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### Abstract

The present technical designer aid is intended to be used after the building system has been designed, hence the structural form is not a cost source. The optimization consists of a problem with multiple goals seeking to improve objectives such as cost which include the plant cost of the boiler and emission systems, fuel costs during heating season and the cost of insulation. Other goals are related with the building heat losses, the overall insulation performance of walls and ceilings. The design variables are the pre-heat power margin, pre-heat time, the factor representing the ratio between the heat supply during office hours and the pre-heat supply, steady state heat loss and transmittance of insulating materials. The model accounts for other factors such as weather factor, occupancy, fuel prices, coefficient of heat absorption and heat transmission coefficients. By using the maximum entropy formalism it is shown that a Pareto solution may be found indirectly by the unconstrained optimization of a scalar function. Keywords: Optimization, Office, Building, Thermal Design, Insulation Performance.

## 1. PROBLEM DEFINITION

Thermal Optimal Design is a problem difficult to solve quantitatively. Some rigorous work has been done in the case of dwellings but only qualitative assessment has been made of Optimal Thermal condition in office buildings since there is the problem of discontinuous usage and heat supply as opposed to the continuous heating policy.

Essentially the problem is that of achieving comfortable thermal conditions in office buildings at minimum cost. The initial cost of the heat supply system to a buildings and the annual cost of the fuel used by that system are dependent upon the design heat loss from the building. The design heat loss is made up of two components; heat loss through the building fabric and heat loss by air exchange or ventilation. Ventilation losses arise from both natural infiltration and random opening of windows by the building users, in the case of the latter, control can be achieved by the installation of non-openable windows which means a mechanical ventilation system must be installed and generally this would imply a totally air conditioned approach to the design.

Thus, in the vast majority of office buildings control of heat losses is largely restricted to the reduction of fabric heat loss. This is usually achieved by the addition of an extra layer of material of high thermal resistance, but this means an increased initial capital expenditure and for this to be acceptable it must be seen to give a desirable rate of return in terms of reduced running costs. The running costs are incurred on a different time scale to initial costs and are subject to different effects - taxation, possible price rises etc. and so their relative importance to initial cost is not obvious. In addition the policy of intermittent heating affords a means of reducing running costs by increased initial expenditure, but this initial expense may be reduced

if insulation is present since power margins will apply to lower design heat losses and the presence of insulation especially as an internal lining "lightens" the building thermally increasing its speed of response to heat input. Thus, the design problem is the complex one of finding those condition of plant output capacity, degree of insulation and heating program which minimize cost, on a common scale, for a given building.

Heating is characterized by the large number of relevant factors and the complexity of their inter-action. Whereas the mechanisms of heat transfer are quite comprehensively quantified in certain circumstances, thermal design involves compromise between scientific rigour and engineering practicality, and for this reason the results of the optimization study will be best interpreted not as explicit values but as conceptual rather than explicitly practical.

## 2. THERMAL MODEL

It has already been indicated that a heating system comprises sub systems of heat production, transmission and emission. The majority of heating systems in office buildings comprise "radiators" (of which 80% of heat emission is by convection) heated by hot water at a certain pressure. A power margin over such a system may be achieved by several methods, individually or in combination. Firstly, it is necessary to use larger boilers, but then an increase in emission rate may be effected by increasing the water flow temperature, increasing the pressure of the water and/or altering the radiators with respect to size, situation, number, nature or type of convection. The relevant one or combination of these effects chosen is particular to a given installation and there are few explicit decision rules governing the choice of one rather than another. Thus for the model it was decided to represent the total heat supply system - production transmission and emission - as a single entity, the heat input. The relevance of the factors comprising this single design parameter will be explained fully during the development of the cost function.

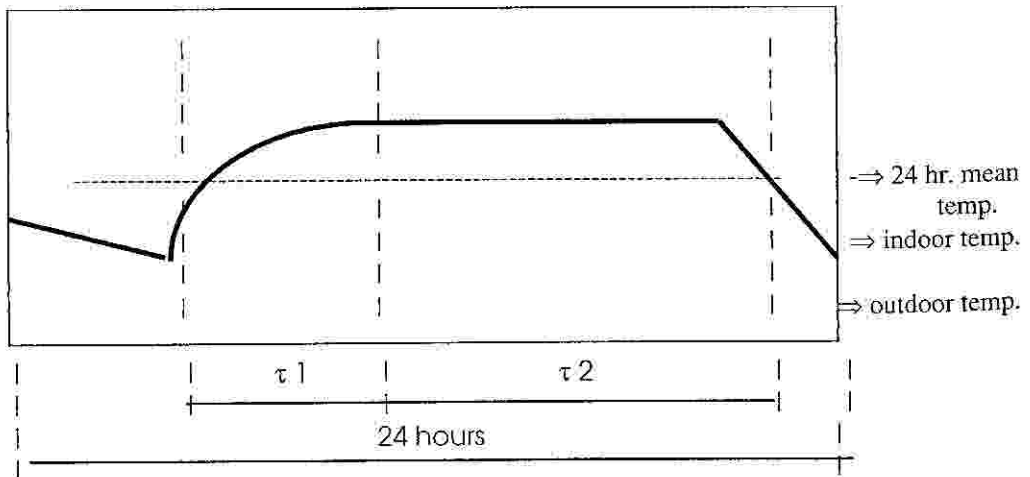


Figure 1 - Variation of temperature

## 2.1. Intermittent Heating

The general form of the temperature profile for a 24 hr. period in an intermittently heated office building is indicated in Fig.1 after Billington. It consists of three portions; the curve due to preheating, the almost constant portion during the occupied period, and the cooling curve after occupancy and heating ceases. The heating curve depends very much upon the plant size which controls the preheating time and rate, and it is these interrelationships formalized into quantitative constraints with which the present section is concerned. The cooling which takes place when heating is discontinued is not strictly exponential. The overall nightly cooling depends on the thermal capacity of the structure, and appears to be independent of the plant thermal capacity. The latter however seems to control the initial rate of cooling. If the thermal capacity of the plant is small or zero, a very rapid drop in air temperature occurs.

The general form of the heating profile required to produce this temperature distribution is plotted in Fig. 2 to the same time scale. It consists of an initial high pulse of heat input during the preheat period and a gradual diminution of supply during the occupied period and then complete shutdown.

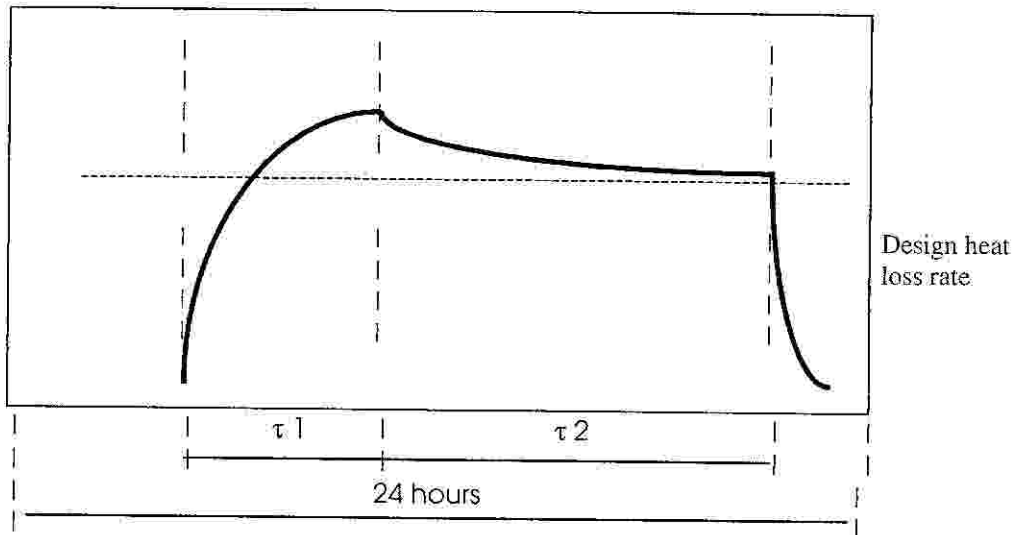


Figure 2 - Variation of heat input

## 2.2. K Value

The parameter K is a measure of the thermal behavior of a building and is titled by Stoeff the equivalent global heat transmission coefficient. It is evaluated by dividing the total fabric hourly heat loss by the total internal surface area per unit temperature difference. The internal surface area for an entire building includes both surfaces of all partitions and intermediate floors

$$K \text{ value} = \frac{\sum_i A_{\text{ext}} u_i}{A_{\text{int}}} \quad (1)$$

Billington has expanded the significance of the K value to the total conventional hourly heat loss divided by internal surface area and this is the sense with which it is used in the present treatment.

### 2.3. Y Value

The Y value expresses the heat transfer to and the temperature fluctuations occurring at the internal surfaces of a building based upon the nature of the building fabric.

In practice, values of Y vary from  $1.5 \text{ W/m}^2 \text{ }^\circ\text{C}$  for insulating materials to  $7.0 \text{ W/m}^2 \text{ }^\circ\text{C}$  for more thermally inert materials. However, Shklover points out that variations between these limits do not have a large effect upon the accuracy of the intermittency calculations and for these reasons average values of Y are advised as adequate. In the common case of an enclosure surrounded by surfaces of dissimilar materials Stoeff advises calculation of a mean value of Y.

### 2.4. Intermittent Supply

The heating profile of Fig.2 is approximated in Shklover's theory by a constant heat pulse of  $W_1$  during the preheat period and  $W_2$  during occupation. If a square heat pulse of  $W_m$  is supplied to a room for a period  $\tau_m$  and then cut off, the incremental temperature change of the internal wall surfaces above external (sol-air) temperature at any time  $\tau$  after the commencement of heat input is given by,

$$\Delta t = \Delta t_{o_{av}} + \frac{W_m \Omega_{\tau m}}{S_o Y} \quad (2)$$

where  $\Delta t_{o_{av}}$  is the increment of the mean daily value of the internal surface temperature related to the main daily heat supply ( $\bar{W}_m \tau_m / 24$ ). Values of the function  $\Omega$  have been derived by Shklover from a basic Fourier analysis of the effect of a discontinuous supply and summary values have been published by both Shklover and Billington, Fig. 3 indicates the variation of  $\Omega$  for a pulse duration of 12 hours.

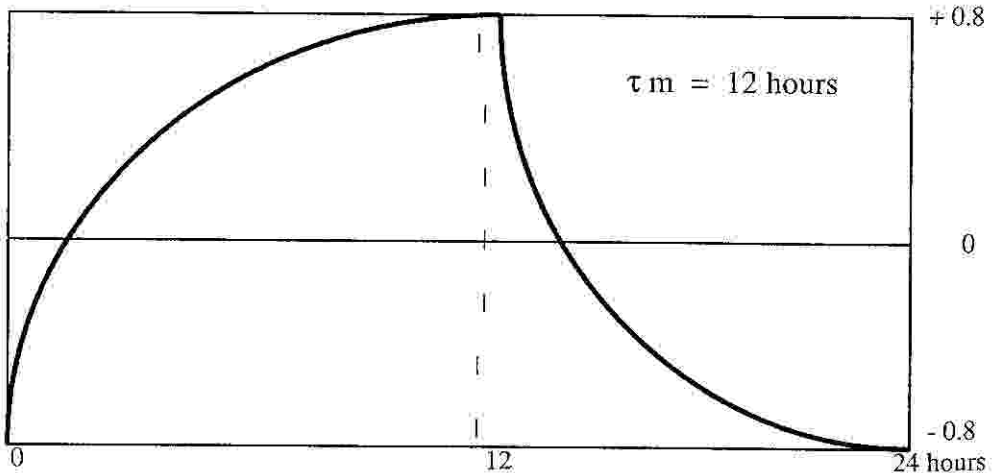


Figure 3

Using this basic relationship of the daily fluctuation in temperature due to a single intermittent heat pulse it is now possible to derive functions relating to the effect of two heating pulses  $W_1$  and  $W_2$  having durations  $\tau_1$  and  $\tau_2$ .

At any instant, there is a balance between the heat taken into a room or building and the heat given out by the air, namely,

$$W_\tau \gamma = \alpha \Delta \tau S_o \quad (3)$$

where  $\gamma$  represents the fractions of the supply heat  $W_\tau$  transmitted by convection from the heating appliance to the air (in the case of warm air heating  $\gamma$  is almost 1 and in the case of radiators it is usually between 0.7 and 0.85) and  $a$  is the coefficient of heat transmission by convection between the inside air and the walls.

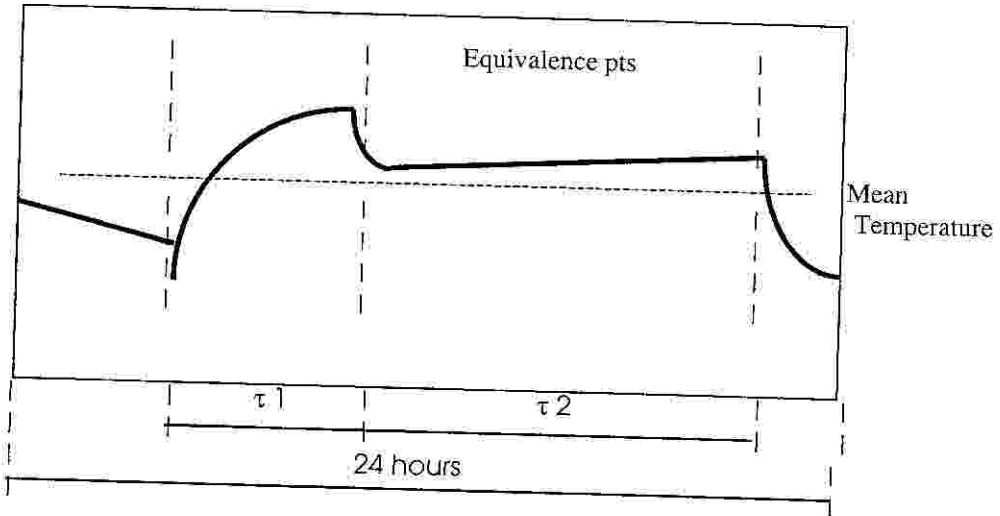


Figure 4 - Temperature curve

To revert to the two part intermittent regime  $W_m$  must be replaced by  $W_1$  and  $W_2$ . Considering the partial values at two points on the temperature profile (Fig.4) one where the time  $\tau$  from the start of heat input  $W_1$  is  $\tau_1$  hours, the time from the start of  $W_2$  is 24 hours and the other where for  $W_1$  it is  $\tau_1 + \tau_2$ , for  $W_2$  it is  $\tau_2$  produces the following equations:

$$\Delta t = \{[(\tau_1 + \psi \tau_2)/24] (1/K - \gamma/2\alpha) + (\Omega^{\tau_1} \tau_1 - \Omega^{24} \tau_1)/Y + \gamma/2\alpha\} W_1/S_0 \quad (4a)$$

$$\Delta t = \{[(\tau_1 + \psi \tau_2)/24] (1/K - \gamma/2\alpha) + (\Omega^{\tau_1 + \tau_2} \tau_1 - \Omega^{\tau_2} \tau_2)/Y + \psi \gamma/2\alpha\} W_1/S_0 \quad (4b)$$

where  $\psi = W_2/W_1$ . The temperature profile of Fig.4 is defined so that these two temperatures are equal, hence, equating (4a) and (4b) gives,

$$\psi = \frac{\Omega^{\tau_1} \tau_1 - \Omega^{\tau_1 + \tau_2} \tau_1 + (\gamma Y/2\alpha)}{\Omega^{\tau_2} \tau_2 - \Omega^{24} \tau_2 + (\gamma Y/2\alpha)} \quad (5)$$

Equations (4a) or (4b) may be rewritten in terms of  $W_1$  as,

$$W_1 = S_0 \Delta t / A \quad (6)$$

where  $A$  refers to the expression between brackets in (4a) or (4b). The value of  $W_1$  thus obtained is the maximum power of the heating supply needed to provide the required internal temperature in the intermittent heating regime outlined. For the same temperature difference  $\Delta t$  to be obtained from a continuous heating regime an output  $W_S$  would be needed where

$$W_S = K S_0 \Delta t \quad (7)$$

Comparing this with equation (6) provides the preheat power margin  $p$  for intermittency

$$p = W_I/W_S = 1/A K \quad (8)$$

## 2.5. Heat Losses

Heat losses are of two kinds: fabrics and ventilation. It was decided to sub-divide fabric losses into several components, each representing a certain external fabric type. The fabric loss  $W_F$  is given by

$$W_F = \sum_{i=1, n} u_i A_i \Delta t \quad (9)$$

where  $n$  = total number of different fabric types.

The heat loss due to ventilation  $W_V$  (for predominantly convective heating) is given by

$$W_V = 0.33 N V (t_{ai} - t_e) \quad (10)$$

since the motivational heat flow is caused by the difference in air temperatures rather than environmental.  $N$  is the rate of air change and  $V$  is the internal volume. However, inside air and environmental temperatures are related by

$$(t_{ai} - t_e) = \Delta t + W_S \gamma / (2 S_o \alpha) \quad (11)$$

where  $W_S$  is the steady state heat loss and

$$W_S = W_F + W_V \quad (12)$$

Amongst the  $n$  types of fabric of the enclosure some will remain constant since no insulation will be added. Thus the summation term in (9) may be replaced,  $W_S$  becoming

$$W_S = \frac{(\sum_{j=1, n-v} u_j A_j + \sum_{i=1, v} u_i A_i + 0.33 N V) \Delta t}{[1 - 0.33 N V \gamma / (2 S_o \alpha)]} \quad (13)$$

where  $v$  = number of fabric types variables in design at this stage.

## 3. DESIGN VARIABLES

There is no unique solution to the problem of model formulation or choice of variables, the same results may be obtained by diverse formulations, but generally there will be a smaller number of possible formulations which achieve a desired balance between their complexity and the results required. The effectiveness of any model must be judged on both theoretical and practical grounds, in the first case it must display the theory and rationale of the design problem it is intended to represent and secondly and perhaps ultimately, the results it provides must pass the test of practicability which means that the model must bear the brunt of shortcomings in the design theory.

The time cycle of the intermittency is defined by both the occupied period and the preheat time, it was decided to regard the occupied period  $\tau_2$  as constant for a given situation, leaving  $\tau_1$  as an independent design variable. The greatest problem of variable definition occurred with the treatment of insulation. The problems arise from the heterogeneous nature of the insulation option and if only one type of insulation in one structural configuration were to be considered (e.g. polystyrene sheets fixed to the inside walls by adhesive) the relevant variable would obviously be the thickness. But this view does not allow for the many other policies of insulation which may be used individually or in combination in a given construction. The variable adopted was the total thermal resistance of the construction  $r_i = u_i^{-1}$ ,  $i = 1, 2, \dots, v$ . The

value of  $v$  - the total number of construction types comprising the external building shell - is building dependent.

In the problem in hand it is better to define intermediate dependent design variables. The effectiveness of thermal insulation of the building must be measured by the design heat loss (steady state) and  $W_S$  becomes an obvious variable which is given by (13) once the  $\tau_1$  (or  $u_1$ ) are known. It is next necessary to represent the heat supply levels in the intermittent mode  $W_1$  and  $W_2$ , each could be used as design variables but an alternative representation is to use the power factor  $p$  given in eq.(8) (so that  $W_1 = p W_S$ ) and the factor  $\psi$  (so that  $\psi = W_2/W_1$ ) which is a function of  $\tau_1$  as seen in eq.(5).

## 4. COSTS

### 4.1. Fuel Cost Function

The annual fuel cost of a heating installation depends upon the nature of usage of the installation throughout the year, usage depends upon the external climate pertaining at any time since heating plant is automatically controlled by thermostats. Estimation of heating season is so complex that various practical design rules have evolved based upon empiricism but which have been found in practice to closely reflect reality. The model allows the user to insert his own values of external temperature and heating season with load factor as problem input data.

The three major types of fuel presently used in office buildings are oil, gas and propane and although rates vary the nature of costing is similar in all cases being a combination of standing charges and fuel usage charges. Standing charges may be regarded as geographically constant for a given fuel type but usage charges vary greatly with location, supplier and many other factors.

The two more important characteristics of the plant are boiler efficiency and plant time constant. The real cost of fuel used in heating depends upon its price, calorific value and the efficiency with which it is used, that is the proportion of the calorific content of the fuel which is actually released by transfer at the boiler. The fuel cost per useful unit of heat is obtained by dividing by this value of efficiency. There is a time lag between ignition of a boiler and the supply of heat to the building which depends on the thermal capacity of the heating system. The parameter which defines this time lag is the plant time constant. Thermal capacities and time lags of the most common heating configurations are available.

The cost of fuel is a recurring cost throughout the life of the building. It cannot be compared directly with the initial capital costs arising from design decisions, but must be factored to reflect the time dependence of money. If the rate of exchange of money now to money one period in the future is as the ratio 1 to  $(1+r)$ , it is possible to compare the initial design cost  $C$  with the resultant long term cost on the same scale  $P$ , giving the Net Present Value (N.P.V.) of the total outlay

$$N.P.V. = C + P = C + [1 - (1+r_0)^{-n}/r_0] A/(1+g) \quad \text{where } r_0 = (r-g)/(1+g) \quad (14)$$

where  $A$  is the annual fuel cost,  $g$  is the % growth rate per annum of fuel cost,  $n$  is the number of years of occupancy.

The fuel cost function, in terms of the system variables has the general form,

$$\text{Fuel Cost} = P [ C_1 + C_2 p W_S (T + \tau_1 + \psi \tau_2) ] \quad (15)$$

where  $C_1$  is the annual standing charge and  $C_2 = WY DW WF FP/E$ .  $WY$  is the heating season (weeks/year),  $DW$  is the occupancy (days/week),  $WF$  is the weather factor related to the heating season,  $FP$  is the fuel price and  $E$  the plant efficiency and  $T$  is the plant time constant.

#### 4.2. Plant Costs

A total plant costing includes the following components; supply, unloading and assembling of boiler, supply and fitting of burner, insulation, thermostats, pumps, flanges, sockets, valves and pipework, supply and installation of heat distribution and emission systems and finally commissioning of the total installation.

Clearly many of the costs mentioned above will have to be met whatever size of boiler or emission system is installed. A survey of manufacturers data reveals little variation in the cost of similar types of boiler and it was decided to use the data of one leading manufacturer rather than to derive market averages.

There are many types of heat emission system and it is possible to consider any type providing the necessary data describing its transfer mechanism is available. In the program it was considered that emission was by the common radiator fed by water under pressure. The distribution system which transferred the water from boiler to radiator including pipework, connections and valves at the radiators was considered constant.

The functions to describe plant costs have the following form,

$$\text{Plant Cost} = a + b(p.W_S) + d/f(p.W_S) \quad (16)$$

where  $a$  and  $b$  are cost coefficients related to the boiler and installation,  $d$  is the average cost of radiators and  $f$  is the average output rating of radiator surface.

#### 4.3. Insulation Costs

For a given fabric type a cost/resistance graph is plotted by testing the addition of each of several standard insulation policies to the basic construction. A filter rejects all those policies inapplicable in a given situation. A function relating cost with resistance is obtained via a regression process, which gives a lower bound to the scatter of points for the given fabric type. An example of a cost/resistance graph for an 11" cavity brick wall plastered on the inside is given in Fig.5.

The device is not perfect, several improvements are possible, for instance, building orientation is not taken into account, nor is the life of the heating plant which will almost certainly be different to the building life.

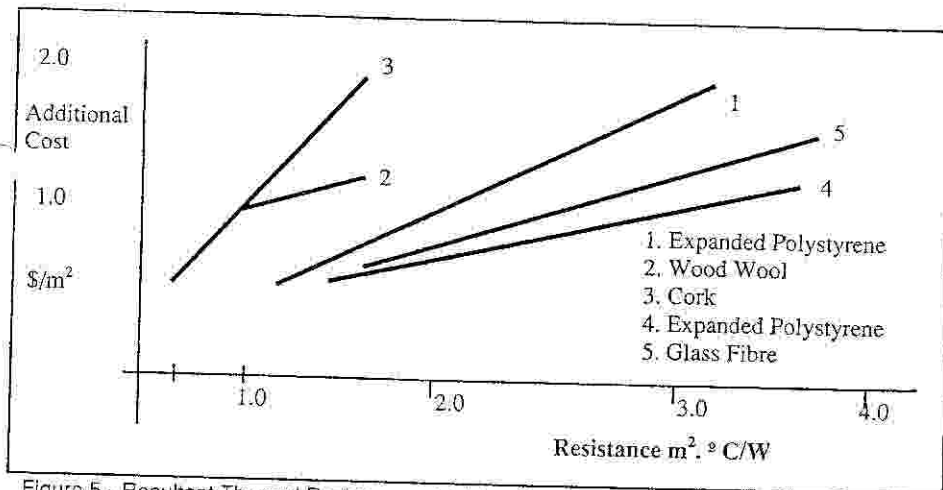


Figure 5 - Resultant Thermal Resistance of Adding Various Forms of Insulation to an 11 ins. cavity Brick Wall



### 5. MULTI-OBJECTIVE FORMULATION

Pareto's economic principle is gaining increasing acceptance to multi-objective optimization problems. In minimization problems a solution vector is said to be Pareto optimal if no other feasible vector exists that could decrease one objective function without increasing at least another one. The optimum vector usually exists in practical problems and is not unique.

The optimization method requires that all the goals should be cast in a normalized form. If some reference total cost  $\underline{C}$  is specified, this goal can be written in the form:

$$g_1(\tau_1, \psi, p, W_S, r_1) = C(\tau_1, \psi, p, W_S, r_1) / \underline{C} - 1 \leq 0 \tag{17a}$$

In addition  $W_2$  must be greater than  $W_S$ , hence

$$g_2(p, \psi) = - p \psi + 1 \leq 0 \tag{17b}$$

The building shell has been designed and it may or may not have some insulation material in its construction. Whatever its nature the building fabric provides some thermal resistance and this must be accounted for in determining any change needed in the overall transmittance of the wall for optimum performance. Thus

$$g_3(r_i) = \frac{r_i}{r_{i(\text{exist})}} + 1 \leq 0 \quad i=1, 2, \dots, v \tag{17c}$$

Condensation occurs upon a surface when the temperature of that surface falls below the dew point temperature of the air (that temperature to which the air must be cooled in order to saturate it, without increase of the water content). It may conveniently be considered in two categories; that which appears on the inside surface of walls and that which occurs within the fabric of the walls (or roof) - interstitial condensation.

Permanent condensation will normally occur on the surfaces of materials of high thermal transmittance and depends, of course, upon the degree of water vapor within the building. In another set of conditions, namely when a period of cold weather is succeeded by warmer and damper weather condensation of a temporary nature may appear. The risk of condensation can be reduced by good insulation, adequate heating and adequate ventilation. Codes of practice recommend the following practice

$$g_4(r_i) = - 1.1 r_i + 1 \leq 0 \quad i=1, 2, \dots, v \tag{17d}$$

To investigate the likelihood of interstitial condensation within a structure it is necessary to plot both temperature and vapor pressure profiles through the construction to determine the existence of critical points indicating condensation. The resultant goals were of the form

$$g_5(r_i) = \frac{r_i}{r_{i(\text{crit})}} - 1 \leq 0 \quad i=1, 2, \dots, v \tag{17e}$$

where  $r_{i(\text{crit})}$  is related with the thickness of the added insulation material to fabric type  $i$ . Simões and Templeman have shown that the solution of this multiobjective optimization may be found indirectly by the unconstrained optimization of the convex scalar function

$$F(x) = \frac{1}{p} \ln \sum_{j=1, j} e^{p g_j} \tag{18}$$

which is both continuous and differentiable and thus considerably easier to solve.  $g_1$  is the cost and  $g_2, \dots, g_j$  are the remaining goals.  $r$  is a positive control parameter initially set by the user.

The strategy adopted was to solve (18) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions  $g_j$  truncated after the linear term. Iterated solutions represent the optimum for the linearized form of function  $F$ , which may be written

$$F(X) = \frac{1}{\rho} \ln \sum_{j=1, J} e^{\rho [g_j(X_0) + \sum_{i=1, NV} \frac{\delta g_j}{\delta x_i} (X_i - X_{0i})]} \quad (19)$$

in which index 0 denotes starting values. The solution of this minimization constitutes the starting design for the next iteration and convergence is attained when the convex objective function decrease from one iteration to the following is smaller than some user-defined value.

## 6. SENSITIVITY ANALYSIS

Iterative optimization algorithms need to know the way a change in the each design variable will affect the requirements expressed as goals. This is the task of the sensitivity analysis. The evolution of the problem depends on a critical way on the accuracy with which these values are computed.

A most important and effective type of sensitivity analysis involves the sensitivity of the optimal policy to the model formulation, for here the significance of optimization as a meaningful design aid is increased dramatically. In the thermal design model there are so many uncertain factors that to formulate a rigid model, optimize it once and present this results as the final optimal policy to be followed would plainly be meaningless. This type of sensitivity analysis allows the designer to become familiar with the factors to which the optimum is sensitive which cannot be done in the absence of optimization for there is then no scale of comparison.