Spectral analysis of scale invariance in the temporal structure of precipitation in Mainland Portugal

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ABSTRACT

The temporal structure of precipitation was investigated by means of spectral analysis of daily precipitation, measured with non-recording gauges, over a period of 54 years. The data were recorded at eight locations in Mainland Portugal. Analysis of the precipitation intensity showed that scaling is present in the data, from one day up to more than one month. Empirical exponents describing the scaling of the energy spectra of the precipitation intensity were determined. Results showed that spectral analysis is able to quantify differences in precipitation variability, taking into account the behaviour observed across scales.

1. INTRODUCTION

Atmospheric processes that produce precipitation operate over a variety of time and space scales, and interact, for example, with surface topography, soil moisture and vegetation. Precipitation is a highly non-linear hydrologic process that exhibits wide variability over a broad range of scales: in time, over intervals of minutes to years; and in space, from less than one to several thousand square kilometres.

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In recent years, scale-invariant approaches have been given considerable attention by the scientific community; they have been used to study diverse types of geophysical processes. In particular, these approaches are being used to study the precipitation process, both in time and in space.

Scaling (or scale-invariance) is a well-known concept in physics. It is based on the invariance of properties across scales. Thus, scaling relates to the absence of a characteristic scale or length in, for example, processes or equations. Scaling is expected to hold from some large (outer or upper) scale down to a small (inner or lower) scale.

In this work the scale-invariant temporal structure of precipitation is investigated with spectral methods. The precipitation data were recorded at eight locations, distributed over Mainland Portugal. The resolution of the data is one day and the length of the records is 54 years.

2. INVESTIGATING SCALE-INVARIANCE WITH SPECTRAL ANALYSIS

Spectral methods are also known as Fourier transform methods (see e.g. Wu, 1973; Box and Jenkins, 1976; Press et al., 1989; Hastings and Sugihara, 1993). The idea behind these methods is that a physical process can be described either in the time domain (by the values of some quantity as a function of time) or in the frequency domain (where the process is specified by giving its amplitude as a function of frequency). The two representations are linked by means of the Fourier transform equations. The Fourier transform can be an efficient computational tool for accomplishing certain manipulations of the data. The related power spectrum, which can be defined as the distribution of variance or power across wavelength or frequency, can be itself of intrinsic interest (see e.g. Press et al., 1989).

Spectral analysis is one approach to the study of the statistical properties of time series. It provides a useful exploratory analysis tool for examining time-series data. Spectral analysis can provide an intuitive frequency-based description of the time series and indicate interesting features such as long memory, presence of high frequency variation and cyclical behaviour (e.g. McLeod and Hipel, 1995). This type of analysis is useful to detect periodicity of short-interval precipitation sequences at a point (Wu, 1973). If a process contains periodic terms, the frequencies of these terms exhibit a number of high and sharp peaks in the spectrum. This indicates that a significant amount of variance is contained in these frequencies (Wu, 1973; Press et al., 1989).

One can use standard spectral methods and analysis to test for scale-invariance. The most familiar consequence of scaling is the power-law behaviour that is expected in the energy (power) spectra of scaling processes (e.g. Mandelbrot, 1982; Schertzer and Lovejoy, 1985, 1987; Ladoy et al., 1991; Lovejoy and Schertzer, 1995):

$$E(\omega) \approx \omega^{-\beta} \qquad (1)$$

where ω is the wave-number, $E(\omega)$ is the energy, and β is the spectral exponent. For temporal processes, the wave-number ω can be approximated by $\omega \sim 1/\tau$, τ being the magnitude of any time interval (i.e., in this application, ω is a frequency).

The type of behaviour expressed in Eq. (1) is expected to occur over a range of wave-numbers and might not be observed for small samples. The energy spectrum is only second-order statistics (i.e. the spectrum is related to the Fourier transform of the autocorrelation function) and, thus, is not particularly robust. When applied to highly intermittent data, large samples may be needed to obtain good estimates of the ensemble average spectra (see e.g. Lovejoy and Schertzer, 1991).

3. THE PRECIPITATION DATA

The precipitation data analysed in this study were recorded at eight locations in Mainland Portugal. Figure 1 shows the location of the measuring stations. The coordinates of the measuring sites and their altitude above mean sea level are indicated in Table 1.

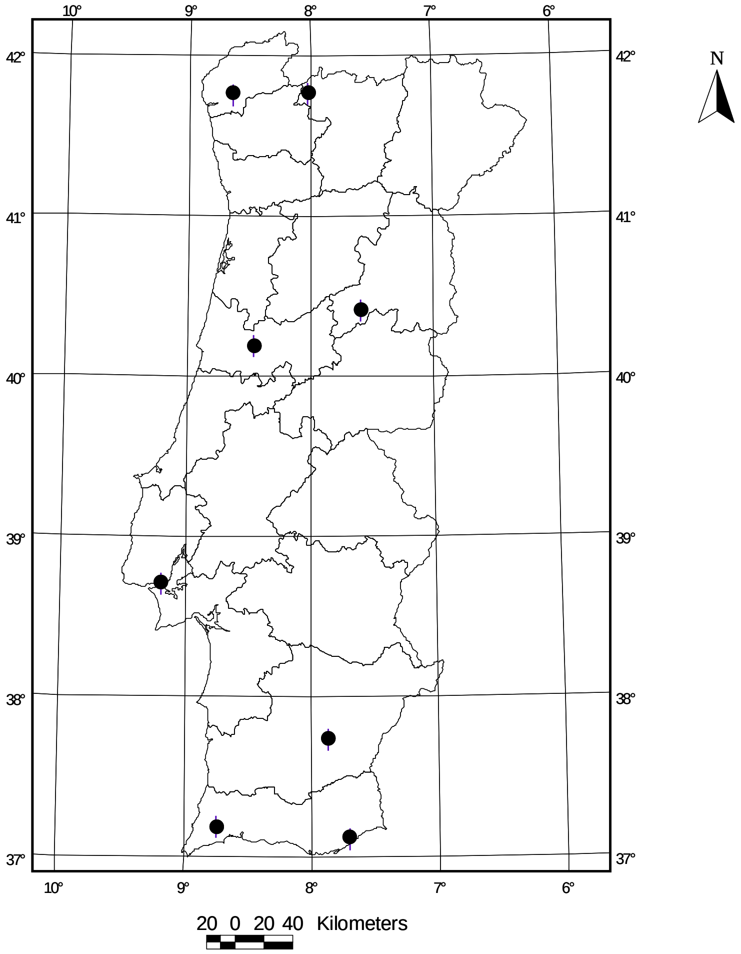

Figure 1 - Map of Mainland Portugal localizing the precipitation measuring stations used in this study (see also Table 1).

Table 1 - Coordinates and altitude above mean sea level of the precipitation measuring sites

Measuring stations	Latitude (N)	Longitude (W)	Altitude (m)
Ponte de Lima	41°46'	8°36'	15
Outeiro do Gerês	41°47'	7°58'	800
Coimbra/Geofísico	40°12'	8°25'	141
Penhas Douradas	40°25'	7°33'	1380
Lisboa/Geofísico	38°43'	9°09'	77
Algodor	37°45'	7°49'	163
Bravura/Barragem	37°12'	8°42'	75
Tavira	37°07'	7°39'	25

Mainland Portugal is located in the transitional region between the sub-tropical anticyclone and the sub-polar depression zones. The most conditioning climate factors in Mainland Portugal are, in addition to latitude, its orography and the effect of the Atlantic Ocean. Although the variation in climate factors is rather small, it is still sufficient to justify significant variations in precipitation. There are marked differences between the northern and southern regions, as well as between the littoral belt and the more in-land regions. While the northwest region of Portugal is one of the wettest spots in Europe, with mean annual precipitation in excess of 2000 mm, average rain amounts in the interior of the Alentejo (in the southern part of Portugal) are of the order of 500 mm and show large interannual variability. As other southern European regions, Portugal is a place of mild Mediterranean climate, with a warm and dry summer period, more pronounced in the southern regions, but with well known vulnerability to climate variability, namely to droughts and desertification in the southern sector.

Two storm types dominate the occurrence of precipitation in Mainland Portugal: convective storms and frontal storms. Convective storms are frequent during the summer season and the early and mid autumn, and are more frequent in the southern regions; frontal storms occur principally in the winter season, and affect more the northern regions.

The precipitation measuring devices are of the 20-14-G type (according to the classification by Sevruk and Klemm, 1989); they have horizontal openings of 200 cm^2 at 1.5m above the ground surface. The gauges were observed daily. The resolution of the measurements is 0.1 mm of precipitation. Trace precipitation of less than 0.1 mm is disregarded and such days are considered dry (zero-precipitation days).

For the eight locations identified in Table 1, Figure 2 shows the monthly precipitation recorded in the years 1941 to 1994 and Figure 3 shows the monthly average precipitation. Table 2 contains some descriptive statistics of precipitation at those locations.

Table 2 - Descriptive statistics of precipitation data from eight locations in Mainland Portugal, for the period 1941-1994.

Type of precipitation data (1941-1994)	Measuring stations							
	Ponte de Lima	Outeiro do Gerês	Coimbra	Penhas Douradas	Lisboa	Algodor	Bravura	Tavira
Mean annual prec. (mm)	1643.6	2233.5	976.9	1692.2	727.6	506.2	692.8	563.1
Coef. of variation (annual prec.)	0.235	0.324	0.244	0.251	0.258	0.298	0.292	0.347
Precipitation in wettest year (mm)	2820.7	4238.2	1651.4	2669.3	1336.0	984.7	1447.0	1165.8
Precipitation in driest year (mm)	994.4	839.1	524.2	1023.9	416.0	244.6	349.0	250.2
Average monthly prec. (mm)	137.0	186.1	81.4	141.0	60.6	42.2	57.5	46.9
Coef. of variation (monthly prec.)	0.901	1.111	0.904	0.987	1.069	1.116	1.163	1.279
Prec. in wettest month (mm)	664.1	1485.0	467.4	802.6	383.3	288.1	440.5	405.2
Prec. in driest months (mm)	0*	0*	0*	0*	0*	0*	0*	0*
Precipitation in wettest day (mm)	137.2	242.0	79.3	159.8	95.6	111.2	97.7	186.0
Coef. of variation (daily prec.)	2.287	2.587	2.454	2.525	2.983	3.525	3.167	3.946

* Daily precipitation below 0.1 mm is disregarded.

The data illustrates the temporal and spatial variability of precipitation in Mainland Portugal. All the cases analysed exhibit a marked seasonal distribution of precipitation during the year. For the wettest and the driest site, respectively, Outeiro do Gerês and Algodor, the number of months that had less than 0.1 mm of precipitation was the following: in Outeiro do Gerês, 18 months (roughly 2.8% of the sample); and in Algodor, 118 months (roughly 18.2% of the sample). The size of each sample was 648 months.

The tendency of the data in Table 2 also illustrates the behaviour of precipitation that result from the singular nature of this process at the small-scale limit. The scale of observation of precipitation is generally larger than the innermost scale (i.e. homogeneity scale). Thus the observations are averages of the densities of the process over the resolution of the measuring devices. The intensity range depends on the resolution level of the observation. The variability of precipitation involves a large dynamic range, which in certain cases leads to catastrophic events, both related to drought and flood situations.

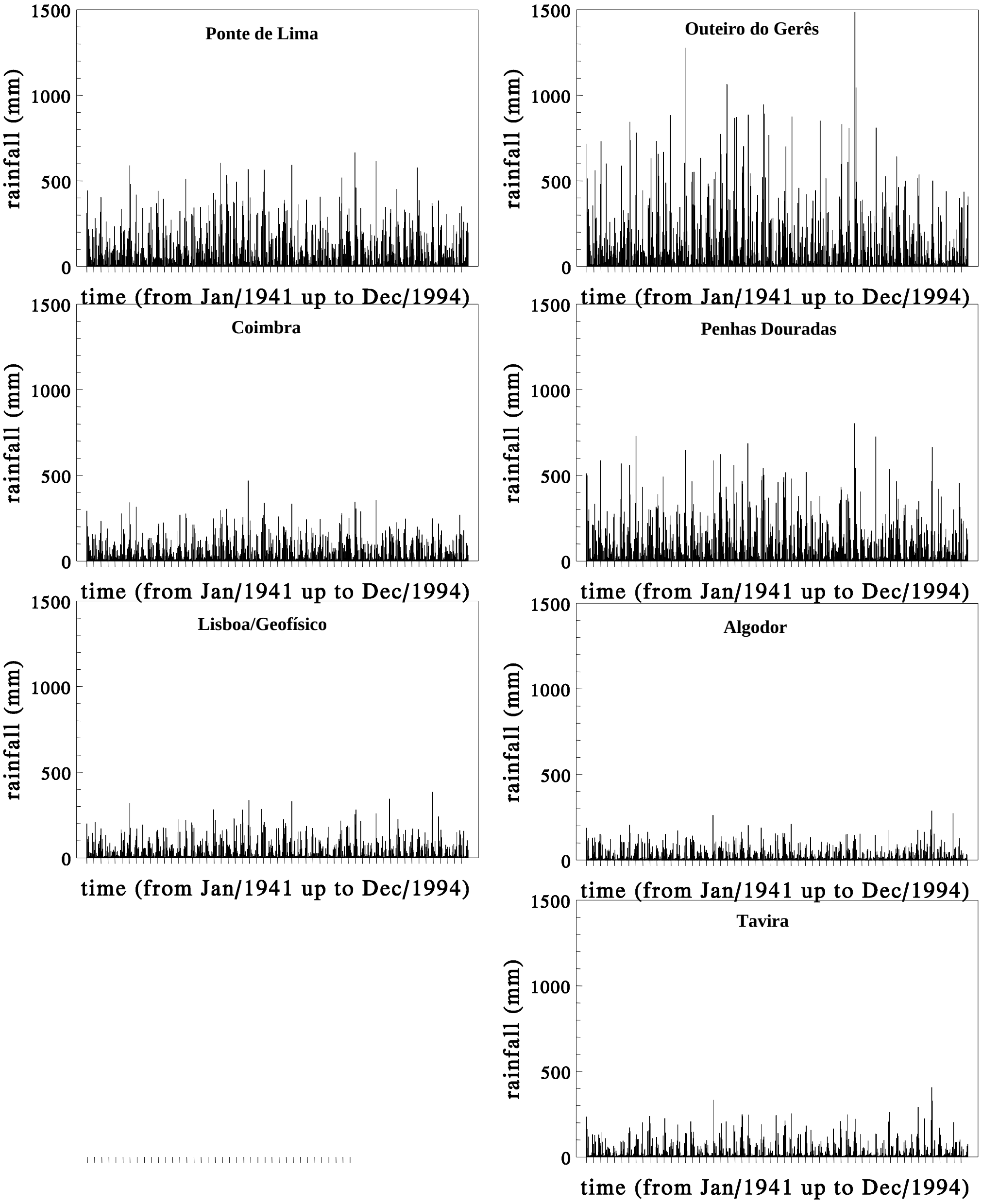

Figure 2 - Monthly precipitation recorded at eight locations in Mainland Portugal, for the years 1941 to 1994.

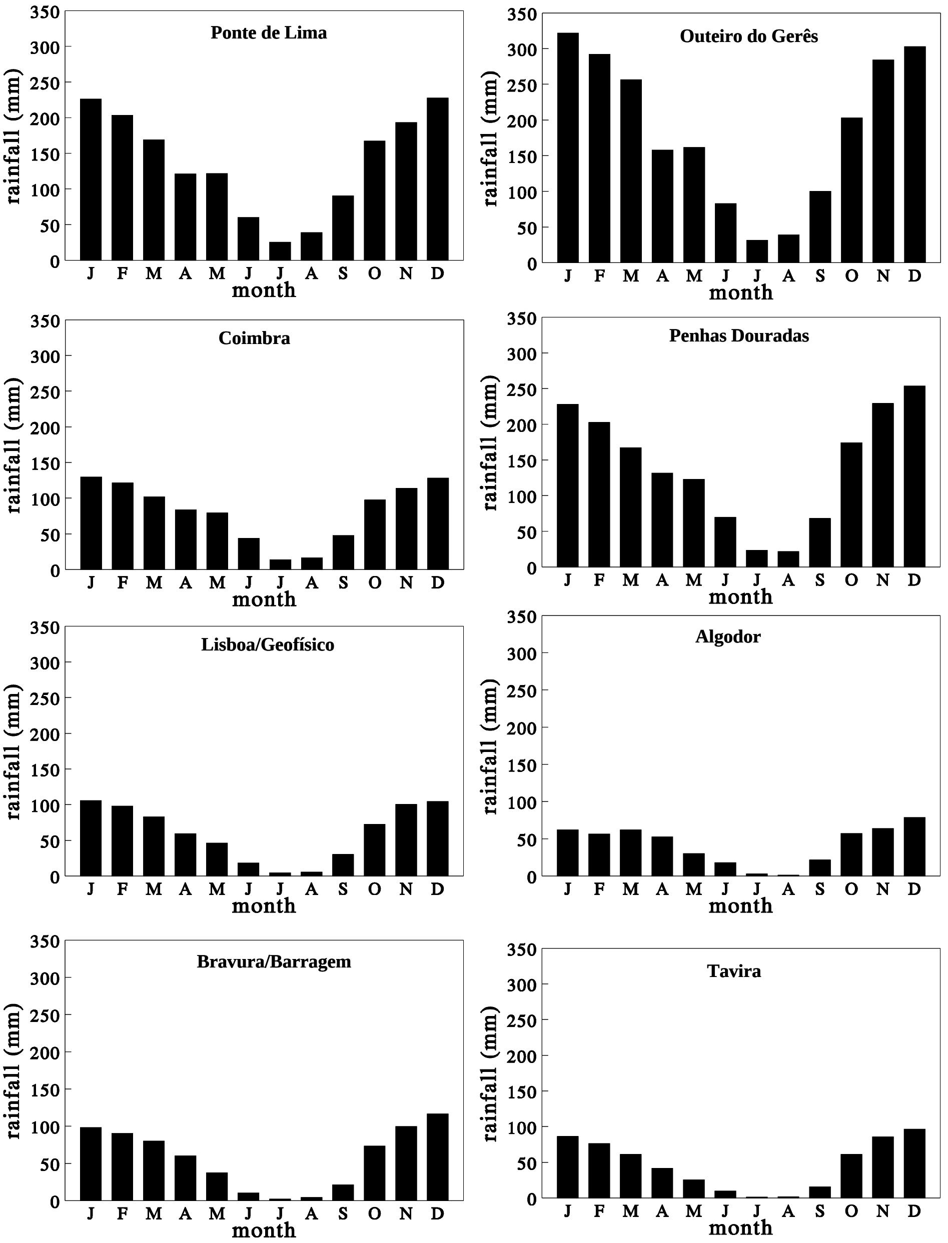

Figure 3 - Monthly average precipitation in eight locations in Mainland Portugal, for the years 1941 to 1994.

4. THE SCALE-INVARIANT BEHAVIOUR OF THE TEMPORAL STRUCTURE OF PRECIPITATION: EXAMPLES FROM PORTUGAL

This section presents results of the investigation of scaling in the temporal structure of precipitation from eight locations in Mainland Portugal (see previous section) using spectral analysis. The data were recorded from 1941 to 1994.

The energy spectra obtained for the daily precipitation from Ponte de Lima, Outeiro do Gerês, Coimbra, Penhas Douradas, Lisboa, Algodor, Bravura and Tavira are plotted in log-log axis in Figure 4. The spectra have been smoothed for high frequencies.

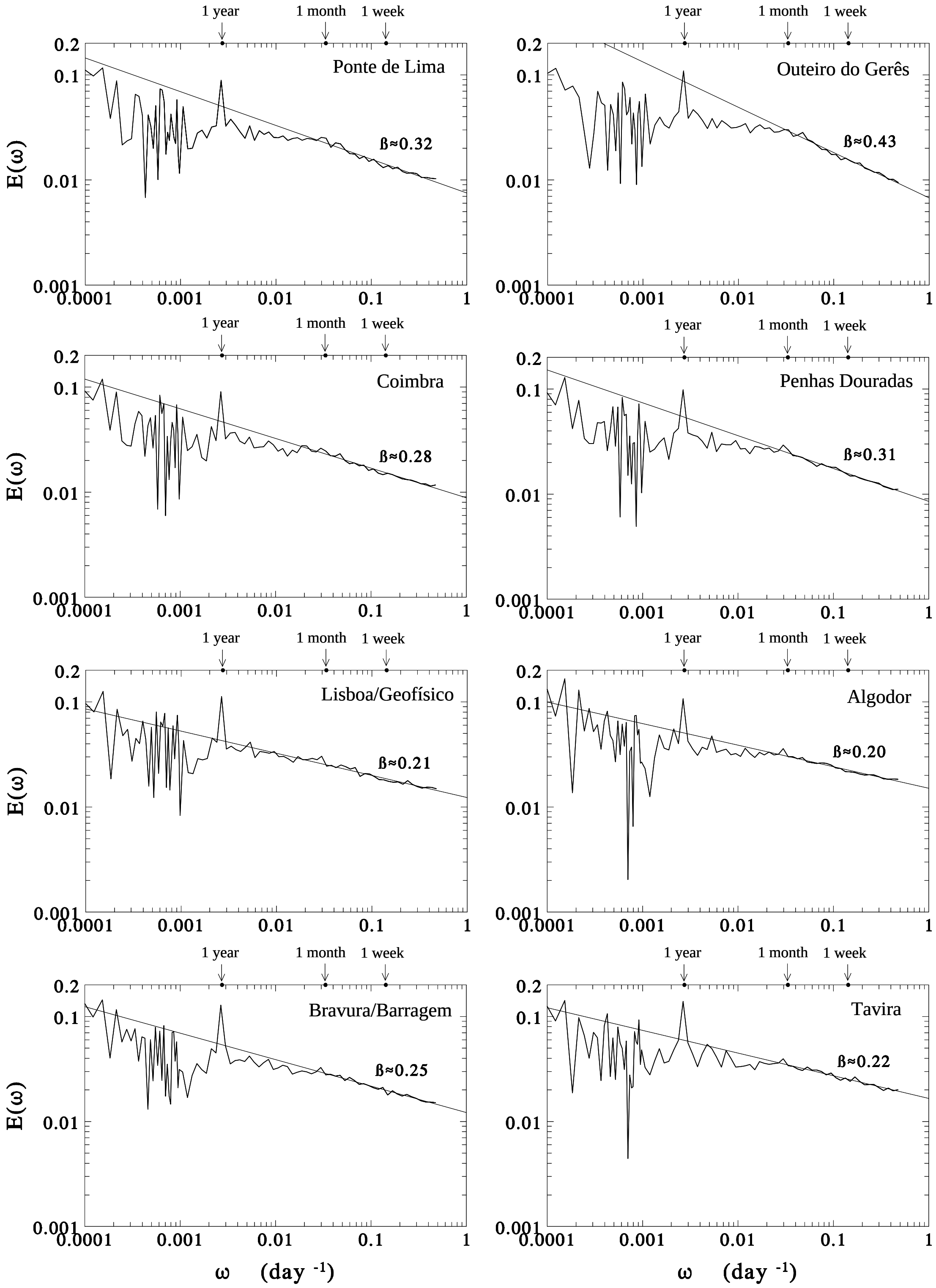

Figure 4 - Energy spectra obtained for daily precipitation recorded at eight locations in Mainland Portugal, from 1941 to 1994.

The spectra exhibit power-law behaviour, confirming the presence of scale invariance in the temporal structure of precipitation over a wide range of scales. The power-law behaviour of the spectra extends from 1 day up to about one-and-a-half months. In certain cases the behaviour suggests that the scaling behaviour extends to even larger scales (see, e.g. the spectrum obtained for the data from Lisboa). The adjacent region of the spectrum exhibits a nearly flat-power behaviour (i.e. $E(\omega)\approx\omega^0$).

To obtain estimates for the spectral exponents β, in Eq. (1), power laws were fitted to the energy spectra with linear regressions of $\log(E(\omega))$ versus $\log(\omega)$, over the range of scales exhibiting scale-invariant (power-law) behaviour. The spectral exponents ß that correspond to the right-hand side scaling regions of the spectra in Figure 4 were estimated as indicated in Table 3. The values of ß are in the range from 0.43, for the data from Outeiro do Gerês, to 0.20, for the data from Algodor. The lower the magnitude of the spectral exponent ß, the higher the variability in the data. As expected, the variability present in the data varies strongly with location.

Table 3 - Spectral exponents obtained for the daily precipitation data, from 1941 to 1994.

Measuring stations	Spectral exponent ß
Ponte de Lima	0.32
Outeiro do Gerês	0.43
Coimbra/Geofísico	0.28
Penhas Douradas	0.31
Lisboa/Geofísico	0.21
Algodor	0.20
Bravura/Barragem	0.25
Tavira	0.22

The spectral peaks at ?≈0.0027 day^{-1} clearly observed in the spectra correspond to the annual cycle frequency associated to the precipitation process. This oscillation seems to 'emerge' on a scaling background, since the spectral slope remains unchanged on both sides of the annual peak.

For rainfall in Europe, Fraedrich and Larnder (1993) reported a flat spectral plateau for time scales from about 3 years down to 3 days. Such type of spectral plateau is not seen in Figure 4 for this range of scales. The scaling regime associated with the range of scales characterized by a flat spectral plateau is expected to govern inter- and intra-seasonal variability (Fraedrich and Larnder, 1993). Fraedrich and Larnder (1993) associated the critical scale (of about 3 days) that they observed in the rainfall spectra with the duration of synoptic events (see also e.g. Ladoy et al., 1991; Tessier et al., 1993). Olsson (1995) reported a similar critical scale at about 3 days. Ladoy et al. (1993) and Tessier et al. (1995), among others, observed the critical scale at around 2 weeks.

In the lowest frequency range (i.e. for the largest time scales), some of the spectra start rising with decreasing frequency (e.g. spectrum obtained for the data from Coimbra, Penhas Douradas, Algodor, Bravura). The behaviour observed for the lowest frequencies is expected to be related to climatic fluctuations (see e.g. Fraedrich and Larnder, 1993). It describes long-term variability. The analysis of the low frequency range should be approached with care because these estimates are based on a small number of long-period cycles. It would be necessary to analyse a larger sample to confirm this spectral break. For studies dealing with spectral analysis of the climatic variability in Portugal see e.g. Antunes et al. (2000).

5. CONCLUDING REMARKS

In this study, spectral analysis of daily precipitation data from eight measuring stations in Mainland Portugal shows that the temporal structure of precipitation exhibits scaling behaviour across a wide range of scales. The data are from both semi-arid and humid regions. Scaling is observed to hold from 1 day up to approximately one-and-a-half months. Other studies have shown that the scaling behaviour of precipitation extends down to scales of the order of minutes (e.g. Olsson, 1995; Lima, 1998; 1999; Lima and Grasman, 1999).

Some of the spectra exhibit a spectral plateau for time scales up to roughly one decade. This plateau is followed by another section (i.e. for even larger time scales), indicating large-scale climatic variability. Similar results have been reported by e.g. Ladoy et al. (1991), Fraedrich and Larnder (1993), Tessier et al. (1996), Svensson et al. (1996).

Results show that spectral analysis is able to quantify differences in precipitation variability, taking into account the behaviour observed across scales. The lower the magnitude of the spectral exponent ß, the higher the variability in the data. The data from the semi-arid region exhibit lower spectral exponents (thus, higher variability) than the data from the humid region, as one would expect.

Spectral methods can be used as exploratory tools for investigating scale-invariant properties in the temporal structure of precipitation. Moreover, spectral analysis, although based only on second order statistics, already gives important information about the structure and dynamics of this highly intermittent hydrological process, which is sometimes not adequately grasped with other approaches.

6. ACKNOWLEDGMENTS

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