Reliability-Based Optimum Design of Glulam Cable-Stayed Footbridges

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Abstract: This work presents a procedure for finding the reliability-based optimum design of cable-stayed bridges. The minimization problem is stated as the minimization of stresses, displacements, reliability, and bridge cost. A finite-element approach is used for structural analysis. It includes a direct analytic sensitivity analysis module, which provides the structural behavior responses to changes in the design variables. An equivalent multicriteria approach is used to solve the nondifferential, nonlinear optimization problem, turning the original problem into sequential minimization of unconstrained convex scalar functions, from which a Pareto optimum is obtained. Examples are given illustrating the procedure.

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Introduction

The optimization of cable-stayed bridges can be stated as that of the minimization of structural cost or volume and the maximum stresses throughout the structure. Additional objectives are aimed at the deflections or displacements and at guaranteeing that the design variables are at least specified minimum values. The work started with shape and sizing optimization by using a (two-dimensional (2D) finite-element model for the analysis. The problem was extended to three-dimensional analysis and the consideration of erection stages under static loading (Negrão and Simões 1997). Seismic effects were considered in the optimization by both a modal-spectral approach and a time-history-based procedure (Simões and Negrão 1999). In most of the previous studies, a grid solution was adopted for modeling the deck, with stiffening girders supporting transverse beams, although box-girder sections were employed (Negrão and Simões 1999). Prestressing design variables were also considered for the problems of optimal correction of cable forces during erection. Deterministic optimization enhanced by reliability performance and formulated within the probabilistic framework is called reliability-based optimum design. These are considered important ingredients in the design of advanced structural systems. Wider applications still exhibit limitations mainly attributed to the deeply nested architecture of this procedure, involving analysis with finite elements, reliability analysis, sensitivity analysis, and optimization.

The behavior of complex structures is often analyzed by means of finite-element analysis (FEA). Stresses and deformations of the structure can be computed given the (deterministic) parameters of loads, geometry, and material behavior. Some structural codes specify a maximum probability of failure within a given reference period (lifetime of the structure). This probability of failure is ideally translated into partial safety factors and combination factors by which variables like strength and load have to be divided or multiplied to find the so-called design values. These design values are to be used as input for a finite-element analysis. The outcome of the calculations is compared with the limit states (for example, collapse or maximum deformation). The structure is supposed to have met the reliability requirements when the limit states are not exceeded. Reality is different. First of all, the (code type) level I method using partial safety factors makes it only plausible that the reliability requirements are met for average structures. Second, safety factors are often based on experience only. A link with the required reliability on a theoretical basis often does not exist. The third aspect is the system behavior of structures. The safety factors are often derived for components of the structure such as girders and columns. A structure as a whole behaves like a system of these components. As a result, depending on the kind of system, the structure can be more or less reliable than its components. The advantage of the code type level I method (using partial safety factors out of codes) is that the limit states are to be checked (by means of an FEA) for only a relatively small number of combinations of variables. A disadvantage is the lack of accuracy. A code type level I method uses partial safety factors that lead to sufficient reliability for average components of structures. These problems can be overcome by using more sophisticated reliability methods such as level II (FORM) and level III (Monte Carlo) reliability methods. The problem with these methods is the considerable computational effort when used in combination with FEA. The combination of directional sampling and a response surface method is a kind of important directional sampling using a smaller number of limit state evaluations. The improvement to the standard directional sampling procedure lies in the use of FEA for the important directions and a response surface for less important directions. In practice this means that, after the response surface is constructed, only a few FEA computations have to be performed.

In this work, first-order reliability methods were used and the

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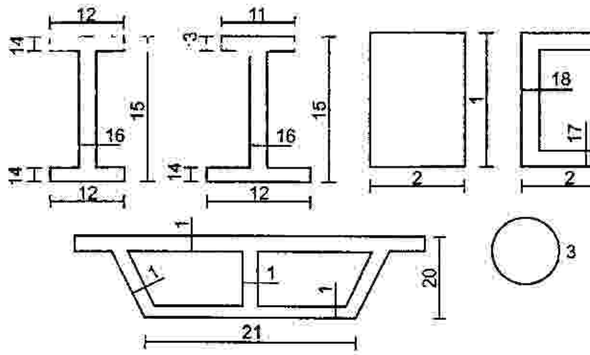

Fig. 1. Types (identified by numbers) of sizing design variables

sensitivity information was obtained analytically. In a forthcoming paper, the advanced simulation method combined with the response surface method will be proposed.

Structural Analysis

The finite-element-based open code MODULEF was used as the basic tool for structural analysis, because code availability was a fundamental requirement for further development. Out of the several element types included in the element library of the program, only the finite element required for two- and three-dimensional (3D) models of cable-stayed bridges were retained and adapted to specific needs. These were 2D and 3D bar and beam (Euler-Bernoulli formulation) elements and four- and eight-noded serendipity plate-membrane (Reissner-Mindlin formulation) elements.

Design Variables

The structural response of a cable-stayed bridge is conditioned by a large number of parameters, concerning cross-sectional shapes and dimensions, overall bridge geometry, applied prestressing forces, deck-to-pylon connections, etc. Some of them play only a limited role on the bridge behavior while others, such as the cable pattern and prestressing forces, are of major importance for both safety and serviceability purposes. Three types of design variables were considered: sizing, shape, and mechanical. Sizing design variables are cross-sectional characteristics of the bar, beam, and plate elements, such as web height, flange width, plate thickness, etc. Changes of such variables do not imply the need for remeshing. Shape design variables produce geometry changes that require a nodal coordinates update or even complete remeshing. Other design variables can be characterized as hybrid, because they define both the box-girder cross-section shape and the deck geometry, requiring coordinates updating only. Finally, the fixed-end prestressing force is a mechanical design variable not related to any geometric quantity. The currently available types are shown in Figs. 1 and 2.

All these types play complementary roles in the process of design optimization. Sizing design variables directly provide for cost/volume decrease. Shape and mechanical design variables have a neglectable direct relation to structural cost but allow for better stress distribution, which in turn leads to further decreases in sizing variables. Prestressing force design variables are essential for achieving acceptable solutions when deflections are considered in the dead-load condition.

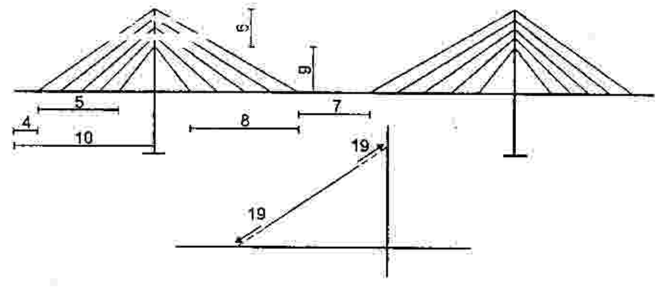

Fig. 2. Types (identified by numbers) of shape design variables

The final behavior of cable-stayed bridges is deeply related t the erection. Among the various methods used for bridge erectio the cantilevering method has become the most popular, due to i suitability for building large spans under strict clearance de mands. For the solution of this problem, it was assumed that th chronological sequence, corresponding to the erection stage se might be thought of as a set of independent substructures, eac corresponding to an erection stage. This is done automatically b the mesh and variable linking generator. The number of righ hand sides will be usually that of the final structure, due to loa combination involving wind, earthquake, and live load, acting i the several positions of the span.

Reliability-Based Optimization

A failure event may be described by a functional relation, th limit state function, in the following way:

$$F=[g(\mathbf{x})\leqslant 0] \quad (1$$

The probability of failure may be determined by the followin integral:

$$P_F=\int_{g(\mathbf{x})<0} f_x(\mathbf{x})dx \quad (2$$

where $f_x(\mathbf{x})$=joint probability density function of the rando variables **x**. This integral is, however, nontrivial to solve. Variou methods for the solution have been proposed, including numerica integration techniques, Monte Carlo simulation, and asymptoti Laplace expansions. Numerical integration techniques become in efficient for increasing dimensions of the vector **x**. Monte Carl simulation techniques may be used, but in the following the focu will be on the first-order second moment methods (FORM which are consistent with the solutions obtained by asymptoti Laplace integral expansions. In the case where the limit stat function $g(\mathbf{x})$ is a linear function of the normally distributed basi random variables **x**, the probability of failure can be written i terms of the linear safety margin M as:

$$P_F=P[g(\mathbf{x})\leqslant 0]=P(M\leqslant 0) \quad (3$$

which reduces to the evaluation of the standard normal distribu tion function

$$P_F=\Phi(-\beta) \quad (4$$

where β=reliability index, given as

$$\beta=\mu_M/\sigma_M \quad (5$$

The reliability index has the geometrical interpretation as th smallest distance from the line (or the hyperplane) forming th boundary between the safe domain and the failure domain. The evaluation of the probability of failure reduces to simple evaluations in terms of mean values and standard deviations of the basic random variables.

When the limit state function is not linear in the random variables **x**, Hasofer and Lind (1974) suggest performing its linearization in the design point of the failure surface represented in normalized space **u**:

$$\mathbf{u}_i=(\mathbf{x}_i-\mu_{x_i})/\sigma_{x_i} \tag{6}$$

As one does not know the design point in advance, this has to be found iteratively in a number of different ways. Provided that the limit state function is differentiable, the following simple iteration scheme may be followed:

$$\alpha i=-\partial g(\beta\alpha)/\partial \mathrm{u}i\left[\sum_{j=1}^{n}\partial g(\beta\alpha)^2/\partial \mathrm{u}i\right]^{-1/2} \tag{7}$$

which will provide the design point $\mathbf{u}^*$ in terms of the hyperdirection cosines, α_i, as well as the reliability index β.

The reliability assessment requires an enumeration of the reliability indices associated with limit state functions to evaluate the structural system probability of failure. Collapse modes are usually correlated through loading and resistances, so an exact evaluation of the probability is impractical, or even impossible, to perform numerically. For this reason, several investigators considered this problem either by finding bounds for p_F or by approximate solutions. In general, the admissible failure probability for structural design is very low. A first estimate of p_F can be found through well-known first-order bounds proposed by Cornell, (1967):

$$\underset{\text{all }k}{\mathrm{Max}}[\Pr(Z_k)]\le p_F\le\sum_{k=1,m}\Pr[Z_k\le 0] \tag{8}$$

The lower bound, which represents the probability of occurrence of the most critical mode (dominant mode), is obtained by assuming the mode failure events Z_k to be perfectly dependent, and the upper bound is derived by assuming independence between mode failure events. Hence, approximation by Cornell's first-order upper bound is very conservative, because it neglects the high correlation between failure modes. Improved bounds can be obtained by taking into account the probabilities of joint failure events such as $P(F_i\cap F_j)$, which means the probability that both events F_i and F_j will simultaneously occur. The resulting closed-form solutions for the lower and upper bounds are as follows:

$$p_F\ge(F_1)+\sum_{i=2}^{m}\max\left\{\left[P(F_i)-\sum_{j=1}^{i-1}P(F_i\cap F_j)\right];0\right\} \tag{9}$$

$$p_F\le\sum_{i=1}^{m}P(F_i)-\sum_{j=2}^{m}\underset{j<i}{\mathrm{Max}}P(F_i\cap F_j) \tag{10}$$

The preceding bounds can be further approximated using Ditlevsen's method (1979) of conditional bounding to find the probabilities of the joint events. This is accomplished by using a Gaussian distribution space in which it is always possible to determine three numbers β_i, β_j, and the correlation coefficient ρ_{ij} for each pair of collapse modes F_i and F_j.

Improved bounds can also be obtained by using Vanmarcke's concept (Vanmarcke 1971) of failure mode decomposition, which takes into account the conditional probability that the $(i-l)$ mode survives given that mode i occurs. By assuming that the probability of occurrence of the ith mode, $P(F_i)=\phi(\beta_i)$, depends on β_i only, the conditional probability $P(S_j/F_i)$ is evaluated in terms of the safety indices β_i and β_j and the coefficient of correlation ρ_{ij} between the failure modes F_i and F_j. A different approximate method that avoids calculating conditional probabilities resulting from conditions leading to failure via pairs of failure modes is the PNET. This method also requires the evaluation of the coefficients of correlation between any two failure modes i and j and is based on the notion of demarcating correlation coefficient ρ_0 assuming those failure modes with high correlation $\rho_{ij}\ge\rho_0$ to be perfectly correlated and those with low correlation $\rho_{ij}<\rho_0$ to be statistically independent. This method is not very convenient, because the solutions will be heavily dependent on the assumed demarcating coefficient ρ_0. A discrete reliability sensitivity analysis is derived and used in the optimization algorithm.

Sensitivity Analysis

The analytic direct method was adopted for the purpose of sensitivity analysis, given the availability of the code, the discrete structural pattern, and the large number of constraints under control. For ordinary linear statics problems, derivatives of kinematic constraints (displacements) are provided by solving a structural system with pseudoloading.

The stress derivatives are accurately determined from the chain derivation of the finite-element stress matrix:

$$\sigma=\mathbf{DB}_e\,\mathbf{u}_e \tag{11}$$

$$\frac{\partial\boldsymbol{\sigma}}{\partial\mathbf{x}_i}=\frac{\partial(\mathbf{DB}_e)}{\partial x_i}\mathbf{u}_e+\mathbf{DB}_e\frac{\partial\mathbf{u}_e}{\partial\mathbf{x}_i} \tag{12}$$

The first term of right-hand side may be directly computed during the computation of element contribution for the global system, on the condition that derivative expressions are preprogrammed and called on that stage. The second term on the right-hand side is somewhat more difficult to compute, because an explicit relation between the displacement vector and design variable set does not exist. Preprogramming and storing the stiffness matrix and right-hand side derivatives in the same way as described for the stress matrix, the displacement derivatives may be computed by the solution of N pseudoload right-hand sides. The stress derivatives are then computed in a straightforward way. The explicit form of matrix derivatives depends on the type of element. For 2D and 3D bar and beam elements, their calculation is a straightforward task. For plate-membrane elements, differentiation of the whole finite-element formulation is required.

Optimization

Pareto's economic principle is gaining increasing acceptance for multiobjective optimization problems. In minimization problems, a solution vector is said to be Pareto optimal if no other feasible vector exists that could decrease one objective function without increasing at least another one. The optimum vector usually exists in practical problems and is not unique. In regard to reliability-based design, several alternative formulations exist. A comprehensive review can be found in Thoft-Christensen (1990). The material cost together with the maximum probability of failure and measures of the structural performance imposed by manufactur-

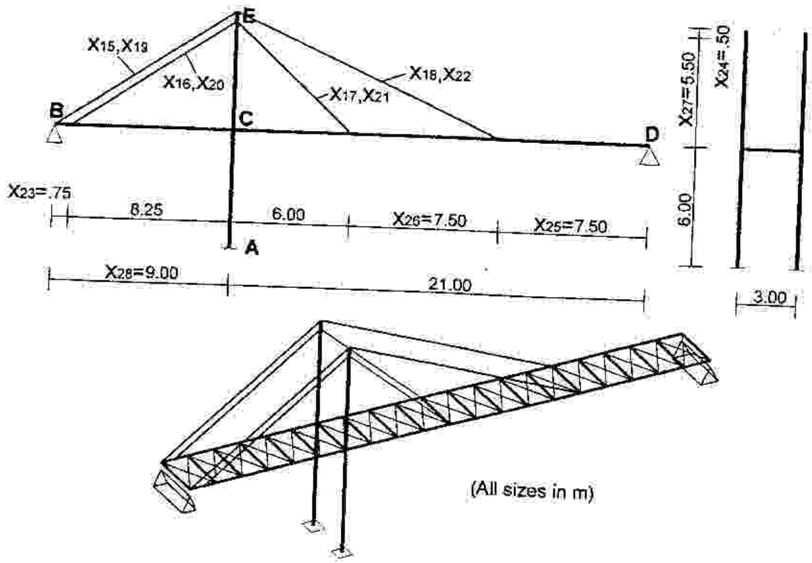

Fig. 3. Geometry of bridge model—starting trial design

ing and technical considerations are the objectives to be minimized. Size, shape, material configuration, and loading parameters may be allowed to vary during the optimization process. Bounds must be set for average cross-sectional and geometric design variables in order to achieve executable solutions and required aesthetic characteristics. The overall objective of cable-stayed bridge design is to achieve an economic and yet safe solution. In this study, it is not intended to include all factors influencing the design economics. One factor conventionally adopted is the cost of material used. A second set of goals arises from the requirement that stresses should be as small as possible. The optimization method requires that all these goals should be cast in a normalized form. Another set of goals arises from the imposition of lower and upper limits on the sizing variables, namely, minimum cable cross sections to prevent topology changes and exequible dimensions for the stiffness girder and pylons cross sections: Similar bounds must be considered for the geometric design variables. Additional bounds are set when geometric design variables are considered, to ensure that no geometry violation occurs when these design variables are updated. Additional goals may be established in order to ensure the desired geometric requirements during the optimization process (mesh discretization, ratios of variation of cable spacing on deck and pylons, etc.). For these, the chosen approach was to initially supply all the necessary information by means of a *geometry coefficients set* describing such conditions.

The objective is to minimize all of these goals over sizing and geometry variables **x**. This problem is discontinuous and nondifferentiable and is therefore hard to solve. However, by using an entropy-based approach, Templeman (1993) has shown that its solution is equivalent to that of an unconstrained convex scalar function, depending only on one control parameter, which may be solved by conventional quasi-Newton methods. This parameter must be steadily increased through the optimization process. The scalar function is very similar to that of Kreisselmeyer-Stainhauser (Haftka and Gurdal 1992), derived for control problems:

$$F(\mathbf{x}) = \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho[g_j(\mathbf{x})]} \right] \tag{13}$$

Eq. (13) is unconstrained and differentiable, which, in theory gives a wide choice of possible numerical solution methods. However, since the goal functions $g_j(\mathbf{x},\mathbf{z})$ do not have explicit algebraic form in most cases, the strategy adopted was to solve Eq. (13) by means of an iterative sequence of explicit approximation models. An explicit approximation can be formulated by taking Taylor series expansions of all the goal functions $g_j(\mathbf{x},\mathbf{z})$ truncated after the linear term. This gives

$$\text{Min } F(\mathbf{x}) = \frac{1}{\rho} \ln\left[\sum_{j=1}^{M} e^{\rho\left(g_{0j}(\mathbf{x}) + \sum_{i=1}^{N} \partial g_{0j}(\mathbf{x})/\partial x_i dx_i\right)} \right] \tag{14}$$

where N and M=respectively, the number of sizing plus geometric design variables and the number of goal functions; and g_{0j} and $\partial g_{0j}/\partial x_i$=goals and their derivatives evaluated for the current design variable vector $(\mathbf{x}_0,\mathbf{z}_0)$, at which the Taylor series expansion is made.

Solving Eq. (14) for particular numerical values of g_{0j} forms only one iteration of the complete solution of Eq. (13). The solution vector $(\mathbf{x}_1,\mathbf{z}_1)$ of such an iteration represents a new design that must be analyzed and gives new values for g_{1j}, $\partial g_{1j}/\partial x_i$ and $(\mathbf{x}_1,\mathbf{z}_1)$, to replace those corresponding to $(\mathbf{x}_0,\mathbf{z}_0)$ in Eq. (14). Iterations continue until changes in the design variables become small. During these iterations the control parameter ρ must not be decreased to ensure that a multiobjective solution is found.

Numerical Example

In order to illustrate the possibilities of the method, a numeric example is presented in this section. The starting trial design is as shown in Fig. 3 and corresponds to an asymmetric two-span cable-stayed footbridge.

Twenty-eight design variables are considered of both sizing, shape, and mechanical types. The former are those shown in Fig.

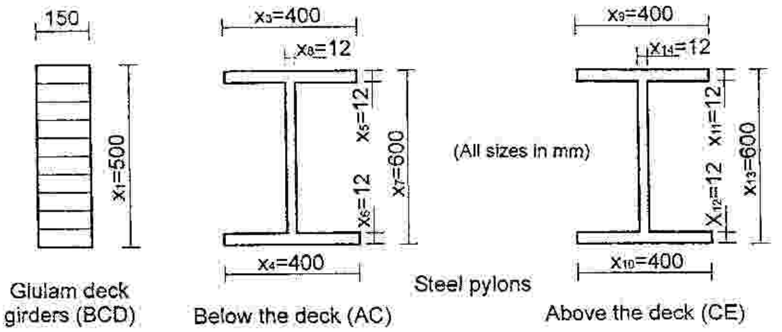

Fig. 4. Starting values of sizing design variables for deck and pylons

4 and concern cross-sectional dimensions for glulam beams and steel pylons and cross-sectional areas for the stays. Shape design variables are as illustrated in Fig. 2, while mechanical design variables are the fixed-end prestressing forces and Young's modulus for glued laminated timber. Starting values of the shape design variables are given in Table 1 and their meaning is drawn from Figs. 2 and 3.

Stiffening girders are made up of strength-graded GL24h glulam. The mechanical characteristics are set according to the Comité Européen de Nationalisation's prEN1194 (CEN 1993). The fact that compressive strength is higher than tensile strength also favors the use of this material in cable-stayed bridges, because compression is the dominating state of stress in this structural system. A fixed width of 150 mm is considered, while the cross-section depth is assigned to Design Variable 1. Given the comparatively thin depth of the laminations, it is assumed a continuous variation of this parameter is possible. The same assumption is made for Young's modulus, which is assigned to Design Variable 2. If the optimum design leads to a value of $E_{0,\text{mean},g}$ not corresponding to one of the glulam grades of prEN1194, an enumerative procedure may be used for the closest grades. This was not the case in the examples presented here, because the algorithm gives an optimum solution made up of GL20h. The sensitivity analysis concerning this type of design variable must account for the change in parameters that direct or indirectly affect the structural cost: the characteristic strength, the mass, and the glulam cost itself. If the improvement of strength, associated to a variation of the Young's modulus, is large enough to compensate for the increase in structural self-weight and in the cost of better—and thus more expensive—glulam beams, the algorithm will lead to superior quality material. For this relation to be properly accounted in the process, approximate relations between Young's modulus and compressive strength, mass, and cost were assumed. These are as follows:

$$f_{c,0,g,k} = 2.15E_{0,\text{mean},g} \tag{15}$$

$$\rho_{g,k} = 0.16 + 0.02E_{0,\text{mean},g} \tag{16}$$

$$€ = €\,50(E_{0,\text{mean},g} - 1) \tag{17}$$

with $f_{c,0,g,k}$ expressed in MPa; $E_{0,\text{mean},g}$ in GPa; $\rho_{g,k}$ in tons; and the cost in Euros, €.

Eqs. (15) and (16) lead to values slightly different from those specified in prEN1194, but the error is not large enough to change the trend towards the decrease or increase of the Young's modulus. Eq. (17) is an estimate that can be rewritten for a specific country and market conditions. The essential goal of this study is to find automatically a reliable solution in agreement with the available data.

A volume unit cost of €10,000 was prescribed for mild steel and one of €25,000 was prescribed for cable steel.

Pylons are assumed to be made up of welded steel asymmetric I-shaped cross sections, the web plan being parallel to the longitudinal plan of the bridge. A fictitious lattice girder simulates the horizontal stiffness provided by the deck surface and required for transverse or eccentric loading.

All stays are assigned a starting cross-sectional area of 3 cm^2 (Design Variables 15–18) and a fixed-end prestressing force of 50 kN (Design Variables 19–22). It must be remarked that these are

Table 1. Initial and Final (Optimized) Values of Design (DesVar) Variables

DesVar	Type[a]	Starting value	Optimized value
X1	1	0.50000	0.54349
X2	22	11.00000	10.00000
X3	11	0.40000	0.20000
X4	12	0.40000	0.20000
X5	13	0.01200	0.01000
X6	14	0.01200	0.01000
X7	15	0.60000	0.30000
X8	16	0.01200	0.01000
X9	11	0.40000	0.20000
X10	12	0.40000	0.20000
X11	13	0.01200	0.01000
[illegible]	14	0.01200	0.01000
X13	15	0.50000	0.30000
X14	16	0.01200	0.01000
X15	3	0.00030	0.00042
X16	3	0.00030	0.00015
X17	3	0.00030	0.00047
X18	3	0.00030	0.00010
X19	19	50.00000	245.61128
X20	19	50.00000	88.70785
X21	19	50.00000	189.93720
X22	19	50.00000	39.50334
X23	5	0.75000	1.54763
X24	6	0.50000	0.30000
X25	7	7.50000	5.25753
X26	8	7.50000	5.64119
X27	9	5.50000	4.00000
X28	10	9.00000	9.36154

Note: Units: variable 2 GPa; 15–18 m^2; 19–22 kN; and others m.
[a]Design variable types as specified in Fig. 1.

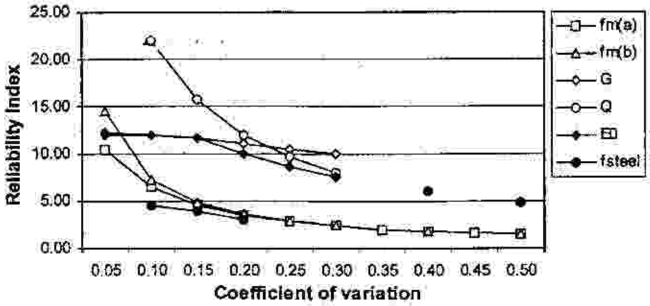

Fig. 5. Parametric analysis on influence of various random parameters

the tensile forces necessary to connect the stays to the anchorage devices in the undeflected structure and not those remaining in the cables after removal of the tensioning equipment, which are smaller due to the cable deformation recovery.

A uniform live load acting on the deck, with the characteristic value of 4 kN/m^2, was considered as the leading action. It was considered to act throughout the whole span or in either the left or right span only. For the sake of simplicity, each of these live-load distributions was assumed to be an independent event. Structural self-weight and a uniform load of 0.5 kN/m^2 on the deck were prescribed as dead loads. The EC1 rules were used to define three load combinations for the purpose of ultimate strength limit state. These correspond to live load acting throughout the whole span or in each side of the pylon only. Coefficients of variation (COV) for the dead and live loads were assigned the values of 0.10 and 0.20, respectively. The corresponding safety factors for ultimate limit state design were 1.35 and 1.50. Given its irrelevancy for the issues focused in this paper, no strength reducing factor k_{mod} was considered.

Prescriptions of ENV 1995-2-Part 2: Bridges (CEN 1997) constraining the values of horizontal and vertical frequencies of the bridge were not included either, because they are currently under implementation in the code. The main problem concerning this issue was the need for using the general procedure described in 7.2.1(5), because the cable-to-deck connection prevents the use of the simplified models listed in Table 7.1 of that standard.

A Gaussian probability density function with a coefficient of variation of 15% was assumed for the glued-laminated timber. Consistent with the traditional ultimate limit state design (level 1 approach), a design bending strength of $f_{m,g,d}=18.5$ MPa was used for the glulam elements. Fe510 was considered for the pylons, and a high grade steel with $f_{s,y,d}=700$ MPa was used for the stay elements. With the usual safety factor of 1.30 for structural timber and the assumed coefficient of variation, these correspond to mean values of 31.9 MPa. Given the controlled production conditions and for the sake of simplicity, COV=0 was generally adopted for the steel, though a range of values was tested in the parametric study, which led to Fig. 5.

The shape and sizing parameters referred to in Table 1 were also initially considered as random variables, with coefficients of variation of 0.01. However, the results have shown no significant differences from the situation in which these parameters were considered as deterministic. Given the strong correlation between the Young's modulus and bending strength, here assumed a[illegible] tally correlated, no simultaneous randomness was allowed these two parameters for the reliability index evaluation.

In this example, the probability of failure concerns cri[illegible] stresses throughout the structure, induced by the loadings. H[illegible] ever, other failure modes or criteria could be used as well, suc[illegible] the excessive deflection or cable understressing.

The initial design was optimized through the use of the n[illegible] ticriteria approach described previously to compare the reliabi[illegible] index and bimodal bounds in both cases. Starting and optimi[illegible] values of design variables are listed in Table 1. The ove[illegible] achieved cost reduction was about 39%. For the starting desig[illegible] minimum reliability index of $\beta=3.47$, with an associate fail[illegible] probability of $\Phi(-\beta)=2.6E-04$ and second-order bounds $2.55E-04\leq P_f\leq 2.57E-04$ were found. The values for the o[illegible] mized solution were $\beta=4.14$; $\Phi=1.75E-05$; and $1.75E-$[illegible] $\leq P_f\leq 3.55E-05$. The bound interval shows that most of [illegible] nearly 1,000 limit states considered are highly correlated.

A parametric analysis was also conducted to evaluate the [illegible]fluence of the various random parameters involved. Fig. 5 su[illegible]marizes these results.

References

Comité Européen de Normalisation (CEN). (1993). "Timber structures-glued laminated timber—strength classes and determination of ch[illegible]acteristic values." *prEN1194*, Brussels, Belgium.

Comité Européen de Normalisation (CEN). (1997). "Eurocode 5: desi[illegible] of timber structures. Part 2: Bridges." *ENV 1995-2*, Brussels, Be[illegible]gium.

Cornell, C. (1967). "Bounds on the reliability of structural systems." *Struct. Div. ASCE*, 93(1), 171–200.

Ditlevsen, O. (1979). "Narrow reliability bounds for structural systems *J. Struct. Mech.*, 7(4),453–472.

Haftka, R. T., and Gurdal, Z. (1992). "Elements of structural optimiz[illegible]tion." *Solid mechanics and its applications*, Kluwer, Dordrecht. Th[illegible] Netherlands.

Hasofer, A. M., and Lind, N. C. (1974). "Exact and invariant second moment code format." *J. Eng. Mech. Div.*, 100(1),111–121.

Negrão, J. H. O., and Simões, L. M. C. (1997). "Optimization of cable stayed bridges with three-dimensional modelling." *Comput. Struct.* 64(1–4),741–758.

Negrão, J. H. O., and Simões, L. M. C. (1999). "Shape and sizing optimization of box-girder decks of cable-stayed bridges." *Proc., 6th Int Conf. on Computer Aided Optimum Design of Structures*, Orlando, Fla., S. Hernandez, A. J. Kassab, and C. A. Brebbia, eds., WIT, Boston, Mass., 323–332.

Simões, L. M. C., and Negrão, J. H. O., (1999). "Optimization of cable-stayed bridges subjected to earthquakes with non-linear behaviour." *Eng. Optimiz.*, 31,457–478.

Templeman, A. B. (1993). "Entropy-based optimization methods for engineering design." *Advanced techniques in the optimum design of structures: Topics in engineering 12*, S. Hernandez, ed., Computational Mechanics, Southampton, U.K, 109–139.

Thoft-Christensen, P. (1990). "On reliability-based structural Optimization." *Reliability and Optimization of Structural Systems: Proc., 3rd IFIP Conf.*, Springer, Berlin, 387–402.

Vanmarcke, E. H. (1971). "Matrix formulation of reliability analysis and reliability based design." *Comput. Struct.*, 3,757–770.