II Pan American Congress of Applied Mechanics
(PACAM) Proceedings
Valparaíso, Chile Jan 2-4, 1991

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RELIABILITY OF SHALLOW CURVED PLATES BY MATHEMATICAL PROGRAMMING

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ABSTRACT

An unified formalism allows the reliability analysis of stretching plates, plates in bending and shallow curved plates discretized in triangular finite elements to be treated in a similar fashion. By using the first order second-moment approximation, the identification of the reliability index associated with the stochastic dominant collapse mode is formulated as a concave quadratic program. A enumerative technique is employed to solve this problem.

INTRODUCTION

The general case of a three-dimensional continuous structure could be first studied and any particular case would then be derived by the introduction of the relevant simplifications, but for the sake of brevity the plate stretching problem is here directly considered.The formulation describing the fundamental relations of the problem reflects the finite element connectivity across the interelement boundaries and it is thereby called kinematic description. The material is assumed stable in Drucker's sense and the convex hypersurface is replaced by a set of hyperplanes. The reliability index of the structure is obtained using the first order second-moment approximation. The identification of the reliability index is formulated as the minimization of a concave quadratic function over a linear domain, the local solutions of which are vertices of the domain. A branch and bound strategy is employed, each node of the combinatorial tree being associated with a linear program.

GOVERNING RELATIONS OF PLATE STRETCHING

Kinematic Description

In the formulation of the finite element method, three distinct levels (i) the infinitesimal element level, (ii) the finite element level and (iii) the structural level - are defined [1]. For plastic collapse, the interelement equilibrium may be achieved if new nodes e are selected to define the independent stress field Γ_σ in terms of the nodal values σ^e as follows:

$$\sigma = \Gamma_\sigma \, \sigma^e \tag{1}$$

The strain-resultant/displacement relations at the infinitesimal element level are (by omitting the initial imposed strain:

$$- \Delta\varepsilon + D^{\sim}\Delta u = 0 \quad (2)$$

where D is the 2nd order differential operator. By dividing the finite element volume into subdomains associated with the control nodes, we have

$$\int_v \Gamma_\sigma^t \Delta\varepsilon \, dv = \Sigma_{j=1,c} \int_{v_j} \Gamma_\sigma^t \Delta\varepsilon \, dv \quad (3)$$

The transposed field matrix Γ_σ^t is assumed to have inside each subdomain the constant value $(\Gamma_\sigma^t)_j$ that it takes at the corresponding c_j control node. The conditions of compatibility at finite element level read,

$$(\Gamma_\sigma^c)^t \Delta g^c - E \Delta u^a = 0$$

where the total nodal strains Δg^c and the compatibility matrix E are given by:

$$\Delta g^c = \Sigma_{j=1,c} \int_{v_j} \Delta\varepsilon \, dv \quad ; \quad E = \int_v \Gamma_\sigma^t D \Gamma_u \, dv \quad (4)$$

At the structural level, if the relevant coordinate transformation matrices T_σ^e, T_u^a are introduced, the compatibility equations become,

$$(T_\sigma^e)^t (\Gamma_\sigma^c)^t T_g^c \Delta g^{cs} - (T_\sigma^e)^t E T_u^a \Delta u^{as} = 0 \quad (5)$$

or in a more compact form,

$$R^s \Delta g^{cs} - E^s \Delta u^{as} = 0 \quad (6)$$

Plasticity Relations

Only the plastic phase of the structural material behavior has to be characterized. The conditions of yielding to occur may be defined by an inequality involving a function of the stress state. If such a hypersurface is replaced by a set of hyperplanes, the yield conditions for every control node,

$$[\, Q^t T_\sigma^t \Gamma_\sigma] \sigma^e - \sigma_* \le 0 \quad (7)$$

If the conditions are stated for all cs nodes, their assemblage can be written,

$$\sigma^{cs} - \sigma_*^{cs} = [\, (Q^{cs})^t \quad (R^s)^t \,] \sigma^{es} - \sigma_*^{cs} \le 0 \quad (8)$$

where R^s is given as in (6). For a stable material, the kinematic variables are defined by an associated flow rule, that for a control node c_j is

$$\Delta g_j = \int_{v_j} \Delta\varepsilon_j \, dv = \int_{v_j} Q \, (\Delta\varepsilon_*)_j \, dv = Q \, (\Delta g_*)_j \quad (9)$$

where $(\Delta g_*)_j$ are the total plastic parameters associated with the subdomain of unspecified volume v_j inside which the material properties (expressed through matrix Q are assumed constant. For all cs control nodes,

$$\Delta g^{cs} = Q^{cs} \Delta g_*^{cs} \quad (10)$$

If the assumption of constant stress field and of constant strain field is introduced, the total plastic dissipation energy ΔD^{cs} becomes,

$$\Delta D^{cs} = (\sigma^{cs})^t \Delta g^{cs} = (\sigma_*^{cs}) \Delta g_*^{cs} \ge 0 \quad (11)$$

For simplicity, superscripts s, c, a and e will be dropped.

RELIABILITY ASSESSMENT

Computation of the reliability index

By adding the finite element strains associated with stresses in half space represented by the same random variables, one has,

$$\Delta g_*^+ = J_g^+ \Delta g_* \quad ; \quad \Delta g_*^- = J_g^- \Delta g_* \quad (12)$$

If the displacements of the point loads (or in the case of uniformly distributed load, deflections of the finite element centroids) linked to dead and live loading, respectively, are summed up,

$$\Delta u_*^d = J_u^d \Delta u \quad ; \quad \Delta u_*^l = J_u^l \Delta u \quad (13)$$

For statistically independent random normal variables, the mathematica ogram that gives the reliability index ß associated with the stochastic most important collapse mode is [2],

$$\min \beta = \frac{[\mu_{\sigma^+}\ \Delta q_*^{+} + \mu_{\sigma^-}\ \Delta q_*^{+} - \mu_d\ \ \Delta u_*^{d} - \mu_1\ \ \Delta u_*^{1}]}{\sqrt{(\sigma_{\sigma^+})^2(\Delta q_*^{+})^2+(\sigma_{\sigma^-})^2(\Delta q_*^{-})^2+(\sigma_d)^2(\Delta u_*^{d})^2+(\sigma_1)^2(\Delta u_*^{1})^2}} \quad (14)$$

subject to the linear incidence equations (12-13), the compatibility relations (6) and sign constraints on the variables. This mathematical program belongs to the class of fractional programming problems and shares its solutions with,

$$\min -1/\beta^2 = -(\sigma_{\sigma^+})^2(\Delta q_*^{+})^2-(\sigma_{\sigma^-})^2(\Delta q_*^{-})^2-(\sigma_d)^2(\Delta u_*^{d})^2-(\sigma_1)^2(\Delta u_*^{1})^2 \quad (15)$$

subject to (12-13),(6), sign constraints and:

$$\mu_{\sigma^+}\ \Delta q_*^{+} + \mu_{\sigma^-}\ \Delta q_*^{+} - \mu_d\ \ \Delta u_*^{d} - \mu_1\ \ \Delta u_*^{1} = 1 \quad (16)$$

that is a quadratic concave minimization.This type of problem cannot be solved by convex programming techniques because of the possibility of nonglobal local minima. The global optimum of these programs gives the plastic deformations for the stochastic most important mechanism and the reduced random variables are,

$$\sigma^{+\prime} = -\ \sigma_{\sigma^+}\ \Delta q_*^{+}\ \beta^2 \quad ; \quad \sigma^{-\prime} = -\ \sigma_{\sigma^-}\ \Delta q_*^{-}\ \beta^2 \quad (17)$$

$$d' = -\ \sigma_d\ \ \Delta u_*^{d}\ \beta^2 \quad ; \quad 1' = -\ \sigma_1\ \ \Delta u_*^{1}\ \beta^2 \quad (18)$$

Branch and bound technique for quadratic concave minimzation

The general nonconvex domain is transformed in the branch and bound (B & B) strategy into a sequence of intersecting convex domains by the use of convex underestimating functions. The two main ingredients are a combinatorial tree where the nodes are associated with linear programs and some upper and lower bounds to the final solution related to each node of the tree. For a quadratic concave function its convex underestimate is the affine function (linear plus a constant) passing through the endpoints of the given function graph. Tight bounds on the nonlinear variables can be found by solving a multiple row linear program.

RELIABILITY ASSESSMENT OF THIN FLEXURAL PLATES

The formulation hold unaltered just by re-interpretation of symbols for the class of above mentioned discrete plastic models.The material is considered to satisfy a yield criterion formulated by Nielsen for reinforced concrete plates. In order to obtain linearized yield conditions, a safe linearization suggested by Wolfensberger which considers an octaedrum is adopted.

RELIABILITY ASSESSMENT OF SHALLOW CURVED PLATES

The mathematical characterization of the plastic behaviour of a general shell would require an yield criterion involving ten stress-resultants and ten strain-resultants. However, as for flexural plates the effect of the two transverse shear forces is generally negligible. Also, since every finite element is shallow, the planar shear forces and the twisting moments are regarded as equal. The kinematic model under consideration can be taken as the supperposition of stretching and bending models and the field functions required for the kinematic and static independent variables may be obtained through the supperposition of the field functions defined for those models.

The yield criterion used considers separately the stretching and bending problems with no interaction between them. Both for the in-plane forces and for the moments, the linearized Nielsen criterion is adopted.

NUMERICAL EXAMPLES

Plate stretching: Concrete pier

The problem consists in determining the reliability index of the dominant mode for the concrete pier of Fig. 1 simply supporting a bridge deck. If any change of geometry is neglected, the pier can be regarded as a plane stress state. Three nodal finite elements are used in order to define the compatibility matrix. A linear displacement field where nodes a are the corner nodes is assumed.

A constant stress field is considered, where the nodal stresses are at a single node anywhere inside the finite element. Thus, the single node e is coincident with the single control node c. Matrix Q is embodied in the yield criterion that considers the two-dimensional stress-space where the yield function is assumed to correspond to the ellipse of Fig.1.This example was solved in ref.[1] for plastic limit analysis. The unsafe linearization is performed by means of the three planes represented. The carrying capacity of the concrete is (μ_s,Ω_s)=(20 kN/m^2,0.11). The live loading acting on the top of the pier is (μ_l,Ω_l)=(8 kN/m^2,0.25). The dead loading due to the bridge deck (μ_d,Ω_d)=(3 kN/m^2,0.10) plus the self-weight of the pier is considered uncorrelated to the live load. The following solution has been found:

$\beta = 2.95$; $\sigma' = -2.17$; $d' = .314$; $l' = 1.973$; $\sigma = 15.226$; $d = 3.094$; $l = 11.946$

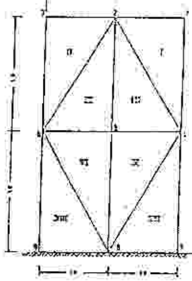
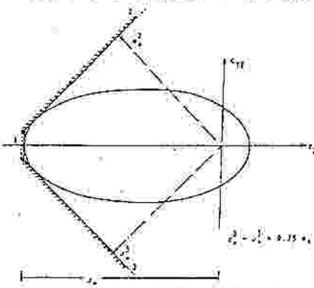


Figure 1 Finite element model



Yield function and linearizing planes

Thin flexural plate: Reinforced concrete floor

Fig. 2 represents the finite element modelling of an octant of a uniformly loaded clamped square plate (l=10m) and a circular sector of a circular plate (d=10m), respectively. The bending moment capacities are $(\mu_{m+},\Omega_{m+})=(\mu_{m-},\Omega_{m-})$ =(100kNm/m,0.05). (μ_d,Ω_d)=(10 kN/m^2,0.10) and (μ_l,Ω_l)=(18 kN/m^2,0.25) are the dead and live transversal loading. The plate discretization is done by means of triangular finite elements with a quadratic deflection field and thus six nodal displacement values must be specified (in these examples: vertical deflections at element corners and mid-side normal rotations) and a constant moment field is considered. These examples were solved in ref.[3] for plastic limit analysis. The solutions,

$\beta = 2.103$; $m^+ = 97.914$; $m^- = 97.914$; $d = 10.438$; $l = 26.870$

$\beta = 3.124$; $m^+ = 96.533$; $m^- = 96.533$; $d = 10.643$; $l = 31.026$

were found for the square and circular plate, respectively.

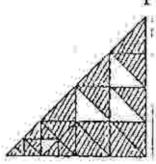
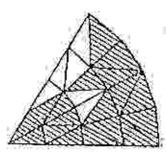


Figure 2 Square plate FE model



Circular Plate FE model

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