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Reliability-based optimum design of conical shells with equidistant and non-equidistant stiffeners

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1. Abstract
Conical shells can be applied in several types of structures such as towers, tanks, submarine and offshore structures. In the present work the following structural characteristics were chosen: steel, slightly conical shell, ring-stiffeners of welded square box section, equidistant (associated with variable shell thickness) and non-equidistant (associated with constant shell thickness) stiffening, external pressure, welding. Design rules of Det Norske Veritas are applied for shell and stiffener buckling constraints.
The design variables are the number of shell segments, the dimensions of ring stiffeners, shell thicknesses in the equidistant stiffeners case or thickness and distances between stiffeners. The cost function includes the cost of material, assembly welding and painting and is formulated according with the fabrication sequence. The optimum design problem involves both discrete and continuous design variables.
Randomness is considered here both in loading and material properties. A level II reliability method (FORM) is employed. Individual reliability constraints related with normal stress due to external pressure in a shell segment and local buckling of the compressed stiffener flange. The overall structural reliability is obtained by using Ditlevsen method of conditional bounding. The costs of the conical shell with equidistant stiffeners is compared with the shell designed assuming constant thickness.
A branch and bound strategy is used to solve the reliability-based optimization. The design variables giving lower bounds on the decision tree can be obtained either by a closed form solution or by continuous optimization and the discrete solutions are found by implicit enumeration. Results are given illustrating the influence of the coefficient of variation of the loading and the probability of failure requirements.
2. Keywords: reliability, optimization, stiffeners , conical, shells

3. Introduction
Conical shells are applied in numerous structures e.g. in submarine and offshore structures, aircraft, tubular structures, towers, tanks, etc. Their structural characteristics are as follows.

- Material: steels, Al-alloys, fibre-reinforced plastics,
- Geometry: slightly conical (transition parts between two circular shells), strongly conical (storage tank roofs), truncated,
- Stiffening: ring-stiffeners, stringers, combined, equidistant, non-equidistant,
- Stiffener profile: flat, box, T-, L-,Z-shape,
- Loads: external pressure, axial compression, torsion, combined,
- Fabrication technology: welding, riveting, bolting, gluing.

Klöppel and Motzel [1] have carried out buckling experiments with truncated unstiffened and ring-stiffened steel conical shell specimens and proposed simple formulae for critical buckling stress.
Rao and Reddy [2] have worked out an optimization procedure for minimum weight of truncated conical shells. Rectangular ring-stiffeners and stringers are used and constraints on shell buckling as well as on natural frequency are considered.
In the book written by Ellinas et al. [3] experimental results and design of stiffened conical shells are treated.
Spagnoli has written a PhD thesis on buckling behaviour and design of stiffened conical shells under axial compression [4]. Rectangular stringers are considered. Later Spagnoli also with co-authors [5,6,7] has published other articles in this field.
Chryssanthopoulos et al. [8] have used finite element method for buckling analysis of stringer-stiffened conical shells in compression.
Singer et al. [9] have given a detailed description of experiments carried out with stiffened conical shell models.
Minimum cost design has been worked out for ring-stiffened circular cylindrical shell in our study [10,11].
In the present study we select the following structural characteristics: steel, slightly conical shell, ring-stiffeners of welded square box section to avoid tripping, both equidistant and non-equidistant stiffening, external pressure,

welding. Design rules of Det Norske Veritas [12,13] are applied for shell and stiffener buckling constraints.
Stresses and displacements can be computed given the deterministic parameters of loads, geometry and material behaviour. Some structural codes specify a maximum probability of failure within a given reference period (lifetime of the structure). This probability of failure is ideally translated into partial safety factors and combination factors by which variables like strength and load have to be divided or multiplied to find the so called design values. The structure is supposed to have met the reliability requirements when the limit states are not exceeded. The advantage of code type level I method (using partial safety factors out of codes) is that the limit states are to be checked for only a small number of combinations of variables. The safety factors are often derived for components of the structure disregarding the system behaviour. The disadvantage is lack of accuracy. This problem can be overcome by using more sophisticated reliability methods such as level II (first order second order reliability method, FOSM [4] and level III (Monte Carlo) reliability methods. In this work FOSM was used and the sensitivity information was obtained analytically. Besides stipulating maximum probabilities of failure for the individual modes, the overall probability of failure which account for the interaction by correlating the modes of failure is considered.
A branch and bound strategy is used to solve the reliability-based optimization. The design variables giving lower bounds on the decision tree can be obtained either by a closed form solution or by continuous optimization and the discrete solutions are found by implicit enumeration. Results are given illustrating the influence of the coefficient of variation of the loading and the probability of failure requirements.

4. Design variables

The variables to be optimized for the non-equidistant stiffening are as follows: shell segments length (L_i) for a given shell thickness (t), dimensions of ring-stiffeners (h_i, t_{ri}). When the distance between stiffeners is identical, segments shell thicknesses (t_i) and dimensions of ring-stiffeners (h_i, t_{ri}) must be determined.
Stiffeners should be used at the ends of the shell and are placed in a small distance from the circumferential welds connecting two segments to allow the inspection of welds, this is marked in Figure 1 by dotted lines. The number of segment (n) is determined given the shell thickness.

5. Constraints

5.1 Overall buckling constraint

According to DNV rules [13], for shell segments between two ring-stiffeners of radii R_i and R_{i+1} the buckling constraint valid for circular cylindrical shells with equivalent radius

$$R_{ei} = \frac{R_{i+1} + R_i}{2\cos\alpha}, \cos\alpha = \frac{1}{\sqrt{\tan^2\alpha + 1}} \tag{1}$$

$$\tan\alpha = \frac{R_{n+1} - R_1}{L_0}, R_{i+1} = L_i \tan\alpha + R_i \tag{2}$$

In the equidistant problem, it is necessary to specify the number n of distances between stiffeners L_i,

$$L_i = \frac{L_0}{n} \tag{3}$$

and the design variable t_i is related to the equivalent thickness,

$$t_{ei} = t_i \cos\alpha \tag{4}$$

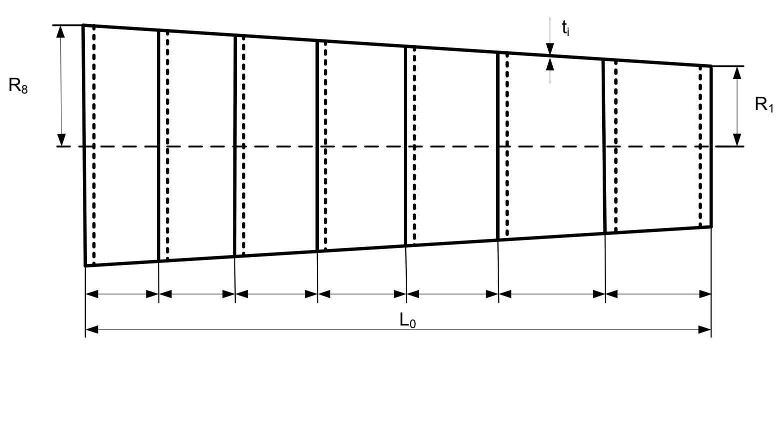

Figure 1:. Main dimensions of the conical shell

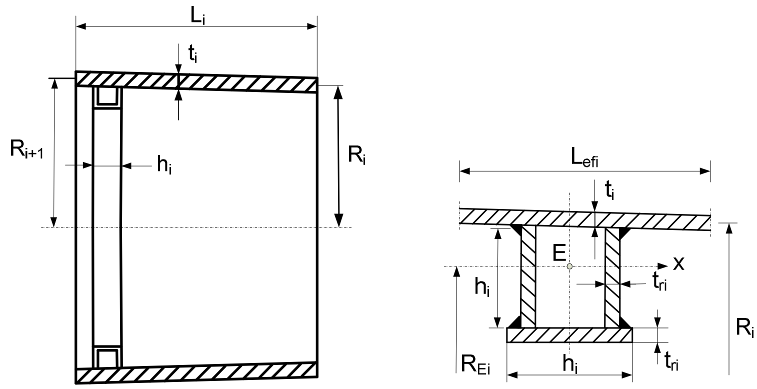

Figure 2: shell segment with the ring-stiffener of welded square box section

For non-equidistant stiffeners $t_i = t$ and L_i becomes a design variable. Here t must be specified and the condition of the sum of Li to be equal to L_0 must be enforced. Eq. (1)-(2) remain valid.
The normal stress due to external pressure in a shell segment should be smaller than the critical buckling stress

$$\sigma_i = \frac{\gamma_b p R_i}{t_{ei}} \le \sigma_{cri} = \frac{f_{y1}}{\sqrt{1+\lambda_i^4}}, \qquad \lambda_i = \sqrt{\frac{f_{y1}}{\sigma_{Ei}}} \tag{5}$$

$$\sigma_{Ei} = \frac{C_i \pi^2 E}{12(1-\nu^2)}\left(\frac{t_{ei}}{L_{ei}}\right)^2, L_{ei} = \frac{L_i}{\cos\alpha} \tag{6}$$

where

$$C_i = 4\sqrt{1+0.023214\frac{L_{ei}^2}{R_{ei}t_{ei}}} \tag{7}$$

The unknowns t_i or L_i are evaluated by using the shell buckling constraint (5).

5.2 Constraint on stiffener buckling
For ring-stiffeners a square box section welded from 3 parts is selected to avoid tripping, which is dangerous failure mode for open-section stiffeners (Fig.1).

The constraint on local buckling of the compressed stiffener flange according to Eurocode 3 [14] is expressed by

$$t_{ri} \ge \delta h_i, 1/\delta = 42\varepsilon, \varepsilon = \sqrt{235/f_y} \tag{8}$$

for $f_y = 355$ MPa $1/\delta = 34$. Eq (8) gives t_{ri} once the height h_i is determined. This dimension can be determined from the stiffener buckling constraint relating to the required moment of inertia of a stiffener section about the axis x of the point E, which is the gravity center of the cross-section including the 3 stiffener parts and the effective part of the shell (Fig.1)

$$I_{xi} \ge I_{reqi} = \frac{\gamma_b p R_i R_{Ei}^2 L_{efi}}{3E}\left[2+\frac{3Ey_{Ei}0.005R_i}{R_{Ei}^2\left(f_{y1}/2-\sigma_i\right)}\right] \tag{9}$$

where

$$I_{xi} = \frac{\delta h_i^4}{6} + 3\delta h_i^2 y_{ri}^2 \frac{L_{efi}t_i}{3\delta h_i^2 + L_{efi}t_i} + \frac{L_{efi}t_i^3}{12}; \quad y_{ri} = \frac{2h_i}{3} + \frac{t_i}{2} \tag{10}$$

$$y_{Ei} = \frac{2\delta h_i^3}{3\delta h_i^2 + L_{efi}t_i} \tag{11}$$

$$L_{efi} = \min\left(L_i, L_{ef0i}\right), L_{ef0i} = 1.56\sqrt{R_i t_i} \tag{12}$$

$$R_{Ei} = R_i - \left(h_i + \frac{t_i}{2} + \frac{\delta h_i}{2} - y_{Ei}\right) \tag{13}$$

The required h_i can be calculated from Eq (9).

6. The cost function

The cost function is formulated according to the fabrication sequence as follows [11].

(1) Forming of 3 plate elements for shell segments into slightly conical shape (K_{F0}).
(2) Welding 3 curved shell elements into a shell segment with GMAW-C (gas metal arc welding with CO_2) butt welds (K_{F1}).
(3) Welding of n+1 ring-stiffeners each from 3 elements with 2 GMAW-C fillet welds (K_{F2}).
(4) Welding of a ring-stiffener into each shell segment with 2 GMAW-C fillet welds (K_{F3}).
(5) Assembly of the whole stiffened shell structure from n shell segments (K_{F4A}).
(6) Welding of n shell segments to form the whole shell structure with n-1 circumferential GMAW-C butt welds (K_{F4W}).
(7) Painting of the whole shell structure from inside and outside (K_P).

The total cost includes the cost of material, assembly, welding and painting

$$K = K_M + K_{F0} + K_{F1} + K_{F2} + K_{F3} + K_{F4} + K_P \tag{14}$$

$$K_M = k_M \rho V, k_M = 1.0\$/kg \tag{15}$$

The volume of the whole structure includes the volume of shell segments (V_{1i}) and ring-stiffeners (V_{ri})

$$V = \sum_{i=1}^{n} V_{1i} + \sum_{i=1}^{n+1} V_{ri} \tag{16}$$

$$K_{F0i} = k_F \Theta e^{\mu}, \mu = 6.8582513 - 4.527217 t_i^{-0.5} + 0.009541996 (2R_{ei})^{0.5},$$

$$K_{F0} = \sum_{i=1}^{n} K_{F0i} \tag{17}$$

where the factor of fabrication difficulty is taken as $\Theta = 3$ and the steel density is $\rho = 7.85x10^{-6}$ kg/mm^3.

$$K_{F1i} = k_F \left[\Theta\sqrt{3\rho V_{1i}} + 1.3x0.152x10^{-3} t_i^{1.9358} x3L_{ei}\right], \quad K_{F1} = \sum_{i=1}^{n} K_{F1i} \tag{18}$$

$$V_{1i} = 2\pi R_{ei} L_{ei} t_i \tag{19}$$

$$K_{F2i} = k_F \left[\Theta\sqrt{3\rho V_{ri}} + 1.3x0.3394x10^{-3} a_{wi}^2 x4\pi (R_i - h_i)\right] \tag{20}$$

where

$$V_{ri} = 4\pi t_{ri} h_i (R_i - h_i/2) + 2\pi t_{ri} h_i (R_i - h_i) \tag{21}$$

and the fillet weld size $a_{wi} = 0.7\delta h_i$.

$$K_{F3i} = k_F \left[\Theta\sqrt{2\rho V_{3i}} + 1.3x0.3394x10^{-3} a_{wi}^2 x4\pi R_i\right], \qquad V_{3i} = V_{1i} + V_{ri} \tag{22}$$

$$K_{F4} = K_{F4A} + K_{F4W}, K_{F4A} = k_F \Theta\sqrt{n\rho V}, \; K_{F4W} = \sum_{i=2}^{n} K_{F4Wi} \tag{23}$$

$$K_{F4Wi} = 1.3 k_F x0.152x10^{-3} t_i^{1.9358} x2\pi R_i \tag{24}$$

$$K_P = K_{P1} + \sum_{i=1}^{n+1} K_{Pi}, K_{P1} = k_P 4\pi \frac{R_{max} + R_1}{2} L_0 \tag{25}$$

$$K_{Pi} = k_P 4\pi h_i (R_i - h_i/2), \; k_P = 2x14.4x10^{-6} \$/mm^2 \tag{26}$$

7. Reliability-based optimization

A failure event may be described by a functional relation, the limit state function, in the following way

$$F = \{g(x) \le 0\} \tag{27}$$

In the case the limit state function g (x) is a linear function of the normally distributed basic random variables x the probability of failure can be written in terms of the linear safety margin M as:

$$P_F = P\{g(x) \le 0\} = P(M \le 0) \tag{28}$$

which reduces to the evaluation of the standard normal distribution function

$$P_F = \Phi(-\beta) \tag{29}$$

where β is the reliability index given as

$$\beta = \mu_M / \sigma_M \tag{30}$$

The reliability index has the geometrical interpretation as the smallest distance from the line (or the hyperplane) forming the boundary between the safe domain and the failure domain. The evaluation of the probability of failure reduces to simple evaluations in terms of mean values and standard deviations of the basic random variables.
When the limit state function is not linear in the random variables x, the linearization of the limit state function in the design point of the failure surface represented in normalised space u. was proposed in Hasofer, A.M. & Lind, N.C. (1974),

$$u_i = \left(x_i - \mu_{x_i}\right) / \sigma_{x_i} \tag{31}$$

As one does not know the design point in advance, this has to be found iteratively. Provided that the limit state function is differentiable, the following simple iteration scheme may be followed:

$$\alpha_i = -\partial g(\beta\alpha) / \partial u_i \left[\sum_{j=1}^{n} \partial g(\beta\alpha)^2 / \partial u_i \right] \tag{32}$$

$$G(\beta\alpha_1, \beta\alpha_2, \ldots \beta\alpha_v) \tag{33}$$

which will provide the design point u* as well as the reliability index β.

The reliability assessment requires an enumeration of the reliability indices associated with the two limit state functions to evaluate the structural system probability of failure. Collapse modes are usually correlated through loading and resistances. By taking into account the probabilities of joint failure events such as $P(F_i \cap F_j)$ which means the probability that both events F_i and F_j will simultaneously occur. The resulting closed-form solutions for the lower and upper bounds are as follows:

$$p_F \geq (F_1) + \sum_{i=2}^{m} Max \left\{ \left[P(F_i) - \sum_{j=1}^{i-1} P(F_i \cap F_j) \right]; 0 \right\} \tag{34}$$

$$p_F \leq \sum_{i=1}^{m} P(F_i) - \sum_{i=2}^{m} \underset{j<i}{Max} P(F_i \cap F_j) \tag{35}$$

The above bounds can be further approximated using Ditlevsen (1979) method of conditional bounding [10] to find the probabilities of the joint events. This is accomplished by using a Gaussian distribution space in which it is always possible to determine three numbers β_1, β_2 and the correlation coefficient ρ_{ij} for each pair of collapse modes F_i and F_j such that if $\rho_{ij} > 0$ (F_i and F_j positively correlated):

$$P(F_i \cap F_j) \geq Max\ \{\Phi(-\beta_j)\ \Phi(-\frac{\beta_i - \beta_j \rho_{ij}}{\sqrt{1-\rho_{ij}^2}})\ ; \Phi(-\beta_i)\ \Phi(-\frac{\beta_j - \beta_i \rho_{ij}}{\sqrt{1-\rho_{ij}^2}})\} \tag{36}$$

$$P(F_i \cap F_j) \leq \Phi(-\beta_j)\ \Phi(-\frac{\beta_i - \beta_j \rho_{ij}}{\sqrt{1-\rho_{ij}^2}}) + \Phi(-\beta_i)\ \Phi(-\frac{\beta_j - \beta_i \rho_{ij}}{\sqrt{1-\rho_{ij}^2}}) \tag{37}$$

In which β_i and β_j are the safety indices of the ith and the jth failure mode and Φ[ ] is the standardized normal probability distribution function.
The probabilities of the joint events $P(F_i \cap F_j)$ in (34) and (35) are then approximated by the appropriate sides of (36) and (37). For example, if F_i and F_j are positively dependent for the lower (34) and upper (35) bounds it is necessary to use the approximations given by the upper (37) and lower (36) bounds, respectively. In this problem the system probability of failure can be represented by the minimum distance β concerning overall buckling given the high mode correlation.

8. Optimization Strategy

The optimization process has the following parts:

(a) design of each shell segment length for a given shell thickness (non-equidistant stiffening) or the thickness for a given segment length (equidistant stiffening) using the shell buckling reliability constraint,
(b) design of ring-stiffeners height h_i (and hence t_{ri}) for each shell segment using the stiffener buckling reliability constraint and the Eurocode expression,
(c) cost calculation for each shell segment and for the whole shell structure.

8.1 Branch and Bound

The problem is non-linear and the design variables are discrete. Given the small number of discrete design variables an implicit branch and bound strategy was adopted to find the least cost solution. The two main ingredients are a combinatorial tree with appropriately defined nodes and some upper and lower bounds to the optimum solution associated the nodes of the tree. It is then possible to eliminate a large number of potential solutions without evaluating them. A partial solution is said to be fathomed if the best completion of the solution can be found or if it can be determined that, no matter how the design variables are assigned to the remaining free members it will be impossible to find a feasible completion of smaller cost than the previous found. If a partial solution is fathomed this means that all possible completions of the partial solution have been implicitly enumerated. When the last node is fathomed the algorithm ends up with the optimum design. Backtracking in the tree is performed so that no solution is repeated or omitted from consideration.

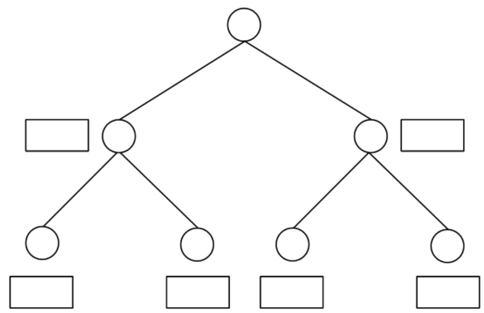

Figure 2: Branch and Bound tree

In the equidistant stiffeners case number of stiffeners n is fixed at the top of the tree, the remaining levels corresponding to t_i associated with h_i and t_{ri}. When n is defined L_i follows from (3) and the minimum values of the design variables t_i are obtained by a reliability-based constraint based on (5) by used a closed form algorithm. h_i and t_{ri} follow by using another reliability constraint based on (9) and (8).
The combinatorial tree covers the cost obtained by employing larger t_i (and smaller h_i and t_{ri}) corresponding to a branching rule. Each node can be branched into new nodes, each of these being associated with larger t_i values. At the last level the resulting minimum discrete solution becomes the incumbent solution (upper bound). Any leaf of the tree whose bound is strictly less than the incumbent is active. Otherwise it is designated as terminated and need not to be considered further. The B&B tree is developed until every leaf is terminated. The branching strategy adopted was breadth first, consisting of choosing the node with the lower bound.
The non-equidistant stiffener case puts put the thickness at the top of the combinatorial tree. It differs from the previous as the sum of length segments must be equal to the defined length of the shell and n is not known at this stage. The adopted strategy was to find the maximum segment lengths L_i which are obtained once the thickness t is defined by using a reliability constraint based on (5). n is found when the number of L_i is not less than L_o. The stiffener design variables h_i and t_{ri} are evaluated when t and L_i known. This procedure was attempted with the starting L_i being evaluated at either end of the shell. The results of these two procedures differ usually less than 0.5%. Alternatively an optimization method involving n-1 design variables L_i was used. The conjugate direction was used to minimize the total cost as there are no constraints involved and h_i and t_{ri} are available. This optimization method leads to no significant improvement on the previously obtained optima. The branch and bound strategy previously described remains valid.

9. Numerical results

Total shell length $L = 15000$, side radii $R_{min} = R_1 = 1850$ and $R_{max} = R_{n+1} = 2850$ mm, , Poisson ratio $\nu = 0.3$, elastic modulus $E = 2.1 \times 10^5$ MPa.
Consistent with the traditional limit state design (level 1 approach), yield stress of steel f_y=355 MPa were considered. With a safety factor for structural steel of 1.10 and an assumed coefficient of variation of 0.10 this corresponds to mean values of 440 MPa. Design and mean values for external pressure intensity are $p = 0.75$ MPa, and 0.3765 MPa, respectively. These are given by assuming a safety factor for loading $\gamma_b = 1.5$ and coefficient of variation of 0.20. The randomness of the Young modulus was not considered for the sake of simplicity.
Although the randomness of Young modulus also plays an important role in the structural reliability, this was not considered here for the sake of simplicity. In this example the probability of failure will be connected with the buckling stresses throughout the structure, the stringer panel buckling and the horizontal displacement of the shell induced by loadings. Gaussian distribution was adopted for the random variables.
In the equidistant stiffener case calculations were carried out for n=4-8. Increasing coefficients of variation for the loading and p_F requirements were defined. The probability of failure is defined for the limit state equation relating the normal stress due to external pressure in each shell segment with the critical buckling stress. The limit state equations governing the ring stiffeners design are highly correlated with the overall buckling requirement and the highest probability of failure is representative of both modes. The total costs are summarized in Table 1.

Table 1 Cost x 10^3. Optima are marked by bold letters

	n=4	n=5	n=6	n=7	n=8
Cov=0.15 p_F=10^{-3}		73.5	73.0	**72.7**	73.6
Cov=0.20 p_F=10^{-4}	80.0	**78.4**	79.2	79.1	
Cov=0.25 p_F=10^{-5}	85.4	84.9	**84.2**	84.5	

The results are similar to those obtained for an increased deterministic loading
In order to characterize the optimum structures, the main data is displayed in Table 2.

Table 2. Main dimensions (in mm) of the optimum designs

n=7, Cost =72.7 10^3			n=5, Cost =78.4 10^3			n=6, Cost =84.2 10^3		
t_i	h_i	t_{ri}	t_i	h_i	t_{ri}	t_i	h_i	t_{ri}
15	110	4	19	119	4	19	131	4
16	118	4	20	133	4	20	143	5
16	128	4	21	146	5	21	156	5
17	136	4	22	160	5	22	169	5
17	146	5	23	174	6	22	184	6
18	155	5	24	188	6	23	198	6
18	164	5				24	211	7
19	173	6						

The solutions for the non-equidistant stiffeners are given in Table 3:

Table 3 Cost x 10^3. Optima are marked by bold letters

	t=16	t=17	t=18	t=19
Cov=0.15 p_F=10^{-3}	74.2	73.8	**72.8**	75.0
Cov=0.20 p_F=10^{-4}	80.1	**78.2**	**78.2**	80.5
Cov=0.25 p_F=10^{-5}	103.2	**86.6**	90.0	90.9

The data corresponding to these optima is represented in Table 4.

Table 4. Main dimensions (in mm) of the optimum designs

t=18, n=6, Cost 72.8 10^3			t=18, n=7, Cost 78.2 10^3			t=17, n=8, Cost 78.2 10^3			t=17, n=9, Cost 86.6 10^3		
L_i	h_i	t_{ri}	L_i	h_i	t_{ri}	L_i	h_i	t_{ri}	L_i	h_i	t_{ri}
2433	108	4	2628	120	4	2415	120	4	2055	134	4
3012	118	4	2458	133	4	2190	133	4	1896	147	5
2686	131	4	2246	145	5	2022	144	5	1772	160	5
2452	142	5	2085	157	5	1892	155	5	1670	174	6
2277	154	5	1958	169	5	1787	167	5	1588	189	6
2140	164	5	1856	181	6	1700	178	6	1518	205	7
			1769	194	6	1627	190	6	1456	224	7
						1042	203	6	1404	248	8
									1315	279	9

10. Conclusions

The optimum design problem is solved for a slightly conical shell loaded in external pressure with equidistant and non-equidistant ring-stiffeners of welded square box section. The reliability-based design minimizes the cost function and fulfils the design requirements. The cost function includes the cost of material, forming of plate elements into shell shape, assembly, welding and painting. The fabrication cost function is formulated according to the fabrication sequence. The forming, welding and painting costs play an important role in the total cost.
The thickness (equidistant stiffeners) or the length of each shell segment (non-equidistant stiffeners) are calculated

from the shell buckling reliability requirements.
The dimensions of ring-stiffeners for each shell segment are determined on the basis of the ring buckling reliability requirements. This constraint is expressed by the required moment of inertia of the ring-stiffener cross-section.
In the case of equidistant stiffeners, there is no general conclusions concerning the optimum number of stiffeners or the trend of the cost of fabrication vs cost of material as the requirements become more demanding. There are close discrete solutions the best of these being selected by the implicit enumeration procedure.
In the case of non-equidistant stiffeners the general conclusion now seems to indicate more stiffeners are required as the cov of loading increases and p_F is reduced. Given the discrete nature of the solutions there are two distinct but very close optimum designs (less than 1%o difference) for loading cov of 0.2 and $p_F=10^{-4}$. The cost data show the ratio material/fabrication costs is reduced as loading and PF requirements increase because more stiffeners are needed to ensure the stability of a thinner shell.
Concerning the cost comparison of both types of design the optimum results for the equidistant stiffeners are usually better than those for non-equidististant for larger loading and p_F requirements given the smaller number of stiffeners involved. This conclusion remains valid with less stringent requirements as it is possible to have now a smaller number of stiffeners in the non-equidistance case. The minimum cost for the intermediate loading do not differ much, although corresponding to different designs.

9. Acknowledgements

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