ENTROPY-BASED CONFIGURATION OPTIMIZATION OF TRUSSES

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SUMMARY

The use of entropy-based algorithms for optimizing the configuration of trusses is investigated in this paper. Two methods are proposed: the multi-level optimization in which the problem size is reduced by iterating between two distinct but coupled design spaces. Alternatively, in the direct design formulation that treats simultaneously sizing and coordinate variables the problem is solved by minimizing a linear convex function over side constraints. Numerical results for examples with stress and displacement constraints are given to illustrate the methods.

1. INTRODUCTION

A problem which has received much attention in structural optimization is the optimization of truss structures. One reason is that these structures are naturally discrete and easily lend themselves for use in testing new optimization algorithms. In most cases, the truss geometry is fixed and only the member sizes are optimized. This problem has been solved under various types of constraints using a variety of methods. A more difficult problem is the optimization of a truss where the nodal coordinates are not fixed but are also design variables along with the member sizes. This problem is much more cumbersome than the pure sizing problem for two reasons. First, the different types of variables have highly different characteristics. The other reason is that the truss configuration variables are much more nonlinear in the behavior constraints than the sizing variables. In 1970, Pedersen [1] presented an iterative technique for the design of trusses using a sequence of linear programs with move limits. Both member sizes and nodal coordinates were used as design variables. The method utilized a sensitivity analysis of the partial derivatives with respect to the member areas and joint coordinates and also accounted for self-weight, stress, Euler buckling and deflection constraints. An alternative multi-level approach was described in ref.[2,3]: The method of feasible directions was used to optimize the structure with fixed geometry and the steepest descent method was employed to move the nodal coordinates to an optimum position. The algorithm used these two techniques, one after another in an iterative fashion keeping the design spaces separate and reducing the number of design variables considered at any one time. Imai and Schmit [4] developed an advanced primal-dual method called the multiplier method, which is an extension of the quadratic penalty function approach. To avoid the severe non-linearity of these truss problems, second-order Taylor-series expansions in terms of the reciprocal sizing and nodal coordinates are used along a search direction. As opposed to the early methods that were not in general well conditioned, their method showed good convergence properties, although requiring many analysis to find a solution. Two methods [5,6] were recently proposed to solve the truss configuration optimization. In both of these methods one replaces the design problem with a sequence of mixed variable approximations of the problem. The convergence properties are controlled by adjusting the convexity and therefore the degree of conservativeness of the approximate subproblem. This paper presents a new class of optimization methods based on informational entropy concepts simple to operate and specially recommended in a personal computer environment. In the first case,the multi-level optimization operates by iterating between two coupled design spaces, one for member sizing variables and one for geometric variables. The optimization phase for the truss sizing reduces to the finding of the parameter which maximizes the (concave) dual volume and from which the Lagrange multipliers and member sizes are evaluated using a simple algebraic expression. Polynomial fitting was employed to move the nodal coordinates to an optimum position. Alternatively, the truss configuration problem is posed in a multicriteria optimization format and a minimax solution is sought. The entropy-based approach to solving minimax optimization formulation transforms the problem to a scalar minimization with just one control parameter.

2. TRUSS CONFIGURATION OPTIMIZATION

Using the displacement method of analysis, the optimal design problem of truss geometry and cross sections can be formulated as follows: Find the cross sectional areas x, the joint coordinates y and the corresponding displacements d, such that:

$$\text{Min } V = l(y)^t x \quad (1a)$$

$$x^L \leq x \quad (1b)$$

$$y^L \leq y \leq y^U \quad (1c)$$

$$d \leq d^U \quad (1d)$$

$$\sigma^L \leq \sigma = S(y)\, d \leq \sigma^U \quad (1e)$$

where V represents the volume (or weight) of the truss and l is the vector of member lengths, which are functions of joint coordinates. Eq.(1b) and (1c) are bounds on the design variables x and y. Design variable linking was used to meet symmetry requirements and to reduce the number of design variables. Eq.(1d) represent displacement constraints and Eq.(1e) are stress constraints. The displacements d are computed for any given design by solving the displacement analysis equilibrium equations:

$$K(x,y)\, d = R \quad (2)$$

The elements of the load vector R are constants and the elements of the stress-transformation matrix S are functions of only the variables y. In general, upper and lower bounds on design variables and stresses are assumed to be constant. If stability of members is considered, the lower bound σ^L can be defined as,

$$\sigma^L = \max \{ \sigma_c, \sigma_b \} \quad (3)$$

in which σ_c is the lower stress limit and σ_b is the allowable stress for Euler buckling given by,

$$\sigma_{bi} = \frac{\pi^2 E}{n\,(l_i / r_G)^2} \quad (4)$$

in which E is the Young's modulus, n is a safety factor, l_i is the length of the member i and r_G is the radius of giration of the cross section. Eq.(4) indicates that the buckling stress depends on the properties of the cross section selected. Since the area of the section is a design variable, it is necessary to express r_G in terms of x. This expression is:

$$r_G = a\, x^b \quad (5)$$

in which a and b are constants. For tubular sections with a nominal diameter to thickness ratio of D/t=10, the buckling stress was given as [2],

$$\sigma_{bi} = \frac{10.1\, \pi\, E\, x_i}{8\, l_i^2} \quad (6)$$

3. MULTI-LEVEL OPTIMIZATION

For the purposes of applying the multi-level strategy to problem (1), the minimum volume (weight) design for the initial geometry is first obtained. Next, when moving each coordinate design variable y slightly, the member areas x must be modified in such a way that the minimum volume design is maintained subject to the requirements that all constraints remain satisfied. That is, the analytic gradient of the objective function is determined with respect to the coordinate variables, such that the optimum design is maintained. In a sense, the optimization proceeds in two separate design spaces. One is the member size variables and the other is the geometric variables.

3.1 Truss sizing problem

The truss sizing problem is usually solved iteratively and this strategy is used here. The optimization problem which must be solved in each cycle of iteration can be stated as:

$$\min\ V = \Sigma_{i=1,N}\ l_i\, x_i \quad (7a)$$

$$\text{st} \quad \Sigma_{i=1,N}\, l_i\, F_{ij}\, \bar{F}_{ik} / (E_i x_i) \leq u_k \quad ; \quad j = 1,...,J \qquad (7b)$$

$$\sigma_i^L \leq \sigma_{ij} = F_{ij} / x_i \leq \sigma_i^U \quad ; \quad k = 1,...,K \qquad (7c)$$

$$x_i \geq x_i^L \quad ; \quad i = 1,...,N \qquad (7d)$$

The N unknown bar sizes x_i, i=1,...,N comprise the design variable vector x. l_i, E_i are the length and elastic modulus, respectively, of the i-th bar. In the displacement constraints (7b) F_{ij} and $\bar{F}_{ik}$ are the force caused by the j-th load case and the virtual force caused by the k-th joint displacement in the i-th bar and u_k is the maximum permissible displacement of the k-th joint. At each optimization all bar forces are known and are assumed to remain constant, so problem (7) can be stated in a simplified form as:

$$\min \quad V = \Sigma_{i=1,N}\, l_i\, x_i \qquad (8a)$$

$$\text{st} \quad \Sigma_{i=1,N}\, c_{ik} / x_i \leq 1 \quad ; \quad k = 1,...,M \qquad (8b)$$

$$\underline{x}_i / x_i \leq 1 \quad ; \quad i = 1,...,N \qquad (8c)$$

The displacement constraints (7b) correspond with Eq.(8b) with c_{ij} , j=1,...M representing general displacement constants evaluated after each analysis. The stress and size constraints in problem (7) have been merged into Eq.(8c); $\underline{x}_i$ is the largest of either x_i^L or the minimum size necessary to satisfy the stress constraints (7c). Problem (8) has the following Lagrangean function,

$$\mathscr{L}(x,\beta) = \Sigma_{i=1,N}\, l_i\, x_i + \Sigma_{k=1,M}\, \mu_k\, [\Sigma_{i=1,N}\, c_{ik}/x_i - 1] + \Sigma_{j=1,N}\, \mu_{M+i}\, [\underline{x}_i/x_i - 1] \qquad (9)$$

Examining the stationarity of $\mathscr{L}(x,\mu)$ with respect to all x_i, i=1,...,N yields equations in x which may be solved algebraically to give:

$$x_i^{[k]} = \{ [\Sigma_{j=1,M}\, c_{ik}\, \mu_j + \underline{x}_i\, \mu_{M+i}] / l_i \}^{1/2} \qquad (10)$$

If an optimum set of multipliers μ^* exists, then the resulting bar sizes x* will also solve problem. Such a set of optimal surrogate multipliers μ^* is, of course, not known "a priori" but found iteratively. The problem then becomes one of developing a method whereby the μ may be iteratively updated towards μ^*, thus solving problem (8). Very many engineering optimization problems essentially consist of iteratively sorting out which ones of many constraints are active at the optimum and which are inactive and then of iteratively estimating values for the active constraint multipliers. Though such a strategy is theoretically valid, changes in the active set between iterations change the optimization problem being solved in a discontinuous way and lead to erratic convergence behavior. The maximum entropy-based algorithms avoid these difficulties by retaining and updating all constraints at all times. Problem discontinuities are not introduced and consequently convergence is smooth. Assuming that the Lagrange multipliers μ_j are given by,

$$\mu_j = \lambda_j\, v_j \qquad (11)$$

where λ_j is a entropy multiplier and v_j is a correction factor, these multipliers may be interpreted probabilistically with each λ_j representing the probability that its corresponding constraint is active at the optimum. With this probabilistic view of the multipliers it is then entirely logical and sensible to calculate most likely or least biased values for them from the Jaynes maximum entropy formalism. An initial set of values for v and λ is chosen such that $v_j^{[o]}=1$ and $\lambda_j^{[o]}=1/(M+N)$, j=1,...,M+N ie: all constraints are equally likely to be active at the optimum.The set of bar cross-sectional areas x obtained from (10) forms an initial design which is analyzed to give bar forces and virtual forces for joint displacements. All bar areas are scaled to ensure that no constraint is violated. The correction factors vector $v^{[1]}$ is assumed a unit vector in this iteration. New estimates of the multipliers $\lambda^{[1]}$ are then obtained by solving the maximum entropy mathematical problem:

$$\text{Max} \quad S = -K\, \Sigma_{j=1,M}\, \lambda_j^{[1]}\, \ln \lambda_j^{[1]} \qquad (12a)$$

$$\text{sa} \quad \Sigma_{j=1,M}\, \lambda_j^{[1]} = 1 \qquad (12b)$$

$$\Sigma_{j=1,M}\, \lambda_j^{[1]}\, g_j(x^{[0]}) = \varepsilon \qquad (12c)$$

$$\lambda_j^{[1]} \geq 0 \qquad (12d)$$

S is the Shannon entropy, K is a positive constant. Equation (12c), that represents the constraints:

$$g_j(x) = c_{ik}/x_i - 1 \qquad \text{for } j = 1,...,M \tag{13a}$$

$$g_j(x) = \underline{x}_i/x_i - 1 \qquad \text{for } j = M+1,...,M+N \tag{13b}$$

has an expected value zero. If the left-hand side had contained $g_j(x^{[1]})$, then the right-hand side would be zero, but since $g_j(x^{[1]})$ values are not yet known $g_j(x^{[o]})$ values are used as the best currently available estimates and this introduces the error term ε into Eq.(12c). The entropy maximization problem has an algebraic solution for $\lambda^{[1]}$:

$$\lambda_j[1] = \frac{\exp[\beta\, g_j(x^{[0]})/K]}{\Sigma_{j=1,M} \exp[\beta\, g_j(x^{[0]})/K]} \tag{14}$$

in which β, the Lagrange multiplier for Eq. (12c) can be found by substituting result (14) into Eq. (12c). However, since ε is nor uniquely known and K is an arbitrary constant, $p=\beta/K$ may be viewed as a penalty parameter used to close the duality gap. Eq.(14) with a selected p yields new constraint activity probabilities $\lambda^{[1]}$. At each iteration, it is necessary to search for the value of p that maximizes the truss volume given by (10) and using the new correction factor and multiplier values. The new design is analyzed by the matrix stiffness method and all bar areas are scaled to ensure that no constraint is violated. The correction factors vector are given by:

$$v^{[2]} = F^{[1]}{}_{\#}\, l = F^{[1]t}\,(F^{[1]}\,F^{[1]t})^{-1}\, l \tag{15}$$

where l represents the member lengths vector and the elements of the matrix F are given by,

$$f_{ij}^{[1]} = \lambda_j\, c_{ij} / x_i^{\,2} \qquad \text{for } j = 1,...,M \qquad i = 1,...,N \tag{16a}$$

$$f_{jj}^{[1]} = \lambda_{M+i}\, \underline{x}_i / x_i^2 \qquad \text{for } j = M+1,...,M+N \tag{16b}$$

Using $g(x^{[1]})$ in place of $g(x^{[0]})$ in Eq.(14) with an appropriate p yields new multipliers $\lambda^{[2]}$. Using $v^{[2]}$ and $\lambda^{[2]}$, values of $x^{[2]}$ follow from Eq.(10) and $V^{[2]}$ from Eq.(8a). Substituting $x^{[2]}$ into Eq.(8b) and (8c) yields values for the constraint functions and all bar areas are scaled to ensure that no constraint is violated. In subsequent iterations, this scaled design and the previous scaled design would be compared and checked against convergence criteria and iterations would be either stopped here or continued. Assuming that convergence has not been achieved, the scaled design enters the optimization phase.Further iterations continue to converge at v^*, λ^* and x^*.which solve both problems (26) and (25). A different iterative two phase algorithm based upon the maximum entropy formalism is proposed in ref.[6]. The constraints are first surrogated and the entropy multipliers are obtained by setting values for p according to an empirical rule.

3.2 Geometry optimization

3.2.1 Direction vector

Assume that the fixed geometry problem has been solved for the current configuration. If a coordinate variable y_k is changed by an amount Δy_k, the volume will change by

$$\Delta V = \partial V/\partial y_k\, \Delta y_k = \Delta y_k\, \Sigma_{i=1,N} \left(l_i \frac{\partial x_i}{\partial y_k} + x_i \frac{\partial l_i}{\partial y_k} + \frac{\partial x_i}{\partial y_k}\, \frac{\partial l_i}{\partial y_k}\, \Delta y_k\right) \tag{17}$$

in which $\partial l_i/\partial y_k$ is the direction cosine of the bar corresponding to the displacement y_k. The last term can be neglected if the variation Δy_k is small. As Eq.(1b),(1d) and (1e) must be satisfied at all times, it follows that for all such constraints:

$$\Delta g_j = dg_j/dy_k\, \Delta y_k = \left(\frac{\partial g_j}{\partial y_k} + \Sigma_{i=1,N} \frac{\partial g_j}{\partial x_i}\, \frac{\partial x_i}{\partial y_k}\right) \Delta y_k \le 0 \tag{18}$$

The derivatives $\partial g_j/\partial y_k$ and $\partial g_j/\partial x_i$ can be computed directly from the expressions of g_j using the procedure described in Section 5. It is necessary to find the unknowns $\partial x_i/\partial y_k$ which minimize the linear function (17) subject to the linear constraints (18). This is a linear programming problem that can be solved by the Simplex method. It is necessary to assume the sign of Δy_k: if the volume is not reduced then the sign of Δy_k is changed and the problem is solved again. If $(\partial V/\partial y_k)\ \Delta y_k$ is still positive, the volume cannot be reduced by changing this coordinate in either direction. In problems where each constraint g_j is considered to be a function of only one area variable x_i such as problems with side constraints on the design variables and stress constraints only, it is not necessary to solve the LP. Assuming that the force redistribution in indeterminate trusses is negligible, Eq.(18) reduces to

$$\Delta g_j = dg_j/dy_k\ \Delta y_k = \left(\frac{\partial g_j}{\partial y_k} + \frac{\partial g_j}{\partial x_i}\frac{\partial x_i}{\partial y_k}\right) \Delta y_k \le 0 \qquad (19)$$

Eq.(19) are a set of upper bounding constraints, each containing a single unknown $\partial x_i/\partial y_k$ and may be solved by a simple comparison procedure.
The preceding steps are repeated for each coordinate variable to yield the gradient of the objective function given by:

$$S^q = [\partial V/\partial y_1\ \Delta y_1 \quad \partial V/\partial y_k\ \Delta y_k \quad \partial V/\partial y_\beta\ \Delta y_\beta]^t \qquad (20)$$

If Y_q is the vector of the independent coordinate variables at iteration q, the new coordinates are found from,

$$Y^{q+1} = Y^q + \alpha^* S^q \qquad (21)$$

in which α^* is the scalar multiplier required to minimize V in direction S^q or to encounter some new constraint of Eq.(1c). If any component is such that a move in this direction would violate a currently active side constraint on the coordinate variable, this component is set to zero.

3.2.2 Polynomial fitting

The step size α^* is calculated by polynomial fitting techniques and the corresponding coordinates of joints are computed by Eq.(21). The truss is reoptimized for this new geometry. A new gradient of the objective function is then determined and the process is repeated until the objective function can no longer be reduced. Polynomial fitting techniques require exact analyses or calculation of the constraint derivatives for several designs. Since more information is used in this class of approximate methods (compared with Taylor series), the quality of the approximations is higher at the expense of more computational effort. Assuming, for example, the quadratic fitting for the truss volume,

$$V(\alpha) = a + b\,\alpha + c\,\alpha^2 \qquad (22)$$

the constants a, b, c can be determined from results of analysis of two or three designs. Assuming the conditions,

$$V^* = a \quad \text{and} \quad \partial V^*/\partial\alpha = b \quad \text{for} \quad \alpha = \alpha^* = 0 \qquad (23a)$$

$$V^{**} = a + b + c \quad \text{for} \quad \alpha = \alpha^{**} = 1 \qquad (23b)$$

and substituting in Eq.(22), one gets:

$$V(\alpha) = V^* + \partial V^*/\partial\alpha\ \ \alpha + (V^{**} - V^* - \partial V^*/\partial\alpha)\ \alpha^2 \qquad (24)$$

This equation is based on two exact analysis and and calculations of the displacement derivatives at a single point. Another possibility, which does not involve evaluation of the derivatives is to use the results of three exact analysis. Substituting the computed values of V* (for $\alpha^*=0$), V** (for $\alpha^{**}=0.5$), V*** (for $\alpha^{***}=1$) into Eq.(22) and solving for a,b and c, one finds:

$$V = V^* + (-3\,V^* + 4\,V^{**} - V^{***})\ \alpha + (2\,V^* - 4\,V^{**} + 2\,V^{***})\ \alpha^2 \qquad (25)$$

Although the optimal step α_{opt} is given by,

$$\alpha_{opt} = -\,b/(2\,c) \qquad (26)$$

an upper limit is imposed to overcome the severe nonlinearity of the problem.

4. DIRECT DESIGN

4.1 Minimax formulation

In the context of the truss configuration problem described above, it is intended to minimize a whole set of goals such as the volume, nodal displacement, etc. by finding an optimal set of cross sectional areas. All these goals need to be cast in a normalized form. If $\underline{V}$ represents a reference volume, eq.(1a) becomes,

$$l(y)^t x \le \underline{V} \quad \Rightarrow \quad g_1(x,y) = \frac{l(y)^t x}{\underline{V}} - 1 \le 0 \tag{27a}$$

The lower bounds on cross-sectional areas (1b) become,

$$g_2(x) = -\frac{x}{x^L} + 1 \le 0 \tag{27b}$$

Similarly, one has for the upper bounds on the nodal displacements(1d),

$$g_3(x,y) = \frac{d}{d^U} - 1 \le 0 \tag{27c}$$

For the upper and lower bounds on the joint coordinates (1c) and stresses (1e):

$$g_4(y) = \frac{y}{y^U} - 1 \le 0 \tag{27d}$$

$$g_5(y) = -\frac{y}{y^L} + 1 \le 0 \tag{27e}$$

$$g_6(x,y) = \frac{\sigma}{\sigma^U} - 1 \le 0 \tag{27f}$$

$$g_7(x,y) = -\frac{\sigma}{\sigma^L} + 1 \le 0 \tag{27g}$$

The problem of finding values for the the cross sectional areas x and joint coordinates y which minimize the maximum of the goals has the form,

$$\text{Min}_{x,y}\ \text{Max}_j\,(g_1, \ldots, g_j \ldots g_7) = \text{Min}_{x,y}\ \text{Max}_{j\in J} < g_j(x,y) > \tag{28}$$

and belongs to the class of minimax optimization.

4.2 Minimax optimization

The method used to solve the minimax optimization problem (27) with goals defined by (27a-g) is a recently developed entropy-based approach. The minimax problem (27) is discontinuous and non-differentiable, both of which attributes makes its numerical solution by direct means difficult. In ref [7] it is shown that the minimax solution may be found indirectly by the unconstrained minimization of a scalar function which is both continuous and differentiable, and is thus considerably easier to solve:

$$\text{Min}_{x,y}\ \text{Max}_{j\in J} < g_j(x,y) > = \text{Min}\ (1/p)\log\{\Sigma_{j=1,J} \exp[p\, g_j(x,y)]\} \tag{29}$$

over variables x, y with a sequence of values of increasingly large positive $p \ge 1$.

4.3 Scalar optimization

The scalar function minimization in the form,

$$\text{Min}\ (1/p)\log\{\Sigma_{j=1,J} \exp[p\, g_j(x,y)]\} \tag{30}$$

is a convex approximation of the criteria, what allows the use of algorithms for convex optimization. The strategy adopted was to solve the implicit optimization problem by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(x,y)$ in problem (27), truncated after the linear term for the sizing variables and the quadratic term for the geometric variables. This gives Eq.(31):

$$\text{Min } (1/p) \log\{ \Sigma_{j=1,J} \exp p\, [g_j(x_o,y_o) + \Sigma_{i=1,N}\left(\frac{\partial g_j}{\partial x_i}\right)_o (x_i-x_o) + \Sigma_{k=1,\beta}\left(\frac{\partial g_j}{\partial y_k}\right)_o (y_k-y_o) +$$

$$+\frac{1}{2} \Sigma_{1=1,N} \Sigma_{k=1,\beta}\left(\frac{\partial^2 g_j}{\partial y_i \partial y_k}\right)_o (y_i-y_o)(y_k-y_o) + \Sigma_{i=1,N} \Sigma_{k=1,\beta}\left(\frac{\partial^2 g_j}{\partial x_i \partial y_k}\right)_o (x_i-x_o)\,(y_k-y_o)]\}$$

Problem (31) is an approximation to problem (30) if values of all the $g_j(x,y)$, $(\partial g_j/\partial x_i)$ and $(\partial g_j/\partial y_k)$ are known numerically. Given such values, problem (11) can be solved directly by any standard unconstrained optimization method. The quasi-Newton routine NAG E04JAF that uses differences on the gradients of the function (30) and its first derivatives to compute approximations to the first and second derivatives, respectively, has solved efficiently the scalar minimization. Problem (31) must be solved iteratively, x_o and y_o being redefined each time as the optimum solution to the preceding problem. Iterations continue until changes in the design variables x,y become small. During these iterations the parameter p must be increased in value to ensure that a minimax optimum solution is found. In the present work, a value of p in the range $10 \le p \le 30$ was used for the first iteration, this value being increased to 100 in subsequent operations. Although the selected move limits are not as critical as in sequential linear programming, it is necessary to ensure that the approximations made to define the explicit problem (31) are realistic.

5. SENSITIVITY ANALYSIS

To formulate and solve the direction finding problem (17)-(18) required for geometry optimization in the multilevel approach or the scalar function minimization (31) used for the direct design, numerical values are required for all the functions $g_j(x,y)$ and their derivatives with respect to the design variables. The truss volume is known explicitly and its first derivatives are:

$$\frac{\partial V}{\partial x_i} = l_i \quad ; \quad \frac{\partial V}{\partial y_k} = \Sigma_{i=1,N}\, x_i \frac{\partial l_i}{\partial y_k} \qquad (32)$$

in which $\partial l_i/\partial y_k$ is the direction cosine of the bar corresponding to the displacement y_k. The second derivative of V with respect to y_k is given by ratio of the square of the direction sine of the bar corresponding to the displacement y_k divided by the member length.

However, member stresses and nodal displacements are implicit functions of x and y. Given some design variables the analysis of the truss will yield numerical values for u. One way of evaluating the derivatives is to calculate them from analytical expressions, as follows. The displacement derivatives $\partial d^o/\partial x_i$ are computed by implicit differentiation of the equilibrium equations:

$$K^o \frac{\partial d^o}{\partial x_i} = -\frac{\partial K^o}{\partial x_i} d^o \qquad (33)$$

Since d^o and K^o are known from analysis of the initial design, solution for $\partial d^o/\partial x_i$ involves only calculation of the r.h.s. vector of Eq.(33) and forward and back substitutions. The stress derivatives $\partial \sigma^o/\partial x_i$ are then determined directly by explicit differentiation,

$$\frac{\partial \sigma^o}{\partial x_i} = S \frac{\partial d^o}{\partial x_i} \qquad (34)$$

The derivatives $\partial d^o/\partial y_k$ and $\partial \sigma^o/\partial y_k$ are computed in a similar manner; however, it should be remembered that the elements of S are functions of the joint coordinates y. The expressions for $\partial d^o/\partial y_k$ and $\partial \sigma^o/\partial y_k$ are:

$$K^o \frac{\partial d^o}{\partial y_k} = -\frac{\partial K^o}{\partial y_k} d^o \qquad (35)$$

$$\frac{\partial \sigma^o}{\partial y_k} = S \frac{\partial d^o}{\partial y_k} + \frac{\partial S^o}{\partial y_k} d^o \qquad (36)$$

To compute $\partial K^o/\partial x_i$, only elements of K associated with member i must be considered. Furthermore, the elements of $\partial K/\partial X_i$ are constant, therefore the computation must not be repeated. To find $\partial K^o/\partial y_k$ and $\partial S^o/\partial y_k$, only elements of K and S associated with the kth joint coordinate must be considered. The second order derivatives with respect to y_k can be calculated in a similar manner,

$$K^o \frac{\partial^2 d^o}{\partial y_k^2} = -\frac{\partial^2 K^o}{\partial y_k^2} d^o - 2 \frac{\partial K^o}{\partial y_k} \frac{\partial d^o}{\partial y_k} \qquad (37)$$

It can be observed that the solution for each of the derivative vectors involves only the calculation of the corresponding r.h.s. vector and forward and back substitutions.

The evaluation of the second order design sensitivities for all the geometric variables is computationally costly. An alternative means is to employ a search direction, which can be obtained by using the procedure described in section 3.2.1. The geometric variables are replaced by a single parameter α, representing the scalar step along a search direction. The second order term of the Taylor series expansion is retained for the displacement response quantities,

$$d = d^o + \frac{\partial d^o}{\partial \alpha} \alpha + \frac{1}{2} \frac{\partial^2 d^o}{\partial \alpha^2} \alpha^2 \qquad (38)$$

which are highly nonlinear with respect to changes in the configuration variables. The derivatives of the displacements and the stresses with respect to α can be obtained in the way described above.

6. NUMERICAL EXAMPLES

6.1 Twenty three bar truss

Fig.1 represents the ground structure and loading of a truss bridge. The horizontal and vertical deflections of the joints were limited to 10 m and 50 mm, respectively. The allowable stress in tension and compression was 0.14 kN/mm^2. The modulus of elasticity was 210 kN/mm^2. For practical reasons, the joints 1, 3, 7 and 9 were only allowed to move horizontally. One of the architectural requirement was that the final design should be symmetrical. Since the joints 5 and 6 were on the symmetry axes, their movement along the horizontal direction was not included in the design variables. The bounds placed on the coordinates of the joints were,

$0 \le y_{H1}, y_{H2} \le 6.$, $6. \le y_{H3}, y_{H4} \le 12.$, $0 \le y_{V6} \le 9.$, $0 \le y_{V2}, y_{V4} \le 10.$ (in m units)

Design variable linking is used to reduce the number of different sizing variables to 5. The supports A and B were fixed and the geometry of the bridge was optimized. The optimum shape shown in Fig.2 was obtained after 7 iterations (in the multicriteria and 11 in the multilevel approach) and it has a volume of 161 m^3.Group areas of this design is given in Table 1. Its geometry is close to a parabolic arch. Since the vertical displacement at joint 5 is at its upper limit, it appears that the bridge design could be improved by combining joints 5 and 6. Regardless of possible weight savings, it may be significant from a cost viewpoint that three members and a joint are eliminated.

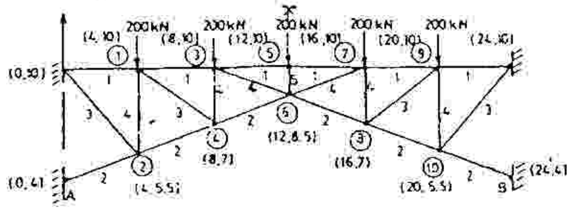

Figure 1 Initial design

Table 1	x_1	x_2	x_3	x_4	Volume
Optimum sizes	12.7 cm^2	3.6 cm^2	2.3 cm^2	15.5 cm^2	17.9 m^3

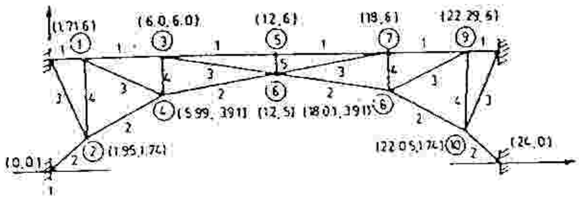

Figure 2 Optimum geometry

6.1 Twenty five bar truss

The bridge truss shown in Fig.3 is subjected to five alternative loading conditions R = 3 x 10^6 N at the five fixed joints of the roadway. The geometrical variables are the coordinates of the five arch joints, which lie initially on a parabolic arch. This optimization problem is treated with specified displacement constraints (10 mm) and with or without stability constraints. The truss variables were linked to enforce symmetry and the problem consists of 13 sizing variables and 5 nodal configuration variables. The structure has the following preassigned parameters:
Fixed bounds on stresses, for the cases in which stability considerations are neglected,

$$\sigma^U = 1.3 \times 10^8 \ N/m^2 \qquad \sigma^L = -1.04 \times 10^8 \ N/m^2$$

Modulus of elasticity,

$$E = 2.1 \times 10^{11} \ N/m^2$$

Table 2 reproduces the optimum volumes obtained corresponding to the layout of Fig.3, 4 and 5.

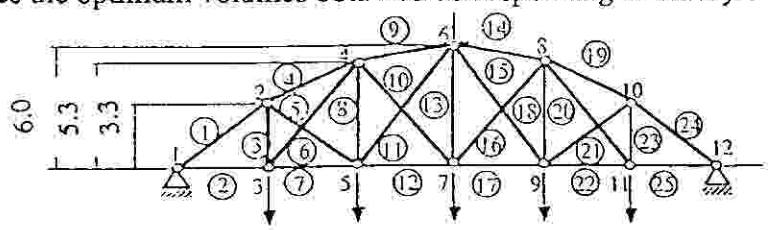

Figure 3 Initial design

Table 2	without stability constraints	with stability constraints
initial configuration	0.449 m^3	0.484 m^3
optimum geometry	0.371 m^3	0.422 m^3

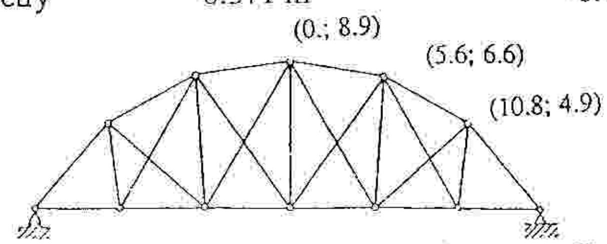

Figure 4 Optimum geometry with displacement and without stability constraints

The convergence is fast (11 iterations in the multicriteria and 17 in the multilevel approaches, respectively) with a majority of the volume change coming from the initial iterations. For a constant allowable compression stress the structure of minimum volume is a funicular polygon. For the data used in the paper the geometric solutions shown have a a smaller volume than the funicular polygon. Compressed member tend to be shorter and the horizontal changes of the positions of the joints are towards the supports.

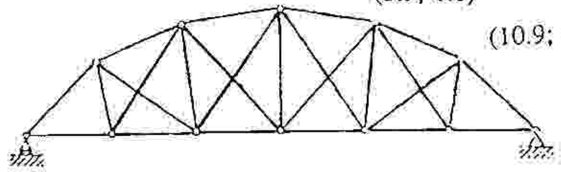

Figure 5 Optimum geometry with displacement and stability constraints

7 CONCLUSIONS

The addition of configuration (shape) variables to the design task creates a particularly difficult problem because of inherent non-linearity. Although the efficiency of the minimizer is not comparable to that in fixed-geometry design, configuration optimization is possible and justified by considerable design improvements achievable by including these variables. Two entropy-based methods are proposed to deal with configuration variables: a multi-level optimization that operates by iterating between two coupled design spaces, one for member sizing variables and one for geometric variables and a direct design including both member sizing and coordinate variables. Unlike optimality criteria and other more recent methods, they do not require an active/passive set strategy. The truss sizing phase is reduced in the former to calculating values for multipliers from an algebraic expression similar in complexity to those used in stress ratio or optimality criteria methods. Although polynomial fitting was employed to move the nodal coordinates to an optimum position, the rate of convergence proved to be heavily dependent on this step. Alternatively, the truss configuration problem is posed in a vector (multicriteria) optimization format and a minimax solution is sought. The minimax solution is determined via the minimization of a convex non-linear scalar function. In order to cope with the high nonlinearity of the response quantities, second-order Taylor series approximations were used. As a result of the amount of information given, the latter approach has shown better convergence properties.The solutions of several test problems indicate that: a) the optimal solution is quite flat with respect to changes in the configuration variables; b) the final lengths of heavily loaded compression members are shorter than their lengths in the initial configuration; c) the optimal solution for combined configuration-sizing problems usually involves a large number of critical constraints and the number of active constraints usually far exceed the number of critical constraints in the corresponding pure sizing optimization.

ACKNOWLEDGEMENTS

The author wish to thank the finantial support given by JNICT (Junta Nacional de Investigação Científica e Tecnológica, Proj. 87 230).

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