CIAO: An integrated software package for analysis and optimization of cable-stayed bridges

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1. Abstract

The trend for the increase of the average span of cable-stayed bridges make desirable the development of computational tools to assist on the preliminary design stage and/or erection control. Programme CIAO, shortname standing for *C*able-stayed bridges *I*ntegrated *A*nalysis and *O*ptimization, is one such tool.

Most of the features and specific problems of such structures were included in the code: Erection control, dead-load deflection condition, seismic loading. Sizing, shape and mechanical design variables are available from a design variable library that controls the procedures for analytic direct discrete sensitivity analysis. Specific sensitivity analysis procedures were developed for the sake of seismic analysis by both the modal-spectral and step-by-step approaches. Pre- and post-processors provide an effective use of the programme and data updating.

The optimization is posed as the problem of minimization of structural cost or volume. Constraints on maximum stresses throughout the structure and minimum stresses in cables, in order to avoid slackening, are considered. The dead load geometry condition, usual in the design of such structures, is met by imposing constraints on the deflection of control nodes of the deck and pylons. The solution of this minimax problem may be shown to be equivalent to that of the minimization of a scalar convex unconstrained function, solvable by conventional quasi-Newton methods.

2. Keywords

Optimization, Cable-stayed bridges, Integrated Package

3. Introduction

Cable-stayed bridges are large and expensive structures whose design may profit from the use of Mathematical Programming tools. The authors have been involved in the development of programme CIAO, shortname standing for *C*able-stayed bridges *I*ntegrated *A*nalysis and *O*ptimization.

The conception of such large and complex structures involves two consecutive levels of decision. Firstly, more or less subjective criteria concerning aspects such as aesthetics or environmental impact are considered. These problems fall within the scope of expertise systems and are not dealt with in here. CIAO competencies thus concern those aspects of design that may be objectively expressed by a numerical merit measure, such as structural erection or maintenance cost. The capabilities of the programme will now be discussed.

4. Structural analysis core

A finite element procedure was chosen as analysis method, given its unique versatility for modelling complex geometries and constitutive laws. The finite element based open code MODULEF was used as the basic tool, because the access to the code was a fundamental requirement for further developments. 2D and 3D bar and beam (Euler-Bernoulli formulation) elements and 4- and 8-noded serendipity plate-membrane (Reissner-Mindlin formulation) elements were selected to allow for both stiffening girder and box-girder modelling.

5. Non-linear behaviour

Concerns with the extents of non-linear effects are necessary in large and slender structures such as cable-stayed bridges. It is generally accepted that the three main sources of non-linearity are of geometric nature: P-δ, large deflections and cable sag effects. The accurate consideration of the first two effects requires a fully non-linear analysis approach, which is beyond the current development stage of the programme. Besides, the requirement of undeflected shape for dead load condition validates the linear model for this part of the load and significantly reduces those effects. Based on this, several authors [1],[2],[3],[4],[5],[6] accept the use of linear analysis. Sag effect may be handled by the multiple straight link approach or catenary formulations, but they also require the use of a fully non-linear approach. The alternative method of the equivalent modulus of elasticity or Ernst method [4],[5],[6],[7],[8] provides accurate solutions (within certain assumptions) while still allowing for the use of a linear approach. The method consists of replacing the true Young modulus by an equivalent value that includes the elastic extension of the material as well as the chord length change due to the loading. Although widely used in actual bridge design and successfully used in a number of previous papers of the authors [9],[10],[11], the method contains certain weaknesses: dynamic cable behaviour cannot be reproduced and different load cases lead to different equivalent modulus. There is no way out for the first problem, because the approach is intrinsically linear. As to the second, either of two procedures may be considered. One consists of using average values for the whole set of load cases, which introduces a further error in the analysis results. Fortunately, this error is usually neglectable for the expected range of variation of the equivalent modulus. The second procedure consists of evaluating the Ernst modulus for each cable and load case. The stresses will then be consistent with the assumed modulus, but at the cost of a complete equation solving for each load case.

6. Erection stages

The optimization of a complete bridge requires some concern with the intermediate structures necessary to build it. Cantilevering method is widely used and was therefore selected for implementation in programme CIAO.
The method consists of erecting balanced segments for both sides of each pylon. The length of each segment is usually that of the cable spacing, so that this is the maximum cantilevered span at any moment of the erection.
The state of stress and deformation change with each new set of stays put in place and prestressed. The final dead load condition will result from successive upwards and downwards deck deflections. The safety assessment for erection stages is usually made by a backward analysis, but this procedure is not well suited for optimization, because it does not simultaneously provide all required stresses and displacements. Instead, it was assumed that the chronological sequence of erection stages might be thought of as a set of independent sub-structures, each corresponding to an erection stage. The overall stiffness matrix is made of uncoupled blocks and thus the number of operations involved in solving the equation system increases only linearly with the number of degrees of freedom.

7. Seismic action

An effective and safe structural design requires that due attention is paid to seismic effects. Modal-spectral approach or step-by-step (time-history) analysis may be used for that. Each method shows comparative advantages and weaknesses and were thus both implemented in CIAO code. Modal analysis may be coded separately from the main stream of the analysis code and be called whenever needed. It gets data concerning mass and stiffness from the main code and delivers dynamic forces and their sensitivities. This *black box* philosophy is advantageous from the point of view of programming and use of resources. The Bathe's subspace iteration method [12] and the Complete Quadratic Combination (CQC) method of Kiureghian *et al* [13] were used.
Time-history approach may be preferable in instances such as those of coalescent modes, because it avoids problems concerning eigenvalues and eigenmodes derivatives. However, conventional F.E. packages handle step-by-step and static load cases separately. Thus, as both results are needed for optimization, it was necessary to adapt the code so as step-by-step and static load cases were analysed in sequence without any switching command. A time-grid selection strategy is used to reduce the amount of data generated. The Newmark direct integration scheme was implemented.

8. Design variables and sensitivity analysis

A number of parameters were grouped in a *design variable types library* from which the user may select the desired set. The currently available types are those shown in Figures 1 and 2. Sizing design variables are cross-sectional characteristics of bar, beam and plate elements, such as web height, flange width, plate thickness, etc. Changes of such variables do not imply the need for remeshing. Shape design variables are numbered 4 to 10 in Figure 2 and require either nodal co-ordinates updating or complete remeshing. Type 19 (fixed-end prestressing forces) is a mechanical design variable not related to any geometric quantity. All these types play complementary roles in the process of design optimization. Sizing design variables directly provide for cost/volume decrease. Shape and mechanical design variables have a neglectable direct relation to structural cost but allow for better stress distributions, which in turn lead to further decreases in sizing variables. Prestressing forces design variables are even essential for achieving acceptable solutions when deflections are included in the constraints set - and they always do, because of the dead load condition. The best structural solutions are achieved when both sizing and shape parameters are combined in the design variable set. For the variable linking between the finite element mesh and the design variables set a *dependency matrix* was used. This array is automatically generated in the pre-processing stage. Its contents are integer coefficients

$$\underline{C}: \{C_{ij} = k\, , i=1,N\, ; j=1,M\} \quad (1)$$

which state that the element i explicitly depends on design variable j, whose type (among those shown in Figures 1-2) is k, $k=0$ when no explicit relation exists. N is the number of elements and M that of the design variables.

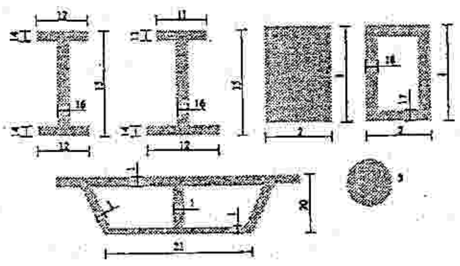

Figure 1 – Sizing and hybrid design variables

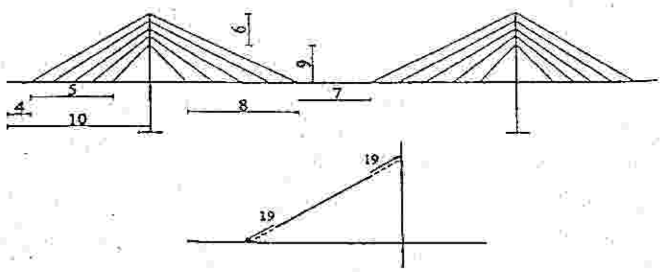

Figure 2 – Shape and mechanical design variables

The analytic direct method was adopted for sensitivity analysis. For ordinary linear statics problems, derivatives of kinematic constraints (displacements) are provided by solving for the pseudo-load system

$$\frac{d\underline{u}}{dx_i} = \underline{K}^{-1}\left(\frac{d\underline{P}}{dx_i} - \frac{d\underline{K}}{dx_i}\underline{u}\right) + \underline{K}^{-1}\,\underline{Q}_{R_i} \tag{2}$$

Derivatives of static constraints (stresses) may thereafter be computed by adding explicit and implicit contributions:

$$\frac{d\sigma}{dx_i} = \left[\frac{d\underline{D}}{dx_i}\underline{B}_e + \underline{D}\frac{d\underline{B}_e}{dx_i}\right]\underline{u}_e + \underline{D}\,\underline{B}_e\frac{d\underline{u}_e}{dx_i} \tag{3}$$

The form of matrix derivatives in equation (3) depends on the element. For bar and beam elements their calculation is straightforward. Plate-membrane elements require the differentiation of the whole formulation, see [14].
The optimized shape of the bridge must comply with the architectural requirements. This means that the freedom for the trial design to evolve for more effective configurations must be limited by geometrical restraints. Some of such restraints were stated as contraints of the objective function, while others influence directly the sensitivity analysis. Special sensitivity analysis procedures were developed for both modal-spectral and time-history analysis. In the latter case, derivatives for timestep incremental quantities result from direct differentiation of the expressions of Newmark algorithm. As to the former, the differentiation of the whole modal-spectral algorithm, including both modal analysis expressions and the modal forces combination by the CQC method, was undertaken. The modal technique of Fox and Kapoor [15],[16] was used for evaluating eigenvector derivatives. However, this gives raise to problems concerning coalescence situations.

9. Optimization

The design enhancement must be done with due attention paid on allowable limits for stresses and deflections. The large number of such implicit constraints makes difficult a conventional NLP solution. An alternative approach is the minimax formulation, which provides an optimum design in the Pareto sense, but these discontinuous and non-differentiable problems are hard to solve. However, by using an entropy-based approach, Templeman [17] has shown that its solution is equivalent to that of an unconstrained convex scalar function, depending on a control parameter ρ:

$$\underset{\underline{x}\in\underline{X}}{\text{Minimize}}\ \max_{i=1,M}\ g_i(\underline{x}) \quad \equiv \quad \underset{\underline{x}\in\underline{X}}{\text{Minimize}}\ \frac{1}{\rho}\ln\sum_{i=1}^{M} e^{\rho g_i(\underline{x})} \tag{4}$$

$$g_1(\underline{x}) = Cost \quad ;\ g_i(\underline{x}) = \sigma_i/\sigma_o - 1\ \ ,\ i=1,NSO \quad ;\ g_j(\underline{x}) = min(0\ ;|\delta_j|/\delta_o - 1)\ \ ,\ j=1,NKO \tag{5}$$

NS(K)O is the *N*umber of *S*tatic or *K*inematic *O*bjectives. For actual numeric implementation, an approximate form of equation (5) is used in which the goals $g(\underline{x})$ are linearized about the starting design values.This procedure requires the use of a sequential strategy in order to bound the error.

$$\underset{\underline{x}\in\underline{X}}{\text{Minimize}}\ \frac{1}{\rho}\ln\sum_{i=1}^{M} e^{\rho\left[g_i(\underline{x}_o) + \sum_{j=1}^{n}\frac{dg_i(\underline{x}_o)}{dx_j}\Delta x_j\right]} \tag{6}$$

10. Conclusion

Conceptual aspects of design require much of hardly reproducible human traces such as intuition and artistic sensibility, while computer achieves algebraic performances far beyond the human capabilities. The authors believe that the presently available computational resources make possible a profitable co-operation between man and machine in structural design. Programme CIAO is intended to play such a role in the preliminary design of cable-stayed bridges.

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