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CFD based evaluation of Scanlan coefficients by using the forced oscillation method

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Abstract
This paper concerns the application of the so-called Finite Volume Method (or Control Volume Method) to the evaluation of Scanlan coefficients used in aeroelastic analysis, using the forced oscillation method. The results obtained by applying this Computational Fluid Dynamics algorithm are presented and compared with some available in the literature, concerning the application to the study of rectangular cross sections.

1 Introduction

Long span bridges are very flexible structures that can be affected by different aeroelastic phenomena, like buffeting, vortex-shedding / lock-in or flutter.
The analysis of the dynamic behaviour of such bridges submitted to wind excitation is usually performed on the basis of experimental tests under physical models in wind tunnels.
Although an increasing tendency for the development of alternative numerical approaches has occurred in the recent past, the application of such numerical methods usually depends on the knowledge of some coefficients (e.g. shape coefficients and Scanlan coefficients), whose evaluation is still made on the basis of experimental tests.
However, an attempt to overcome such limitation can be made by using different types of algorithms from Computational Fluid Dynamics (*CFD*), that permit the numerical simulation of the wind flow around the deck cross-sections.
According to this context, the main objective of this paper is to present one application of the so-called Finite Volume Method to the evaluation of several Scanlan coefficients adopted in aeroelastic analysis by using the forced oscillation method. The results obtained by applying this *CFD* algorithm, implemented in the computer code *PCAMVF*, are presented and compared with some available in the literature, concerning its application to the study of rectangular cross sections.

2 Fluid Flow Simulation

The computer program *PCAMVF*, based on the Finite Volume Method [1-3], is suitable to simulate incompressible and isotherm bidimensional unsteady fluid flows around obstacles. It is assumed that the flow domain may be discretised in a control volume mesh, whose faces have orthogonal directions. Differential forms of the general transport equations are discretised using a hybrid differentiation scheme. To reduce false diffusion, the quick differentiation scheme is also used in deferred correction context. Alternate value fields are avoided on the basis of a staggered grid approach. Solution procedures for transient calculations are implemented adapting under-relaxation factors depending on time increment. The high Reynolds number $k-\varepsilon$ turbulence model is applied to simulate the flow turbulence [4-7].
The iterative solution procedures for every time increment are the *TDMA* line-by-line solver of the governing mass, momentum and turbulence conservation algebraic equations of unsteady turbulent flow, and the *SIMPLE* algorithm to ensure correct linkage between pressure and velocity.
The convergence criterion for pressure-correction equations is set up by

$$\frac{1}{n}\sum_{n}\frac{\left\|{}^{i}b\right\|}{\rho U}\leq 10^{-4} \tag{1}$$

where n is the number of control volumes, ${}^{i}b$ is the source term at the i^{th} iteration, ρ is the fluid density and U is the flow velocity out of domain.
For the remaining equations, the convergence criterion is given by

$$\frac{1}{n}\sum_{n}\frac{\left\|{}^{i}\phi-{}^{i-1}\phi\right\|}{\phi_{inlet}}\leq 10^{-4} \tag{2}$$

where ${}^{i}\phi$ is the field value calculated at the i^{th} iteration and ϕ_{inlet} is the field value in the inlet domain.

3 Aeroelastic Analysis

3.1 Scanlan Model

A structural system is submitted to several forces when immersed in a wind flow [8,9]. They depend on three fundamental effects:

- External flow instability, by velocity field flutuations in external domain;
- Internal flow instability, by structural geometry and flow characteristics;
- Structural movements.

Those forces are called self-excited (or aeroelastic) forces when these structural movements play an important role in terms of forces characteristics. Initially, Scanlan proposed an analytical model to describe those aeroelastic forces: he considered that external flow is permanent and structural movements are incipient and periodic with constant frequency. For a slender horizontal structure, aeroelastic forces were only dependent on three kinematic variables of cross-section: rotation, vertical displacement and angular velocity. Afterwords, this aeroelastic forces model was generalised to all displacements and velocities of cross-section [10,11].

Let us take into account a cross-section of a slender horizontal structure (B = along-flow dimension) immersed in a wind flow, which is normal to the span (U = mean velocity; ρ = fluid density). If the section is assumed to be in oscillatory motion at an angular frequency ($\omega = 2\pi f$), the generalised aeroelastic forces model can be expressed by a linear relation between the aeroelastic forces (F_{a1} = drag; F_{a2} = lift and F_{a12} = moment) and the movements (a = displacements and $\dot{a}$ = velocities) with regard to the three orthogonal directions (Ox_1 = horizontal; Ox_2 = vertical and Ox_{12} = angular)

$$F_a = CF \,.\, PHA^* \,.\, a \tag{3}$$

where

$$F_a^{\,T} = \begin{bmatrix} F_{a1} & F_{a2} & F_{a12} \end{bmatrix}; \qquad CF = \frac{\rho U^2 B}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & B \end{bmatrix}; \qquad a^T = \begin{bmatrix} \dot{a}_2 & \dot{a}_{12} & a_{12} & a_2 & \dot{a}_1 & a_1 \end{bmatrix} \tag{4}$$

$$PHA^* = \begin{bmatrix} \frac{KP_5^*}{U} & \frac{KBP_2^*}{U} & K^2P_3^* & \frac{K^2P_6^*}{B} & \frac{KP_1^*}{U} & \frac{K^2P_4^*}{B} \\ \frac{KH_1^*}{U} & \frac{KBH_2^*}{U} & K^2H_3^* & \frac{K^2H_4^*}{B} & \frac{KH_5^*}{U} & \frac{K^2H_6^*}{B} \\ \frac{KA_1^*}{U} & \frac{KBA_2^*}{U} & K^2A_3^* & \frac{K^2A_4^*}{B} & \frac{KA_5^*}{U} & \frac{K^2A_6^*}{B} \end{bmatrix} \tag{5}$$

In the above equations P_i^*, H_i^* and A_i^* are the Scanlan coefficients (or flutter derivatives), and $K=B\omega/U$ is the reduced frequency.

In Wind Engineering practice, it is assumed that Scanlan coefficients have the advantage of being purely aerodynamic. This means that they depend only on the geometrical shape of the cross-section and reduced frequency (structural movements). Instead of this reduced frequency, they are commonly presented in graphic form as a function of the reduced velocity $U_r=U/fB=2\pi/K$.

Using Fourier transform, the loading model (eq. 3) can be transformed into the frequency domain as

$$\tilde{F}_{a1} = \frac{\rho U^2 BK^2}{2}\left[\left(P_4^* + iP_1^*\right)\frac{\tilde{A}_1}{B} + \left(P_3^* + iP_2^*\right)\tilde{A}_{12} + \left(P_6^* + iP_5^*\right)\frac{\tilde{A}_2}{B}\right] \tag{6}$$

$$\tilde{F}_{a2} = \frac{\rho U^2 BK^2}{2}\left[\left(H_4^* + iH_1^*\right)\frac{\tilde{A}_2}{B} + \left(H_3^* + iH_2^*\right)\tilde{A}_{12} + \left(H_6^* + iH_5^*\right)\frac{\tilde{A}_1}{B}\right] \tag{7}$$

$$\tilde{F}_{a12} = \frac{\rho U^2 B^2 K^2}{2}\left[\left(A_4^* + iA_1^*\right)\frac{\tilde{A}_2}{B} + \left(A_3^* + iA_2^*\right)\tilde{A}_{12} + \left(A_6^* + iA_5^*\right)\frac{\tilde{A}_1}{B}\right] \tag{8}$$

where frequency functions $\tilde{F}$ and $\tilde{A}$ represent the Fourier transforms of the time functions f and a.

3.2 Identification of Scanlan Coefficientes by the Forced Oscillation Method

Forced oscillation method was used by Nakamura [12] to identify Scanlan coefficientes. Let us take into account a cross-section oscillating in sinusoidal motion along one direction with a particular amplitude (a_0) and frequency (ω), the displacement (a) and velocity ($\dot{a}$) being defined by

$$a = a_0 \cos(\omega t); \qquad \dot{a} = -a_0 \omega \sin(\omega t) \tag{9}$$

Further, it is said that the aeroelastic forces (F_a) can also be described by a sinusoidal function with an amplitude (F_{a0}) and the same frequency, given by

$$F_a = F_{a0} \cos(\omega t + \varphi_L) \tag{10}$$

where φ_L represents the phase angle in relation to motion.
If the cross-section oscillates in the horizontal direction (Ox_1), the corresponding Scanlan coefficients can be estimated, taking into account the forces model in the frequency domain (equations 6 to 8), by

$$P_4^* + iP_1^* = \frac{2}{\rho U^2 K^2}\frac{\tilde{F}_{a1}}{\tilde{A}_1}; \qquad H_6^* + iH_5^* = \frac{2}{\rho U^2 K^2}\frac{\tilde{F}_{a2}}{\tilde{A}_1}; \qquad A_6^* + iA_5^* = \frac{2}{\rho U^2 BK^2}\frac{\tilde{F}_{a12}}{\tilde{A}_1} \tag{11}$$

Now, considering that the cross-section oscillates in the vertical direction (Ox_2), it is possible to calculate related Scanlan coefficients by

$$P_6^* + iP_5^* = \frac{2}{\rho U^2 K^2}\frac{\tilde{F}_{a1}}{\tilde{A}_2}; \qquad H_4^* + iH_1^* = \frac{2}{\rho U^2 K^2}\frac{\tilde{F}_{a2}}{\tilde{A}_2}; \qquad A_4^* + iA_1^* = \frac{2}{\rho U^2 BK^2}\frac{\tilde{F}_{a12}}{\tilde{A}_2} \tag{12}$$

In the same way, if the cross-section oscillates in the angular direction (Ox_{12}), the associated Scanlan coefficients can be calculated by

$$P_3^* + iP_2^* = \frac{2}{\rho U^2 BK^2}\frac{\tilde{F}_{a1}}{\tilde{A}_{12}}; \qquad H_3^* + iH_2^* = \frac{2}{\rho U^2 BK^2}\frac{\tilde{F}_{a2}}{\tilde{A}_{12}}; \qquad A_3^* + iA_2^* = \frac{2}{\rho U^2 B^2 K^2}\frac{\tilde{F}_{a12}}{\tilde{A}_{12}} \tag{13}$$

The computer program for fluid flow simulations *PCAMVF* was adapted to compute Scanlan coefficientes considering the above equations.

3.3 Simulation of Structural Movements in Fluid Flow

Consider an obstacle imerse in a bidimensional fluid flow with the corresponding movements being characterized by the displacement functions a_{ij} in correspondence with Ox_{ij} directions.
These movements can be indirectely modeled by changing the velocity components (v_1 and v_2) of fluid flow at external inlet boundary domain. For example, one obstacle translation a_i, matching with Ox_i axis, can be modeled by specifying the velocity components of fluid flow at inlet boundary domain through

$$v_j \rightarrow v_j - \dot{a}_i \delta_{ij} \tag{14}$$

where $\dot{a}_i$ is the velocity of the obstacle translation.
On the other hand, one obstacle rotation a_{12}, matching with Ox_{12} axis, can be modeled by specifying the velocity components of fluid flow at inlet boundary domain by

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \rightarrow T \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{15}$$

where T is the transformation matix

$$T = \begin{bmatrix} \cos a_{12} & \sin a_{12} \\ -\sin a_{12} & \cos a_{12} \end{bmatrix} \tag{16}$$

In this previous case, the aeroelastic forces have to be determined according to Ox_{ij} axes, which represent general directions for structural analysis and for drag, lift and moment aeroelastic forces. This can be done by modifying the aeroelastic forces obtained through the transformation

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \leftarrow T^T \cdot \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{17}$$

4 CFD based Evaluation of Scanlan Coefficients

In order to compute Scanlan coefficients, it was considered three fluid flow simulations around a rectangle (*B = 4m*; *D = 1m*): one considering laminar flow (*L500* with *Re=5E2*), and two others considering turbulent flows (*T5E4*, with *Re = 5E4*, and *T5E6*, with *Re = 5E6*). It was adopted a time interval of *2s*, *2E-2s* and *2E-4s* for the corresponding time increments. The air at standard conditions was the considered fluid.
The fluid flow mesh (see Figure 1) was built using *88x53* control volumes (with a minimum dimension of *5E-2m* and a maximum of *77E-2m*). Some static results (*Re* = Reynolds number; *CF* = force coefficient, *St* = Strouhal number and U_{r_equiv} = reduced velocity equivalent to Strouhal number) are presented in Table 1.
Every simulation were performed assuming four phases: in the first one, the cross-section is fixed and the flow velocity out of domain is incresed to the corresponding Reynolds number simulated; then, if necessary, some parameters (e.g. time interval) are adapted; afterwards, the cross-section is forced to move under sinusoidal motion (equation 9) and it is expected that a stable condition about aeroelastic forces is reached; at last it is stored aeroelastic forces and movements to compute Scanlan coefficients (throughout *20 000* time increments or five waves of forced movements at least). This computation considers the maximum waves of forced movements registered.

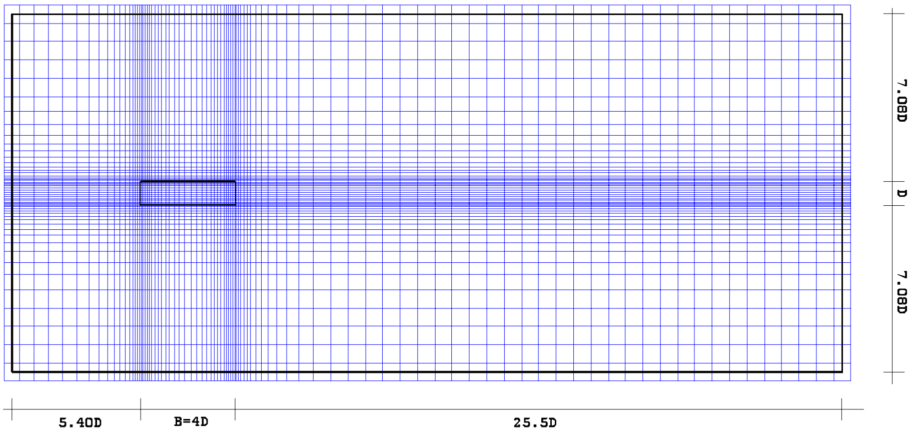

Figure 1: Control volume mesh for fluid flow simulation.

Table 1 – Static results for fluid flow around a Rectangle (*B/D=4*).

	Simulation	*L500*	*T5E4*	*T5E6*
Re	$\rho UD/\nu$	*5E2*	*5E4*	*5E6*
CF_1	$F_{a1}/0{,}5\rho DU^2$	*1,34±0,19*	*1,38±0,043*	*1,22±0,0*
CF_2	$F_{a2}/0{,}5\rho BU^2$	*±0,51*	*±0,46*	*±0,0*
CF_{12}	$F_{a12}/0{,}5\rho DBU^2$	*±0,33*	*±0,31*	*±0,0*
St	fD/U	*0,134*	*0,142*	*0,127*
U_{r_equiv}	U/fB	*1,87*	*1,76*	*1,97*

Moreover, it is necessary to set some limitations for time interval and forced movements. In order to get a good temporal discretization of high frequency movements, it was set up that one period of forced movement has at least to be described in *500* time increments. Therefore, the time increment step has to be not greater than $0{,}002BU_r/U$.

On the other hand, the evaluation of Scanlan coefficients has to consider incipient movements. This consideration can be done indirectly by limiting velocity amplitude of forced movement. In this case, the maximum velocity of forced movement was set up to *3%* of flow velocity out of domain. Therefore, the maximum displacement amplitude has to be $0{,}005BU_r$ for translation movement or *0,03 rad* for angular movement.

Figures 2 to 10 present the most significant Scanlan coefficients taken when the cross-section was oscillating in Ox_1, Ox_2 or Ox_{12} direction. The maximum values of the remaining Scanlan coefficients are indicated in Table 2.

Through experimental tests performed in a wind tunel, Nakamura [12] has evaluated some results, which can be used to compare with some computed equivalent coefficients based on *PCAMVF* program. The wind tunnel used was *3m* height and the cross-section had *D=0,09m* (*3%* of free height). The Reynolds number used was between *1E4* and *6E4*. The Strouhal number calculated was *0,118*. One of the forced oscillation amplitudes was set up to *0,06D* with frequencies varing from *0,5Hz* to *7Hz*. Those presented values can be translated in terms of Scanlan coefficient H_1^*. In Figure 11 are presented the tansformed Nakamura values and the values computed by *PCAMVF* program. In this case, it was set up the Reynolds number of fluid flow equal to *2E4*. The remaining adimensional parameters were modeled.

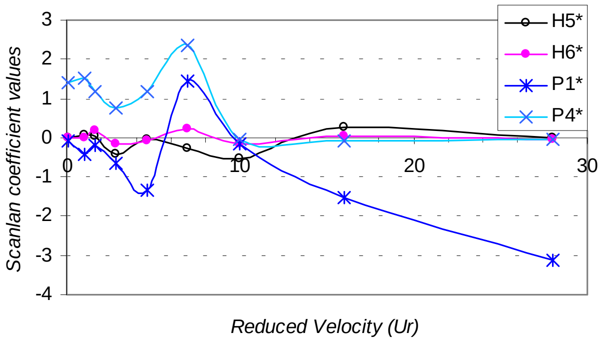

Figure 2: *L500* simulation; Ox_1 oscillation.

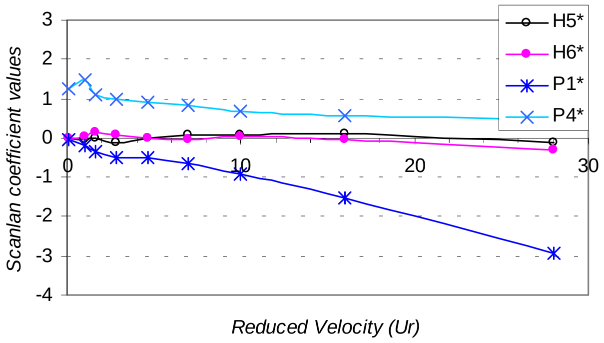

Figure 3: *T5E4* simulation; Ox_1 oscillation.

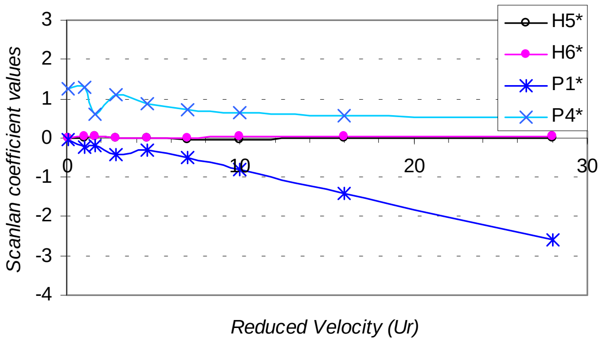

Figure 4: *T5E6* simulation; Ox_1 oscillation.

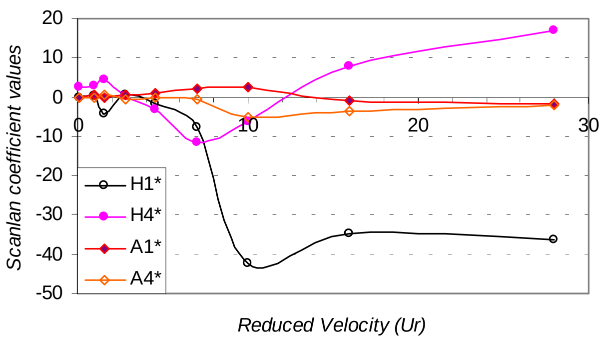

Figure 5: *L500* simulation; Ox_2 oscillation.

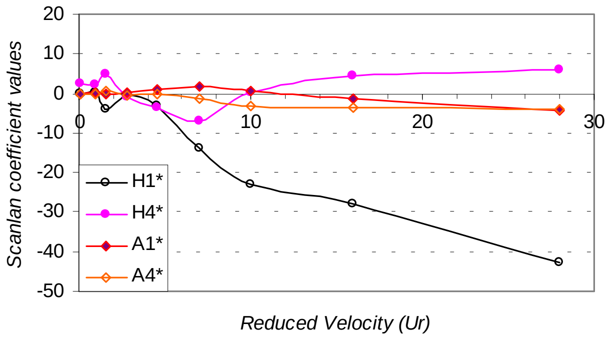

Figure 6: *T5E4* simulation; Ox_2 oscillation.

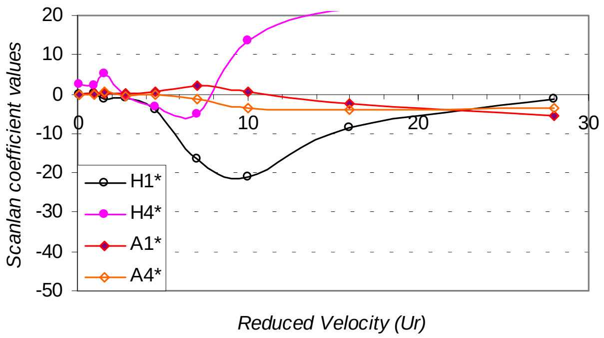

Figure 7: *T5E6* simulation; Ox_2 oscillation.

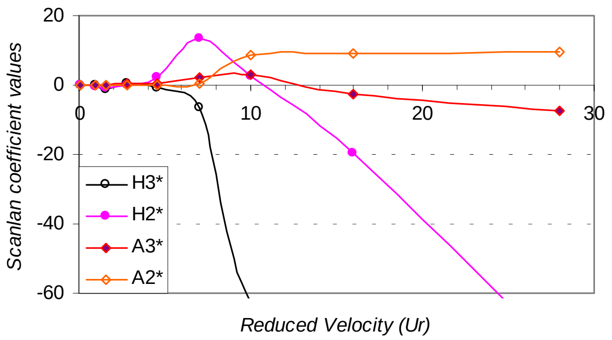

Figure 8: *L500* simulation; Ox_{12} oscillation.

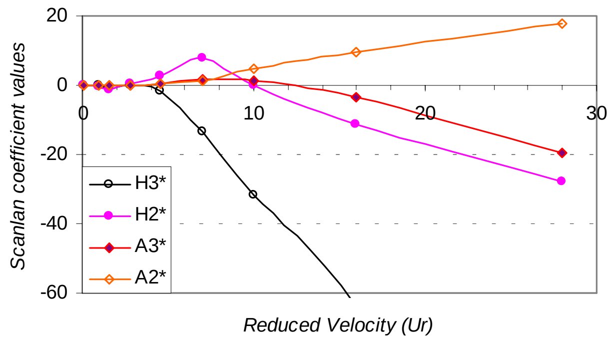

Figure 9: *T5E4* simulation; Ox_{12} oscillation.

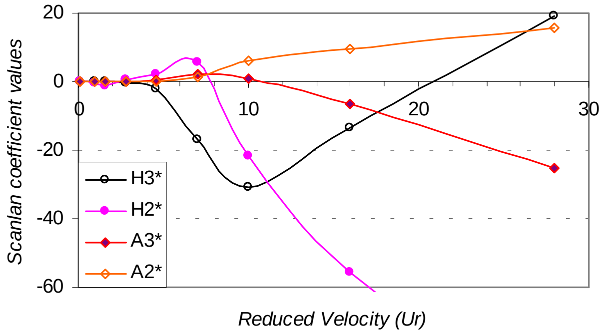

Figure 10: *T5E6* simulation; Ox_{12} oscillation.

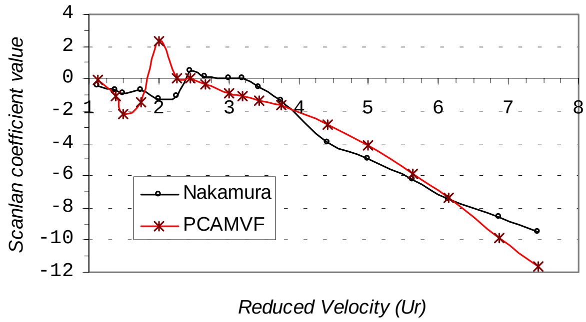

Figure 11: Nakamura simulation; Ox_2 oscillation.

Table 2 – Maximum values of smaller Scanlan Coefficients.

Coeff \ Simulation	*L500*	*T5E4*	*T5E6*
A_5^*	*-0,105 (U_r=2,8)*	*-0,036 (U_r=28)*	*0,005 (U_r=1,6)*
A_6^*	*0,145 (U_r=2,8)*	*-0,068 (U_r=28)*	*-0,005 (U_r=10)*
P_5^*	*-0,10 (U_r=28)*	*-0,07 (U_r=28)*	*0,02 (U_r=28)*
P_6^*	*1,05 (U_r=28)*	*-0,06 (U_r=28)*	*0,01 (U_r=28)*
P_3^*	*-1,37 (U_r=10)*	*-0,33 (U_r=28)*	*0,07 (U_r=28)*
P_2^*	*1,91 (U_r=10)*	*0,27 (U_r=28)*	*-0,02 (U_r=28)*

The results presented in Figures 2 to 11 and Table 2 allow to draw the following particular conclusions:

- The small values associated to coefficients presented in Table 2 indicate that the aeroelastic moment is independent from horizontal oscillations, and vertical or angular motions do not influence aeroelastic drag;
- Aeroelastic drag is only dependent on horizontal oscillations;
- Aeroelastic lift is significantly influenced by vertical and angular motions and is weakly conditioned by horizontal oscillations;
- Aeroelastic moment is strongly dependent of angular oscillations and it is moderately influenced by vertical motions;
- Although the calculated coefficients can be softly changed by several simulation parameters (e.g. time interval registered, wave number of forced oscillation considered), it is possible to conclude that the flow velocity has a large influence on several values, mainly for bigest reduced velocities;
- Except for *L500* simulation at $U_r=7$, the P_1^* coefficient is always less than zero. If this cross-section belongs to a long span structure, this negative value contributes to increment aeroelastic damping and, consequently, the total structural damping (or dynamic stability) in horizontal direction increases;
- The H_1^* coefficient has positive values at about $U_r=1.0$ and almost zero at about $U_r=2.0$. In Figure 11, it is seen that the maximum value happens at about $U_r=2.0$. This means that the lift

increases when the frequency oscillations is close to Strouhal frequency, which can amplify the vertical oscillation amplitude or else can origin instability on structural system;

- The H_1^* coefficient becomes strongly negative at higher reduced velocities, which contributes positively for vertical aeroelastic damping of this cross-section and also for dynamic stability. However, *T5E6* simulation shows that this contribution can be smaller than it is expected considering *T5E4* simulation;
- The A_2^* coefficient becomes positive at about $U_r=2.0$. This means that the aeroelastic angular damping becomes negative, which can lead to the structure instabilization by incresing angular oscillations amplitude;

In Figure 11 are compared the results calculated by the PCAMVF program and those presented by Nakamura [12]. Ignoring several differences bettween experimental tests and this numerical simulation (e.g. real dimensions of cross-section, velocities at boundary domain, forced oscillation frequency, flow Reynolds number), this graphic shows a good agreement of values for higher reduced velocities and considerable differences when reduced velocity is close to equivalent Strouhal number.

5 CONCLUSIONS

In forced oscillation method context, the evaluations of Scanlan Coefficients were made by applying a Computational Fluid Dynamics algorithm (Finite Volume Method) to simulate the fluid flow around a rectangular cross-section. It is considered one laminar and two turbulent fluid flow simulations to evaluate all Scanlan Coefficients. These computed results are compared with some referred by Nakamura, which were obtained from experimental tests in a wind tunnel.
Although the Scanlan Coefficients depend on some parameters like oscillation amplitude, recording time interval, forced oscillation wave number considered and fluid flow simulation (e.g. mesh, differention scheme used, deferred correction), it is possible to conclude that the flow velocity simulation has a high influence on those coefficients, mainly for higher reduced velocities.
From the aeroelastic point of view, and assuming that this rectangular cross-section belongs to a long span structure, the main conclusions about these results presented herein are:

- The horizontal dynamic stability is incremented by aeroelastic damping;
- The vertical oscillation amplitude increases or else it may become unstable when the frequency oscillation is close to the Strouhal frequency;
- The structure may become unstable in terms of rotation for higher reduced velocities.

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