicit solution without assuming country symmetry[26].

Before getting the specific relations for each functional form we can derive some general results that are independent from the utility function used.

Start by considering that the steady state stock bond holding is zero ($b_{SS}^H = 0$ and $b_{SS}^F = 0$). This implies that equations (3.50) and (3.54) in the NSSS are:

$$\begin{cases} \lambda_{BC,t}^H = \beta.\left[\lambda_{BC,t+1}^H.(1+i_t)\right] - 2\psi b_t^H \\ \lambda_{BC,t}^F = \beta.\left[\lambda_{BC,t+1}^F.(1+i_t)\right] - 2\psi b_t^F \end{cases} \Leftrightarrow \begin{cases} \frac{\lambda_{BC,t}^H + 2\psi b_t^H}{\beta.\lambda_{BC,t+1}^H} = (1+i_t) \\ \frac{\lambda_{BC,t}^F + 2\psi b_t^F}{\beta.\lambda_{BC,t+1}^F} = .(1+i_t) \end{cases} \Leftrightarrow \tag{3.72}$$

$$\Leftrightarrow \begin{cases} \frac{1}{\beta} + \frac{2\psi b_t^H}{\beta.\lambda_{BC,t+1}^H} = (1+i_t) \\ \frac{1}{\beta} + \frac{2\psi b_t^F}{\beta.\lambda_{BC,t+1}^F} = (1+i_t) \end{cases} \Leftrightarrow \begin{cases} \frac{1}{\beta} + \frac{2\psi b_t^H}{\beta.\lambda_{BC,t+1}^H} = \frac{1}{\beta} + \frac{2\psi b_t^F}{\beta.\lambda_{BC,t+1}^F} \\ ----------- \end{cases} \Leftrightarrow$$

$$\Leftrightarrow b_t^H = b_t^F$$

Together with the no net bonds condition ($b_t^H + b_t^F = 0$) leads to:

$$\begin{cases} \frac{1}{\beta} = (1+i_t) \\ b_t^H = b_t^F = 0 \end{cases} \tag{3.73}$$

This means that equation (3.61)[27] can be written as:

$$\sum_{\tau=t+1}^{+\infty} \left(\frac{1}{1+i_t}\right)^{\tau-t} pv_\tau^H.(1-\varphi^V)^{\tau-t-1} = \frac{w_t^H}{A_t^{H,RD}} \Leftrightarrow \tag{3.74}$$

$$\Leftrightarrow \frac{1}{1+i_t}.pv_\tau^H.\frac{1}{1-\frac{1-\varphi^V}{1+i_t}} = \frac{w_t^H}{A_t^{H,RD}} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{\varphi^V + i_t}.pv_\tau^H. = \frac{w_t^H}{A_t^{H,RD}} \Leftrightarrow w_t^H = A_t^{H,RD}\frac{1}{\varphi^V + i_t}.pv_\tau^H.$$

[26]For symmetry we mean that the home-bias level, the steady state of the total factors productiveness, the populations, the intertemporal and intratemporal elasticities and varieties death rate are equal across countries.

[27]Making use of the result of equation (3.124) of appendix 3.C in steady state.

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and the same for equation (3.62):

$$w_t^F = A_t^{F,RD} \frac{1}{\varphi^V + i_t}.pv_\tau^F \tag{3.75}$$

if we input the previous result into equation (3.56), we get:

$$\begin{aligned} \frac{pv_t^H(s)}{n_H} &= \frac{1}{\theta - 1}.A_t^{H,RD} \frac{1}{\varphi^V + i_t}.pv_\tau^H.h_t^H(s) \Leftrightarrow \\ &\Leftrightarrow \frac{(\theta - 1)\left(\varphi^V + i_t\right)}{n_H.A_t^{H,RD}} = h_t^H(s) \end{aligned} \tag{3.76}$$

and:

$$\frac{(\theta - 1)\left(\varphi^V + i_t\right)}{n_F.A_t^{F,RD}} = h_t^F(s) \tag{3.77}$$

implying from the symmetry of countries:

$$h_t^H(s) = h_t^F(s) \tag{3.78}$$

Introducing the result of equation (3.77) on $H_{H,t} = \int_0^{v_{H,t}} h_{H,t}(s)ds + h_{H,t}^{RD}$ and knowing that in NSSS $v_{H,t} = \frac{A_t^{H,RD}.n_H.h_t^{H,RD}}{\varphi^V}$ we get:

$$h_{H,t} = \left(\frac{(\theta - 1)\left(i_t + \varphi^V\right)}{n_H.A_t^{H,RD}} \frac{n_H.A_t^{H,RD}}{\varphi^V} + 1 \right) h_{H,t}^{RD} \tag{3.79}$$

from the fact that the sum of hours in both sectors is equal to total hours worked we get:

$$h_{H,t}^{RD} = \frac{\varphi^V}{(\theta - 1)\, i_t + \theta.\varphi^V}.H_{H,t} \tag{3.80}$$

From equations (3.55) and (3.56) we get that wages are:

$$\begin{aligned} \frac{\theta - 1}{\theta} A_t^H &= \frac{w_t^H}{p_{H,t}(s)} \\ \frac{\theta - 1}{\theta} A_t^F &= \frac{w_t^F}{p_{F,t}(s)} \end{aligned} \tag{3.81}$$

which implies, from symmetry, that $\frac{w_t^H}{p_{H,t}(s)} = \frac{w_t^F}{p_{F,t}(s)}$.
From this equality and equation (3.78) we get from (3.56) and (3.58):

$$\frac{pv_t^H}{p_{H,t}(s)} = \frac{pv_t^F}{p_{F,t}(s)} \tag{3.82}$$

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Profits are:

$$\begin{aligned}\pi_H^{RD} &= \frac{v_{H,t}}{n_H}.pv_t^H - w_t^H.h_t^{H,RD} = \\ &= \left(\frac{A_{H,t}^{RD}}{\varphi^V}.pv_t^H - w_t^H\right).h_{H,t}^{RD} = \left(\frac{A_{H,t}^{RD}}{\varphi^V}.pv_t^H - w_t^H\right).\frac{\varphi^V}{(\theta-1)\,i_t + \theta.\varphi^V}.H_{H,t}\end{aligned} \tag{3.83}$$

$$\begin{aligned}\pi_F^{RD} &= \frac{v_{F,t}}{n_F}.pv_t^F - w_t^F.h_t^{F,RD} = \\ &= \left(\frac{A_{F,t}^{RD}}{\varphi^V.}.pv_t^F - w_t^F\right).h_{F,t}^{RD} = \left(\frac{A_{F,t}^{RD}}{\varphi^V}.pv_t^F - w_t^F\right).\frac{\varphi^V}{(\theta-1)\,i_t + \theta.\varphi^V}.H_{F,t}\end{aligned} \tag{3.84}$$

which implies that:

$$\frac{\pi_H^{RD}}{p_H(s)} = \frac{\theta-1}{\theta}A_t^H.\frac{i_t}{(\theta-1)\,i_t + \theta.\varphi^V}.H_{H,t} \tag{3.85}$$

$$\frac{\pi_F^{RD}}{p_F(s)} = .\frac{\theta-1}{\theta}A_t^F.\frac{i_t}{(\theta-1)\,i_t + \theta.\varphi^V}.H_{F,t} \tag{3.86}$$

So we can express all variables in terms of the own country tradable goods price as functions of the total labour supply, but this will depend on the functional form used for the utility function

3.3.1 The utility function

The functional form that we use for the utility function is:

$$U(C_{H,t};L_{H,t}) = \frac{(C_{H,t})^{1-\rho}}{1-\rho} - k.\frac{(H_{H,t})^{1+\varpi}}{1+\varpi} = \frac{(C_{H,t})^{1-\rho}}{1-\rho} - k.\frac{(1-L_{H,t})^{1+\varpi}}{1+\varpi} \tag{3.87}$$

where $\varpi \geq 0$ and $\frac{1}{\varpi}$ is the Frisch intertemporal elasticity of substitution. This utility function implies that equations (3.47) , (3.48), (3.51) and (3.52) are:

$$(C_{H,t})^{-\rho} = \lambda_{BC,t}^H.P_{H,t} \tag{3.88}$$

$$k.(1-L_{H,t})^{\varpi} = \lambda_{L,t}^H \tag{3.89}$$

$$(C_{F,t})^{-\rho} = \lambda_{BC,t}^F.P_{F,t} \tag{3.90}$$

$$k.(1-L_{F,t})^{\varpi} = \lambda_{L,t}^F \tag{3.91}$$

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From appendix 3.D we get that $H_{H,t} = H_{F,t}$ and therefore:

$$H_{H,t} = \left(\frac{\left(\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} \left(\left(\gamma_H^H\right)^\theta . + \left(\gamma_H^F\right)^\theta .(1-T)^{\theta-1} \right) \right)^{\frac{\rho-1}{(1-\theta)\rho}}}{\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)i_t+\theta.\varphi^V}. \right) . \left(\frac{\theta-1}{\theta} A_t^H \right)^{1-\frac{1}{\rho}} . (k)^{\frac{1}{\rho}}} \right)^{\frac{\rho(1-\theta)}{\varpi(1-\theta)+1-\rho\theta}} \tag{3.92}$$

Having derived the NSSS for $H_{H,t}$ we can also obtain the value of all the other variables in terms of the prices of the tradable goods[28].

A deficiency of this utility function in this framework is that it is not in line with a stable growth path. In fact $H_{H,t}$ depends on $\frac{\left(n_H.A_t^{H,RD}\right)^{\frac{1}{1-\theta}}}{A_t^H}$, which implies that $H_{H,t}$ is negatively correlated with the productivity levels in both sectors and population size. One way to fix this would be to set $\frac{\left(n_H.A_{SS}^{H,RD}\right)^{\frac{1}{1-\theta}}}{A_{SS}^H}$ equal to a constant. However this would imply that with an increase in the productivity level of the consumption goods sector the productivity in the R&D would have to decrease, leading to a reduction in the number of varieties. A second approach would be to relate the utility function to population and technological progress. So if we consider $\left(\frac{\left(n_H.A_{SS}^{H,RD}\right)^{\frac{1}{1-\theta}}}{A_{SS}^H} \right)^{\frac{\rho-1}{\rho}} . \frac{1}{k^{\frac{1}{\rho}}}$ equal to a constant, than $k = k_1. \left(\frac{\left(n_H.A_{SS}^{H,RD}\right)^{\frac{1}{1-\theta}}}{A_{SS}^H} \right)^{\rho-1}$. As k controls the level of negative utility driven from working, then it would decrease with technological progress and population size. As for technological progress we can reason that when it increases, not only are the workers more productive, but also the work effort decreases, reducing the unpleasantness of working. As for population increase, we know that this would lead to more varieties produced, therefore (and if we think that different sectors require different abilities from workers, that these workers have the same abilities endowment but their preferences over them differ) the variety of work available would increase and the representative agent would be capable to match better their personnel ability usage preference with the jobs available. In this way the level of unpleasantness from working would decrease.

Moreover, we can check that $\frac{\partial H_{h,T}}{\partial T} > 0$ which implies that :

$$\frac{\partial v_{H,t}}{\partial T} = \underset{(+)}{\frac{\partial v_{H,T}}{\partial h_{H,t}^{RD}}} . \underset{(+)}{\frac{\partial h_{H,t}^{RD}}{\partial H_{H,t}}} . \underset{(+)}{\frac{\partial H_{h,T}}{\partial T}} > 0 \tag{3.93}$$

[28] From before we saw that we can determine relative prices but not the level of the individual prices. To determine this level we considered that there is a worldwide constant supply of money . For details see appendix 3.E

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Therefore an increase in trade costs will increase the number of varieties produced.

Here we should think that if it is in line with trade theory, mainly this claims that if we open the country to trade the number of varieties accessible to consumers increases but the number of varieties produced worldwide decreases. Trade theory take the extreme cases of autarchy versus free trade and the previous result is consistent with that. However, for intermediate levels of trade costs the previous model implies that if we decrease trade costs the number of varieties available in the market will diminish. But the number of varieties available is not the same as access to them. We might think that if the trade cost decreases the acessibility of foreign goods increases and hence the agent's utility increases. A more suitable index to summarize the access of agent to different varieties would be the value of the consumption bundle $C_{H,t}$, therefore we expect that the lower the trade cost the higher the value of this bundle. Figure 3.2 shows the results for different values of θ and ϖ.Furthermore, it also shows that lower trade costs reduce the amount of labour in the economy for the functional form of the utility function used.

3.4 Results and comparison with data

In this section we will analyze the results from the model for some selected parameters and compare with results taken from the data. Therefore the next subsection will present the estimation for a selected number of countries; the following describes the flow of events and the last presents the results of the model for a number of parameterizations.

3.4.1 Some stylized facts

In order to compare the moments from the model we calculated the cross-correlation and standard deviations from quarterly data of the G7 countries. The data collected were those of GDP (both at constant and current prices), Consumption (at constant prices), civil employment and net trade (at current prices) , taken from the OECD Statistical Compendium, Main Economic Indicators 2005/2 and the terms of trade (taken from Datastream, relying on the national sources).

We first calculated the ratio of Net trade/GDP using both measures at current prices and, afterwards, we applied a band pass filter to this ratio, to the TOT, to the civil employment and to the GDP and the Consumption series at constant prices as described by Artis et al.(2003) retaining the fluctuations between 6 and 32 quarters following Baxter and King(1999).

From table 3.1 we can see that consumption and labour are strongly positively correlated

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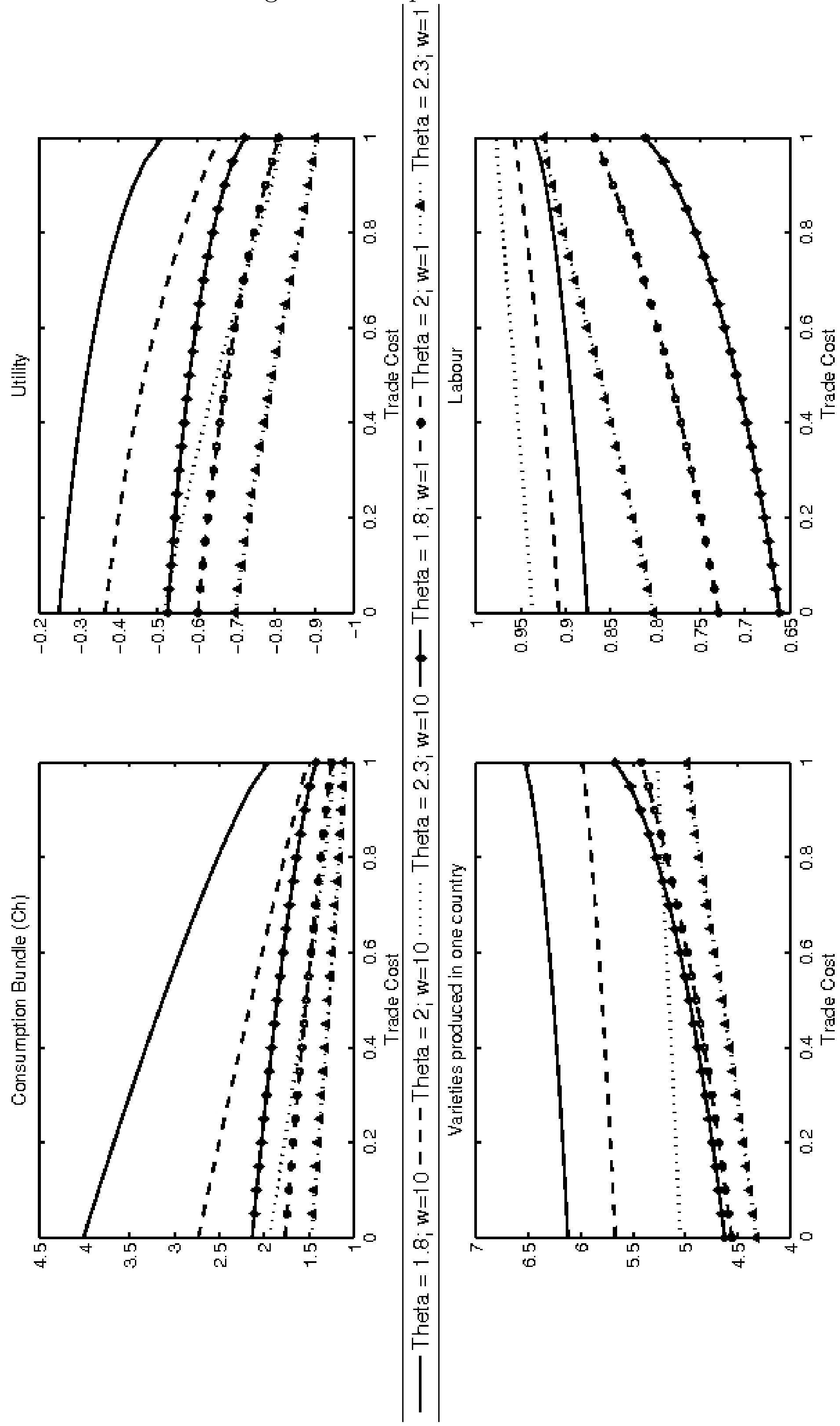
Figure 3.2: Impact of trade costs



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with output. As for the terms of trade, defined as $\frac{\text{Price of Exports}}{\text{Price of Imports}}$ we can see that the correlation with output varies from being strongly negative as for Japan after 1994, Italy and post-92 Germany to positive values as for USA and Canada [29]. For UK, France and West Germany the correlation was weakly negative. As for the ratio of net trade over GDP (nx) we realize that the correlation with GDP is negative for all countries, with the exception of Canada.

Comparing the relative volatilities we can see that consumption, labour and nx are less volatile than output (exceptions are Italy for consumption and labour, UK for consumption and Canada for nx) as the terms of trade are on average 2.5 times more volatile than output.

From tables 3.2 and 3.3 we can see that usually GDP and Consumption are positively cross-correlated, being that on the majority of the cases the cross-correlations of GDP between countries is higher than the one for consumption.

As noted in Ambler, Cardia and Zimmerman (2000) from these data two anomalies are apparent that standard DSGE models have trouble replicating. The first is the quantity anomaly. This refers to the fact that consumption appears in the data to be less correlated across countries than output. Moreover in standard models as those of Backus, Kydland and Prescott there is usually a negative cross correlation of output and labour and a positive one across consumption, as in the data both are usually positively cross correlated.

The second is the price anomaly, the TOT[30] are in DSGE models strongly negative correlated with output. This is, after an increase in home output home prices decrease. Moreover the volatility is quite low. Reducing the elasticity of substitution increases the volatility to the values in data but at expense of a counterfactual low volatility of the balance of trade with respect to output.

As for getting an idea of what are the moments across countries for product varieties we can use the number of new registered trademarks even if this number does not coincide exactly with the number of varieties in the economy. Some companies when launching a new product do not register the trademark or extend the use of an existing trademark to the new product. Also, some firms register a trademark without ever using it and others register more than one trademark (words, logos, 3D mark, sound, etc.), associated with a single product. Therefore, as with patents[31], trademarks are not an exact representation

[29] We should note that the prices used include oil prices. A better indicator would be to have the terms of trade without these.

[30] In this model terms of trade are defined as is usual in trade literature: $\frac{\text{Price of Exports}}{\text{Price of Imports}}$, and not as usually defined in IRBC models $\left(\frac{\text{Price of Imports}}{\text{Price of Exports}}\right)$. In the IRBC papers they usually depict a strong positive correlation between TOT and GDP, which translating to our definition should be read as negative.

[31] For the use of patents as R&D indicator see Griliches et al(1986).

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Table 3.1: Within countries moments

	USA	Jap		Can	UK	Fra	Ita.	WGer	Ger
Period	70Q1-04Q4	63Q1-99Q4	94Q1-04Q4	86Q1-04Q4	70Q1-02Q2	78Q1-04Q4	80Q1-04Q4	70Q-90Q4	92Q1-04Q4
Corr(y,c_c)	0,875	0,791	0,664	0,722	0,822	0,730	0,623	0,620	0,585
Corr(y,h)	0,867	0,562	0,726	0,777	0,663	0,809	0,408	0,549	0,441
Corr(y,tot)	0,181	0,011	-0,798	0,146	-0,055	-0,126	-0,413	-0,134	-0,728
Corr(y,nx)	-0,436	-0,319	-0,528	0,153	-0,475	-0,422	-0,513	-0,210	-0,389
std(c_c)/std(y)	0,793	0,816	0,569	0,507	1,107	0,938	1,271	0,789	0,913
std(h)/std(y)	0,643	0,309	0,413	0,498	0,799	0,764	1,053	0,752	0,595
std(tot)/std(y)	1,891	4,539	3,029	2,849	2,078	3,409	3,884	2,977	2,713
std(nx)/std(y)	0,252	0,556	0,573	1,166	0,722	0,849	0,857	0,601	0,613

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Table 3.2: Cross correlations for GDP and Consumption - 1st set

Country		Jap(1)		Can	UK	Fra	Ita	Ger	W Ger
USA	GDP	0,441	0,181	0,735	0,673	0,079	0,324	0,272	0,754
	C_c	0,461	-0,588	0,749	0,567	-0,077	-0,088	0,276	0,648
	H	0,295	0,518	0,771	0,675	-0,004	-0,199	0,326	0,346
	Period	(70Q1:99Q4)	(94Q1:04Q4)	(86Q1:04Q4)	(70Q1:02Q2)	(78Q1:04Q4)	(80Q1:04Q4)	(92Q1:04Q4)	(70Q1:90Q4)
Jap(1)	GDP			-0,299	0,404	-0,130	0,242		0,725
	C_c			0,175	0,663	0,346	0,087		0,464
	H			-0,373	0,195	0,080	0,486		0,283
	Period			(86Q1:99Q4)	(70Q1:99Q4)	(78Q1:99Q2)	(80Q1:99Q4)		(70Q1:90Q4)
Jap(2)	GDP			-0,156	0,191	-0,150	0,105	0,254	
	C_c			-0,457	-0,516	-0,408	-0,750	0,005	
	H			-0,391	0,002	-0,372	-0,021	-0,095	
	Period			(94Q1:04Q4)	(94Q1:02Q2)	(94Q1:04Q4)	(94Q1:04Q4)	(94Q1:04Q4)	
Can	GDP				0,915	0,367	0,417	0,623	
	C_c				0,872	0,143	0,024	-0,100	
	H				0,899	0,266	-0,291	0,141	
	Period				(86Q1:02Q2)	(86Q1:04Q4)	(86Q1:04Q4)	(92Q1:04Q4)	

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Table 3.3: Cross correlations for GDP and Consumption - 2nd set

Country		Jap	Can	UK	Fra	Ita	Ger	W Ger
UK	GDP				0,327	0,406	0,822	0,526
	C_c				0,213	-0,154	0,101	0,451
	H				0,231	-0,102	0,455	0,157
	Period				(78Q1:02Q2)	(80Q1:02Q2)	(92Q1:02Q2)	(70Q1:90Q4)
Fra	GDP					0,620	0,858	0,031
	C_c					0,378	0,625	-0,132
	H					0,420	0,619	-0,011
	Period					(80Q1:04Q4)	(92Q1:04Q4)	(78Q1:90Q4)
Ita	GDP						0,886	0,148
	C_c						0,138	0,327
	H						0,422	0,566
	Period						(92Q1:04Q4)	(80Q1:90Q4)

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of the ideal indicator. Even with these limitations the main advantage of using registered trademarks are their simplicity and availability; even if it is not an exact representation of market differentiation, it should be a close indicator. Therefore we will use the new trademarks data as a proxy variable for the introduction of new products[32].

The data used were registered trademarks in a given year by/or in the name of a resident from the World Intellectual Property Organization (WIPO). Data for Italy and unified Germany are not available and for the remaining countries are only available at annual frequencies. Therefore the moments reported below are for annual data where we applied the band pass filter to frequencies of 1,5 to 8 years (corresponding to 6 to 32 quarters).

Table 3.4: Within countries moments of new trademarks

	USA	Jap	Can	UK	Fra	WGer
Period	1970:2001	1963:2000	1986:2001	1970:2001	1970:1995	1970:1989
Corr(y,tm)	-0,274	-0,247	0,1529	-0,124	0,204	0,378
Corr(tot,tm)	0,058	-0,213	-0,0306	0,053	0,3954	-0,148
std(tm)/std(y)	8,703	2,310	4,3156	8,905	18,706	2,970
std(tm)/std(tot)	5,427	2,462	3,312	4,739	5,980	0,956

From table 3.4 we can see that the correlations with output or the terms of trade do not present a clear sign and averaging both correlations across countries they are almost zero.

From the volatility we can see that the registration of new trademarks is more (in some cases much more) volatile than output and (in most cases) the terms of trade. An interesting curiosity is that the higher is the ratio of volatility of trademarks in respect to output or terms of trade the more positive is the correlation between the terms of trade and the introduction of trademarks, however this sample is too small to say anything definitive about that.

From table 3.5 we can see that the negative entries are in the majority, however US, UK and Japan have a positive cross-correlation of new trademarks registered as, also, West Germany with Canada and France. Furthermore the overall average is close to zero. From these values it seems that we cannot arrive at any general conclusion about its behaviour in this sample.

[32] Although there is some theoretical studies about trademarks there was almost no empirical studies using trademarks until recently. The main reasons were that trademark offices have only recently began to make these data public and national offices have paid little attention to collecting such data in any homogeneous form. Two recent studies that look at trademark data are Luini and Mangano (2004) and Mendonça et al.(2004) The first try to correlate trademark deposits with economic activity and the second discusses the merits of using these data as an innovation indicator.

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Table 3.5: Cross correlations of new registred trademarks

Country		Jap	Can	UK	Fra	W Ger
USA		0,539	-0,210	0,398	-0.353	-0,388
	Period	1964-2000	1975-2001	1967-2001	1964-1995	1964-1989
Jap			-0,382	0,3602	-0,374	-0,254
	Period		1975-2000	1967-2000	1963-1995	1963-1989
Can				-0,404	-0,136	0,573
	Period			1975-2001	1975-1995	1975-1989
UK					-0,234	-0,234
	Period				1967-1995	1967-1989
Fra						0,491
	Period					1963-1989

3.4.2 Flow of events

The sequence of events in the model is the following: (i) at the beginning of time t the number of varieties is pre-set (the varieties are state variables), (ii) shocks hit the productivity of the different sectors; (iii) consumers solve their utility maximization problem as producers solve their profit maximization problem (labor in the different sectors, production of consumption goods and of new varieties, bond contracts sold and consumption levels are set, and prices are fixed). (iv) The production of new varieties extends the support of consumption goods varieties to be produced in the next period.

3.4.3 Results from the model

The model was solved using a numerical method, namely a perturbation method around a second order linearization purposed by Schmitt-Grohe and Uribe (2002) and the results were drawn from 1000 simulations of 100 periods each[33].

Before presenting the results of the model we should parameterize it[34]. Most of the

[33] Tha random generator processor for the shocks was seeded in order to have the same series of shocks over the different simulations. Data from the simulations were filtered as the real data.

[34] We showed before that the model had a unique solution, but not if it was stable around the steady state. Even if the stability issue is not derived exactly, from the simulations we noted that for some combinations of parameters the model would not be stable. These combinations had a common feature: all of them lead to high profits on the R&D sector and a shock to the model would trigger a mechanism where one country would increase the varieties indefinitely (leading the number of them and the price of the final goods to infinity).

These combinations would be a low θ, a slow death rate of the shocks in the R&D sector and a slow death rate of varieties (φ^V).

parameters of the model are standard and we can find in the related literature sensible values for each of them. The parametrization of the *standard* parameters was:

$\beta = 0.99$ (implying a steady state interest rate of 1.01%, therefore we assume each period is one quarter)

$\rho = 2$;

$\theta = 1.2$;and 1.7[35];

$A^{*;*}_{ss} = 1$ and $k = 1$;

$n_h = n_F = 1$;

$\varpi = 10$ and 1 which implies Frisch elasticities of 0.1 and 1, respectively;

$bias$ =0.2 ($\gamma_H^H = \gamma_F^F = 0.7429$. and $\gamma_H^F = \gamma_F^H = 0.3667$) and 0 ($\gamma_H^F = \gamma_F^H = \gamma_H^H = \gamma_F^F = 0.5612$);

$T = 0.0$ and 0.1;

$\psi = 0.00037$ following Schmitt-Grohe and Uribe (2003);

However φ^V and the stochastic processes are harder to define.

The first is not a standard parameter in other models, at least considering that it describes the death rate of products. For instance, Ghironi and Melitz(2005) have set a similar parameter to 0.025, however they have parameterized it according to the death rate of companies, not products. Therefore we can think that products have a higher death rate than companies . Bayus(1998) measured the life time of personal computer brand models. His estimates of the average lifetime of brand models was 3.52 years for established companies and 3.89 years for new entrants. Souza, Wagner and Bayus (2004) in a theoretical model designed to assess the optimal time between the launch of new products obtained estimates that ranged from 2 to 52 quarters; for a significant number of tested values in that paper the estimated period was around 10 quarters. From these studies, we selected a $\varphi^V = 0.075$.(average lifetime of 13.3 quarters[36]) which seems to fall between the values reported in the papers cited .

As for the stochastic processes we have to take into account what is the R&D sector. In a *substrictus sensus* we can think that the R&D sector in the model is what we are usually told R&D is, this is, a number of scientists and engineers trying to invent new technologies.

[35]In our framework this would imply that the elasticity of substitution across any two goods is 1.2. or 1.7. Although the value is much smaller than the usual values used in other papers for substitutability across goods of the same country it is in line with the values found for the elasticity across goods of different countries: see footnote 5 in Bergin and Glick(2004) or Ruhl(2003).

[36]I conducted a sensitivity test on this value, but the results were qualitatively invariant to the ones presented in the next sections, see appendix 3.F

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However in the model the R&D sector is responsible for the introduction of new varieties. That differentiation of products can come from new technologies (being the result of the traditional R&D), the assembling of old technologies in a new way (being part of product development), or just due to a subjective differentiation of the product (in this case branding and marketing would be responsible for the new variety). The approach we take about what is R&D is of fundamental importance. If we take R&D in the *substrictus sensus* then the amount of labour devoted to it will be a small percentage of total labour ; however if we consider it in the *latus sensus* then R&D would include not only scientists and engineers (the traditional view of R&D) but also all other workers that are part of the differentiation process of goods (marketers, lawyers, managers, etc.) In this way the total amount of labour devoted to product differentiation can be a considerable part of the total labour in the economy[37], still we need to offer some alternative cases to examine.

3.4.3.1 Case 1: Shocks just in the TFP of the consumption goods sector

For the first parametrization we followed the Backus et al. (1994) parameterization of the stochastic shocks and consider that there are no shocks in the R&D sector:

$$\log(A_t) = (1 - B).\log(A_{SS}) + B * \log(A_{t-1)} + \varepsilon_t$$

with:

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.906 & 0.088 & 0 & 0 \\ 0.088 & 0.906 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\begin{aligned} E(\varepsilon_t^H, \varepsilon_t^F) &= 0,258 \text{ and } E(\varepsilon_t^H, \varepsilon_{t-j}^F) = 0, \ j \neq 0 \\ var(\varepsilon_t^H) &= var(\varepsilon_t^F) \neq 0 \\ var(\varepsilon_t^{H,RD}) &= var(\varepsilon_t^{F,RD}) = 0 \end{aligned}$$

As in their model, we do not expect that this model will replicate the data, but just use it as a benchmark to measure up the following parameterizations of the stochastic shocks. Therefore, from tables 3.6 and 3.7 we can see that we have a number of moments that are at

[37] See a similar discussion in Aghion and Hewitt (1999) pp. 436-448.

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odds with the empirical data. For the within country correlations: the ones between output/ labour and output/NX have in the model the inverse sign relative to the ones that we find in data. As for the one between output/TOT it is strongly negative in the model but from the data we can only find such strong negative relations for Germany and Japan during the nineties. For the other countries is either mildly negative or positive.

As for the cross correlations between countries, the correlations between outputs are too low, the ones between consumption are too high the ones of hours worked are too high when compared with data. Furthermore the cross-country correlations of consumption are in the data less correlated than the cross country correlations of output than in the model. This fact, is due to the fact that in the model there are two mechanism operating, first consumers can sell/buy bonds in order to smooth consumption, that can be seen in the IRF of net trade balance in figure 3.3, as the shock hits the final goods sector of home country the production of these goods increase due to the shock but also because labor in that sector increases, however home county will increase the value of exports relative to imports as the foreign country will finance it by selling bonds. Second the Cole and Obstfeld (1991) effect is operating, the TOT decrease, making home goods relatively cheaper than foreign ones, and therefore, even if foreigners would not finance their increase in consumption by selling bonds they could increase consumption because, now, for the same quantity of foreign goods they would get more home goods.

As for the varieties indicators the main difference with the one obtained from data is that the volatility of those relative to GDP is too low.

Even if most of the moments are at odds with data, we should take into account two issues: home bias vs. trade costs effects and the impact of different values of the elasticity of substitution.

First the introduction of a home bias works differently than trade costs when reducing the consumption correlation gap. In fact when considering the consumption as the nominal expense deflated by the CPI, the cost of transportation is included in the consumption aggregate. In this case consumption is in fact composed of three components: home goods, foreign goods and transport services. Therefore when the country not hit by a positive shock in the final goods sector wants to risk share consumption it will increase the amount of foreign goods and transport services consumed. In this way they will have an amount of consumption expenditure identical to the one that had the positive shock, and correlations between expenditure do not suffer a great variation as when there are no trade costs. If we look to the volume of goods consumed $(c_{H,t}^{H} * v_{H,t} + (1-T).c_{H,t}^{F} * v_{H,t})$[38], the country not

[38] Correlations not included in the tables.

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Table 3.6: Estimated moments for case 1- 1st set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Within Country moments										
corr(y,c_c)	0.619	0.689	0.755	0.829	0.767	0.840	0.854	0.631	0.693	0.764
	(0.145)	(0.125)	(0.102)	(0.075)	(0.1)	(0.072)	(0.066)	(0.142)	(0.123)	(0.100)
corr(y,h)	-0.39	-0.579	-0.677	-0.801	-0.475	-0.664	-0.719	-0.385	-0.581	-0.678
	(0.195)	(0.154)	(0.128)	(0.089)	(0.179)	(0.132)	(0.116)	(0.197)	(0.154)	(0.128)
corr(y,tot)	-0.794	-0.727	-0.670	-0.619	-0.778	-0.708	-0.656	-0.792	-0.727	-0.670
	(0.087)	(0.109)	(0.126)	(0.141)	(0.092)	(0.115)	(0.13)	(0.087)	(0.109)	(0.126)
corr(y,nx)	0.793	0.725	0.669	0.468	0.777	0.706	0.651	0.790	0.725	0.662
	(0.087)	(0.11)	(0.129)	(0.188)	(0.093)	(0.116)	(0.130)	(0.087)	(0.110)	(0.126)
std(c_c)/std(y)	0.630	0.646	0.724	0.743	0.664	0.681	0.747	0.631	0.646	0.724
	(0.115)	(0.109)	(0.11)	(0.1)	(0.099)	(0.086)	(0.090)	(0.113)	(0.109)	(0.108)
std(h)/std(y)	0.328	0.237	0.060	0.059	0.334	0.244	0.061	0.312	0.237	0.060
	(0.069)	(0.046)	(0.01)	(0.009)	(0.067)	(0.043)	(0.011)	(0.070)	(0.045)	(0.011)
std(tot)/std(y)	1.088	1.259	0.947	1.020	1.159	1.306	0.980	1.059	1.258	0.946
	(0.156)	(0.210)	(0.172)	(0.196)	(0.172)	(0.224)	(0.181)	(0.157)	(0.210)	(0.172)
std(nx)/std(y)	0.365	0.113	0.3	0.039	0.399	0.177	0.323	0.405	0.130	0.342
	(0.052)	(0.019)	(0.054)	(0.008)	(0.059)	(0.030)	(0.060)	(0.060)	(0.022)	(0.062)

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Table 3.7: Estimated moments for case 1- 2nd set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Cross countries correlations										
corr(y(h),y(f))	-0.273	-0.081	0.076	0.203	-0.223	-0.027	0.113	-0.273	-0.080	0.076
	(0.212)	(0.227)	(0.225)	(0.218)	(0.217)	(0.228)	(0.224)	(0.212)	(0.227)	(0.225)
corr(c_c(h),c_c(f))	0.997	0.998	0.995	0.979	0.904	0.884	0.921	0.994	0.998	0.992
	(0.001)	(0.00)	(0.003)	(0.011)	(0.046)	(0.055)	(0.039)	(0.003)	(0.001)	(0.004)
corr(h(h),h(f))	0.899	0.991	0.987	0.993	0.951	0.99	0.994	0.899	0.991	0.987
	(0.048)	(0.004)	(0.007)	(0.00)	(0.024)	(0.006)	(0.004)	(0.048)	(0.005)	(0.007)
Varieties indicators (calculated on annual based values)										
corr(y,nv)	-0.759	-0.649	-0.796	-0.782	-0.750	-0.705	-0.831	-0.755	-0.650	-0.796
	(0.141)	(0.18)	(0.109)	(0.125)	(0.144)	(0.160)	(0.094)	(0.147)	(0.182)	(0.109)
corr(tot,nv)	0.250	0.04	0.947	0.027	0.213	0.097	0.933	0.254	0.042	0.169
	(0.277)	(0.292)	(0.027)	(0.285)	(0.281)	(0.29)	(0.039)	(0.275)	(0.292)	(0.034)
std(nv)/std(y)	0.365	0.221	0.169	0.046	0.373	0.223	0.131	0.346	0.217	0.187
	(0.07)	(0.051)	(0.035)	(0.016)	(0.072)	(0.048)	(0.025)	(0.073)	(0.050)	(0.015)
std(nv)/std(tot)	0.361	0.188	0.187	0.051	0.348	0.187	0.141	0.352	0.188	0.948
	(0.110)	(0.059)	(0.015)	(0.025)	(0.106)	(0.059)	(0.014)	(0.109)	(0.059)	(0.027)
corr(nv(h),nv(f))	0.858	0.987	-0.873	0.947	0.896	0.971	-0.786	0.849	0.987	-0.873
	(0.09)	(0.11)	(0.077)	(0.052)	(0.069)	(0.022)	(0.121)	(0.097)	(0.011)	(0.077)

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Figure 3.3: Shock in the consumption sector of "home" country, according to case 1. (1st Set)

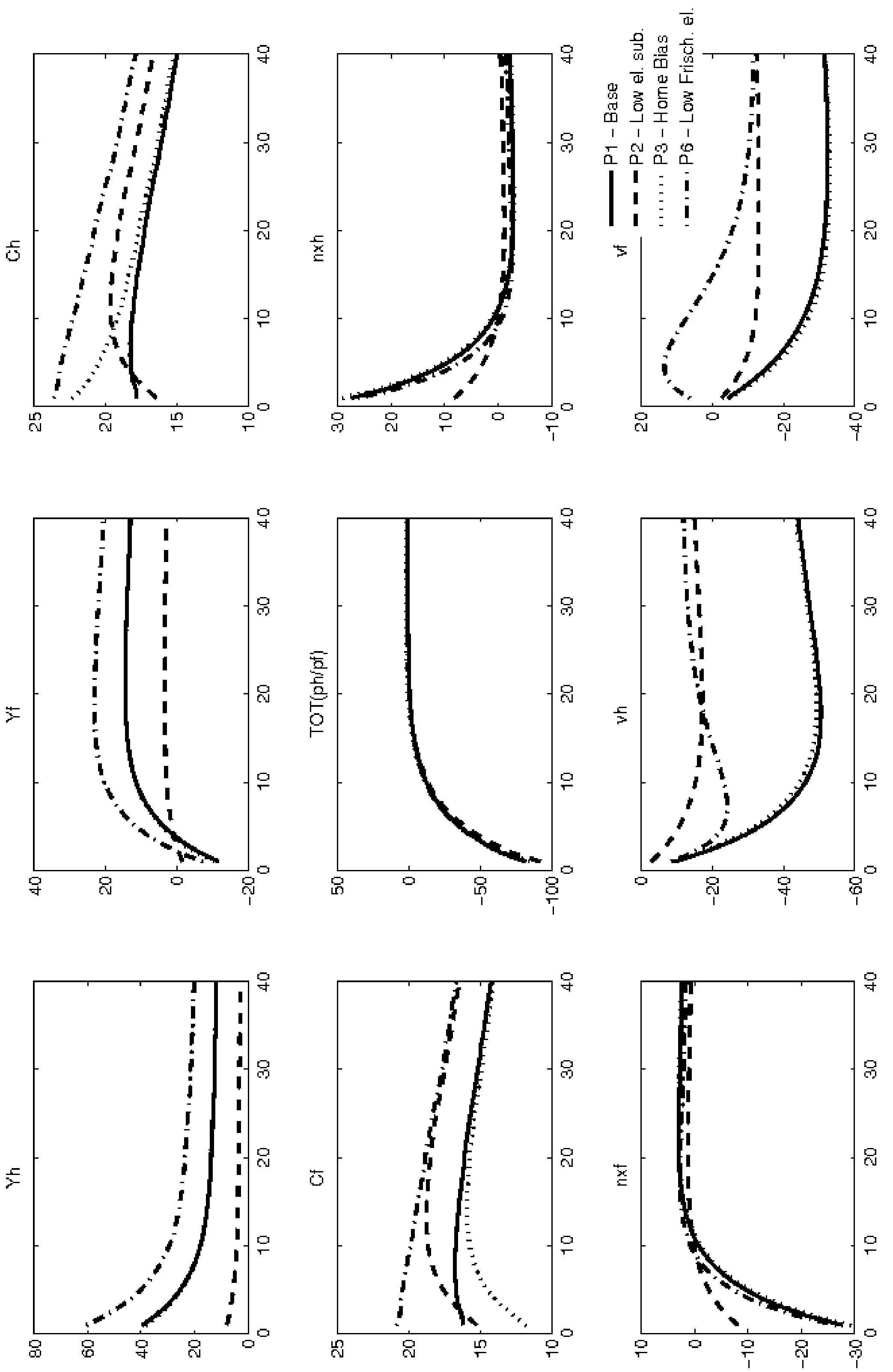

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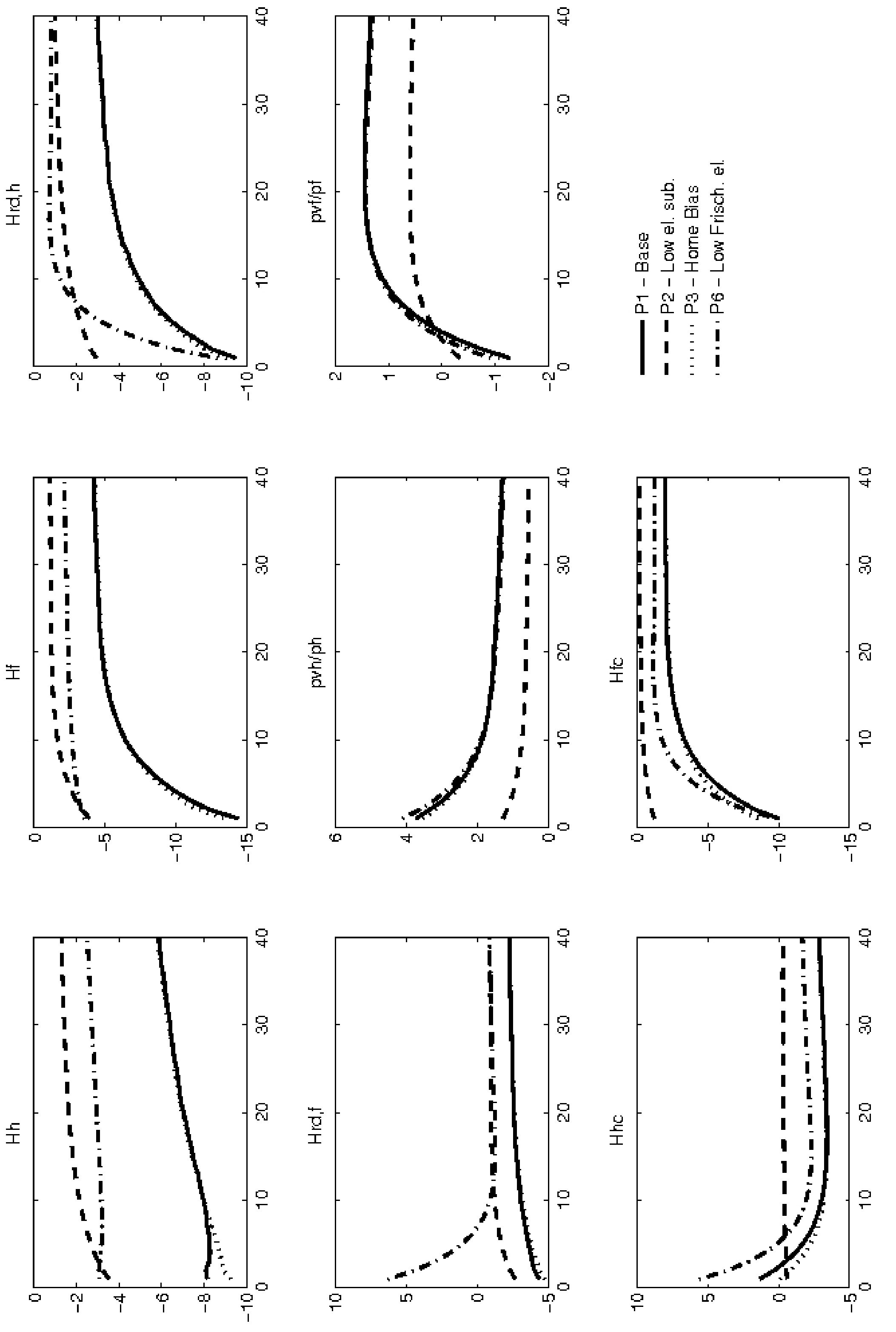
Figure 3.4: Shock in the consumption sector of "home" country, according to case 1. (2nd Set)



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hit by the positive shock will allocate a higher share of its expenditure to transportation services, reducing in this way the amount of effective consumption, in this case the cross correlations of consumptions between countries will be lower. With home bias both measures of consumption have almost the same variation, as the differences of shares between home and foreign goods consumed are due to the preferences of consumers and not to the need to buy an extra service: international transport.

A second issue is that a low Frisch elasticity combined with a high elasticity of substitution leads to a negative cross-correlation between new varieties. If we look at the impulse response functions [39](figures 3.3 and 3.4) we can note that in this case after a positive shock in the home sector of the consumption goods sector, there is an increase in workers in that sector and a decrease in the R&D sector. As in the foreign country (in spite of the productivity of the final goods sector increase afterwards) there is a decrease in the workers in the final good sector and an increase in the R&D sector. This means that after a shock the countries specialize in the sector in which they have a relative comparative advantage (a transitory Ricardian comparative advantage).

For low values of the elasticity of substitution the intertemporal substitution effect of allocating effort to R&D through time is not strong enough to counterbalance the income effect that leads to a reduction on hours worked in both sectors, affecting more the final good than the R&D. For that reason we see that hours worked in both sectors and in both countries decrease. Anyway, even if weak, the re-allocation of hours worked in R&D produces the result that the share of hours dedicated to it is lower in the country that was hit by the positive productivity shock than in the other one.

3.4.3.2 Case 2. Shocks just in the TFP of the R&D ("creativity" shocks) without international contamination

Due to the lack of data on new products introduced in the market for different economies we have to resort to judicious choices of the relevant parameters. First, we will suppose that the death rate of the shocks is twice as high as the death rate of shocks in the TFP of the consumption goods sector considered in the previous case.

$$\log(A_t) = (1 - B).\log(A_{SS}) + B * \log(A_{t-1)} + \varepsilon_t$$

[39] The IRFs depict relative response, therefore a 100 response is equal to one standard deviation of the shock (100%).

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with:

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix}$$

and:

$$\begin{aligned} E(\varepsilon_t^{H,RD}, \varepsilon_t^{F,RD}) &= 0, \forall\, j \in I \\ var(\varepsilon_t^H) &= var(\varepsilon_t^F) = 0 \\ var(\varepsilon_t^{H,RD}) &= var(\varepsilon_t^{F,RD}) \neq 0 \end{aligned}$$

From tables 3.8 and 3.9 the first striking fact, when comparing with the shocks in the previous case is that the volatilities of all the variables, relative to GDP are higher. This is due to the reduced response of output when the shocks are in the creative process rather than in the production of final goods.

In the previous case the home GDP response to a positive shock was unambiguously positive as the foreign GDP response was negative at the impact of the shock, increasing afterwards . In the home country this was because the increase in production of final goods but also because the decrease in R&D production due to the relocation of labour to the final goods sector was compensated in part by a high value of varieties in terms of final goods (remember that we are measuring output in terms of final goods). In the foreign country after the shock in the home country, the reverse happens. At the impact there is a decline in labor overall and a shift of labor to the R&D sector, decreasing production of final goods and as varieties worth less in terms of final goods GDP declines. Afterwards not only overall labor increase but also the value of new varieites increase leading to an increase of GDP measured in terms of final goods.

In the current case the positive responses are smaller and may even be negative. This is because the relocation of labour to the R&D sector and more production in that sector is mitigated by the lower value of varieties (in terms of final goods). In order to home GDP have a positive response (after a positive shock in the R&D productivity) the increase in R&D production has to be big enough to compensate the decrease in production in the final goods sector, as well as, the lower value of the produced varieties. The response is negative when we have a low Frisch and substitution elasticities. For the low Frisch elasticity almost all of the responses are due to the reallocation of labour between sectors and not from the

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Table 3.8: Estimated moments for case 2 - 1st set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Within Country moments										
corr(y,c_c)	-0.074	0.764	0.081	0.457	-0.259	0.754	0.455	-0.099	0.762	0.117
	(0.157)	(0.070)	(0.211)	(0.154)	(0.178)	(0.102)	(0.167)	(0.161)	(0.072)	(0.212)
corr(y,h)	0.325	0.121	-0.768	-0.479	0.478	0.256	-0.780	0.325	0.120	-0.768
	(0.188)	(0.224)	(0.090)	(0.181)	(0.168)	(0.194)	(0.088)	(0.188)	(0.224)	(0.091)
corr(y,tot)	-0.101	-0.248	-0.921	-0.853	-0.014	-0.236	-0.936	-0.101	-0.251	-0.921
	(0.182)	(0.207)	(0.021)	(0.061)	(0.198)	(0.212)	(0.020)	(0.182)	(0.207)	(0.021)
corr(y,nx)	0.535	0.385	0.933	0.723	0.335	0.112	0.917	0.535	0.383	0.931
	(0.156)	(0.211)	(0.02)	(0.083)	(0.194)	(0.241)	(0.025)	(0.157)	(0.212)	(0.021)
std(c_c)/std(y)	0.945	1.308	0.636	0.857	1.237	1.553	0.798	0.939	1.305	0.631
	(0.133)	(0.118)	(0.143)	(0.175)	(0.245)	(0.221)	(0.158)	(0.142)	(0.122)	(0.142)
std(h)/std(y)	2.487	1.838	0.2	0.228	2.739	1.745	0.225	2.489	1.834	0.200
	(0.512)	(0.426)	(0.028)	(0.043)	(0.544)	(0.361)	(0.032)	(0.512)	(0.425)	(0.028)
std(tot)/std(y)	4.417	3.879	3.285	3.438	4.875	3.915	3.759	4.429	3.883	3.293
	(0.825)	(0.877)	(0.154)	(0.341)	(1.036)	(0.940)	(0.190)	(0.829)	(0.877)	(0.154)
std(nx)/std(y)	2.156	0.925	1.35	0.464	2.423	1.299	1.570	2.447	1.053	1.531
	(0.404)	(0.217)	(0.059)	(0.044)	(0.523)	(0.331)	(0.079)	(0.459)	(0.248)	(0.067)

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Table 3.9: Estimated moments for case 2 - 2nd set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Cross countries correlations										
corr(y(h),y(f))	0.256	0.673	-0.932	-0.615	0.626	0.885	-0.910	0.260	0.674	-0.932
	(0.225)	(0.141)	(0.033)	(0.157)	(0.155)	(0.057)	(0.043)	(0.224)	(0.141)	(0.033)
corr(c_c(h),c_c(f))	0.941	0.927	0.938	0.714	0.486	0.543	0.595	0.978	0.938	0.973
	(0.030)	(0.412)	(0.03)	(0.161)	(0.205)	(0.209)	(0.162)	(0.011)	(0.035)	(0.013)
corr(h(h),h(f))	-0.240	-0.423	-0.105	-0.204	-0.189	-0.292	-0.067	-0.238	-0.421	-0.103
	(0.207)	(0.183)	(0.218)	(0.227)	(0.210)	(0.202)	(0.219)	(0.207)	(0.184)	(0.218)
Varieties indicators (calculated on annual based values)										
corr(y,nv)	0.138	-0.129	-0.426	-0.738	0.284	-0.095	-0.901	0.140	-0.130	-0.899
	(0.270)	(0.272)	(0.245)	(0.13)	(0.257)	(0.235)	(0.056)	(0.270)	(0.271)	(0.056)
corr(tot,nv)	0.806	0.809	0.82	0.736	0.797	0.778	0.813	0.806	0.809	0.820
	(0.100)	(0.102)	(0.096)	(0.131)	(0.106)	(0.115)	(0.101)	(0.100)	(0.102)	(0.096)
std(nv)./std(y)	9.946	4.727	5.228	3.93	10.909	4.609	5.728	9.960	4.725	5.230
	(2.958)	(1.423)	(0.672)	(0.738)	(3.239)	(1.206)	(0.741)	(2.963)	(1.420)	(0.672)
std(nv)./std(tot)	2.379	1.387	1.549	1.17	2.383	1.393	1.504	2.375	1.384	1.547
	(0.406)	(0.241)	(0.261)	(0.327)	(0.425)	(0.258)	(0.264)	(0.406)	(0.241)	(0.261)
corr(nv(h),nv(f))	-0.406	-0.377	-0.426	-0.168	-0.348	-0.292	-0.377	-0.405	-0.376	-0.426
	(0.247)	(0.253)	(0.245)	(0.285)	(0.258)	(0.268)	(0.255)	(0.247)	(0.253)	(0.245)

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Figure 3.5: Shock in the R&D sector of "home" country, according case 2. (1st Set)

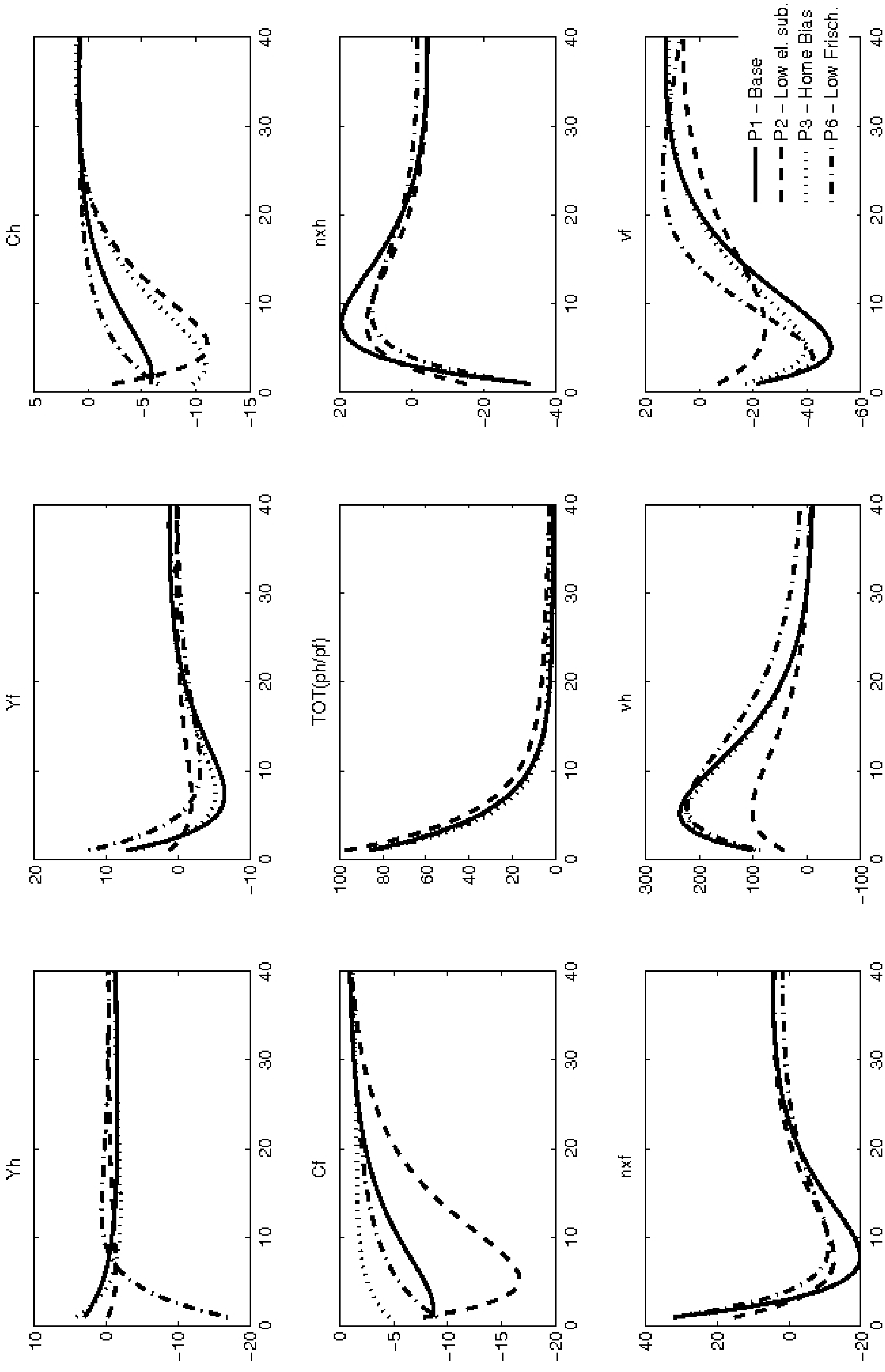

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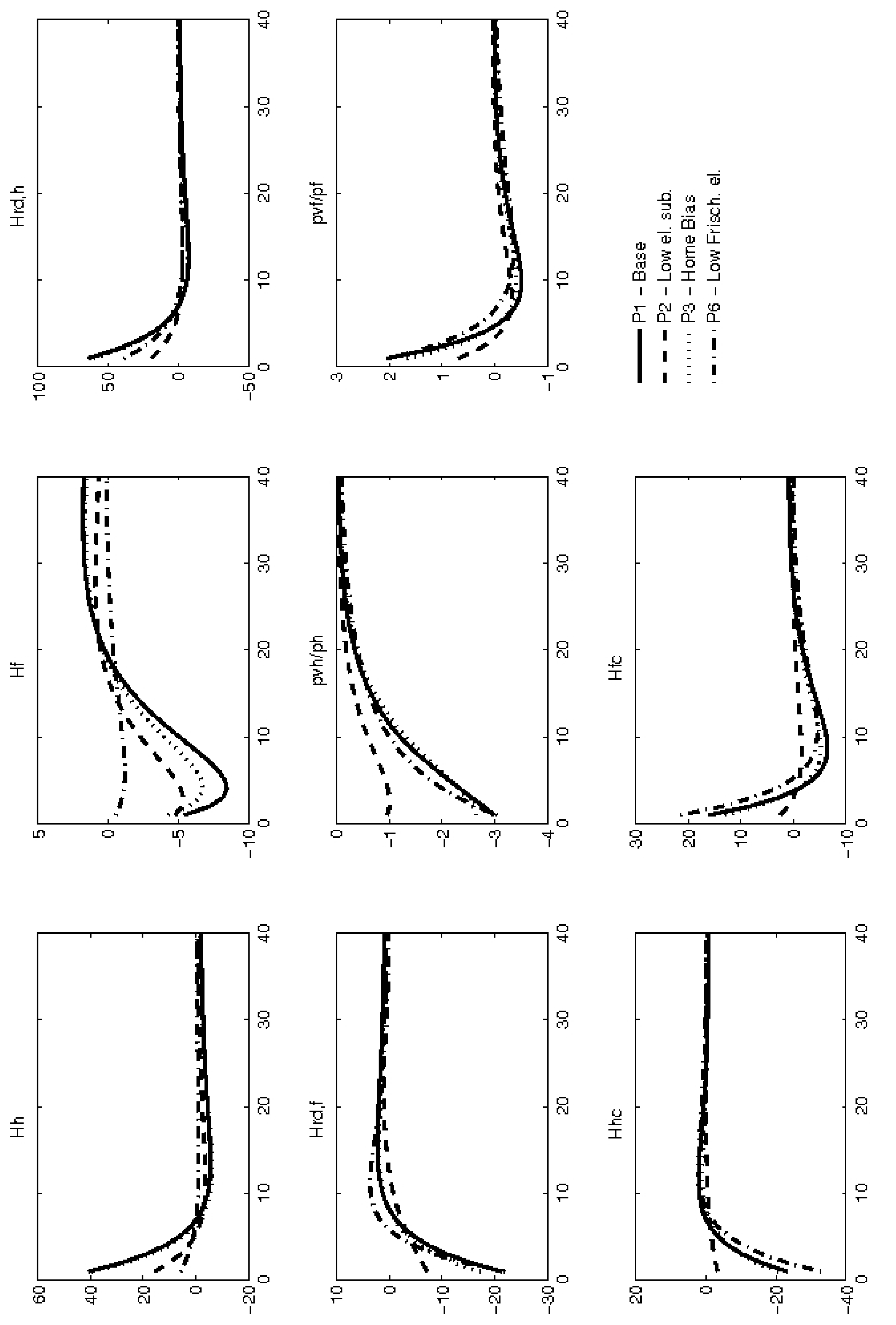
Figure 3.6: Shock in the R&D sector of "home" country, according case 2. (2nd Set)



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variation of hours worked, the increase production in R&D is completely surpassed by the decrease in final goods and the decrease in the value of new varieties and, therefore, after a positive shock in R&D productivity we have a negative response of output. As for the low elasticity of substitution the preference for varieties is so high that in fact in the steady state most people are working in R&D, therefore the positive shock leads to a very marginal reallocation of hours worked, but anyway to a decrease in the relative value of varieties (even if smaller than in other cases), with production levels in each sector almost unaltered as the value of production in R&D diminishes, leading to a marginal negative response of output.

The foreign GDP also reverses the responses of the previous case. Initially increases as labor in final good sector increases due to a shift from the R&D sector to the final goods sector, although labor overall decrease, and because the price of new varieties increase compensating the decrease in production in the R&D sector. Afterwards those initially responses the variables revert to the initially values, leading to a decline on foreign GDP, and even reaching a lower level than at the moment of the shock. This might happen because the number of varieties is inferior to the steady state, labor moves from the final goods sector to the R&D sector leading to a decline in final goods production and an increase in varieites, but as the price of varieites is declining the increase in production in R&D does not compensante in terms of value the decline in the final good sector, so measured foreign GDP in terms of equivalent final goods will be for a period lower than at the moment of the shock.

A second difference is in the response of consumption. In this case consumption decreases after a positive shock. This is due to two factors: first, home consumers reduce consumption at moment zero, to invest in R&D expenditure and, hence more varieties. Second, the agents do a trade off between quantity and diversity. After a positive shock, they prefer less quantity of each good (and overall) to consume a bigger array of goods. In the foreign country (except in the case of home-bias) the negative response of consumption is even higher. That is due to the fact that the price of home goods is higher relative to foreign ones (the home country produces less of each variety and foreign country produces more, therefore the TOT of home country increase), therefore foreign consumers in order to import home goods and benefit from the increased number of varieties have to give away a higher quantity of own produced goods, therefore the measured consumption decreases.

With the introduction of a home bias, each country takes more utility from its own goods. In this case the home country will have to reduce consumption in order to finance R&D, as it cannot finance it by selling bonds and continuing to consume foreign goods (nor do home agents derive that much utility from foreign goods, nor are foreign agents interested

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because they cannot take as much utility from the increased number of varieties). As we will see this latter fact is going to be crucial when comparing the cross-correlations of GDP and consumption.

A third result is that the cross correlations of output, completely invert the correlations found on the previous table and are more extreme. When the Frisch elasticity is high the correlation is positive. As we said, the responses are mainly due to reallocation of labour across sectors, that leads to a reduction of the GDP of the country hitten by the positive shock and an increase of GDP in the other one. When the Frisch elasticity of labour is high, the increase in hours worked in the country hit by the shock allows that we have more people in the R&D sector without sacrificing much production in the other sector, and, therefore the output response is positive, hence a positive cross country correlation of output. As for the cross-country correlations of consumption we can see that those are usually lower than the ones found in the previous case, reflecting to some extent that consumers cannot use the risk sharing mechanisms as well as before because as the shock is in the R&D sector it turns more acute the decision between quantity and diveristy. However in most cases the variation on this mix is not strong enough to revert the comparision between the cross country correlations of GDP and consumption. In fact, without home bias, foreign consumers continue to value diversity high enough to export increased quantities of goods to have access to more varieites. Is only when there is a home bias and therefore each country value more the goods it produces, that home country financing of R&D has to be done at the expense of consumption, therefore they prefer diversity to quantitiy. In the foreign country agents do not care about the increased diversity of goods from the home country and expands consumption of own goods in terms of quantitiy, hence prefering quantitiy to diversity. In this case the cross country correlation of consumption is reduced to the point that this one is lower than the cross country correlation of output (5th and 6th columns of table 3.9), producing therefore the consumption correlation puzzle. However as noted before, the volatilities of the variables relative to output are quite high.

As for the correlations of the TOT and net exports with output they are respectively negative[40] and positive, as before. However in many cases the estimated value is not significant, and therefore, together with higher volatilities of those variables relative to output there is some degree of success in replicating the price anomaly. The case described be-

[40] In Corsetti et al.(2005) they got a positive long run impact of these kind of shocks in TOT $\left(\frac{p_h(s)}{p_f(s)}\right)$ and in relative output, which would suggests a positive correlation between those variables. However that result is the long-run steady state response to permanent shocks. As for temporary shocks the TOT jump immediately to a higher level decreasing afterwards and output has the tendency, in most cases, to decrease slightly when the shock hits the economy increasing afterwards before returning to equilibirum. These initial movements are the responsible for finding in the cases presented negative correlations.

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fore that replicates the consumption correlation puzzle is also the one that has the lower correlation of TOT with output.

However the high volatilities of consumption and hours worked relative to output , the negative output-consumption correlation (for high values of the elasticity of substitution) and the negative cross country correlation of hours worked are at odds with the data and the previous apparent success in replicating the anomalies is achieved at expenses of obtaining other counterfactual moments.

3.4.3.3 Case 3. Shocks just in the R&D sector ("creativity" shocks) with strong negative contamination

In this case we will consider that a country that has a shock in its creativeness process will have a negative impact on the introduction of new varieties by the other country.

We will also consider that this advantage (disadvantage) is long lasting, so the subsystem $A_t^{*,RD} = (1-B) * A_{t-1}^{*,RD} + \varepsilon_t^{*,RD}$ is near to a unit root.

$$\log(A_t) = B * \log(A_{t-1}) + \varepsilon_t$$

with:

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \ \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & -0.19 \\ 0 & 0 & -0.19 & 0.8 \end{bmatrix}$$

and

$$\begin{aligned} E(\varepsilon_t^{H,RD}, \varepsilon_{t-j}^{F,RD}) &= 0 \ , \forall \ j \ \in I \\ var(\varepsilon_t^H) &= var(\varepsilon_t^F) = 0 \\ var(\varepsilon_t^{H,RD}) &= var(\varepsilon_t^{F,RD}) \neq 0 \end{aligned}$$

The idea is that if one country has a positive(negative) shock and introduces in that period more(less) varieties than normal it will stay *one step ahead* (behind) in the innovation race[41], therefore the scope of creativity on the other country decreases (increases) as there are less (more) market opportunities available. Figure 3.7 shows this idea. In period one both countries create the same number of varieties (in the scheme the home country explores the

[41] The innovation race is characterized by a continuous upgrading of existing products; there are no innovative *jumps.*

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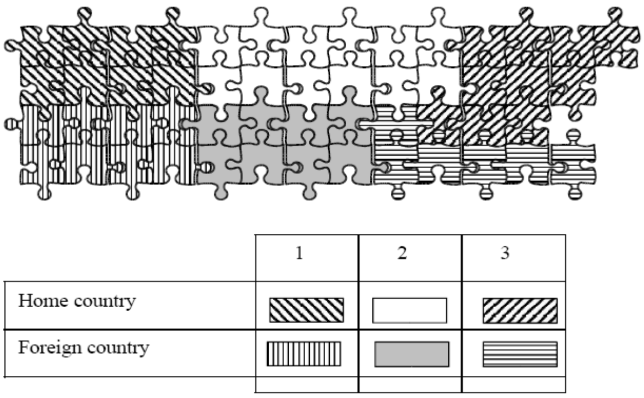
first two lines of the puzzle as the foreign country explores the two bottom lines), therefore the initial range of possibilities on both countries is equal in period 2 (adding puzzle pieces in front of the previous). In period two, the home country has a positive creative shock, therefore it not only introduces more varieties but also increases the range of possibilities to create in the subsequent period (in the figure more space to add puzzle pieces). When it takes advantage of that increased scope (in period three it adds puzzle pieces in the third row) it reduces the creativeness range for the foreign country (in the figure it can just add puzzle pieces in the fourth line).

Figure 3.7: Contamination schematics



From the IRFs (figures 3.8 and 3.9) the first fact worth noting is that after the initial positive shock both countries increase the amount of hours worked in R&D. If this increase is normal in the country hit by the positive shock, the other one will increase because it knows that in the future the productivity will be lower. There is kind of "rush to innovation", where the foreign country tries not to lose too much in relation to the home country. The other main difference is the response of home output, that in this case is always negative; this is due to the fact that the relative price of varieties in terms of the final good decreases more than in the previous case, making that the value of R&D output less valuable in terms of final goods. This increased devaluation in the value of varieties is due to a much higher persistence that increases not only the response of varieties in both countries but keeps them away from the long run steady state values for a longer period.

From tables 3.10 and 3.11 we can realize that most of the moments are at odds with data, the only exception being the output - hours worked correlation for high values of the Frisch elasticity and the cross country correlation of hours worked (both are positive). The

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Table 3.10: Estimated moments for case 3 - 1st set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Within Country moments										
corr(y,c_c)	-0.579	0.041	-0.461	0.461	-0.526	0.091	-0.454	-0.585	0.031	-0.476
	(0.100)	(0.213)	(0.158)	(0.154)	(0.151)	(0.237)	(0.171)	(0.102)	(0.215)	(0.156)
corr(y,h)	0.469	0.354	-0.189	-0.175	0.555	0.631	-0.023	0.474	0.365	-0.156
	(0.159)	(0.156)	(0.190)	(0.172)	(0.136)	(0.084)	(0.193)	(0.158)	(0.154)	(0.192)
corr(y,tot)	-0.670	-0.689	-0.892	-0.643	-0.679	-0.730	-0.918	-0.676	-0.694	-0.895
	(0.102)	(0.116)	(0.028)	(0.116)	(0.108)	(0.109)	(0.025)	(0.101)	(0.114)	(0.027)
corr(y,nx)	0.751	0.709	0.964	0.730	0.678	0.624	0.953	0.750	0.707	0.963
	(0.092)	(0.113)	(0.014)	(0.101)	(0.109)	(0.127)	(0.018)	(0.091)	(0.113)	(0.013)
std(c_c)/std(y)	0.940	1.177)	1.071	1.395	0.897	1.156	1.133	0.924	1.171	1.065
	(0.103)	(0.249	(0.19)	(0.24)	(0.146)	(0.270)	(0.221)	(0.105)	(0.250)	(0.190)
std(h)/std(y)	1.311	0.904	0.164	0.262	1.408	0.920	0.189	1.302	0.895	0.181
	(0.238)	(0.164)	(0.036)	(0.047)	(0.234)	(0.114)	(0.040)	(0.236)	(0.161)	(0.040)
std(tot)/std(y)	2.794	2.871	3.056	2.912	3.291	3.154	3.911	2.806	2.865	3.091
	(0.403)	(0.471)	(0.17)	(0.433)	(0.506)	(0.516)	(0.225)	(0.403)	(0.469)	(0.168)
std(nx)/std(y)	1.615	0.982	1.825	0.754	1.684	1.050	2.124	1.786	1.071	2.033
	(0.227)	(0.160)	(0.087)	(0.108)	(0.257)	(0.173)	(0.114)	(0.250)	(0.174)	(0.095)

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Table 3.11: Estimated moments for case 3 - 2nd set

θ	1,7	1,2	1,7	1,2	1,7	1,2	1,7	1,7	1,2	1,7
T	0	0	0	0	0	0	0	0,1	0,1	0,1
bias	0	0	0	0	0,2	0,2	0,2	0	0	0
ϖ	1	1	10	10	1	1	10	1	1	10
Cross country correlations										
corr(y(h),y(f))	-0.166	-0.076	-0.908	-0.272	-0.027	-0.082	-0.883	-0.174	-0.087	-0.914
	(0.210)	(0.23)	(0.043)	(0.218)	(0.214)	(0.230)	(0.053)	(0.209)	(0.230)	(0.040)
corr(c_c(h),c_c(f))	0.205	0.38	0.012	0.563	0.556	0.424	0.143	0.237	0.379	0.021
	(0.218)	(0.211)	(0.218)	(0.184)	(0.159)	(0.203)	(0.212)	(0.214)	(0.211)	(0.218)
corr(h(h),h(f))	0.939	0.926	0.992	0.63	0.961	0.834	0.903	0.946	0.938	0.994
	(0.028)	(0.034)	(0.004)	(0.143)	(0.018)	(0.073)	(0.044)	(0.024)	(0.028)	(0.003)
Varieties indicators (calculated on annual based values)										
corr(y,nv)	-0.271	-0.424	-0.905	-0.701	-0.208	-0.382	-0.910	-0.274	-0.426	-0.901
	(0.259)	(0.223)	(0.048)	(0.151)	(0.262)	(0.207)	(0.047)	(0.258)	(0.222)	(0.051)
corr(tot,nv)	0.721	0.74	0.846	0.756	0.698	0.651	0.839	0.719	0.737	0.841
	(0.133)	(0.13)	(0.081)	(0.11)	(0.143)	(0.161)	(0.087)	(0.134)	(0.131)	(0.083)
std(nv)./std(y)	6.308	3.17	5.23	5.703	6.537	2.746	5.574	6.254	3.134	5.233
	(1.667)	(0.799)	(0.567)	(1.212)	(1.707)	(0.632)	(0.612)	(1.660)	(0.786)	(0.587)
std(nv)./std(tot)	2.055	1.076	1.642	1.897	1.820	0.873	1.384	2.029	1.066	1.624
	(0.404)	(0.208)	(0.243)	(0.326)	(0.375)	(0.193)	(0.214)	(0.400)	(0.207)	(0.243)
corr(nv(h),nv(f))	-0.148	-0.154	-0.515	-0.357	-0.065	0.082	-0.464	-0.143	-0.145	-0.507
	(0.276)	(0.275)	(0.216)	(0.251)	(0.280)	(0.279)	(0.229)	(0.276)	(0.275)	(0.218)

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Figure 3.8: Shock in the R&D sector of "home" country, according case 3. (1st Set)

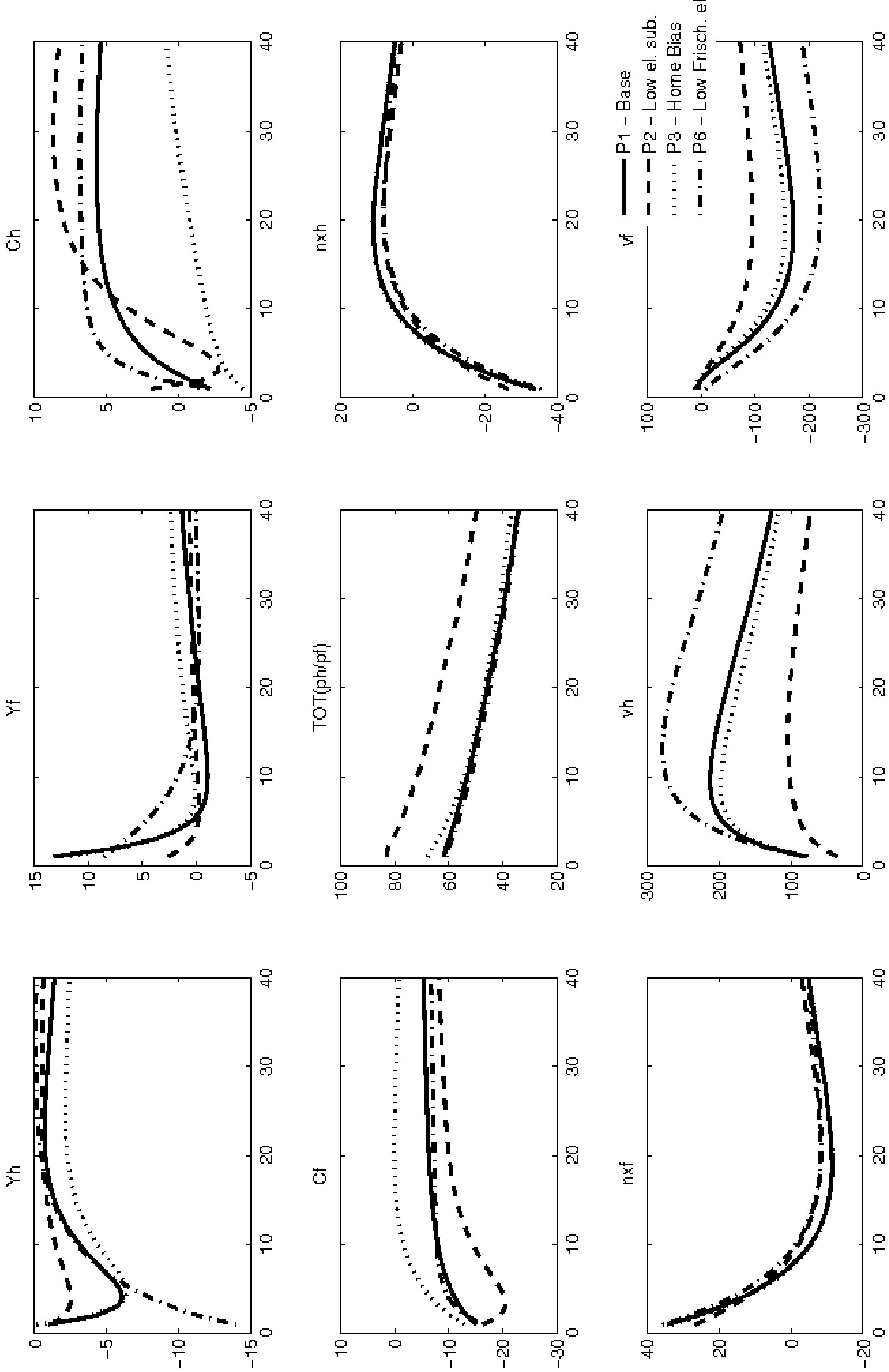

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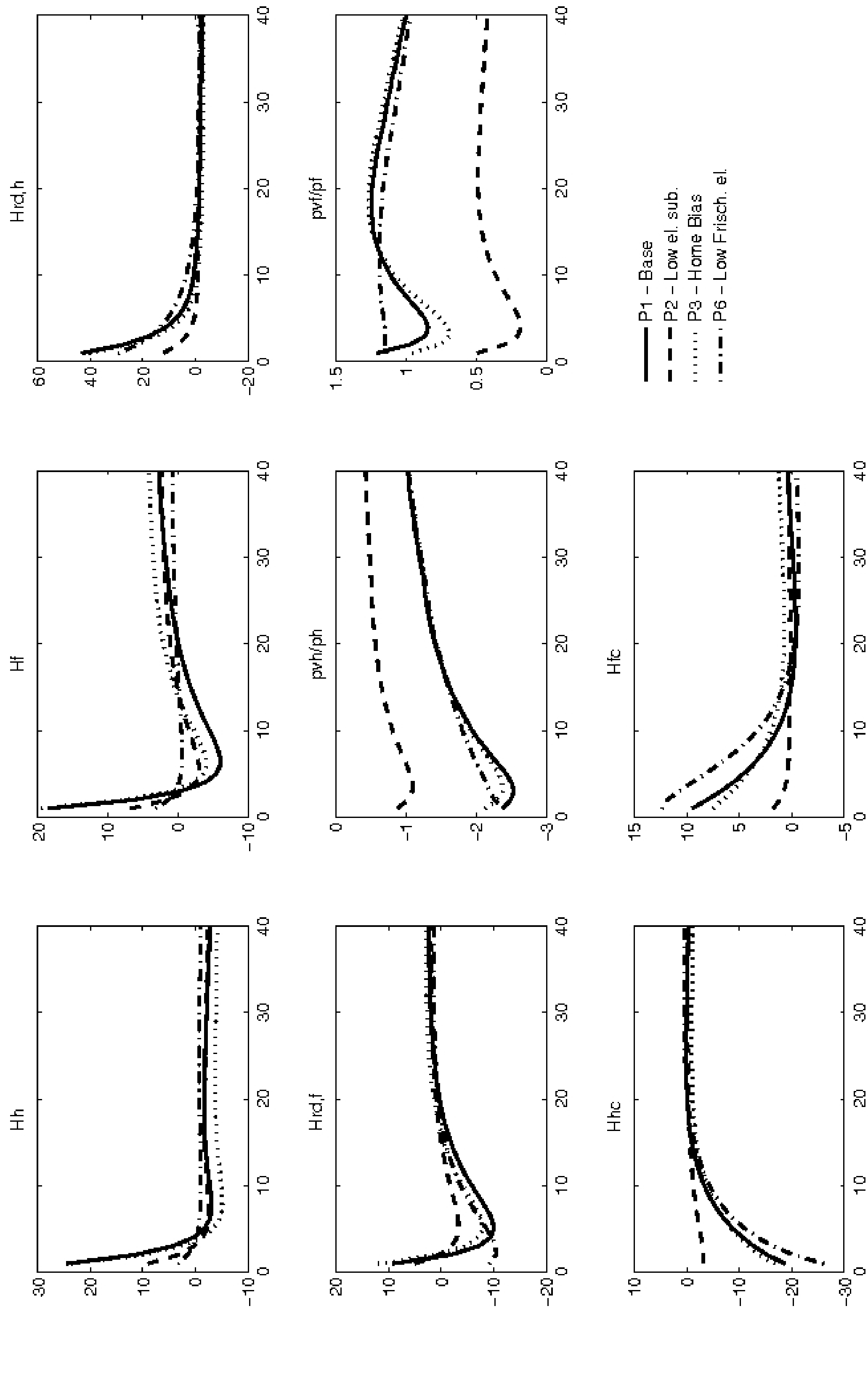
Figure 3.9: Shock in the R&D sector of "home" country, according case 3. (2nd Set)



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remaining correlations are wrongly signed or weaker/stronger than in the data. As for the relative size of the volatilities, the output volatility is low which produces the result that consumption and hours worked are much more volatile in respect to output in the model than they are in the data.

3.4.3.4 Case 4 - Shocks in both sectors

Finally we tried some combinations of the stochastic shocks in both sectors. Before proceeding to comment the results we should advert to the reason why a certain combinations of both might solve the consumption correlation puzzle and the price anomaly.

Comparing the IRFs (figures 3.3 and 3.4) from the shock in the consumption goods sector with the IRFs (figures 3.5, 3.6, 3.8 and 3.9) from the shock in the R&D sector we can see that on all the variables the shocks lead to variable responses that are in general the opposite, however the intensity of the response differs. This difference in intensity might be such that for certain combinations of the correlation of the shocks and of the relative volatility of them might be able to replicate some of the puzzles found in data. The next two subsections will combine the shocks in the consumption sector according to case 1 with the two studied for the R&D sector (case 2 and case 3).

Case 4.1 - Case 1 and 2 The parameterizations tested for the stochastic process were:

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.906 & 0.088 & 0 & 0 \\ 0.088 & 0.906 & 0 & 0 \\ 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0.8 \end{bmatrix}$$

$$\begin{aligned} E(\varepsilon_t^H, \varepsilon_t^F) &= 0,258 \text{ and } E(\varepsilon_t^H, \varepsilon_{t-j}^F) = 0,\ j \neq 0 \\ E(\varepsilon_t^{H,RD}, \varepsilon_{t-j}^{F,RD}) &= 0.,\ \text{for } \forall\ j \in\ I. \\ var(\varepsilon_t^H) &= var(\varepsilon_t^F) \neq 0 \\ var(\varepsilon_t^{H,RD}) &= var(\varepsilon_t^{F,RD}) \neq 0 \\ & \text{no trade costs and} \\ E(\varepsilon_t^*, \varepsilon_t^{*,RD}) &= 0,5\,;\ 0.85 \\ \frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)} &= 3;1 \end{aligned}$$

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In this parameterization the two added parameters were the correlation between shocks on both sectors and the relative volatility of them. As for the first we considered a positive correlation as part of the shocks can be the outcome of events common to both sectors. As for the second we tested for equal volatilities and for a stronger volatility in the R&D sector. Anyway, in principle the creative process of developing a new variety should be more volatile than the process of producing it.

From tables 3.12 and 3.13 we can see that the cross country correlations for output and hours worked are negative (with the exception of either the Frisch elasticity or the elasticity of substitution are low), and furthermore the cross country correlations of consumption are higher than those for output. Neither of these results is in line with the correlations found in the data where the cross correlations are positive and that of output is higher than that of consumption.

As for the within country correlations and those regarding the new varieties the ones that follow closer the data are the cases where the volatility of the shock in both sectors is equal. However, even in this case the TOT are less volatile in the model than in the data.

When we increase the volatility of the shocks in the R&D sector we increase the volatility of the TOT but reduce the ouptut-consumption correlation and increase the volatility of the hours worked when the Frisch elasticity ($\frac{1}{\varpi}$) is low. When the Frisch elasticity is higher, we get values for those moments closer to the data, however the output-hours worked correlation is negative.

Overall, we can see that we are never able to replicate the consumption correlation puzzle and even if we can replicate the price puzzle that is done at the expense of conterfactual moments for other variables.

Case 4.2 - Case 1 and 3 The parameterizations tested were:

$$A_t = \begin{bmatrix} A_t^H \\ A_t^F \\ A_t^{H,RD} \\ A_t^{F,RD} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_t^F \\ \varepsilon_t^H \\ \varepsilon_t^{H,RD} \\ \varepsilon_t^{F,RD} \end{bmatrix} \text{ and } B = \begin{bmatrix} 0.906 & 0.088 & 0 & 0 \\ 0.088 & 0.906 & 0 & 0 \\ 0 & 0 & 0.8 & -0.19 \\ 0 & 0 & -0.19 & 0.8 \end{bmatrix}$$

From tables 3.14 and 3.15 we can see that in columns one to three and in the last two the cross country correlation of consumption is lower than the one of output. The necessary condition seem to be that shocks in both sectors have to be highly correlated. Furthermore for equally sized shocks in both sectors the elasticity of substitution has to be low (1,2) but as the volatility of shocks in R&D relative to those in the final goods sector increases the consumption correlation puzzle is replicated for higher values of elasticity of substitution.

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Table 3.12: Estimated moments for case 4.1 - 1st set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
$bias$	0.2	0	0.2	0.2	0.2	0.2	0.2
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Within Country moments							
corr(y,c_c)	0.184	0.469	0.801	0.543	0.306	0.474	0.621
	(0.230)	(0.187)	(0.087)	(0.172)	(0.210)	(0.156)	(0.145)
corr(y,h)	0.660	0.649	-0.199	0.500	0.435	0.615	0.419
	(0.117)	(0.116)	(0.195)	(0.163)	(0.180)	(0.140)	(0.182)
corr(y,tot)	0.484	0.490	0.024	0.002	0.040	0.395	-0.002
	(0.169)	(0.166)	(0.218)	(0.219)	(0.217)	(0.186)	(0.216)
corr(y,nx)	0.047	0.111	0.160	0.211	0.183	-0.046	0.045
	(0.187)	(0.184)	(0.225)	(0.181)	(0.202)	(0.198)	(0.194)
std(c_c)/std(y)	0.467	0.430	0.768	0.497	0.617	0.897	0.648
	(0.108)	(0.090)	(0.108)	(0.099)	(0.139)	(0.164)	(0.113)
std(h)/std(y)	1.107	1.140	0.162	0.714	1.147	1.220	0.629
	(0.165)	(0.167)	(0.032)	(0.131)	(0.222)	(0.215)	(0.138)
std(tot)/std(y)	1.096	1.162	1.523	0.470	1.656	1.535	0.746
	(0.201)	(0.211)	(0.339)	(0.103)	(0.361)	(0.310)	(0.150)
std(nx)/std(y)	0.844	0.841	0.825	0.510	0.939	0.856	0.500
	(0.147)	(0.142)	(0.184)	(0.091)	(0.191)	(0.177)	(0.099)

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Table 3.13: Estimated moments for case 4.1 - 2nd set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
$bias$	0.2	0	0.2	0.2	0.2	0.2	0.2
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^{*})}$	3	3	3	1	3	3	1
$E(\varepsilon_t^{*},\varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Cross country correlations							
corr(y(h),y(f))	-0.069	-0.102	0.609	-0.158	-0.067	0.226	0.016
	(0.225)	(0.228)	(0.147)	(0.220)	(0.222)	(0.215)	(0.224)
corr(c_c(h),c_c(f))	0.692	0.903	0.946	0.907	0.717	0.425	0.603
	(0.134)	(0.046)	(0.027)	(0.043)	(0.124)	(0.216)	(0.153)
corr(h(h),h(f))	-0.333	-0.395	-0.075	-0.318	-0.251	-0.325	-0.263
	(0.195)	(0.186)	(0.226)	(0.201)	(0.200)	(0.199)	(0.211)
Varieties indicators (calculated on annual based values)							
corr(y,nv)	0.739	0.691	0.331	0.698	0.475	0.632	0.646
	(0.124)	(0.133)	(0.228)	(0.141)	(0.221)	(0.179)	(0.174)
corr(tot,nv)	0.769	0.786	0.768	0.332	0.698	0.742	0.300
	(0.123)	(0.112)	(0.123)	(0.251)	(0.148)	(0.131)	(0.254)
std(nv)./std(y)	4.327	4.384	3.516	2.600	4.482	3.589	2.090
	(0.765)	(0.791)	(0.928)	(0.490)	(1.145)	(0.842)	(0.477)
std(nv)./std(tot)	4.216	4.011	2.558	6.007	2.872	2.480	3.573
	(0.775)	(0.700)	(0.471)	(1.624)	(0.597)	(0.471)	(0.983)
corr(nv(h),nv(f))	-0.413	-0.466	-0.364	-0.441	-0.393	-0.362	-0.395
	(0.233	(0.221)	(0.243)	(0.227)	(0.238)	(0.243)	(0.237)

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Table 3.14: Estimated moments for case 4.2 - 1st set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
$bias$	0.2	0	0.2	0.2	0.2	0.2	0.2
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Within Country moments							
corr(y,c_c)	0.520	0.446	0.818	0.809	0.434	0.467	0.809
	(0.159)	(0.167)	(0.071)	(0.077)	(0.184)	(0.173)	(0.080)
corr(y,h)	0.342	0.353	-0.517	0.002	0.222	0.391	-0.100
	(0.186)	(0.187)	(0.144)	(0.215)	(0.211)	(0.187)	(0.207)
corr(y,tot)	0.005	-0.009	-0.038	-0.328	-0.377	-0.263	-0.281
	(0.202)	(0.204)	(0.227)	(0.178)	(0.187)	(0.190)	(0.186)
corr(y,nx)	-0.279	-0.245	-0.168	-0.232	0.204	-0.114	-0.364
	(0.200)	(0.203)	(0.221)	(0.214)	(0.213)	(0.201)	(0.192)
std(c_c)/std(y)	0.767	0.790	1.025	0.690	0.754	1.204	0.878
	(0.140)	(0.151)	(0.124)	(0.087)	(0.150)	(0.237)	(0.112)
std(h)/std(y)	0.962	0.983	0.122	0.589	0.870	1.033	0.641
	(0.183)	(0.186)	(0.020)	(0.123)	(0.185)	(0.211)	(0.139)
std(tot)/std(y)	1.416	1.322	1.457	0.793	1.734	2.340	1.184
	(0.312)	(0.300)	(0.352)	(0.161)	(0.353)	(0.489)	(0.254)
std(nx)/std(y)	0.768	0.843	0.881	0.296	0.863	0.970	0.479
	(0.162)	(0.180)	(0.204)	(0.063)	(0.181)	(0.202)	(0.099)

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Table 3.15: Estimated moments for case 4.2 - 2nd set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
$bias$	0.2	0	0.2	0.2	0.2	0.2	0.2
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Cross country correlations							
corr(y(h),y(f))	0.455 (0.168)	0.483 (0.161)	0.640 (0.134)	0.128 (0.212)	0.127 (0.209)	0.484 (0.171)	0.298 (0.199)
corr(c_c(h),c_c(f))	0.299 (0.210)	0.252 (0.219)	0.294 (0.212)	0.565 (0.160)	0.539 (0.162)	-0.057 (0.232)	0.101 (0.227)
corr(h(h),h(f))	0.918 (0.036)	0.869 (0.054)	0.905 (0.048)	0.896 (0.046)	0.935 (0.028)	0.810 (0.085)	0.814 (0.085)
Varieties indicators (calculated on annual based values)							
corr(y,nv)	0.502 (0.216)	0.497 (0.217)	0.238 (0.264)	0.466 (0.229)	0.173 (0.277)	0.131 (0.255)	0.255 (0.262)
corr(tot,nv)	0.547 (0.166)	0.525 (0.162)	0.689 (0.121)	0.160 (0.217)	0.573 (0.180)	0.587 (0.169)	0.356 (0.201)
std(nv)./std(y)	4.439 (1.138)	4.727 (1.220)	3.124 (0.958)	2.628 (0.672)	3.848 (1.090)	3.121 (0.828)	1.769 (0.506)
std(nv)./std(tot)	2.667 (0.608)	2.984 (0.673)	1.997 (0.377)	2.334 (0.611)	2.068 (0.473)	1.207 (0.276)	1.332 (0.338)
corr(nv(h),nv(f))	-0.098 (0.275)	-0.179 (0.269)	-0.448 (0.226)	-0.103 (0.276)	-0.088 (0.276)	0.043 (0.279)	0.028 (0.280)

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Focusing on those columns we can check what happens to the price anomaly. First we see that in all those columns (1 to 3, 6 and 7) the values reported for the corr(y,TOT) are not significantly different from zero. Anyway those closer to zero are the first three columns. (High elasticity of substitution, high correlation between the shocks of the two sectors, relative high volatility of the shocks hitting R&D when compared with the volatility of the shocks of the other sector). Looking to the relative volatilities we can see that in all of them the TOT are more volatile than output as the net exports are slightly less volatile as output. Also here they go in the direction indicated by data, even if the volatility of the TOT is not as high as in the data in general and the one of net trade is not as low as the one found for US but relatively equal to the ones found for other countries.

Furthermore in two cases the correlation output/hours worked is negative. One is when the elasticity of substitution is low with equal sized shocks and the other when the Frisch elasticity ($\frac{1}{\varpi}$) is low. That keeps the first two columns and the 6th, as possible cases even if the correlations hours worked/GDP are lower than in data and the cross country correlation of hours worked is positive but quite high. Therefore it seems that, although correctly signed, there is an issue in the labour market that is not well captured by the model.

Finally looking at the annual moments in respect of new varieties the use of registered trademarks as proxy was quite uninformative about correlations, as they seem to be quite diverse. The only indication was that the volatility of them should be higher than the volatility of output. In the model all the parameterizations replicate the volatility size of new varieties, but show a positive correlation of this variable with output and TOT which is at odds with data, even if in some cases the zero is not away more than 2 std of the value found.

In conclusion we might say that with a high Frisch elasticity, volatility of the shocks in R&D higher than the shocks in the other sector and with a strong correlation between the shocks in the two sectors that the stochastic processes of cases 1 and 3 seem to, at the same time, replicate the consumption correlation puzzle and the price anomaly[42]. There are only two results that do not connect well with the data. The first has to do with the labour market where the cross-country correlations of labour, and the correlation of this one with output, although correctly signed, are too strong or weak, respectively, when compared with data (even that if we increase the wage elasticity of labor - lower ϖ - to values around 2 we get closer to the data moments). The second has to do with the correlation between new varieties and the ratio of output to the TOT. In these cases both are positive in the model,

[42] As said before, different values for φ^V do not change the results qualitatively. For further details see appendix 3.F

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but from data we should expect something closer to zero and/or larger variances.

3.5 Conclusion and future directions

Many contributions have attempted to solve the consumption correlation puzzle and the price anomaly, however most of them were directed to one of the two issues. In this paper we proposed a model where we allowed for the existence of shocks in different sectors, one in the total productivity of final goods production and the other in R&D creativity with the aim of solving, simultaneously, both puzzles.

As for choice of the parameters this was done from a calibration exercise (focusing in adjusting the theoretical moments to the data values) but keeping them inside the bounds of what the literature assumes as reasonable for those that are standard. For the non-standard ones, this is the volatility and the stochastic process of creative shocks, the time-series available for the creative-destructive process of varieties from industrial surveys are too sparse in time and heterogeneous across countries to be used . Therefore we calibrated them for a number of different alternatives.

The results showed that those two kinds of shocks alone are not able to replicate most of the (unconditional) moments of the data and both have, with different magnitudes, the same problems. However, even if the impact of shocks on different variables leads to similar moments, the fact is that the relative response of the variables is not equal in each case, for which reason, when we allowed both shocks to coexist we found that we could replicate most of the moments from the data, including the correlation puzzle and the price anomaly. This was so for reasonable values of the standard parameters together with a strong correlation between the shocks in R&D and the productivity shocks in the final sector and with negative contamination across countries in the stochastic process of the R&D shocks.

Even if we get some degree of success in replicating the moments from the data, the international commovements of investment , the home-bias puzzle, an analysis of the impact of outsourcing variety production (rather than just exporting the final product) and the effects of pricing to market were not addressed by the previous model. Therefore future research would be to extend the current framework to introduce capital and international capital markets to tackle the first two issues, to consider patents internationally tradable to tackle the third issue, and to consider that the elasticity of substitution/markup to be a function of an "accessibility index to varieties" to tackle the fourth issue. A final issue would be to use the data from industrial surveys, where we will have to collect and homogenize them across time and countries, to get some reasonable estimates of the R&D stochastic

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process from the data and to check whether the parameterizations of the model that can replicate the moments from data can be considered reasonable or not.

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Appendices

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Appendix 3.A Obtaining the consumption price index and the demand equations

So the problem for the home country is:

$$\min_{c_{H,t}^{H};c_{H,t}^{F}} \int_0^{v_{H,t}} p_{H,t}(s).c_{H,t}^{H}(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c_{H,t}^{F}(s)ds$$

subject to:

$$C_{H,t} = \left(\int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \right)^{\frac{\theta}{\theta-1}}$$

The Lagrangian is:

$$\begin{aligned} \pounds \left(c_{H,t}^H(s); c_{H,t}^F(s)\right) &= \int_0^{v_{H,t}} p_{H,t}(s).c_{H,t}^H(s)ds + \int_0^{v_{F,t}} p_{F,t}(s).c_{H,t}^F(s)ds + \\ &+\lambda_{H,t}. \left(C_{H,t} - \left(\begin{array}{c} \int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \\ + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \end{array} \right)^{\frac{\theta}{\theta-1}} \right) \end{aligned} \tag{3.94}$$

$\frac{\partial \pounds}{\partial c_{H,t}^H(s)} = 0$

$$p_{H,t}(s). - \lambda_{H,t} \frac{\theta}{\theta-1}. \left(\begin{array}{c} \int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \\ + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \end{array} \right)^{\frac{\theta}{\theta-1}-1} \frac{\theta-1}{\theta}.\gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}-1} = 0 \tag{3.95}$$

$\frac{\partial \pounds}{\partial c_{H,t}^F(s)} = 0$

$$p_{F,t}(s). - \lambda_{H,t} \left(\begin{array}{c} \int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \\ + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \end{array} \right)^{\frac{\theta}{\theta-1}-1} \gamma_H^F(1-T). \left((1-T).c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}-1} = 0 \tag{3.96}$$

and,

$$C_{H,t} = \left(\int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \right)^{\frac{\theta}{\theta-1}} \tag{3.97}$$

Simplifying equations (3.95) and (3.96), we get:

$$p_{H,t}(s). - \lambda_{H,t}. \left(\begin{array}{c} \int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \\ + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \end{array} \right)^{\frac{1}{\theta-1}} .\gamma_H^H. \left(c_{H,t}^H(s)\right)^{-\frac{1}{\theta}} = 0 \tag{3.98}$$

$$p_{F,t}(s). - \lambda_{H,t} \left(\begin{array}{c} \int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \\ + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds \end{array} \right)^{\frac{1}{\theta-1}} \gamma_H^F(1-T). \left((1-T).c_{H,t}^F(s)\right)^{-\frac{1}{\theta}} = 0 \tag{3.99}$$

Transforming equation (3.97) into:

$$(C_{H,t})^{\frac{1}{\theta}} = \left(\int_0^{v_{H,t}} \gamma_H^H. \left(c_{H,t}^H(s)\right)^{\frac{\theta-1}{\theta}} ds + \int_0^{v_{F,t}} \gamma_H^F. \left((1-T)c_{H,t}^F(s)\right)^{\frac{\theta-1}{\theta}} ds\right)^{\frac{1}{\theta-1}} \tag{3.100}$$

and replacing into (3.98) and (3.99), we get:

$$p_{H,t}(s). - \lambda_{H,t}..\gamma_H^H. (C_{H,t})^{\frac{1}{\theta}} \left(c_{H,t}^H(s)\right)^{-\frac{1}{\theta}} = 0 \tag{3.101}$$

$$p_{F,t}(s). - \lambda_{H,t} (C_{H,t})^{\frac{1}{\theta}} \gamma_H^F(1-T). \left((1-T).c_{H,t}^F(s)\right)^{-\frac{1}{\theta}} = 0 \tag{3.102}$$

Solving for $c_{H,t}^H(s)$ and $c_{H,t}^F(s)$:

$$c_{H,t}^H(s) = \left(\frac{p_{H,t}(s).}{\lambda_{H,t}..\gamma_H^H.}\right)^{-\theta} C_{H,t} \tag{3.103}$$

$$c_{H,t}^F(s) = \left(\frac{p_{F,t}(s)}{\lambda_{H,t}.\gamma_H^F.(1-T)}\right)^{-\theta} \frac{C_{H,t}.}{(1-T).} \tag{3.104}$$

replacing into equation (3.97) and re-arranging:

$$\lambda_{H,t} = \left(\int_0^{v_{H,t}} \left(\gamma_H^H\right)^\theta . (p_{H,t}(s))^{1-\theta} ds + \int_0^{v_{F,t}} \left(\gamma_H^F\right)^\theta . \left(\frac{p_{F,t}(s)}{(1-T)}\right)^{1-\theta} ds\right)^{\frac{1}{1-\theta}} \tag{3.105}$$

Therefore $\lambda_{H,t}$ is the home consumption price index, that we will denote by $P_{H,t}$:

$$P_{H,t} = \left(\int_0^{v_{H,t}} \left(\gamma_H^H\right)^\theta . (p_{H,t}(s))^{1-\theta} ds + \int_0^{v_{F,t}} \left(\gamma_H^F\right)^\theta . \left(\frac{p_{F,t}(s)}{(1-T)}\right)^{1-\theta} ds\right)^{\frac{1}{1-\theta}} \tag{3.106}$$

and the demand equations by the home agent are:

$$c_{H,t}^H(s) = \left(\frac{p_{H,t}(s).}{P_{H,t}..\gamma_H^H.}\right)^{-\theta} C_{H,t} \tag{3.107}$$

$$c_{H,t}^F(s) = \left(\frac{p_{F,t}(s)}{P_{H,t}.\gamma_H^F.(1-T)}\right)^{-\theta} \frac{C_{H,t}.}{(1-T).} \tag{3.108}$$

Similar equations hold for the foreign representative household.

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Appendix 3.B Solving the household intertemporal problem

$$\pounds\left(C_{H,t};L_{H,t}:H_t^H;b_t^H\right)=E_0\left[\sum_{t=0}^{+\infty}\beta^t\left[\begin{array}{c} U(C_{H,t};L_{H,t})+\\ +\lambda_{BC,t}^H.\left(\begin{array}{c} w_t^H.H_t^H+b_t^H+\pi_t^H-\\ -\int_0^{v_{H,t}}p_{H,t}(s).c_{H,t}^H(s)ds-\\ -\int_0^{v_{F,t}}p_{F,t}(s).c_{H,t}^F(s)ds-\\ -b_{t-1}^H.(1+i_{t-1})+\psi(b_t^H-b_{SS}^H)^2\end{array}\right)-\\ -\lambda_{L,t}^H\left(L_{H,t}+H_{H,t}-1\right).\end{array}\right]\right] \tag{3.109}$$

Note that:

$$\pi_t^H=\int_0^{v_{H,t}}\pi_t^H(s)ds+\pi_t^{H,RD}=\pi_t^{H,RD} \tag{3.110}$$

As we saw in section 2.3, $\pi_t^H(s)=0$.

Also:

$$\int_0^{v_{H,t}}p_{H,t}(s).c_{H,t}^H(s)ds+\int_0^{v_{F,t}}p_{F,t}(s).c_{H,t}^F(s)ds= \tag{3.111}$$

$$=\int_0^{v_{H,t}}p_{H,t}(s).\left(\frac{p_{H,t}(s).}{\lambda_{H,t}..\gamma_H^H.}\right)^{-\theta}C_{H,t}ds+\int_0^{v_{F,t}}p_{F,t}(s).\left(\frac{p_{F,t}(s)}{\lambda_{H,t}.\gamma_H^F.(1-T)}\right)^{-\theta}\frac{C_{H,t}.}{(1-T).}ds=$$

$$=\int_0^{v_{H,t}}\left(p_{H,t}(s)\right)^{1-\theta}.\left(\gamma_H^H\right)^{\theta}\left(P_{H,t}\right)^{\theta}C_{H,t}ds+\int_0^{v_{F,t}}\left(\frac{p_{F,t}(s)}{1-T}\right)^{1-\theta}.\left(\gamma_H^F\right)^{\theta}\left(P_{H,t}\right)^{\theta}C_{H,t}ds=$$

$$=\left(\int_0^{v_{H,t}}\left(p_{H,t}(s)\right)^{1-\theta}.\left(\gamma_H^H\right)^{\theta}ds+\int_0^{v_{F,t}}\left(\frac{p_{F,t}(s)}{1-T}\right)^{1-\theta}.\left(\gamma_H^F\right)ds\right)\left(P_{H,t}\right)^{\theta}C_{H,t}=$$

using equation (3.5), we obtain:

$$=\left(P_{H,t}\right)^{1-\theta}\left(P_{H,t}\right)^{\theta}C_{H,t}=P_{H,t}.C_{H,t}$$

Replacing equations (3.110) and (3.111) in (3.109), we obtain:

$$\pounds\left(C_{H,t};L_{H,t}:H_t^H;b_t^H\right)=E_0\left[\sum_{t=0}^{+\infty}\beta^t\left[\begin{array}{c} U(C_{H,t};L_{H,t})+\\ +\lambda_{BC,t}^H.\left(\begin{array}{c} w_t^H.H_t^H+b_t^H+\pi_t^{H,RD}-\\ -P_{H,t}.C_{H,t}-b_{t-1}^H.(1+i_{t-1})+\\ \psi(b_t^H-b_{SS}^H)^2\end{array}\right)-\\ -\lambda_{L,t}^H\left(L_{H,t}+H_{H,t}-1\right).\end{array}\right]\right] \tag{3.112}$$

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The FOCs at any time t are:

$$\begin{aligned}
\frac{\partial U}{\partial C_{H,t}} &= \lambda_{BC,t}^{H}.P_{H,t} \\
\frac{\partial U}{\partial L_{H,t}} &= \lambda_{L,t}^{H} \\
\lambda_{BC,t}^{H}.w_{t}^{H} &= \lambda_{L,t}^{H} \\
\lambda_{BC,t}^{H} &= \beta E_{t}\left[\lambda_{BC,t+1}^{H}.(1+i_{t})\right] - 2\psi(b_{t}^{H} - b_{SS}^{H})
\end{aligned} \tag{3.113}$$

plus the restrictions.

Replacing $\lambda_{BC,t}^{H}$ in the fourth equation by the value from the first equation we get the Euler equation:

$$\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}} = \beta E_{t}\left[\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}.(1+i_{t})\right] - 2\psi(b_{t}^{H} - b_{SS}^{H}) \tag{3.114}$$

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Appendix 3.C Deriving the R&D arbitrage condition

The R&D arbitrage condition states that the marginal cost of financing the production of an extra variety has to be paid by the marginal increase of future revenues.

The future flow of nominal profits expected at time t, valued at t is:

$$E_t\left(\overline{\Pi}_{H,t}^{RD}+\sum_{\tau=t+1}^{+\infty}\frac{\overline{\Pi}_{H,\tau}^{RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\right)=E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}}pv_t^H(s)ds-w_t^H.\overline{H}_t^{H,RD}\right)+\\+\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{v_{H,\tau}}pv_\tau^H(s)ds-w_\tau^H.\overline{H}_\tau^{H,RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right) \tag{3.115}$$

Note that i_t^H is the interest rate that a home consumer will use when he decides to postpone consumption, this is, we can see this as a game between the home agent today against himself tomorrow. If he wants to anticipate/postpone consumption from tomorrow/today to today/tomorrow the agent would sell/buy a bond to himself at that interest rate. The Euler equation would be similar to equation (3.10) but with adjustment costs equal to zero:

$$\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}=\beta E_t\left[\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}.(1+i_t^H)\right] \tag{3.116}$$

The arbitrage R&D condition will give that:

$$\frac{\partial}{\partial v_{H,t+1}}\left(E_t\left(\overline{\Pi}_{H,t}^{RD}+\sum_{\tau=t+1}^{+\infty}\frac{\overline{\Pi}_{H,\tau}^{RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\right)\right)=0 \tag{3.117}$$

But from equation (3.23) $dv_{H,t+1}=d\overline{Y}_t^{H,RD}$, we can rewrite the previous equation as:

$$\frac{\partial}{\partial\overline{Y}_t^{H,RD}}\left(E_t\left(\overline{\Pi}_{H,t}^{RD}+\sum_{\tau=t+1}^{+\infty}\frac{\overline{\Pi}_{H,\tau}^{RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\right)\right)=0 \tag{3.118}$$

implying:

$$\frac{\partial}{\partial\overline{Y}_t^{H,RD}}\left(E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}}pv_t^H(s)ds-w_t^H.\overline{H}_t^{H,RD}\right)+\\+\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{v_{H,\tau}}pv_\tau^H(s)ds-w_\tau^H.\overline{H}_\tau^{H,RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right)\right)=0 \tag{3.119}$$

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Replacing $v_{H,\tau}$ by the relation implied by equation (3.23) :

$$\frac{\partial}{\partial \overline{Y}_t^{H,RD}}\left(E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}} pv_t^H(s)ds - w_t^H.\overline{H}_t^{H,RD}\right)+ \\ +\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{(1-\varphi^V)v_{H,\tau-1}+\overline{Y}_{\tau-1}^{H,RD}} pv_\tau^H(s)ds - w_\tau^H.\overline{H}_\tau^{H,RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right)\right)=0$$

Replacing $v_{H,\tau-1}$:

$$\frac{\partial}{\partial \overline{Y}_t^{H,RD}}\left(E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}} pv_t^H(s)ds - w_t^H.\overline{H}_t^{H,RD}\right)+ \\ +\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{(1-\varphi^V)\left((1-\varphi^V)v_{H,\tau-2}+\overline{Y}_{\tau-2}^{H,RD}\right)+\overline{Y}_{\tau-1}^{H,RD}} pv_\tau^H(s)ds - w_\tau^H.\overline{H}_\tau^{H,RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right)\right)=0 \tag{3.120}$$

and continuing replacing $v_{H,\tau-i}$ backwards:

$$\frac{\partial}{\partial \overline{Y}_t^{H,RD}}\left(E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}} pv_t^H(s)ds - w_t^H.\overline{H}_t^{H,RD}\right)+ \\ +\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{\sum_{i=1}^{+\infty}(1-\varphi^V)^{i-1}\overline{Y}_{\tau-i}^{H,RD}} pv_\tau^H(s)ds - w_\tau^H.\overline{H}_\tau^{H,RD}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right)\right)=0 \tag{3.121}$$

Replacing $\overline{H}_t^{H,RD}$ by the relation implied by equation (3.22):

$$\frac{\partial}{\partial \overline{Y}_t^{H,RD}}\left(E_t\left(\begin{array}{c}\left(\int_0^{v_{H,t}} pv_t^H(s)ds - w_t^H.\frac{\overline{Y}_t^{H,RD}}{A_t^{H,RD}}\right)+ \\ +\sum_{\tau=t+1}^{+\infty}\frac{\int_0^{\sum_{i=1}^{+\infty}(1-\varphi^V)^{i-1}\overline{Y}_{\tau-i}^{H,RD}} pv_\tau^H(s)ds - w_\tau^H.\frac{\overline{Y}_\tau^{H,RD}}{A_\tau^{H,RD}}}{\prod_{j=t}^{\tau-1}(1+i_j^H)}\end{array}\right)\right)=0 \tag{3.122}$$

which gives the R&D arbitrage condition:

$$E_t\left[\sum_{\tau=t+1}^{+\infty}\frac{1}{\prod_{j=t}^{\tau-1}(1+i_j^H)}pv_\tau^H.(1-\varphi^V)^{\tau-t-1}\right]=\frac{w_t^H}{A_t^{H,RD}} \tag{3.123}$$

Note that, if we use equation (3.116):

$$\frac{1}{1+i_t^H}=\beta E_t\left[\frac{\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}}{\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}}\right] \tag{3.124}$$

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and

$$E_t\left[\frac{1}{\prod_{j=t}^{\tau-1}(1+i_j)}\right] = E_t\left[\begin{array}{c}\beta\frac{\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}}{\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}}.\beta\frac{\frac{\partial U}{\partial C_{H,t+2}}.\frac{1}{P_{H,t+2}}}{\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}}... \\ ...\beta\frac{\frac{\partial U}{\partial C_{H,\tau}}.\frac{1}{P_{H,\tau}}}{\frac{\partial U}{\partial C_{H,\tau-1}}.\frac{1}{P_{H,\tau-1}}}\end{array}\right] = \tag{3.125}$$

$$= \beta^{\tau-t}E_t\left[\frac{\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}}{\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}}.\frac{\frac{\partial U}{\partial C_{H,t+2}}.\frac{1}{P_{H,t+2}}}{\frac{\partial U}{\partial C_{H,t+1}}.\frac{1}{P_{H,t+1}}}...\frac{\frac{\partial U}{\partial C_{H,\tau}}.\frac{1}{P_{H,\tau}}}{\frac{\partial U}{\partial C_{H,\tau-1}}.\frac{1}{P_{H,\tau-1}}}\right] = \beta^{\tau-t}E_t\left[\frac{\frac{\partial U}{\partial C_{H,\tau}}.\frac{1}{P_{H,\tau}}}{\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}}\right]$$

equation (3.123) would turn into:

$$E_t\left[\sum_{\tau=t+1}^{+\infty}\beta^{\tau-t}\frac{\frac{\partial U}{\partial C_{H,\tau}}.\frac{1}{P_{H,\tau}}}{\frac{\partial U}{\partial C_{H,t}}.\frac{1}{P_{H,t}}}pv_\tau^H.(1-\varphi^V)^{\tau-t-1}\right] = \frac{w_t^H}{A_t^{H,RD}} \Leftrightarrow \tag{3.126}$$

$$\Leftrightarrow E_t\left[\sum_{\tau=t+1}^{+\infty}\beta^{\tau-t}\frac{\frac{\partial U}{\partial C_{H,\tau}}}{\frac{\partial U}{\partial C_{H,t}}}\frac{pv_\tau^H}{P_{H,\tau}}.(1-\varphi^V)^{\tau-t-1}\right] = \frac{w_t^H}{A_t^{H,RD}.P_{H,t}} \Leftrightarrow$$

Appendix 3.D Showing that parameter symmetry in countries leads to equal pricing and real variables in the NSSS

Using equations (3.49) and (3.81) we can rewrite equation (3.89) as:

$$k.(1-L_{H,t})^{\varpi}=p_H(s).\frac{\theta-1}{\theta}A_t^H\lambda_{BC,t}^H\Leftrightarrow k.\left(H_{H,t}\right)^{\varpi}=p_H(s).\frac{\theta-1}{\theta}A_t^H\lambda_{BC,t}^H \qquad (3.127)$$

and so:

$$(C_{H,t})^{-\rho} = \frac{P_{H,t}}{p_H(s)}.k.\left(H_{H,t}\right)^{\varpi}\frac{\theta}{\theta-1}\frac{1}{A_t^H}\Longleftrightarrow \qquad (3.128)$$

$$\Longleftrightarrow C_{H,t}=\left(\frac{P_{H,t}}{p_H(s)}.k.\left(H_{H,t}\right)^{\varpi}\frac{\theta}{\theta-1}\frac{1}{A_t^H}\right)^{-\frac{1}{\rho}}$$

Using equation (3.63) in the steady state:

$$w_{H,t}*H_{H,t}+\pi_{H,t}^{RD}=P_{H,t}*C_{H,t}$$

replacing the per capita profits :

$$w_{H,t}*H_{H,t}+\frac{\theta-1}{\theta}A_t^H.\frac{i_t}{(\theta-1)\,i_t+\theta.\varphi^V}.H_{H,t}.p_H(s)=P_{H,t}*C_{H,t}\Longleftrightarrow$$

$$\Longleftrightarrow\left(1+\frac{i_t}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).w_{H,t}.H_{H,t}=P_{H,t}*C_{H,t}\Longleftrightarrow$$

$$\Longleftrightarrow\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).w_{H,t}.H_{H,t}=P_{H,t}*C_{H,t}$$

replacing $C_{H,t}$ by the relation of equation (3.128):

$$\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).w_{H,t}.H_{H,t}=P_{H,t}*\left(\frac{P_{H,t}}{p_H(s)}.k.\left(H_{H,t}\right)^{\varpi}\frac{\theta}{\theta-1}\frac{1}{A_t^H}\right)^{-\frac{1}{\rho}}\Longleftrightarrow$$

$$\Longleftrightarrow\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).\frac{w_{H,t}}{p_H(s)}.H_{H,t}*\left(k.\left(H_{H,t}\right)^{\varpi}\frac{\theta}{\theta-1}\frac{1}{A_t^H}\right)^{\frac{1}{\rho}}=\left(\frac{P_{H,t}}{p_H(s)}\right)^{1-\frac{1}{\rho}}\Longleftrightarrow$$

$$\Longleftrightarrow\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).\left(\frac{w_{H,t}}{p_H(s)}\right)^{1-\frac{1}{\rho}}.(k)^{\frac{1}{\rho}}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\frac{P_{H,t}}{p_H(s)}\right)^{1-\frac{1}{\rho}}\Leftrightarrow$$

$$\Leftrightarrow\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)\,i_t+\theta.\varphi^V}.\right).\left(\frac{w_{H,t}}{p_H(s)}\right)^{1-\frac{1}{\rho}}.(k)^{\frac{1}{\rho}}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\frac{P_{H,t}}{p_H(s)}\right)^{1-\frac{1}{\rho}}$$

Calling $\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)i_t+\theta.\varphi^V}.\right).\left(\frac{w_{H,t}}{p_H(s)}.\right)^{1-\frac{1}{\rho}}.(k)^{\frac{1}{\rho}}=\Omega_{S1}$

$$\Omega_{S1}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\frac{P_{H,t}}{p_H(s)}\right)^{\frac{\rho-1}{\rho}}$$

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Replacing the price index given by equation (3.41):

$$\Omega_{S1}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\left(\int_{0}^{v_{H,t}}\left(\gamma_{H}^{H}\right)^{\theta}.ds+\int_{0}^{v_{F,t}}\left(\gamma_{H}^{F}\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}ds\right)^{\frac{1}{1-\theta}}\right)^{1-\frac{1}{\rho}}$$

which by replacing the number of varieties by it's NSSS value gives:

$$\Omega_{S1}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\begin{array}{c}\frac{A_{t}^{H,RD}.n_{H}}{(\theta-1)i_{t}+\varphi^{V}.\theta}H_{H,t}.\left(\gamma_{H}^{H}\right)^{\theta}.+\\+\frac{A_{t}^{F,RD}.n_{F}}{(\theta-1)i_{t}+\varphi^{V}.\theta}H_{F,t}.\left(\gamma_{H}^{F}\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}}$$

Identically relationship holds for the foreign country:

$$\Omega_{S1}.\left(H_{F,t}\right)^{\frac{\rho+\varpi}{\rho}}=\left(\begin{array}{c}\frac{A_{t}^{F,RD}.n_{F}}{(\theta-1)i_{t}+\varphi^{V}.\theta}H_{F,t}.\left(\gamma_{F}^{F}\right)^{\theta}.+\\+\frac{A_{t}^{H,RD}.n_{H}}{(\theta-1)i_{t}+\varphi^{V}.\theta}H_{H,t}.\left(\gamma_{F}^{H}\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}}$$

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Now we can solve the two equations in order to $H_{H,t}$ and $H_{F,t}$:

$$\begin{cases} \Omega_{S1}.\left(H_{H,t}\right)^{\frac{\rho+\varpi}{\rho}} = \left(\begin{array}{c} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_H^H\right)^{\theta}.+ \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_H^F\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \\ \Omega_{S1}.\left(H_{F,t}\right)^{\frac{\rho+\varpi}{\rho}} = \left(\begin{array}{c} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_F^F\right)^{\theta}.+ \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_F^H\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \end{cases} \Leftrightarrow \tag{3.129}$$

$$\Leftrightarrow \begin{cases} \Omega_{S1} = \left(\begin{array}{c} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_H^H\right)^{\theta}.+ \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_H^F\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \left(H_{H,t}\right)^{-\frac{\rho+\varpi}{\rho}} \\ \Omega_{S1} = \left(\begin{array}{c} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_F^F\right)^{\theta}.+ \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_F^H\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \left(H_{F,t}\right)^{-\frac{\rho+\varpi}{\rho}} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \left(\begin{array}{c} \left(\begin{array}{c} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_H^H\right)^{\theta}.+ \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_H^F\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \left(H_{H,t}\right)^{-\frac{\rho+\varpi}{\rho}} = \\ = \left(\begin{array}{c} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_F^F\right)^{\theta}.+ \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_F^H\right)^{\frac{1}{\theta}}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array}\right)^{\frac{\rho-1}{(1-\theta)\rho}} \left(H_{F,t}\right)^{-\frac{\rho+\varpi}{\rho}} \end{array}\right) \Leftrightarrow \tag{3.130}$$

$$\Leftrightarrow \left(\frac{H_{H,t}}{H_{F,t}}\right)^{\frac{\rho+\varpi}{\rho}} = \left(\frac{\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^{\theta}.+\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_H^F\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_F^F\right)^{\theta}+\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}\right)^{\frac{\rho-1}{(1-\theta)\rho}}$$

By other side using equations (3.67) and (3.68), and the assumption of equal populations:

$$\begin{cases} c_{H,t}^H(s)+c_{F,t}^H(s)=A_t^H.h_t^H(s) \\ c_{H,t}^F(s)+c_{F,t}^F(s)=A_t^F.h_t^F(s) \end{cases} \Rightarrow (\text{as } h_t^H(s)=h_t^F(s)) \Rightarrow c_{H,t}^H(s)+c_{F,t}^H(s)=c_{H,t}^F(s)+c_{F,t}^F(s) \tag{3.131}$$

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Using the demand equations (3.43) to (3.46) we get:

$$\left(\frac{p_{H,t}(s).}{P_{H,t}..\gamma_{H}^{H}.}\right)^{-\theta} C_{H,t} + \left(\frac{p_{H,t}(s)}{P_{F,t}.\gamma_{F}^{H}.(1-T)}\right)^{-\theta} \frac{C_{F,t}.}{(1-T).} = \tag{3.132}$$

$$= \left(\frac{p_{F,t}(s)}{P_{H,t}.\gamma_{H}^{F}.(1-T)}\right)^{-\theta} \frac{C_{H,t}.}{(1-T).} + \left(\frac{p_{F,t}(s).}{P_{F,t}..\gamma_{F}^{F}.}\right)^{-\theta} C_{F,t} \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} \left(\frac{p_{H,t}(s).}{P_{H,t}..\gamma_{H}^{H}.}\right)^{-\theta} - \\ -\left(\frac{p_{F,t}(s)}{P_{H,t}.\gamma_{H}^{F}.(1-T)}\right)^{-\theta} \frac{1}{(1-T).} \end{bmatrix} C_{H,t} = \begin{bmatrix} \left(\frac{p_{F,t}(s).}{P_{F,t}..\gamma_{F}^{F}.}\right)^{-\theta} - \\ -\left(\frac{p_{H,t}(s)}{P_{F,t}.\gamma_{F}^{H}.(1-T)}\right)^{-\theta} \frac{1}{(1-T).} \end{bmatrix} C_{F,t}$$

From earlier we know that:

$$\frac{C_F}{C_H} = \left(\left(\frac{H_H}{H_F}\right)^{\varpi} \frac{\frac{P_H}{p_H(s)}}{\frac{P_F}{p_F(s)}}\right)^{\frac{1}{\rho}}$$

replacing in the previous equation and simplifying we get:

$$\frac{\left[\left(\gamma_{H}^{H}.\right)^{\theta} - \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}\left(\gamma_{H}^{F}\right)^{\theta}(1-T)^{\theta-1}\right]}{\left[\left(\gamma_{F}^{F}\right)^{\theta} - \left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}\left(\gamma_{F}^{H}\right)^{\theta}.(1-T)^{\theta-1}\right]} = \left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}} \left(\frac{\frac{P_H}{p_H(s)}}{\frac{P_F}{p_F(s)}}\right)^{\frac{1}{\rho}-\theta}$$

replacing in the previous equation $\frac{P_F}{p_F(s)}$ and $\frac{P_H}{p_H(s)}$ by:

$$\frac{P_H}{p_H(s)} = \begin{pmatrix} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_{H}^{H}\right)^{\theta} + \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_{H}^{F}\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{pmatrix}^{\frac{1}{(1-\theta)}}$$

$$\frac{P_F}{p_F(s)} = \begin{pmatrix} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta} H_{F,t}.\left(\gamma_{F}^{F}\right)^{\theta} + \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta} H_{H,t}.\left(\gamma_{F}^{H}\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{pmatrix}^{\frac{1}{(1-\theta)}}$$

we get:

$$\frac{\left[\left(\gamma_{H}^{H}.\right)^{\theta} - \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}\left(\gamma_{H}^{F}\right)^{\theta}(1-T)^{\theta-1}\right]}{\left[\left(\gamma_{F}^{F}\right)^{\theta} - \left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}\left(\gamma_{F}^{H}\right)^{\theta}.(1-T)^{\theta-1}\right]} = \tag{3.133}$$

$$= \left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}} \left(\frac{\left(\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}.\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_{H}^{H}\right)^{\theta}.+\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_{H}^{F}\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\right)}{\left(\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_{F}^{F}\right)^{\theta}.+\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}.\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_{F}^{H}\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\right)}\right)^{\frac{1-\theta\rho}{\rho(1-\theta)}}$$

Under symmetry (equal parameters across countries) we can see that in equation (3.129) and (3.133) that if $\frac{H_F}{H_H}=\frac{p_{F,t}(s)}{p_{H,t}(s)}=1$ both equations are transformed into an identity. The question is to know if these relationships are the unique ones that solve both equations at the same time. So the system is:

$$\begin{cases} \left(\frac{H_{H,t}}{H_{F,t}}\right)^{\frac{\rho+\varpi}{\rho}} = \left(\frac{ \begin{array}{c} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+ \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array} }{ \left(\begin{array}{c} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_F^F\right)^\theta+ \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array} \right) } \right)^{\frac{\rho-1}{(1-\theta)\rho}} \\ \left(\begin{array}{c} \frac{\left[\left(\gamma_H^H.\right)^\theta-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}\left(\gamma_H^F\right)^\theta(1-T)^{\theta-1}\right]}{\left[\left(\gamma_F^F\right)^\theta-\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}\left(\gamma_F^H\right)^\theta.(1-T)^{\theta-1}\right]}= \\ =\left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}}\left(\frac{ \left(\begin{array}{c} \frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}.\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+ \\ +\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array} \right) }{ \left(\begin{array}{c} \frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}.\left(\gamma_F^F\right)^\theta.+ \\ +\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}.\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \end{array} \right) } \right)^{\frac{1-\theta\rho}{\rho(1-\theta)}} \end{array} \right) \end{cases}$$

Under symmetry $\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}=\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}=\frac{A_t^{F,RD}.n_F}{(\theta-1)i_t+\varphi^V.\theta}=\frac{A_t^{H,RD}.n_H}{(\theta-1)i_t+\varphi^V.\theta}$ we get:

$$\begin{cases} \left(\frac{H_{H,t}}{H_{F,t}}\right)^{1+\frac{\varpi}{\rho}} = \left(\frac{\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}} \right)^{\frac{\rho-1}{(1-\theta)\rho}} \\ \frac{\left[\left(\gamma_H^H.\right)^\theta-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}\left(\gamma_H^F\right)^\theta(1-T)^{\theta-1}\right]}{\left[\left(\gamma_F^F\right)^\theta-\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}\left(\gamma_F^H\right)^\theta.(1-T)^{\theta-1}\right]} = \left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}} \left(\frac{\left(\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\right)}{\left(\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}\right)} \right)^{\frac{1-\theta\rho}{(1-\theta)\rho}} \end{cases}$$

Let´s consider that $\frac{H_{H,t}}{H_{F,t}}>1$ (same reasoning holds for $\frac{H_{H,t}}{H_{F,t}}<1$)
From the first condition this can be rewritten has:

$$\left(\frac{H_{H,t}}{H_{F,t}}\right)^{\left(1+\frac{\varpi}{\rho}\right)\frac{1}{(\rho-1)}} = \left(\frac{\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}} \right)^{\frac{1}{(1-\theta)\rho}}$$

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Assuming that $\theta > 1$, $\rho > 1$ and $\varpi \geq 0$ we have that: $\left(1+\frac{\varpi}{\rho}\right)(\rho-1) > 0$ and $\frac{1}{(1-\theta)\rho} < 0$ then

$$\left(\frac{\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}\right)^{\frac{1}{(1-\theta)\rho}} > 1 \Rightarrow$$

$$\Rightarrow 0 < \left(\frac{\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}\right) < 1 \Rightarrow$$

(as both terms are positive we can skip the left inequality)

$$\Rightarrow \frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^\theta.+\left(\gamma_H^F\right)^\theta.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}$$

$$<$$

$$\left(\gamma_F^F\right)^\theta+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^\theta.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1} \Rightarrow$$

(assuming symmetry in the parameters $\left[\begin{array}{c}\left(\gamma_H^H\right)^\theta=\left(\gamma_F^F\right)^\theta=A>0\\ \left(\gamma_H^F\right)^\theta.(1-T)^{\theta-1}=\left(\gamma_F^H\right)^\theta.(1-T)^{\theta-1}=B>0\end{array}\right]$

$$\Rightarrow A\left(\frac{H_{H,t}}{H_{F,t}}-1\right) < B\left(\frac{H_{H,t}}{H_{F,t}}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}\right) \Rightarrow$$

assuming if there is a bias it is a home one, therefore $\left(\gamma_H^H\right)^\theta \geq \left(\gamma_H^F\right)^\theta \Rightarrow A \geq B$ a necessary condition (but not sufficient) for the above inequality to hold is:

$$\left(\frac{H_{H,t}}{H_{F,t}}-1\right) < \frac{H_{H,t}}{H_{F,t}}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta-1}-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta} \Rightarrow$$

$$\Rightarrow \frac{H_{H,t}}{H_{F,t}}\left(1-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta-1}\right) < 1-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta} \Rightarrow$$

as $\frac{H_{H,t}}{H_{F,t}} > 1$ a necessary condition is:

$$\left(1-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta-1}\right) < 1-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta} \Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta-1} > \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta} \Rightarrow$$

$$\Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta-1-1+\theta} > 1 \Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{2(\theta-1)} > 1 \Rightarrow \frac{p_{F,t}(s)}{p_{H,t}(s)} > 1$$

(the last result is due to $\theta > 1 \Rightarrow 2(\theta-1) > 0$)

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So from the first relationship we've that if $\frac{H_{H,t}}{H_{F,t}} > 1 \Rightarrow \frac{p_{F,t}(s)}{p_{H,t}(s)} > 1$.

Let's turn to the second relationship. Form the first one we know that:

$$\left(\frac{H_{H,t}}{H_{F,t}}\right)^{\left(1+\frac{\varpi}{\rho}\right)(\rho-1)} = \left(\frac{\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_H^H\right)^{\theta}.+\left(\gamma_H^F\right)^{\theta}.\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}{\left(\gamma_F^F\right)^{\theta}+\frac{H_{H,t}}{H_{F,t}}.\left(\gamma_F^H\right)^{\theta}.\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{1-\theta}(1-T)^{\theta-1}}\right)^{\frac{1}{(1-\theta)\rho}}$$

replacing in the second one we get:

$$\frac{\left[\left(\gamma_H^H.\right)^{\theta}-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}\left(\gamma_H^F\right)^{\theta}(1-T)^{\theta-1}\right]}{\left[\left(\gamma_F^F\right)^{\theta}-\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}\left(\gamma_F^H\right)^{\theta}.(1-T)^{\theta-1}\right]} = \left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}}\left(\frac{H_{H,t}}{H_{F,t}}\right)^{\left(1+\frac{\varpi}{\rho}\right)\frac{(1-\theta\rho)}{(\rho-1)}} = \left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}+\left(1+\frac{\varpi}{\rho}\right)\frac{(1-\theta\rho)}{(\rho-1)}}$$

Checking that the exponent of the rhs is negative:

$$\frac{\varpi}{\rho}+\left(1+\frac{\varpi}{\rho}\right)\frac{(1-\theta\rho)}{(\rho-1)} = \frac{\varpi(\rho-1)}{\rho(\rho-1)}+\left(\frac{\rho+\varpi}{\rho}\right)\frac{(1-\theta\rho)}{(\rho-1)} = \frac{\varpi(\rho-1)+(\rho+\varpi)(1-\theta\rho)}{\rho(\rho-1)} =$$

$$= \frac{\varpi\rho-\varpi+\rho+\varpi-\rho\theta\rho-\varpi\theta\rho}{\rho(\rho-1)} = \frac{\varpi\rho+\rho-\rho\theta\rho-\varpi\theta\rho}{\rho(\rho-1)} =$$

$$= \frac{\varpi+1-\theta\rho-\varpi\theta}{(\rho-1)} = \frac{\varpi(1-\theta)+1-\theta\rho}{(\rho-1)}$$

(under the previous assumptions)

$$\begin{matrix}\varpi(1-\theta)+1-\theta\rho<0\\(\rho-1)>0\end{matrix} \Rightarrow \frac{\varpi(1-\theta)+1-\theta\rho}{(\rho-1)}<0$$

We had from the initial assumption that $\frac{H_H}{H_F} > 1$. Therefore:

$$\left(\frac{H_H}{H_F}\right)^{\frac{\varpi}{\rho}+\left(1+\frac{\varpi}{\rho}\right)\frac{(1-\theta\rho)}{(\rho-1)}} < 1 \Rightarrow \frac{\left[A-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}B\right]}{\left[A-\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}B\right]} < 1 \Rightarrow$$

$$\Rightarrow A-\left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta}B < A-\left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta}B \Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta} > \left(\frac{p_{H,t}(s)}{p_{F,t}(s)}\right)^{-\theta} \Rightarrow$$

$$\Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta} > \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{\theta} \Rightarrow \left(\frac{p_{F,t}(s)}{p_{H,t}(s)}\right)^{-\theta-\theta} > 1 \Rightarrow \frac{p_{F,t}(s)}{p_{H,t}(s)} < 1$$

From the second relationship we take that if $\frac{H_{H,t}}{H_{F,t}} > 1 \Rightarrow \frac{p_{F,t}(s)}{p_{H,t}(s)} < 1$, which contradicts the relationship taken from the first equation.

As the same reasoning can be made when $\frac{H_{H,t}}{H_{F,t}} < 1$ we see that the only solution is when $\frac{H_{H,t}}{H_{F,t}} = \frac{p_{F,t}(s)}{p_{H,t}(s)} = 1$.

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Appendix 3.E The worldwide money demand and price level in the steady state

In order to anchor prices we assume that there is only one worldwide currency and that the money supply - M^S- and circulation speed coefficient - v - are constant at all periods. These simplifications in the monetary market do not have any influence on the real side of the economy,as the economic model is a flex-price one and money's only purpose is to be a unit of account.

Worldwide money demand resorts to the quantity theory of money

$$M^d v = GDP_{H,t}^{Nom} + GDP_{F,t}^{Nom}$$

For simplicity assume that $v = 1$. Therefore:

$$GDP_{H,t}^{Nom} + GDP_{F,t}^{Nom} = M^d = M^S$$

Replacing $GDP_{H,t}^{Nom}$ and $GDP_{F,t}^{Nom}$:

$$\overline{H}_t^H.w_t^H + \overline{\Pi}_{H,t}^{RD} + \overline{H}_t^F.w_t^F + \overline{\Pi}_{F,t}^{RD} = M^S$$

Dividing by country population (that is equal in both countries):

$$H_t^H.w_t^H + \pi_{H,t}^{RD} + H_t^F.w_t^F + \pi_{F,t}^{RD} = \frac{M^S}{n_*}$$

From symmetry we, also, that total labour supply, nominal wages and profits are equal across countries. Therefore:

$$H_t^*.w_t^* + \pi_{*,t}^{RD} = \frac{M^S}{2n_*}$$

Replacing profits and wages:

$$\left(H_t^*.\frac{\theta-1}{\theta}A_t^H + \frac{\theta-1}{\theta}A_t^H.\frac{i_t}{(\theta-1)\,i_t + \theta.\varphi^V}.H_{*,t} \right) p_*(s) = \frac{M^S}{2n_*}$$

We get that:

$$p_*(s) = \frac{M^S}{2n_*}.\frac{1}{H_t^*.\frac{\theta-1}{\theta}A_t^*\left(1 + .\frac{i_t}{(\theta-1)i_t+\theta.\varphi^V}\right)}$$

Replacing labour in the utility function we have that the price level is:

$$p_H(s) = \frac{M^S}{2n_H} . \frac{1}{\left(\frac{\left(\frac{\left(\gamma_H^H\right)^{\theta} .+ \left(\gamma_H^F\right)^{\theta} .(1-T)^{\theta-1}}{(\theta-1)i_t + \varphi^V .\theta} \right)^{\frac{\rho-1}{(1-\theta)\rho}}}{\left(\frac{\theta.(i_t+\varphi^V)}{(\theta-1)i_t+\theta.\varphi^V} . \right) . \left(\frac{\theta-1}{\theta} \right)^{1-\frac{1}{\rho}} .(k_1)^{\frac{1}{\rho}}} \right)^{\frac{\rho(1-\theta)}{\varpi(1-\theta)+1-\rho\theta}} . \frac{\theta-1}{\theta} A_t^H \left(1 + . \frac{i_t}{(\theta-1)i_t+\theta.\varphi^V} \right)}$$

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Appendix 3.F Sensibility analysis to φ^V

This appendix shows the results when we vary the value of φ^V (the dead rate of varieties). The values tested for φ^V are 0.036 (around 28 quarters or 7 years) and 0.125 (8 quarters or 2 years). The first section shows the results for cases 1 (shocks in the final good sector), 2 (shocks in the R&D sector without negative contamination) and 3 (shocks in the R&D sector with negative contamination). The second section shows the results for case 4.2 (shocks in the final good sector and in the R&D sector with negative contamination).

3.F.1 Cases 1, 2 and 3

The simulations in this case were done for the base values of the different parameters corresponding to the first columns of tables 3.6, 3.8 and 3.10 of the third chapter: $\beta = 0.99$, $\rho = 2$, $\theta = 1,7$, bias =0, T=0, $\varpi = 1$ and $\psi = 0,00037$.

From tables F1 and F2 we can verify that the different values of φ^V do not cause many significant changes and from the IRFs depicted in figures (F1) to (F6), the responses of the different variables have the same shape and for the those variables that are not connected directly with the R&D sector they are similar in terms of response magnitude.

In case 1 the biggest differences in terms of moments when φ^V decreases (less rotation of varieties) are the increases in the correlations between H_H and H_F (from 0.779 to 0.974), GDP and *new varieites* (from 0.573 to 0.908), the decrease in the correlation between new varieties in both countries (from 0.975 to 0.452) and the increase in the std. deviation of terms of trade relative to GDP (from 0.992 to 1,123). In the IRFs - figures (F1) and (F2) - we can realize that when there is a slower rotation of varieties in the market (φ^V =0.036) that there is a more pronounced impact in v_H , bigger response of workers at the R&D sector in the home country, a smaller response at the moment of the shock but bigger variation in v_F and smaller responses in H_F^{RD} and $\frac{pv_H}{p_H(s)}$.

The more pronounced impact in v_H when φ^V is small is due to the fact that now varieties have a longer expected life time, therefore in the steady state there will be more varieties and consequently more workers in the R&D sector. Therefore, for an equal shock, more workers move at the impact from one sector to other leading to a reduction on new varieties produced. The number of varieties produced take longer to adjust (as the dead rate takes longer to reduce the initial number of varieties in the market) but also the relative variation to get to the optimal level is also bigger. As for the foreign country as the adjustment in the

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home country is longer, the foreign country takes some time to be fully hit by the variations in the home country and therefore the response at the impact of H_F^{RD} and of υ_F is smaller.

In case 2, when φ^V decreases the relative volatility of GDP to all other variables (except net trade balance) decreases, moreover the cross-country correlations of GDP and hours worked increase and the correlation between TOT and GDP decrease. In fact for higher values of φ^V the cross country correlation of GDPs is negative and the one between GDP and TOT is positive.

In terms of IRFs - figures (F3) and (F4) - we can realize that the only difference is that foreign output and foreign hours worked have smaller responses for lower levels of varieties rotation (lower φ^V). This smaller response to foreign shocks is the responsible for the lower levels of volatility of GDP. On the other hand when φ^V increases, as the GDP response to foreign shocks is bigger that lead to the negative cross country correlation of GDPs and the positive correlation between GDP and TOT as the initial reduction is deeper[43].

Finally in case 3 the reduction in φ^V leads to a reduction in the correlation of output and consumption, as well as consumption is less volatile and hours worked are more volatile than output . In terms of the IRFs the biggest difference (other than the adjustment and reaction of varieties due to the different dead rates) are the relative prices of varieties to the ones of final goods.

Overall there are no qualitatively big differences as the different cases have with sightly different magnitudes the same problems, when we confront the results with the data.

[43] Remember that for computing the moments we are filtering the data, using the frequnecies between 8 and 32 quarters, therefore the movements of the variables at the impact have a very limited impact in the measured moments.

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Table F1: Estimated moments for cases 1, 2 and 3 - 1st set

Case	1			2			3		
φ^V	0,036	0,075	0,125	0,036	0,075	0,125	0,036	0,075	0,125
Within Country moments									
corr(y,c_c)	0.617	0.619	0.625	-0.609	-0.074	0.327	-0.814	-0.579	-0.264
	(0.146)	(0.145)	(0.145)	(0.094)	(0.157)	(0.170)	(0.054)	(0.100)	(0.139)
corr(y,h)	-0.493	-0.39	-0.281	0.295	0.325	0.390	0.429	0.469	0.472
	(0.179)	(0.195)	(0.300)	(0.203)	(0.188)	(0.165)	(0.171)	(0.159)	(0.152)
corr(y,tot)	-0.797	-0.794	-0.785	-0.336	-0.101	0.139	-0.709	-0.670	-0.573
	(0.084)	(0.087)	(0.089)	(0.182)	(0.182)	(0.167)	(0.100)	(0.102)	(0.118)
corr(y,nx)	0.795	0.793	0.785	0.458	0.535	0.657	0.742	0.751	0.732
	(0.085)	(0.087)	(0.090)	(0.175)	(0.156)	(0.124)	(0.096)	(0.092)	(0.096)
std.(c_c)/std.(y)	0.629	0.630	0.632	1.040	0.945	0.851	0.924	0.940	1.021
	(0.114)	(0.115)	(0.115)	(0.104)	(0.133)	(0.155)	(0.073)	(0.103)	(0.140)
std.(h)/std.(y)	0.318	0.328	0.300	3.041	2.487	1.915	1.618	1.311	1.108
	(0.068)	(0.069)	(0.069)	(0.650)	(0.512)	(0.353)	(0.306)	(0.238)	(0.196)
std.(tot)/std.(y)	1.123	1.088	0.992	4.961	4.417	3.649	2.883	2.794	2.896
	(0.164)	(0.156)	(0.149)	(0.998)	(0.825)	(0.601)	(0.419)	(0.403)	(0.441)
std.(nx)/std.(y)	0.374	0.365	0.342	2.083	2.156	2.104	1.587	1.615	1.680
	(0.055)	(0.052)	(0.051)	(0.420)	(0.404)	(0.350)	(0.230)	(0.227)	(0.242)

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Table F2: Estimated moments for cases 1, 2 and 3 - 2nd set

Case	1			2			3		
φ^V	0,036	0,075	0,125	0,036	0,075	0,125	0,036	0,075	0,125
Cross countries correlations									
corr(y(h),y(f))	-0.291	-0.273	-0.252	0.519	0.256	-0.077	-0.119	-0.166	-0.140
	(0.209)	(0.212)	(0.215)	(0.168)	(0.225)	(0.243)	(0.214)	(0.210)	(0.209)
corr(c_c(h),c_c(f))	0.996	0.997	0.998	0.953	0.941	0.950	0.376	0.205	0.202
	(0.002)	(0.001)	(0.001)	(0.023)	(0.030)	(0.028)	(0.190)	(0.218)	(0.225)
corr(h(h),h(f))	0.974	0.899	0.779	-0.083	-0.240	-0.417	0.858	0.939	0.948
	(0.014)	(0.048)	(0.093)	(0.219)	(0.207)	(0.181)	(0.069)	(0.028)	(0.023)
Varieties indicators (calculated on annual based values)									
corr(y,nv)	-0.908	-0.759	-0.573	0.139	0.138	0.147	-0.214	-0.271	-0.330
	(0.076)	(0.141)	(0.214)	(0.286)	(0.270)	(0.253)	(0.270)	(0.259)	(0.243)
corr(tot,nv)	0.503	0.250	0.014	0.800	0.806	0.813	0.732	0.721	0.730
	(0.222)	(0.277)	(0.290)	(0.107)	(0.100)	(0.091)	(0.133)	(0.133)	(0.124)
std.(nv)/std.(y)	0.422	0.365	0.300	13.060	9.946	7.288	7.649	6.308	5.890
	(0.064)	(0.07)	(0.076)	(3.921)	(2.958)	(2.082)	(2.097)	(1.667)	(1.467)
std.(nv)/std.(tot)	0.404	0.361	0.323	2.676	2.379	2.152	2.453	2.055	1.781
	(0.113)	(0.110)	(0.101)	(0.485)	(0.406)	(0.344)	(0.485)	(0.404)	(0.336)
corr(nv(h),nv(f))	0.452	0.858	0.975	-0.340	-0.406	-0.478	-0.143	-0.148	-0.210
	(0.244)	(0.09)	(0.022)	(0.262)	(0.247)	(0.227)	(0.278)	(0.276)	(0.266)

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Figure F1: Case 1 - Sensibility to φ^V. (1st Set)

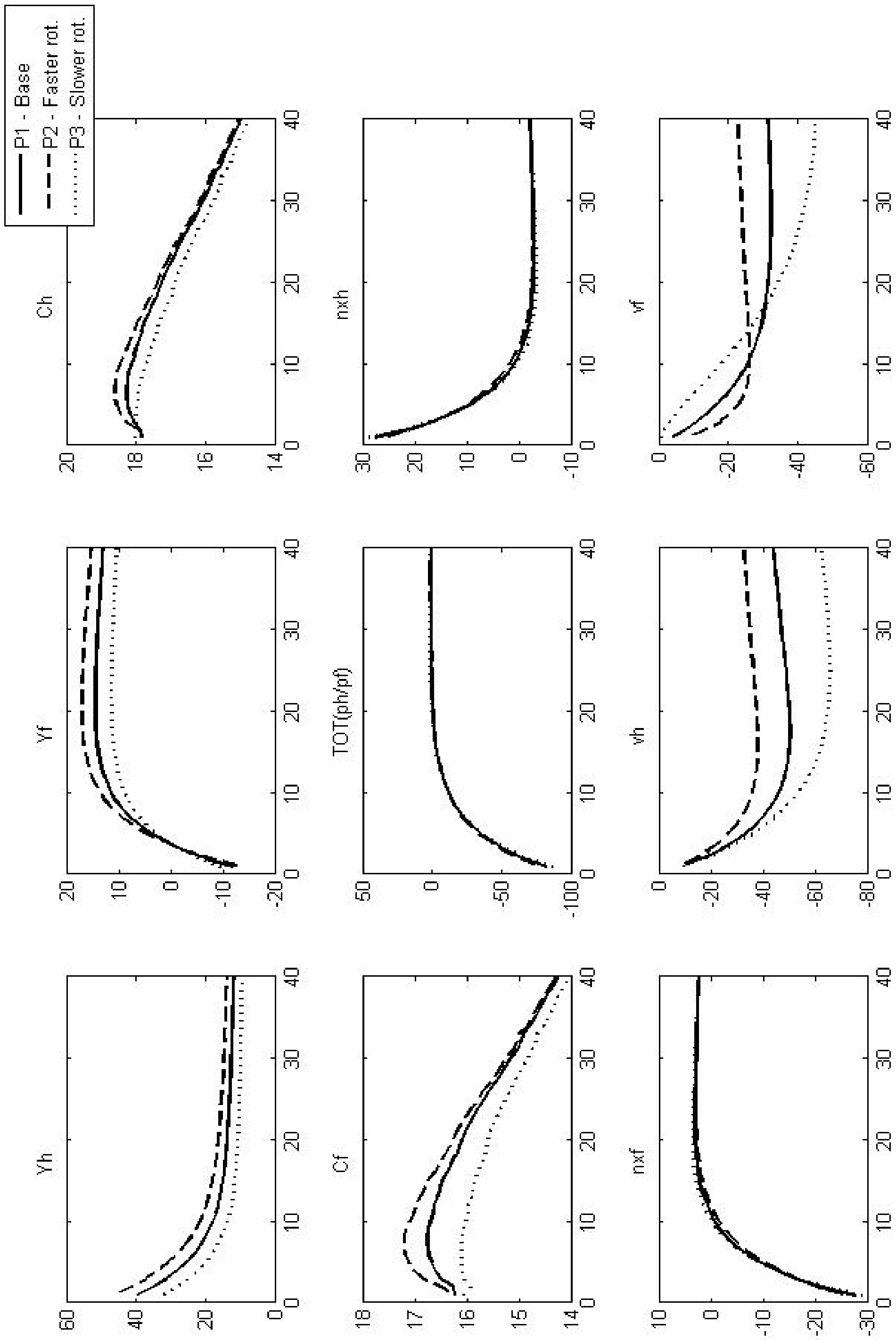

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Figure F2: Case 1 - Sensibility to φ^V. (2nd Set)

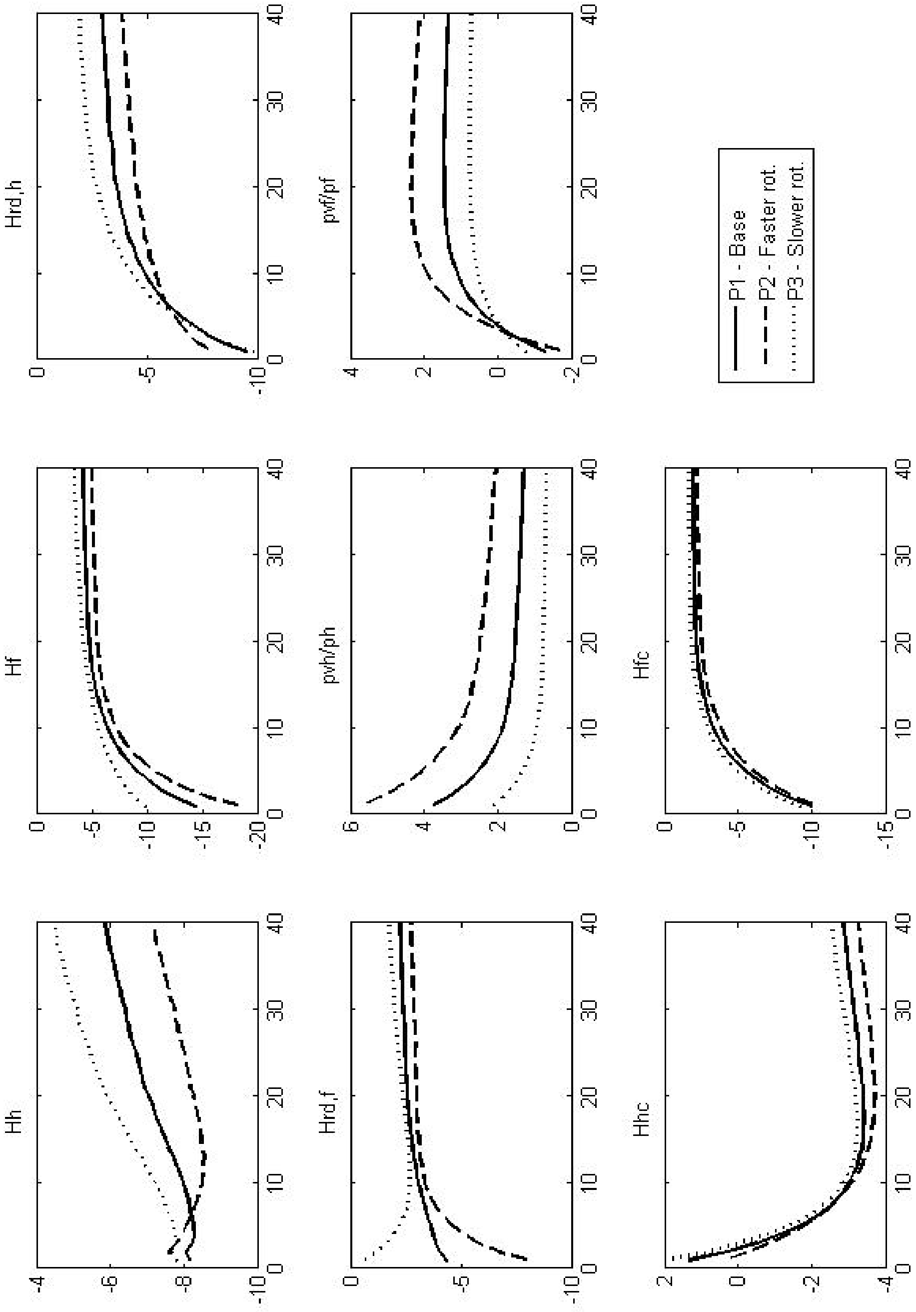

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Figure F3: Case 2 - Sensibility to φ^V. (1st Set)

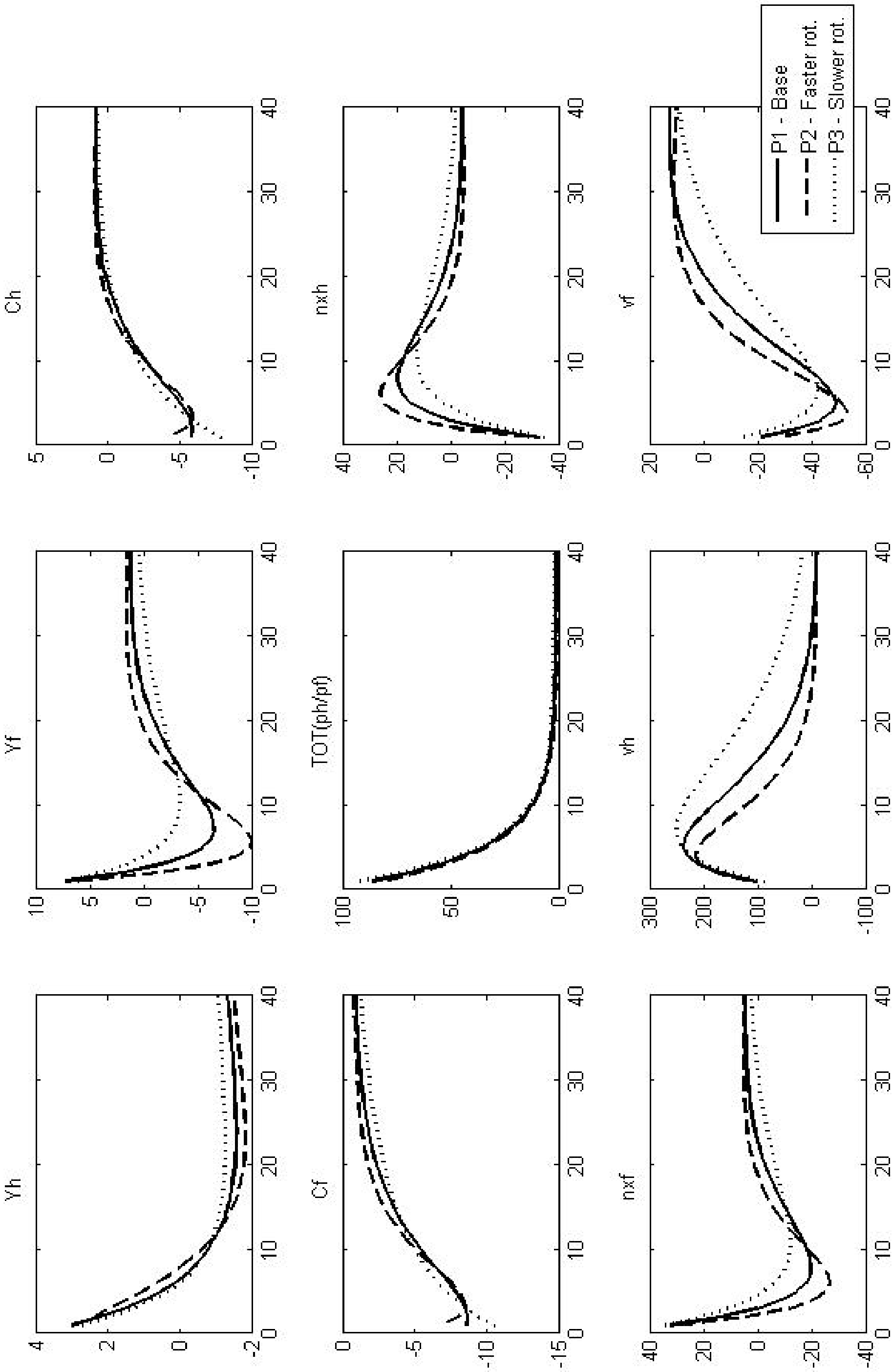

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Figure F4: Case 2 - Sensibility to φ^V. (2nd Set)

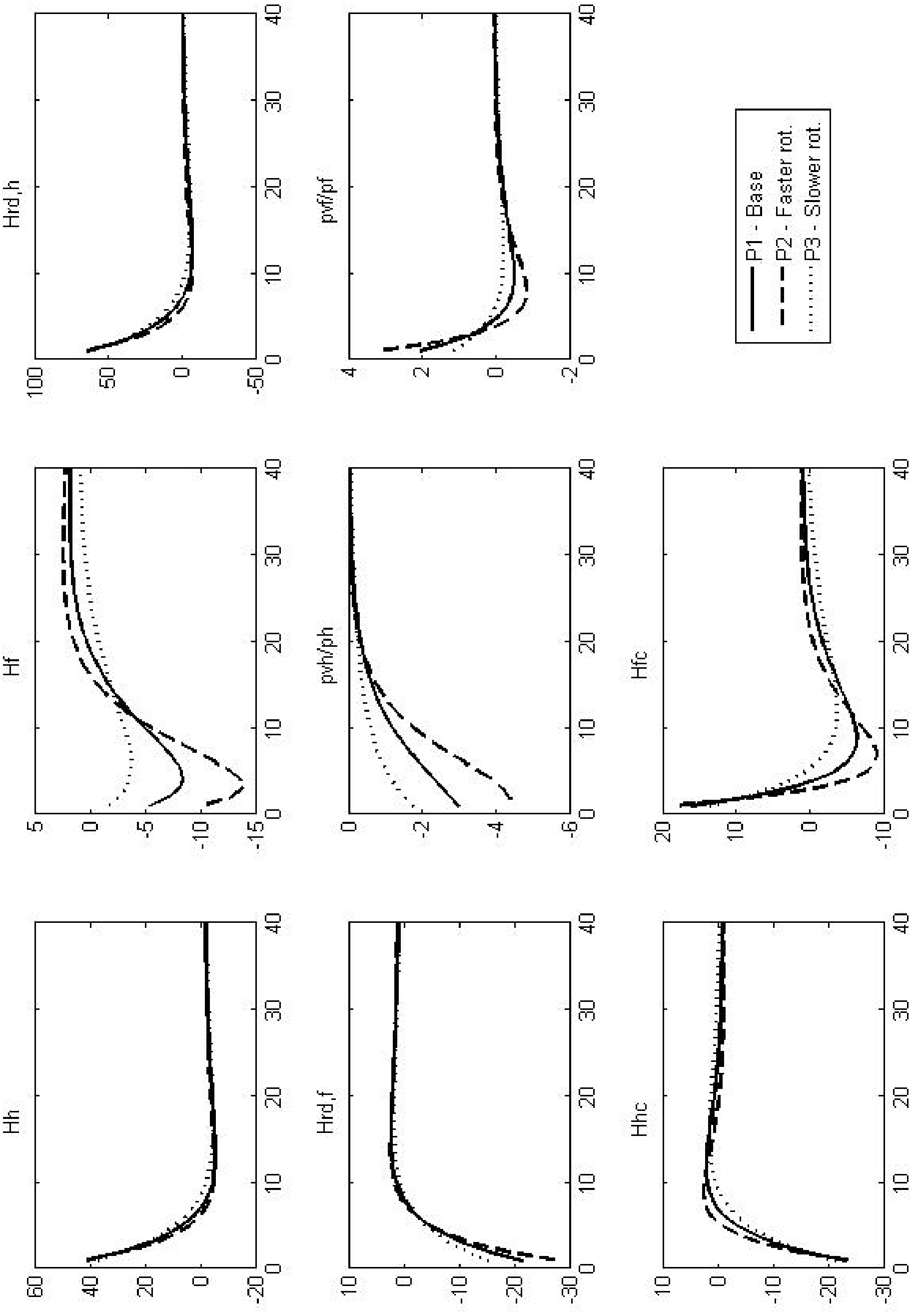

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Figure F5: Case 3 - Sensibility to φ^V. (1st Set)

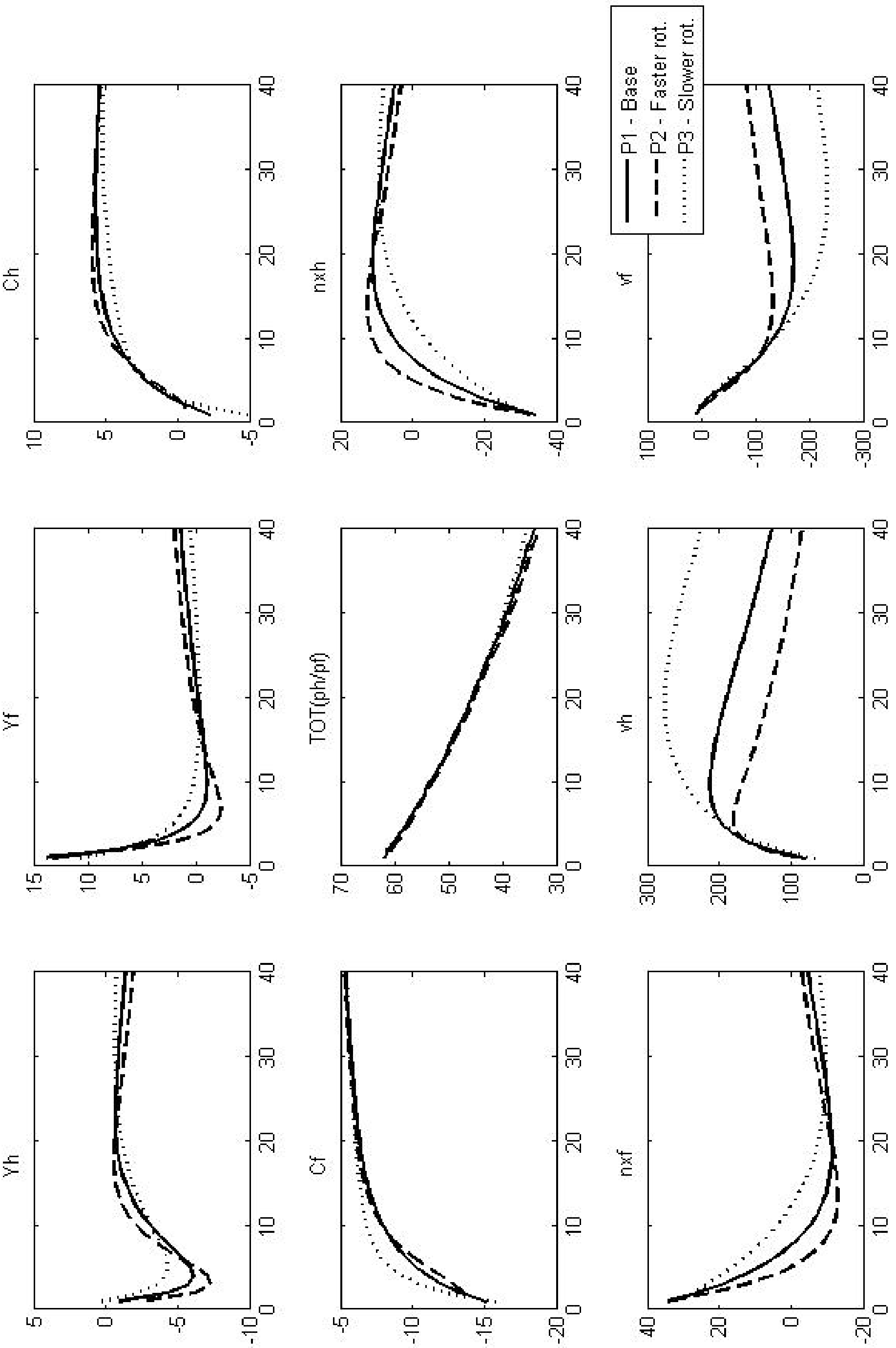

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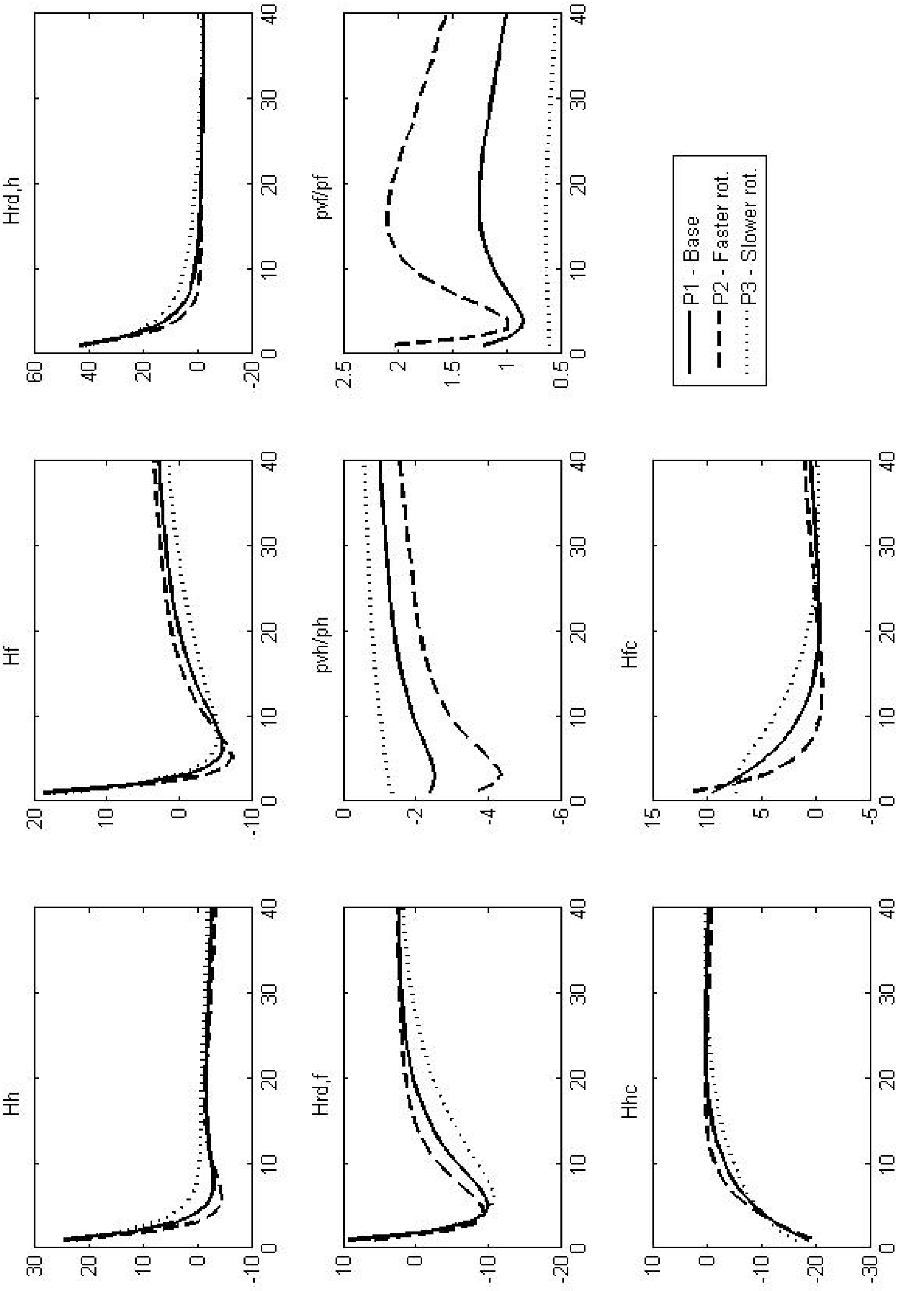
Figure F6: Case 3 - Sensibility to φ^V. (2nd Set)



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3.F.2 Case 4.2

Tables F3 and F4 depict the results with $\varphi^V = 0.036$ and tables F5 and F6 with $\varphi^V = 0.125$ for case 4.2 . Overall, there are not great changes in the moments, as they keep relative close when compared to the base case. However when φ^V increases we can realize that the correlations between GDP and Consumption or TOT have a tendency to increase, the correlations between GDP and hours worked have a tendency to decrease and GDP is less volatile than the other variables (with the exception of hours worked) .

Has for the cross-country correlations there isn't a tendency to increase or decrease, however the cross-country correlations of output and consumption in the first two columns are closer than what appeared in the base case in tables 3.14 and 3.15 . That's due to the fact that when φ^V decreases the cross-country correlations of consumption increase and when φ^V increase the cross-country correlations of output decrease. In fact if we go beyond the values depicted we get results that revert the order of magnitudes for high values of θ. An interesting fact is that the parameters in column three can also replicate the quantity anomaly for $\varphi^V = 0.036$.

As for the new varieties the volatility relative to GDP and TOT decrease with increases in the dead rate of varieties.

Table F3: Estimated moments for case 4.2 with $\varphi^V = 0.036$ - 1st set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
φ^V	0,036	0,036	0,036	0,036	0,036	0,036	0,036
$bias$ / T	0.2 / 0	0 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Within Country moments							
corr(y,c_c)	0.423 (0.168)	0.344 (0.175)	0.831 (0.063)	0.762 (0.089)	0.359 (0.195)	0.415 (0.182)	0.809 (0.075)
corr(y,h)	0.451 (0.169)	0.453 (0.170)	-0.373 (0.173)	0.134 (0.211)	0.277 (0.206)	0.418 (0.176)	0.0623 (0.208)
corr(y,tot)	-0.039 (0.205)	-0.089 (0.205)	-0.117 (0.224)	-0.361 (0.176)	-0.380 (0.187)	-0.168 (0.200)	-0.308 (0.185)
corr(y,nx)	-0.218 (0.202)	-0.205 (0.205)	-0.115 (0.220)	-0.125 (0.212)	0.2197 (0.211)	-0.160 (0.201)	-0.337 (0.192)
std.(c_c)/std.(y)	0.708 (0.129)	0.715 (0.133)	0.926 (0.100)	0.663 (0.087)	0.727 (0.148)	0.913 (0.182)	0.743 (0.090)
std.(h)/std.(y)	1.004 (0.175)	1.003 (0.190)	0.117 (0.022)	0.613 (0.127)	0.972 (0.202)	1.129 (0.226)	0.651 (0.137)
std.(tot)/std.(y)	1.317 (0.292)	1.192 (0.262)	1.264 (0.302)	0.796 (0.162)	1.698 (0.346)	1.930 (0.403)	1.025 (0.218)
std.(nx)/std.(y)	0.678 (0.145)	0.719 (0.152)	0.692 (0.161)	0.284 (0.061)	0.807 (0.175)	0.750 (0.160)	0.355 (0.074)

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Table F4: Estimated moments for case 4.2 with $\varphi^V = 0.036$ - 2nd set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
φ^V	0,036	0,036	0,036	0,036	0,036	0,036	0,036
$bias$ / T	0.2 / 0	0 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Cross country correlations							
corr(y(h),y(f))	0.474	0.488	0.570	0.160	0.166	0.491	0.281
	(0.165)	(0.161)	(0.154)	(0.211)	(0.206)	(0.169)	(0.202)
corr(c_c(h),c_c(f))	0.434	0.423	0.568	0.617	0.626	-0.058	0.232
	(0.188)	(0.191)	(0.161)	(0.148)	(0.140)	(0.229)	(0.220)
corr(h(h),h(f))	0.872	0.813	0.954	0.871	0.887	0.959	0.962
	(0.054)	(0.075)	(0.023)	(0.056)	(0.048)	(0.022)	(0.022)
Varieties indicators (calculated on annual based values)							
corr(y,nv)	0.505	0.485	0.201	0.469	0.207	0.327	0.362
	(0.217)	(0.222)	(0.271)	(0.229)	(0.276)	(0.249)	(0.249)
corr(tot,nv)	0.567	0.548	0.678	0.209	0.590	0.572	0.300
	(0.168)	(0.166)	(0.126)	(0.219)	(0.181)	(0.177)	(0.213)
std.(nv)./std.(y)	4.788	5.043	3.328	2.632	4.420	3.677	2.013
	(1.229)	(1.318)	(1.035)	(0.688)	(1.250)	(1.019)	(0.564)
std.(nv)./std.(tot)	3.104	3.545	2.435	2.953	2.419	1.680	1.746
	(0.713)	(0.808)	(0.470)	(0.753)	(0.552)	(0.393)	(0.450)
corr(nv(h),nv(f))	-0.069	-0.142	-0.443	-0.057	-0.071	0.055	0.043
	(0.278)	(0.275)	(0.227)	(0.281)	(0.280)	(0.281)	(0.281)

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Table F5: Estimated moments for case 4.2 with $\varphi^V = 0.125$ - 1st set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
φ^V	0,125	0,125	0,125	0,125	0,125	0,125	0,125
$bias$ / T	0.2 / 0	0 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Within Country moments							
corr(y,c_c)	0.641	0.607	0.855	0.838	0.544	0.644	0.851
	(0.133)	(0.136)	(0.059)	(0.072)	(0.159)	(0.128)	(0.068)
corr(y,h)	0.196	0.231	-0.623	-0.108	0.141	0.352	-0.204
	(0.204)	(0.203)	(0.124)	(0.214)	(0.215)	(0.197)	(0.206)
corr(y,tot)	0.086	0.130	0.052	-0.300	-0.339	-0.258	-0.278
	(0.199)	(0.202)	(0.226)	(0.184)	(0.192)	(0.193)	(0.190)
corr(y,nx)	-0.241	-0.181	-0.179	-0.104	0.201	-0.183	-0.384
	(0.198)	(0.201)	(0.223)	(0.209)	(0.215)	(0.205)	(0.196)
std.(c_c)/std.(y)	0.804	0.835	1.024	0.698	0.777	1.389	0.969
	(0.134)	(0.145)	(0.117)	(0.087)	(0.143)	(0.229)	(0.112)
std.(h)/std.(y)	0.891	0.903	0.128	0.553	0.763	0.960	0.630
	(0.177)	(0.178)	(0.020)	(0.117)	(0.165)	(0.201)	(0.142)
std.(tot)/std.(y)	1.452	1.392	1.457	0.769	1.727	2.324	1.183
	(0.314)	(0.307)	(0.350)	(0.156)	(0.360)	(0.488)	(0.257)
std.(nx)/std.(y)	0.850	0.917	0.892	0.362	0.889	1.018	0.520
	(0.173)	(0.178)	(0.194)	(0.073)	(0.190)	(0.211)	(0.108)

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Table F6: Estimated moments for case 4.2 with $\varphi^V = 0.125$ - 2nd set

θ	1,7	1,7	1,7	1,7	1.7	1,2	1,2
φ^V	0,125	0,125	0,125	0,125	0,125	0,125	0,125
$bias$ / T	0.2 / 0	0 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0	0.2 / 0
ϖ	1	1	10	1	1	1	1
$\frac{var(\varepsilon_t^{*,RD})}{var(\varepsilon_t^*)}$	3	3	3	1	3	3	1
$E(\varepsilon_t^*, \varepsilon_t^{*,RD})$	0.8	0.8	0.8	0.8	0.5	0..8	0.8
Cross country correlations							
corr(y(h),y(f))	0.344 (0.187)	0.334 (0.189)	0.640 (0.134)	0.054 (0.218)	0.072 (0.213)	0.515 (0.165)	0.291 (0.201)
corr(c_c(h),c_c(f))	0.325 (0.204)	0.221 (0.224)	0.263 (0.213)	0.596 (0.150)	0.565 (0.156)	0.140 (0.224)	0.198 (0.218)
corr(h(h),h(f))	0.897 (0.044)	0.830 (0.069)	0.841 (0.0783)	0.861 (0.060)	0.9181 (0.035)	0.603 (0.153)	0.656 (0.142)
Varieties indicators (calculated on annual based values)							
corr(y,nv)	0.489 (0.218)	0.509 (0.211)	0.313 (0.250)	0.464 (0.227)	0.148 (0.277)	-0.019 (0.247)	0.177 (0.265)
corr(tot,nv)	0.539 (0.157)	0.522 (0.153)	0.672 (0.124)	0.240 (0.593)	0.568 (0.176)	0.617 (0.152)	0.355 (0.193)
std.(nv)./std.(y)	4.009 (1.028)	4.219 (1.065)	2.887 (0.849)	2.046 (0.537)	3.376 (0.955)	2.603 (0.688)	1.511 (0.441)
std.(nv)./std.(tot)	2.399 (0.533)	2.604 (0.567)	1.861 (0.356)	2.406 (0.608)	1.846 (0.416)	1.043 (0.226)	1.165 (0.287)
corr(nv(h),nv(f))	-0.180 (0.264)	-0.270 (0.254)	-0.437 (0.226)	-0.202 (0.264)	-0.160 (0.267)	-0.074 (0.273)	-0.094 (0.273)

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Part IV

Conclusion

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In conclusion we think that the work presented in the previous chapters has largely fulfilled the objectives stated in the introduction and gives new insights into a number of problems viewed in a different perspective from what has been done before.

As for the first objective, to characterize business cycle synchronization across countries, instead of splitting the world into regions and then trying to find the common components specific to each we used a different approach without making any prior assumption about regional composition. First we estimated the common components and then we checked which common components could be associated with which countries groups, thus forming regions.

When we estimated the common components we found that we should consider four common components to characterize business cycle synchronization across countries and is notably the more important in high income countries.

As for the other common components the third common component affects mostly middle income countries; and the second and fourth bridge different countries without a distinctive geographical, economical or cultural similarity. Therefore we can define two groups of countries as regions in *substrictus sensus (* this is, countries for which their business cycle is affected by a distinctive common component), being the high income and the middle income countries. As for the low income countries they seem detached from the world economy, their business cycle being mostly due to idiosyncratic variations.

Even if we can only find two distinctive *substrictus sensus* regions we can find further subdivisions regarding the dependence structure from the different factors. In the high income countries we found evidence of two groups that we labeled the French-Japanese and the German-American groups. These two groups are evident if we speak about similar structure or commovement and therefore they form regions in *latus sensus.* Furthermore these two groups seem to form a world core in terms of commovement as the remaining countries seem to form a periphery. In the middle income countries we found a South East Asian group that has a similar dependence structure but does not commove. This leads us to consider that those countries may be starting to form a group but their integration is still not strong enough to classify them as a region in *latus sensus.*

As for the second objective stated in the introduction, to measure how much consumption risk sharing is done and in what way across countries, previous investigations have just managed to estimate aggregate values for countries groups. We tried to estimated individual values for each country using a factor model to gain power in order to be able to make inference over the results. Even if the estimated variance of the estimators is not as small

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as in panel methods, it is small enough in order to obtain statistically significant estimators which were difficult to get when individual estimation was done directly on the data.

Even if the aggregate results found are not different from those existing in the literature (asset markets are marginally important, if we aggregate the data the world value is zero), capital depreciation contributes to unsmooth consumption (-8% at world level), international transfers are not important (0% for the world) and the bulk of consumption is done through the savings/credit market mechanism (at the world level this accounts for 39%)), the fact is that when we looked to the individual estimations of the different channels these aggregations hide important differences among countries. Smoothing through the asset market channels ranges from -17.4% to 16.4%, whilst international transfers (with the exception of Germany and Portugal) are zero for most countries and the savings channel ranges from 2.5% to 82.9%.

This differences led us to try to relate the different values found for the asset market channel and the savings/credit market channel to different economic and financial characteristics of the countries.

As for the asset market channel, it is positively related with the importance of the world component in the country cycles and negatively (when we take out Belgium, Netherlands and Ireland) related to the degree of openness without any more evidence of other economic or financial factors. These results can be considered unexpected. As for the correlation with the importance of the world component in the country cycle, we could assume that the less important is the world component in the national cycle, the more the country has to gain from engaging in risk sharing through portfolio diversification, but we found the inverse relationship. As for the correlation to the degree of openness we could think that the more open is a country, the more integrated it is and therefore it would have easier access to international asset markets; however, this hypothesis is not only not confirmed but, if anything, we found the inverse relationship. Furthermore, we can think that the absence of correlation with financial indicators can, also, be seen as unexpected as this might indicate that the reasons for the differences through this channel are insensitive to financial market structure. However we can also think that the indicators used do not capture all the relevant differences in the financial markets which can explain the heterogenous values found for this channel.

For the saving channel we found a negative relationship with population size (when we do not consider US) indicating that smaller countries smooth more through this channel. This can be seen as expected, as smaller countries have a smaller impact on international credit markets and can lend/borrow to/from more agents. As for financial indicators we found that the estimators for this channel are correlated with one indicator of market efficiency (net

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interest margin) and one of market structure (concentration). More efficient banks (lower net interest margin) and a more concentrated market structure would lead to a more efficient consumption smoothing through the savings/credit channel.

As the final contribution of this thesis we proposed a different channel that could solve the consumption correlation puzzle and the price anomaly simultaneously. This channel would be shocks to the R&D sector that, together with shocks in the total productivity sector of final goods produced some encouraging results.

In the model used, the parameters were chosen from a calibration exercise (focusing in adjusting the theoretical moments to the data values) but keeping them inside the bounds of what the literature assumes reasonable for those that are standard. For the non-standard ones, this is the volatility and the stochastic process of creative shocks in the R&D sector, the time-series available for the creative-destructive process of varieties from industrial surveys are too sparse in time and heterogeneous across countries to be used . Therefore we calibrated them for a number of different alternatives.

The first set of results showed that those two kinds of shocks alone are not able to replicate most of the (unconditional) moments of the data and both have, with different magnitudes, the same problems. However, from this set of results we could see that even if the impact of shocks on the different variables leads to similar moments, the fact is that the relative response is not equal in each case, for which reason, when we allowed both shocks to coexist we found that we could replicate most of the moments from the data, including the correlation puzzle and the price anomaly, which was our aim. This was so for reasonable values of the standard parameters together with a strong correlation between the shocks in R&D and the productivity shocks in the final sector and with negative contamination across countries in the stochastic process of the R&D shocks.

Overall we managed to get some success in the objectives set forward in the introduction; however there are questions that this thesis did not address that deserve further research.

As for the characterization of the business cycle synchronization across countries we did not check the causes of it, as the model does not try to identify the factors with particular economic activities or structures. Moreover, previous studies using factors were unable to find a single variable that could explain the factors variation and therefore identification of what the factors are depicting is still a open question.

As for the different channels of consumption smoothing we should be careful when interpreting the correlations between the estimators and the different indicators. These relations should be read more as indications of what is causing the heterogeneity found across coun-

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tries on the smoothing mechanisms and which directions deserve further research, than as the definite causes of those differences.

Finally, in the last chapter even if we get some degree of success in replicating the moments from the data, the international commovements of investment, the home-bias puzzle, an analysis of the impact of outsourcing variety production (rather than just exporting the final product) and the effects of pricing to market were not addressed by the model. Future research could be extend the current framework to tackle those issues. Also, the model calls for future empirical research using data from industrial surveys, where we will have to collect and homogenize them across time and countries, to get some reasonable estimates of the R&D stochastic process from the data and to check whether the parameterizations used in the model can be considered reasonable when compared with those found in the data.

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