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Information Entropy Applications in Structural Optimization

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ABSTRACT

Two recently developed entropy-based algorithms for the optimization of elastic structures are presented in this work. They are simple to operate and efficient, suggesting their potential use. The first algorithm, which is applied to a structural sizing problem, consists of maximizing a function of a single variable until convergence. Surrogate multipliers are obtained by maximizing Shannon's entropy function. In the second case, the maximum entropy formalism solves a multicriteria optimization via the unconstrained minimization of a nonlinear convex scalar function. Numerical results are given to illustrate the methods.

INTRODUCTION

Methods of optimization currently available seem to have entered a diminishing returns phase in respect of further research potential. Some radically different directions and new approaches are needed for the further development of engineering optimization techniques. Annealing is the physical process of heating up a solid until it melts followed by cooling it down until it crystallizes in a state with a perfect lattice. During the process the free energy of the solid is minimized. Practice shows that the cooling must be done carefully in order not to get trapped in locally optimal lattice structures with crystal imperfections. In nonlinear optimization one can define a similar process by establishing a correspondence between the cost function and the free energy and between the solution and the physical states. Entropy is a natural measure of the amount of disorder (or information) in a system. Entropy is viewed in information theory as a quantitative measure of the information content of a system. In the case of optimization, the entropy can be interpreted as a measure for the degree of optimality.

The main purpose of this paper is to present two recently developed entropy-based techniques. The first method, based on the the maximum entropy principle, is applied to structural sizing problems. Unlike optimality criteria and other more recent algorithms, it does not require an active/passive set strategy. This methodology considers simultaneously all the constraints, assigning to them different weights according to the probabilities given by Shanon's entropy function. The optimization phase reduces to the finding of the parameter which maximizes the (concave) dual volume. The Lagrange multipliers and member sizes are evaluated in terms of this parameter by using a simple algebraic expression.

Entropy is used implicitly in another algorithm in which a group of objectives is minimized. A Pareto solution of this vector problem is obtained by the scalar minimization of a nonlinear convex function involving one control parameter.

Examples are given to demonstrate the potentialities of these methods. The first is a grillage, demonstrating efficient size optimization for this type of structure. The second example is a reliability based configuration optimization of a truss showing the state of the art in discrete element shape optimization.

ENTROPY IN OPTIMIZATION PROCESSES

Constrained Nonlinear Programming

Entropy can be used to deduce desired results when only limited information is available. The general inequality constrained nonlinear programming problem,

$$\text{Min} \quad f(x) \qquad i = 1,\dots,n \tag{1a}$$

$$\text{st} \quad g_j(x) \le 0 \quad \text{or} \quad g_j(x) + s_j = 0 \qquad j = 1,\dots,m \tag{1b}$$

was examined in Ref.[1]. An initial point was chosen and information is calculated about the objective and constraint functions, typically their numerical values and gradients at the design point. This numerical information was then used in a mathematical programming algorithm to infer where the next trial point should be placed so as to get closer to the constrained optimum of the problem. The new trial generates more information from which another point is inferred and eventually the solution is reached by this process of gathering better information and using it in an inference-based algorithm. The essence of the method consisted in transforming problem (1) into an equivalent surrogate form,

$$\text{Min} \quad f(x) \tag{2a}$$

$$\text{st} \quad \Sigma_{j=1,M}\ \lambda_j\ g_j(x) = 0 \tag{2b}$$

$$\Sigma_{j=1,M}\ \lambda_j = 1 \tag{2c}$$

$$\lambda_j \ge 0 \tag{2d}$$

and using maximum entropy to obtain least biased estimates of the optimum values of the surrogate multipliers λ_j. In this two phase method the absence of an explicit surrogate dual objective function is overcome by introducing the Shanon entropy as a means of forcing iterations towards a saddle point. Each estimate lead to a new problem in the space of the x variables and generated information upon which to base an improved estimate of the optimum surrogate multipliers.

Truss sizing problem

An initial set of bar cross-sectional areas is chosen to form an initial design which is analyzed to give bar forces and virtual forces for joint displacements. These forces are assumed to remain constant and an optimization problem is set up and solved to give new bar sizes. The structure is reanalyzed with the new bar sizes which are then scaled to ensure feasibility of the new design. Convergence checks are carried out on bar sizes and forces and iterations terminate if the changes are acceptably small. If convergence is not achieved a new optimization is set up with the new bar forces and solution proceeds iteratively until convergence is achieved. The optimization problem which must be solved in each cycle of iteration can be stated as:

$$\min \quad V = \Sigma_{i=1,N}\ l_i\, x_i \tag{3a}$$

$$\text{st} \quad \Sigma_{i=1,N}\ l_i\ F_{ij}\, E_{ik} / (E_i\, x_i) \le u_k \quad ; \quad j = 1,\dots,J \tag{3b}$$

$$\sigma_i^L \le \sigma_{ij} = F_{ij} / x_i \le \sigma_i^U \quad ; \quad k = 1,\dots,K \tag{3c}$$

$$x_i \ge x_i^L \quad ; \quad i = 1,\dots,N \tag{3d}$$

The N unknown bar sizes x_i, i=1,...,N comprise the design variable vector x. l_i, E_i are the length and elastic modulus, respectively, of the i-th bar. In the displacement constraints (3b) F_{ij} and E_{ik} are the force caused by the j-th load case and the virtual force caused by the k-th joint displacement in the i-th bar and u_k is the maximum permissible displacement of the k-th joint. At each optimization all bar forces are known and are assumed to remain constant, so problem (3) can be stated in a simplified form as:

$$\min \; V = \Sigma_{i=1,N} \; l_i x_i \tag{4a}$$

$$\text{st} \quad \Sigma_{i=1,N} c_{ik}/x_i \leq 1 \qquad ; \quad k = 1,...,M \tag{4b}$$

$$\underline{x}_i/x_i \leq 1 \qquad ; \quad i = 1,...,N \tag{4c}$$

The displacement constraints (3b) correspond with Eq.(4b) with c_{ij}, j=1,...M representing gen displacement constants evaluated after each analysis. The stress and size constraints in problem (3) have been merged into Eq.(4c); $\underline{x}_i$ is the largest of either x_i^L or the minimum size necessary to satisfy the stress constraints (3c). Problem (4) has the following Lagrangean function,

$$\mathcal{L}(x,\beta) = \Sigma_{i=1,N} \, l_i x_i + \Sigma_{k=1,M} \; \mu_k [\Sigma_{i=1,N} c_{ik}/x_i - 1] + \Sigma_{j=1,N} \, \mu_{M+i} [\underline{x}_i/x_i - 1] \tag{5}$$

Examining the stationarity of $\mathcal{L}(x,\mu)$ with respect to all x_i, i=1,...,N yields equations in x which may be solved algebraically to give:

$$x_i^{[k]} = ([\Sigma_{j=1,M} c_{ik} \, \mu_j + \underline{x}_i \, \mu_{M+i}] / l_i)^{1/2} \tag{6}$$

If an optimum set of multipliers μ^* exists, then the resulting bar sizes x* will also solve the problem. Such a set of optimal surrogate multipliers μ^* is, of course, not known "a priori" but found iteratively. The problem then becomes one of developing a method whereby the μ may be iteratively updated towards μ^*, thus solving problem (4). Very many engineering optimization problems essentially consist of iteratively sorting out which ones of many constraints are active at the optimum and which are inactive and then of iteratively estimating values for the active constraint multipliers. Though such a strategy is theoretically valid, changes in the active set between iterations change the optimization problem being solved in a discontinuous way and lead to erratic convergence behavior. The maximum entropy-based algorithms avoid these difficulties by retaining and updating all constraints at all times. Pro a discontinuities are not introduced and consequently convergence is smooth. Assuming that the Lagrange multipliers μ_j are given by,

$$\mu_j = \lambda_j v_j \tag{7}$$

where λ_j is a entropy multiplier and v_j is a correction factor, these multipliers may be interpreted probabilistically with each λ_j representing the probability that its corresponding constraint is active at the optimum. With this probabilistic view of the multipliers it is then entirely logical and sensible to calculate most likely or least biased values for them from the Jaynes maximum entropy formalism. An initial set of values for v and λ is chosen such that $v_j^{[o]}=1$ and $\lambda_j^{[o]}=1/(M+N)$, j=1,...,M+N ie: all constraints are equally likely to be active at the optimum.The set of bar cross-sectional areas x obtained from (6) forms an initial design which is analyzed to give bar forces and virtual forces for joint displacements. All bar areas are scaled to ensure that no constraint is violated. The correction factors vector $v^{[1]}$ is assumed a unit vector in this iteration. New estimates of the multipliers $\lambda^{[1]}$ are then obtained by solving the maximum entropy mathematical problem:

$$\text{Max} \quad S = -K\,\Sigma_{j=1,M}\, \lambda_j^{[1]} \ln\lambda_j^{[1]} \tag{8a}$$

$$\text{st,} \quad \Sigma_{j=1,M}\, \lambda_j^{[1]} = 1 \tag{8b}$$

$$\Sigma_{j=1,M}\, \lambda_j^{[1]}\, g_j(x^{[0]}) = \varepsilon \tag{8c}$$

$$\lambda_j^{[1]} \geq 0 \tag{8d}$$

S is the Shannon entropy, K is a positive constant. Equation (8c), that represents the constraints:

$$g_j(x) = c_{ik}/x_i - 1 \qquad \text{for } j = 1,...,M \tag{9a}$$

$$g_j(x) = \underline{x}_i/x_i - 1 \qquad \text{for } j = M+1,...,M+N \tag{9b}$$

has an expected value zero. If the left-hand side had contained $g_j(x^{[1]})$, then the right-hand side would be zero, but since $g_j(x^{[1]})$ values are not yet known $g_j(x^{[0]})$ values are used as the best currently available estimates and this introduces the error term ε into Eq.(8c). The entropy maximization problem has an algebraic solution for $\lambda^{[1]}$:

$$\lambda_j^{[1]} = \frac{\exp[\beta\; g_j(x^{[0]})/K]}{\Sigma_{j=1,M}\; \exp[\beta\; g_j(x^{[0]})/K]} \tag{10}$$

in which β, the Lagrange multiplier for Eq.(12c), can be found by substituting result (10) into Eq. (8c). However, since ε is not uniquely known and K is an arbitrary constant, $p=\beta/K$ may be viewed as a penalty parameter used to close the duality gap. Eq.(10) with a selected p yields new constraint activity probabilities $\lambda^{[1]}$. At each iteration, it is necessary to search for the value of p that maximizes the truss volume given by (6) and using the new correction factor and multiplier values. The new design is analyzed by the matrix stiffness method and all bar areas are scaled to ensure that no constraint is violated. The correction factors vector are given by:

$$v^{[2]} = F^{[1]}{}_{\#}\, l = F^{[1]t}\, (F^{[1]}\, F^{[1]t})^{-1}\, l \tag{11}$$

where l represents the member lengths vector and the elements of the matrix F are given by,

$$f_{ij}^{[1]} = \lambda_j\, c_{ij} / x_i^{\;2} \qquad \text{for } j = 1,...,M \qquad i = 1,...,N \tag{12a}$$

$$f_{jj}^{[1]} = \lambda_{M+i}\, \underline{x}_i / x_i^{\;2} \qquad \text{for } j = M+1,...,M+N \tag{12b}$$

Using $g(x^{[1]})$ in place of $g(x^{[0]})$ in Eq.(10) with an appropriate p yields new multipliers $\lambda^{[2]}$. Using $v^{[2]}$ and $\lambda^{[2]}$, values of $x^{[2]}$ follow from Eq.(6) and the dual volume $V^{[2]}$ from Eq.(4a). Substituting $x^{[2]}$ into Eq.(4b) and (4c) yields values for the constraint functions and all bar areas are scaled to ensure that no constraint is violated. In subsequent iterations, this scaled design and the previous scaled design would be compared and checked against convergence criteria and iterations would be either stopped here or continued.

Alternative entropy-based formulation

In ref.1 it is proposed an alternative solution scheme which combines the two phases into a single phase consisting of solving an unconstrained problem. The Lagrangean of problem (2) is augmented with an entropy term and the stationarity conditions reduce to:

$$\text{Min}_{\,x}\; f(x) + 1/\rho\, \ln \Sigma_{j=1,m} \exp[\rho\,\alpha\; g_j(x)] \tag{13}$$

that must be solved for an increasing positive parameter $\rho\,\alpha$. A different entropy-based procedure more appropriate for shape optimization will be proposed next.

MINIMAX OPTIMIZATION

Minimax problems are discontinuous and non-differentiable, both of which attributes make its numerical solution by direct means difficult. Ref.2 explores the relationships between the minimax optimization problem and the scalar optimization function and extends the equivalences to general multicriteria optimization. Specifically it is shown that a minimax problem can be solved indirectly by minimizing a continuous differentiable scalar optimization problem. The Shannon/Jaynes maximum entropy principle plays a keyrole in these classes of problems, hence the characterization of these methods as entropy-based. In this section some of the theory behind this approach to minimax optimization is briefly described.

For any set of real, positive numbers Uj, j=1, ..., J, and real $\rho \geq q \geq 1$, Jensen's inequality states that,

$$(\Sigma_{j=1,m} \; U_j^{\rho})^{1/\rho} \leq (\Sigma_{j=1,m} \; U_j^{q})^{1/q} \tag{14}$$

Inequality (14) means that the p-th norm of the set U decreases monotonically as its order, p, increases. Another important property of the p-th norm is its limit as ρ tends towards infinity:

$$\lim_{\rho \to \infty} (\Sigma_{j=1,m} U_j^{\rho})^{1/\rho} = \text{Max}_{j=1,m} < U_j > \tag{15}$$

Consider the minimax optimization problem,

$$\text{Min}_x \; \text{Max}_{j=1,m} < g_j(x) > \tag{16}$$

and Jensen's inequality. Let $U_j = \exp\,[g_j(x)]$, j=1,...,m thus ensuring that $U_j > 0$, for all positive $g_j(x)$. Then,

$$(\Sigma_{j=1,m} \; U_j^{\rho})^{1/\rho} = [\Sigma_{j=1,m} \; \exp[\rho \; g_j(x)] \;]^{1/\rho} \tag{17}$$

And from (14),

$$\lim_{\rho \to \infty} [\Sigma_{j=1,m} \; \exp[\rho \; g(x)]]^{1/\rho} = \text{Max}_{j=1,m} < g_j(x) > \tag{18}$$

Taking logarithms of both sides and noting that,

$$\log \lim(f) = \lim \log(f) \quad \underline{\text{and}} \quad \log \text{Max}(f) = \text{Max} \log(f) \tag{19}$$

Eq.(18) becomes,

$$\lim_{\rho \to \infty} (1/\rho) \log\{ \Sigma_{j=1,m} \exp[\rho \; g(x)]\} = \text{Max}_{j=1,m} < g_j(x) > \tag{20}$$

sult (20) holds for any set of vectors g(x), including that set which results from minimizing both sides of (16) over x. Thus (20) can be extended to:

$$\text{Min}_x \; \text{Max}_{j=1,m} < g_j(x) > = \text{Min}_x \; (1/\rho) \log\{ \Sigma_{j=1,m} \exp[\rho \; g_j(x)]\} \tag{21}$$

with increasing ρ in the range $1 \leq \rho \leq \infty$. Result (21) shows that a Pareto solution of the minimax optimization problem can be obtained by the scalar minimization,

$$\text{Min}_x \; (1/\rho) \; \log\{ \Sigma_{j=1,m} \; \exp[\rho \; g_j(x)]\} \tag{22}$$

with a sequence of values of increasingly large positive $\rho \geq 1$.

Truss configuration optimization

The Pareto optimal design of truss geometry and cross sections consists of minimizing a whole set of goals by finding an optimal set of cross sectional areas x, joint coordinates y and corresponding displacements d. All these goals (volume, nodal displacement, etc.) need to be cast in a normalized form. If $\underline{V}$ represents a reference volume, the volume is reduced if,

$$l(y)^t x \leq \underline{V} \quad \Rightarrow \quad g_1(x,y) = \frac{l(y)^t x}{\underline{V}} - 1 \leq 0 \tag{23a}$$

where the member lengths are functions of joint coordinates. Lower bounds on cross-sectional areas are imposed to avoid topology changes,

$$g_2(x) = -\frac{x}{x^L} + 1 \leq 0 \tag{23b}$$

Similarly, one has for the upper and lower bounds on the joint coordinates:

$$g_3(y) = \frac{y}{y^U} - 1 \leq 0 \quad ; \quad g_4(y) = -\frac{y}{y^L} + 1 \leq 0 \tag{23c}$$

The displacements d are computed for any given design by solving the displacement analysis equilibrium equations. The elements of the load vector R are constants and the elements of the stiffness matrix K are functions of both the variables x and y. The criterium concerning upper bounds on the nodal displacements is,

$$g_5(x,y) = \frac{d(x,y)}{d^U} - 1 \leq 0 \tag{23d}$$

For the upper and lower bounds on the stresses:

$$g_6(x,y) = \frac{\sigma = S(y)\, d(x,y)}{\sigma^U} - 1 \leq 0 \; ; \; g_7(x,y) = -\frac{\sigma = S(y)\, d(x,y)}{\sigma^L} + 1 \leq 0 \tag{23e}$$

where the elements of the stress-transformation matrix S are functions of only the variables y.

Design variable linking is used to meet symmetry requirements and to reduce the number of design variables. In general, upper and lower bounds on design variables and stresses are assumed to be constant. If stability of members is considered, the lower bound σ^L can be defined as,

$$\sigma^L = \max\,[\,\sigma_c, \sigma_b] \tag{24}$$

in which σ_c is the lower stress limit and σ_b is the allowable stress for Euler buckling. For tubular sections with a nominal diameter to thickness ratio of D/t=10, the buckling stress can be given as,

$$\sigma_{bi} = \frac{10.1\,\pi\, E\, x_i}{8\, l_i(y)^2} \tag{25}$$

which depends on both the sizing and geometric variables.

The problem of finding values for the the cross sectional areas x and joint coordinates y which minimize the maximum of the goals has the form,

$$\text{Min}_{x,y}\ \text{Max}_j\,(g_1, ..., g_j \cdots g_7) = \text{Min}_{x,y}\ \text{Max}_{j \in J} < g_j(x,y) > \tag{26}$$

and belongs to the class of minimax optimization.

Reliability-based elastic design

Consistent with a first-order second-moment reliability approach, the minimum statistical information required for the evaluation of the optimum solution is: (a) The mean values of the loads, the coefficients of variation of the loads Ω_L and the coefficients of correlation between pairs of loads; (b) The coefficients of variation of the admissible stresses Ω_σ and the coefficients of correlation between pairs of admissible stresses. Assuming that the vector σ represents the elastic envelope stress-resultant coefficients obtained by the deterministic analysis and $\mu_{\sigma U}$, $\mu_{\sigma L}$ are the mean elastic capacities, the probability of unserviceability of individual sections is given by,

$$P_{sj} = P[\, \sigma^U_j - \sigma_j \leq 0\,] \quad \text{or} \quad P[\, \sigma_j - \sigma^L_j \leq 0\,] \quad ; \qquad j=1,..,m \tag{27}$$

It is assumed that safety with regard to unserviceability of the section j depends only on the reli- ity index $\beta_j = \Phi^{-1}(P_{sj})$, that is defined as the shortest distance from the origin to the admissible stress surface in the reduced random variables coordinate system:

$$\beta_j = \frac{\mu_{\sigma U_j} - \mu_{\sigma_j}}{\sqrt{(\mu_{\sigma U_j}\, \Omega_\sigma)^2 + (\mu_{\sigma_j}\, \Omega_L)^2}} \tag{28a}$$

or

$$\beta_j = \frac{\mu_{\sigma_j} - \mu_{\sigma L_j}}{\sqrt{(\mu_{\sigma L_j}\, \Omega_\sigma)^2 + (\mu_{\sigma_j}\, \Omega_L)^2}} \tag{28b}$$

For completely correlated elastic capacities, the probability of failure is,

$$P_s = \max_{j=1,m} P_{sj} \tag{29a}$$

and for uncorrelated elastic capacities,

$$P_s = \max_{j=1,m} (\Sigma_{k=1,v_j} P_{sk}) \tag{29b}$$

where v_j represents the number of critical sections corresponding to all considered loading schemes.

. ming that the nodal coordinates are deterministic, the reliability-based optimization problem consists of member size selection for given probabilities of failure against unserviceability P_{s*}. If $\underline{V}$ represents an (average) reference volume, (23a) becomes,

$$g_1(x,y) = \frac{l(y)^t \mu_x}{\underline{V}} - 1 \leq 0 \tag{30a}$$

Similarly, one has for the bounds on design variables:

$$g_2(x) = -\frac{\mu_x}{x^L} + 1 \leq 0 \tag{30b}$$

$$g_3(y) = \frac{y}{y^U} - 1 \leq 0 \quad ; \quad g_4(y) = -\frac{y}{y^L} + 1 \leq 0 \tag{30c}$$

By defining $\beta_* = \Phi^{-1}(P_{s*})$, individual stress bounds are given by,

$$g_5(x,y) = -\frac{\beta(x,y)}{\beta_*} + 1 \leq 0 \tag{30d}$$

Similar expressions can be employed to satisfy the overal probability of failure against unserviceability and the upper bounds on the nodal displacements specified in probabilistic terms. This formulation give a Pareto solution to the multi-objective reliability-based optimization.

Scalar Function Optimization

Problem (22) is unconstrained and differentiable which, in theory, gives a wide choice of possible numerical solution methods. However, since the goal functions $g_j(x)$ do not have explicit algebraic form in most cases, the strategy adopted was to solve (22) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(x,y)$, truncated after the linear term. The quality of the approximation improves by considering the quadratic term for the geometric variables. This gives Eq.(31):

$$\text{Min } (1/\rho)\log\{\Sigma_{j=1,J} \exp \rho[g_j(x_o,y_o) + \Sigma_{i=1,N}\frac{\partial g_j}{\partial x_i}\Big|_o (x_i-x_o)+\Sigma_{k=1,\beta}\frac{\partial g_j}{\partial y_k}\Big|_o (y_k-y_o) +$$

$$+\frac{1}{2}\Sigma_{1=1,N}\Sigma_{k=1,\beta}\frac{\partial^2 g_j}{\partial y_i \partial y_k}\Big|_o (y_i-y_o)(y_k-y_o) + \Sigma_{i=1,N}\Sigma_{k=1,\beta}\frac{\partial^2 g_j}{\partial x_i \partial y_k}\Big|_o (x_i-x_o)(y_k-y_o)]\}$$

Problem (36) is an approximation to problem (27) if values of all the $g_j(x,y)$, $(\partial g_j/\partial x_i)$ and $(\partial g_j/\partial y_k)$ are known numerically. Given such values, problem (36) can be solved directly by any standard unconstrained optimization method. This problem must be solved iteratively, x_o and y_o being redefined each time as the optimum solution to the preceding problem. Iterations continue until changes in the design variables x,y become small. During these iterations the parameter ρ must be increased in value to ensure that a minimax optimum solution is found. In the present work, a constant value of $\rho = 100$ was used.

The choice of a large control parameter ρ for a very unfeasible design point may cause overflow problems. To overcome this situation (22) can be replaced by,

$$\text{Min}_x \; \{ g_M(x) + (1/\rho)\; \log(\Sigma_{j=1,m} \exp[\rho\, [g_j(x)-g_M(x)]\}) \tag{32}$$

where $g_M(x)$ is the largest of the goals $g_j(x)$, j=1,..m and ρ is a positive constant.

Sensitivity Analysis

To formulate and solve the scalar function minimization (31) used for the direct design, numerical values are required for all the functions $g_j(x,y)$ and their derivatives with respect to the design variables. The truss volume is known explicitly and its first derivatives are:

$$\frac{\partial V}{\partial x_i} = l_i \quad ; \quad \frac{\partial V}{\partial y_k} = \Sigma_{i=1,N}\, x_i \frac{\partial l_i}{\partial y_k} \tag{33}$$

in which $\partial l_i / \partial y_k$ is the direction cosine of the bar corresponding to the displacement y_k. The second derivative of V with respect to y_k is given by ratio of the square of the direction sine of the bar corresponding to the displacement y_k divided by the member length.

The derivatives of the reliability indices β with respect to the design variables require an approximation of the member stresses and nodal displacements which are implicit functions of x and y. One way of evaluating the derivatives of σ and d is to calculate them from analytical expressions, as follows. The displacement derivatives $\partial d^o / \partial x_i$ are computed by implicit differentiation of the equilibrium equations:

$$K^o \frac{\partial d^o}{\partial x_i} = -\frac{\partial K^o}{\partial x_i} d^o \tag{34}$$

Since d^o and K^o are known from analysis of the initial design, solution for $\partial d^o / \partial x_i$ involves only calculation of the r.h.s. vector of Eq.(34) and forward and back substitutions. The stress derivatives $\partial \sigma^o / \partial x_i$ are then determined directly by explicit differentiation,

$$\frac{\partial \sigma^o}{\partial x_i} = S \frac{\partial d^o}{\partial x_i} \tag{35}$$

The derivatives $\partial d^o / \partial y_k$ and $\partial \sigma^o / \partial y_k$ are computed in a similar manner; however, it should be remembered that the elements of S are functions of the joint coordinates y. The expressions for $\partial d^o / \partial y_k$ and $\partial \sigma^o / \partial y_k$ are:

$$K^o \frac{\partial d^o}{\partial y_k} = -\frac{\partial K^o}{\partial y_k} d^o \tag{36}$$

$$\frac{\partial \sigma^o}{\partial y_k} = S \frac{\partial d^o}{\partial y_k} + \frac{\partial S^o}{\partial y_k} d^o \tag{37}$$

To ipute $\partial K^o / \partial x_i$, only elements of K associated with member i must be considered. Furthermore, the elements of $\partial K / \partial X_i$ are constant, therefore the computation must not be repeated. To find $\partial K^o / \partial y_k$ and $\partial S^o / \partial y_k$, only elements of K and S associated with the kth joint coordinate must be considered.

The second order derivatives with respect to y_k can be calculated in a similar manner,

$$K^o \frac{\partial^2 d^o}{\partial y_k^2} = -\frac{\partial^2 K^o}{\partial y_k^2} d^o - 2 \frac{\partial K^o}{\partial y_k} \frac{\partial d^o}{\partial y_k} \tag{38}$$

It can be observed that the solution for each of the derivative vectors involves only the calculation of the corresponding right hand side vector and forward and back substitutions.

NUMERICAL EXAMPLES

Least volume design of a grillage
The existence of relative minima in grillage structures can easily be found in two-dimensional design problems. Consider the grillage shown in Fig.1 subjected to a uniformly distributed load of 1.0 on longitudinal beams.The grillage has eight design variables. Moments of inertia and section moduli are derived from the areas using the relationships of eq.(39). Lower bounds on design variables are 5.0. Upper and lower bounds on normal stresses are ± 20

$$I_i = 0.3563\, x_i^{2.65} \quad ; \quad W_i = 0.4899\, x_i^{1.82} \tag{39}$$

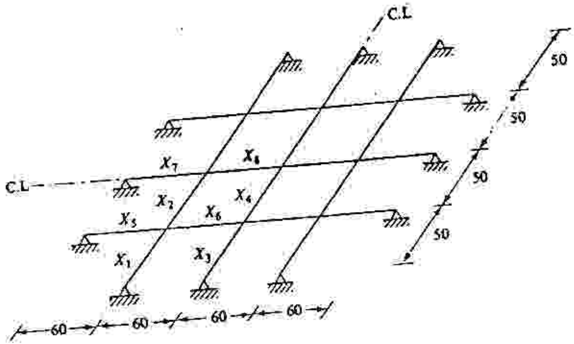

Figure 1

The iteration history is shown in Fig.2, where it can be seen that convergence is achieved with only six analysis.

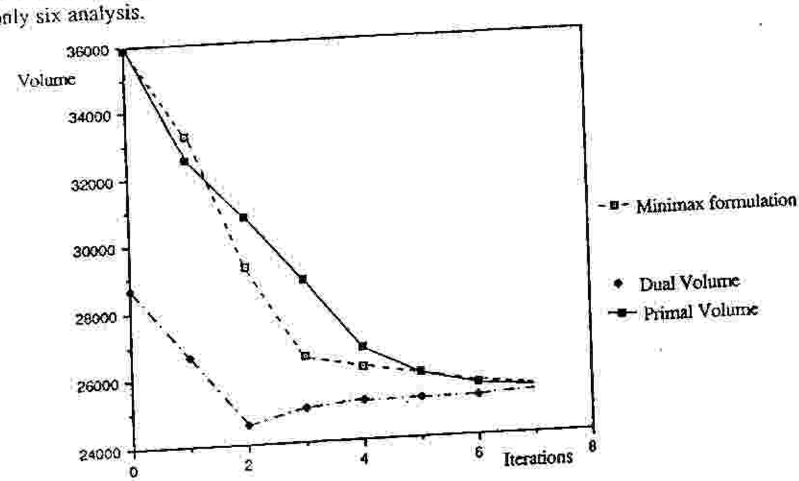

Figure 2

The sensitivity analysis is required in the minimax formulation as opposed to the first algorithm, that only needs the information concerning the current design point. On the other hand, convergence is faster in the second solution scheme. The grillage has several local optima and both methods lead to different solutions according to the starting point and the schedule for the control parameter selection.

Reliability-based truss configuration

Fig.3 shows a 47-bar planar tower subject to three independent loading conditions. This structure is designed for optimum geometry and this reliability-based problem is formulated based on probabilistic requirements for the sizing variables. The average loads are described in Table 1, where $\Omega_L = 0.20$.

	Load condition 1		Load condition 2		Load condition 3	
Joint	17	22	17	22	17	22
Load, x direction	4.5	0.	0.	4.5	4.5	4.5
Load, y direction	-10.5	0.	0.	-10.5	-10.5	-10.5

Average lower and upper bounds on stresses are $\mu_{\sigma}L = -20$, $\mu_{\sigma}U = 27$ and $\Omega_{\sigma} =$ 0 . The members were assumed to be tubular with a constant ratio of diameter-to-wall thickness of 10. Euler buckling was prohibited for all members. The modulus of elasticity was taken as 3×10^4 and the material density, $\rho = 3 \times 10^{-4}$. A minimum allowable area of 10^{-6} was specified. Joints 15, 16, 17 and 22 were held stationary in space and joints 1 and 2 were required to lie on the x axis. Symmetry is imposed and there are a total of 27 independent area variables and 17 independent coordinate variables. The specified probability of failure against unserviceability is 1.5 10^{-3} and the members are assumed uncorrelated.

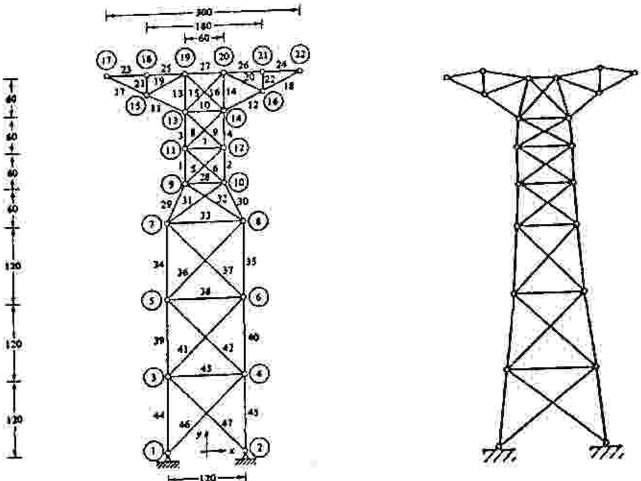

Figure 3

The iteration history is represented in Fig.4, showing a smooth convergence. The solution drawn in Fig.3 required a total of 9 iterations to converge, although results within a 6% margin of error were reached after five analyses. The difference in the geometries obtained after the fifth iteration show that the design space is rather flat in the vicinity of the Pareto solution. As opposed to the algorithms more conventionally used, the minimax formulation is not so heavily dependent on the specified move limits. Buckling constrained problems in which the forces in the members with the minimum allowable areas change as a result of geometry changes may lead to erratic convergence behavior. The minimax formulation prevents this occurrence because it does not look for active constraints (such as the lower bounds imposed on the sizing variables) but rather considers all objectives simultaneously.

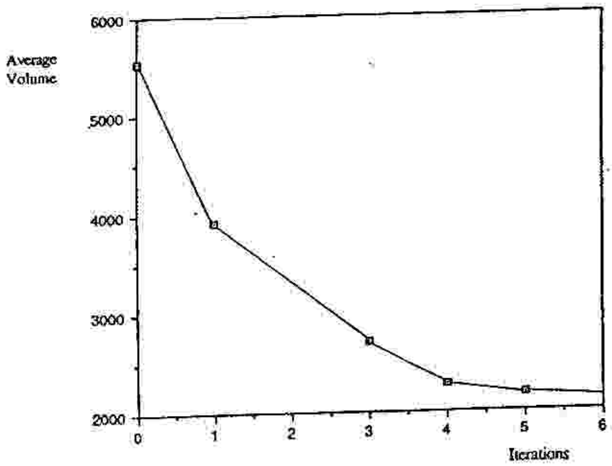

Figure 4

CONCLUSIONS

This paper presents a new class of optimization methods based on informational entropy concepts that are under current development. They are a radically different alternative to existing methods possessing distinctive features and advantages. One of these entropy-based methods, which is applied to structural sizing problems, is computationally extremely simple to implement. The optimization phase is reduced to calculating values for multipliers from an algebraic expression similar in complexity to those used in stress ratio or optimality criteria methods. Unlike optimality criteria and other more recent methods, it does not require an active/passive set strategy. The assignment is done on the basis of the application of Shannon's maximum entropy principle, used to measure the uncertainty in a random process.

Alternatively, the maximum entropy principle is applied to vector and Pareto optimization. Specifically it was shown that the minimax problem can be solved by minimizing a continuous differentiable unconstrained function. The number of iterations required to obtain optimum solutions is small, what makes this algorithm competitive with respect to other more sophisticated methods.

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