Multicriteria Optimum Design of Prestressed Concrete Bridge Girders

Y.B.Miao and L.M.C.Simões

Dept. of Civil Engineering, University of Coimbra
3049 Coimbra, Portugal

ABSTRACT

This paper describes an application of a direct design method for optimum design of reinforced and prestressed concrete structures. The approach integrates multicriteria optimization strategy with commercial finite element package. The design problem is posed directly as a multicriteria optimization with goals of minimum cost, stresses and displacements. Most of constraints and some of objective functions are governed by the FEM equilibrium equations in static analysis. The multicriteria problem is folded into a single envelop function based on maximum entropy formulation, where pareto optimum achieve a comprise among the objectives and constraints and represent more meaningful and rational results in practice. The validity of the proposed design method is examined by means of numerical examples.

Key Words: Multicriteria Optimization, Maximum Entropy Formulation, Iterative Methods, ADINA

INTRODUCTION

In the last three decades, much work has been done in the structural optimization, in addition to considerable development in optimization theory[1]. However, structural optimization has not been used extensively for civil engineering structures [3](as opposed to mechanical or aerospace structures). It is said that the main reason to cause the situation is the difficulty in choosing the meaningful objective that includes all relevant criteria. Unlike mechanical or aerospace structures, it is not easy and obvious to find one dominating criterion in the civil engineering structures but several (possibly conflicting) criteria such as: minimum cost, maximum safety, minimum stresses, minimum deflections and so forth. The advantage of multicriteria is that it simultaneously considers all competing objectives and convergence to a Pareto solution for which it is not possible to improve one merit function without seriously impairing others. It is very nature and suitable for practical design of civil engineering structures.

The maximum entropy formulation, first published by Jaynes in 1957[5], is recognized as a fundamental concept in information theory. It determines a least biased possibility for a problem in a random process via maximum entropy direction. In recent years, it has emerged as an important and powerful in a wide variety of different fields, and it has found applications in many disciplines throughout science and technology as well as in the structural optimization [8]. In the present paper, the maximum entropy formulation is used to archive the Pareto solution of a multicriteria optimization problem.

The optimization of prestressed beams, which possesses a large number of constraints and occasionally conflicting objectives, is dealt within this work. The problem is posed as a multicriteria optimization with goals of minimum cost, stresses and displacements. The problem is solved through the minimization of a convex function involving one control parameter obtained by folding all goals and constraints into a single envelope. The single objective optimization problem is approximated by first order Taylor's series expansion of structural response and then solved by the public domain Non-Linear Programming NLP program TN/GAMS[6] based on the truncated-Newton algorithm. All structural responses are gained by using commercial finite element package (ADINA). The numerical examples illustrated the application of such direct design method of a prestressed beam. The design variables consist of the areas of prestressing and mild steel, overall depth and depth and web thickness. The actual cost construction is composed of prestressing, mild steel, concrete and formwork costs, and other possible objectives such as initial camber, minimum depth of girder were introduced also.

MULTICRITERIA OPTIMIZATION PROBLEM AND PARETO OPTIMUM

The multicriteria (milticriterion, multiobjective, vector) optimization problem may be formulated as follows: to determine a vector of design variables that satisfy the constraints and minimize a vector of objective function. Mathematically, this can be stated as follow:

$$\text{Min} \quad \mathbf{Z} = \mathbf{f}(\mathbf{x}) = [\, f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_m(\mathbf{x})] \qquad \mathbf{x} \in \Omega \tag{1}$$

where $\mathbf{f}$ = vector of objective functions; f_i = component objective function ($i=1,2,\ldots,m$); Ω = feasible set to which $\mathbf{x}$ belongs and is a subset of R^n

$$\Omega = \{\mathbf{x} \in R^n : \mathbf{g}(\mathbf{x}) \leq 0,\ \mathbf{h}(\mathbf{x}) = 0\} \tag{2}$$

Usually there exists no unique point which would give an optimum for all m criteria simultaneously. The concept of Pareto Optimum is introduced as a solution to multicriteria optimization.

A vector $\mathbf{x}^* \in \Omega$ is Pareto optimum for problem (1) if and only if there exists no $\mathbf{x} \in \Omega$ such as $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$, for $i = 1,2,\ldots,m$ with $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one j. In other word, $\mathbf{x}^*$ is a Pareto optimum if there is no feasible solution $\mathbf{x}$ which decrease some objective without causing a simultaneous seriously increase of at least another objective function.

SOLUTION OF MULTICRITERIA OPTIMIZATION PROBLEM: MAXIMUM ENTROPY FORMULATION APPROACH

Several approach, already applied in operation research and control theory, have been proposed in the literature for the solution of multicriteria optimization problems [2]. In the study, the approach based on the Maximum Entropy Formulation are accepted, because of its concrete foundation.

Entropy is most commonly known in the physics in connection with the second law of thermodynamics - the entropy law - which states that entropy, or amount of disorder, in any closed conservative thermodynamics system tends to maximum. A fundamental step in using entropy in new context unrelated to thermodynamics was provided by Shannon [7] who realized that entropy could be used to measure other types of disorder by using the following algebraic form to measure the amount of uncertainty in any discrete probability distribution.

$$S = -k \sum p_i \ln p_i \tag{3}$$

where p_i is the probability of event i , k is a constant

The Multicriteria optimization process could be considered as a deductive process. Given a set of objective function ***f*** and some constraint functions, the process commences without any numerical information. An initial point is then chosen and information is calculated about the gradient of objective function and constraint function. The numerical information is then used to determine the current situation to infer where the next trial point should be placed via the maximum entropy direction. $\mathbf{x}_k = D(\mathbf{x}_{k-1})$

The Kreisselmeier-Steinhauser function [4] defined in (4) could be considered the measure of entropy in the multicriteria optimization process of (1). The most rational direction of the process maybe the maximum KS norm direction, so that the multicriteria optimization problem (1) was replaced by maximizing a single KS norm function (4).

$$KS(f^*,g) = \frac{1}{\rho} ln(\sum e^{\rho f_i^*} + \sum e^{\rho g_j}) \tag{4}$$

$$f^* = \frac{f - f_{k-1}}{f_{k-1}} \tag{5}$$

where $\boldsymbol{f^*}$ is set of gradient of objective functions.

It has been proved [4] that for any positive value of ρ, the *KS* norm is always more positive than the most positive constraint.

$$Min(f^*,g) \leq KS(f^*) \leq Min(f^*,g) - \frac{ln(m+n)}{\rho} \tag{6}$$

Because of the behaviour of the *KS* norm, it is obvious that when ρ becomes sufficiently large, the result of

$$\text{Minimum } [KS(\boldsymbol{f^*}, \boldsymbol{g})] \tag{7}$$

is the Pareto-optimal solution of multicriteria optimization problem (1). A parameter study had been done (FIG.3), and ρ=10-50 were assumed in the study. Therefore the multicriteria unconstraint optimization problem (1) can be solved by minimizing a single continuous and differentiable function (7).

Until now, the multicriteria optimization problem has been converted so far into a single objective function optimization problem.

SOLUTION OF SINGLE OBJECTIVE OPTIMIZATION PROBLEM

After the multicriteria optimization is transferred into an equivalent single-objective problem based on the maximum entropy formulation, the latter may be solved using standard non-linear programming(NLP) constrained by the equilibrium equations of finite element methods. The problem could be formulated as follows.

Minimize $\quad KS(\boldsymbol{f}(\mathbf{x}), \mathbf{g}(\mathbf{x}))$ (8a)

Such that: $\quad \mathbf{h}(\mathbf{x}) = \mathbf{K}\mathbf{u}(\mathbf{x}) - \mathbf{P} = 0$ (8b)

$\quad \mathbf{x}^{lower} \leq \mathbf{x} \leq \mathbf{x}^{upper}$ (8c)

where $\mathbf{h}$ = vector of equal constraints, $\mathbf{K}$ = stiffness matrix, $\mathbf{P}$ = load cases, $\mathbf{u}$ = vector of displacements, $\mathbf{x}$, $\mathbf{x}^{lower}$, $\mathbf{x}^{upper}$ = vectors of design variables and corresponding lower and upper bounds.

It is obviously that equations(8a-c) are very complex non-linear programming program. As the computational expensive results of (8b), it isn't suitable to use the standard NLP directly. The iterative method was employed in the study. At the start of each iteration, the nonlinear KS norm are linearized at the current point $\mathbf{x}_k$ using first-order Taylor's series expansion, i.e.

Minimize	$KS(x) = KS(\mathbf{x}_k) + KS'(\mathbf{x}_k)\,(\,\mathbf{x} - \mathbf{x}_k\,)$	(9)
Such that:	$\mathbf{h}(\mathbf{x}) = \mathbf{Ku}(\mathbf{x}) - \mathbf{P} = 0$	(8b)
	$\mathbf{x}_k - \Delta(\mathbf{x}_k - \mathbf{x}_{low}) \le \mathbf{x} \le \mathbf{x}_k + \Delta(\mathbf{x}_{upper} - \mathbf{x}_k)$	(10)

where Δ is a parameter, 0.1 - 0.3 was assumed; and

$$KS'(x) = \frac{\partial KS}{\partial g}\cdot\frac{\Delta g}{\Delta x} + \frac{\partial KS}{\partial f}\cdot\frac{\partial f}{\partial x} \qquad (11)$$

It is easy to calculate (11), because all components in equation are implicit expression but $\Delta g/\Delta x$ and some $\partial f_j/\partial x$. The overall finite difference approach was implemented to calculate the sensitivity information.

The iterative procedure stop until it archives its stop criteria. The stop criteria include three criterion: first when the error $\mathbf{e}_k \equiv \mathbf{x}_k - \mathbf{x}$ is small enought; second if the error no longer decreasing or decreasing too slowly, and third when the number of iteration is over the limit amount of iteration. The approach has a very strong trend to convergence to the feasible, if the initial point is in the infeasible domain. As it uses the linear approximate, it is possible to archives infeasible domain from feasible domain, especially in the optimum point. In the case, we set

$$\mathbf{x} = \omega\mathbf{x} + (1-\omega)\,\mathbf{x}_k \qquad (12)$$

Until it archive feasible domain.
where ω is a parameter, $\omega \in (0,2.0)$. $\omega = 0.618$ was assumed.

In general minimizing (9)(8b)(10) is a linear programming problem that may be solved by a variety of techniques. The public domain optimization program GAMS/TN had been adopted as a black box to solve the maximum KS.

NUMERICAL EXAMPLE

<u>DESIGN VARIABLES</u>

The prestressed concrete slab box girders with the cross section, tendon layout and loading in Fig. is to be designed for two objective functions:(1) Minimum the total cost;(2) minimum the initial camber. The nine design variables are the girder depth b, the areas of prestressed steel and reinforcement A_{ps} and A_s, the tendon eccentricity at support section e_e and the thicknesses top and bottom slabs and web t_t, t_b and t_w, etc.

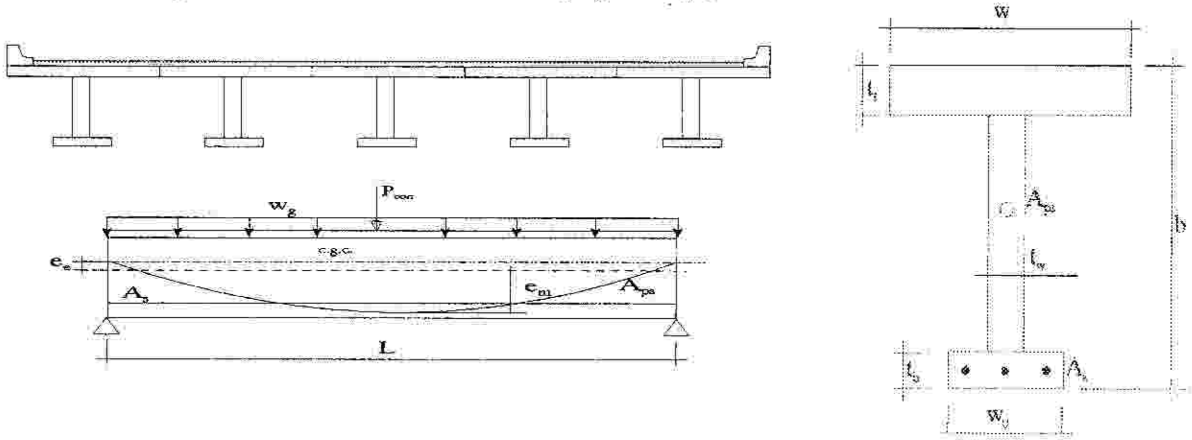


FIG.1 Structure Geometry

x_1:	b	x_2:	$e_e + e_m$
x_3:	half of straight tendon	x_4:	t_b

x_5:	w_0	x_6:	$2t_w$
x_7:	t_t	x_8:	A_{ps}
x_9:	A_s		

CONSTRAINTS:

The commercial finite element package ADINA was used to simulate structural responses under the static design loads. The concrete is modeled using 9-node plane strain elements and the prestressed tendon and reinforcement steel by 3-node truss elements (Fig.2). In addition to its own weight, the girder carries 9.343 kN/m.lane uniform design load, and concentrated design force 80 kN/lane for midspan and 115 kN for quadspan. Instead of practical design code, a general two dimensional under principle stresses failure criteria of concrete material in tension and compression in two principle stresses directions described in Fig.2 are employed as constraints. For each selected point in the concrete elements, following constraints were set (FIG.2):

$$\sigma_1/f_{cc} + 1.0 > 0.0$$
$$\sigma_2/f_{cc} + 1.0 > 0.0$$
$$1.0 - \sigma_1/f_{ct} > 0.0$$
$$1.0 - \sigma_2/f_{ct} > 0.0$$
$$+ \sigma_2/f_{cc} - \sigma_1/f_{ct} + 1.0 > 0.0$$
$$- \sigma_2/f_{ct} + \sigma_1/f_{cc} + 1.0 > 0.0$$

Similarly, for each prestressed tendon and mild steel elements:

$$1.0 - \sigma_s/f_{yk} > 0.0$$
$$1.0 - \sigma_{ps}/f_y > 0.0$$

The concrete (C50/60) has $f_{cc} = 0.3333\times10^8$ N/m^2, $f_{ct} = -0.2800\times10^7$ N/m^2, $\rho_c = 0.2450\times10^5$ N/m^3, $E_c = 0.37\times10^{11}$ N/m^2, $E_{ci} = 0.34\times10^{11}$ N/m^2, $\gamma = 0.2$. Prestressing Steel has $f_{ps} = .1785\times10^{10}$ N/m^2, $f_{yk} = f_{ps} / \gamma_{ps} = .1552\times10^{10}$ N/m^2, E = 2.0×10^{11} N/m. The mild steel has $f_s = .4500\times10^9$ N/m^2, $f_y = f_s / \gamma_s = .3910\times10^9$ N/m^2. Prestress losses of 15% between transfer and service are assumed.

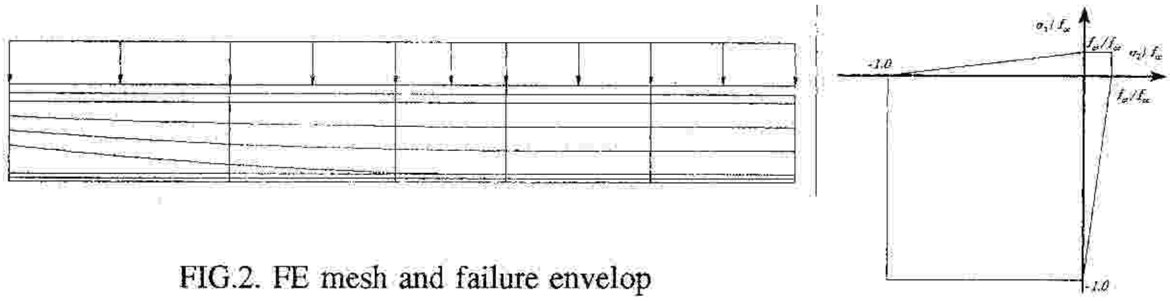


FIG.2. FE mesh and failure envelop

The absolute lower and upper bounds were set according to geometrical constraints, and some practical experience. For example, the reinforcement area for a reinforced concrete beam should be

$$0.15\% < 100A_s/A_c < 4.0\% \quad (16)$$

The minimum reinforcement is provided mainly to control thermal and shrinkage cracking, while the maximum steel is determined largely from the practical need to achieve adequate compaction of the concrete around reinforcement. In addition to the relative bounds (10), the follows absolute bounds are set:

x_1:	(*span*/24, *span*/8)	x_2:	(x_4+thickness, x_2-x_7-*thickness*)
x_3:	(*span*/9, 2**span*/9)	x_4:	(0.5*thickness*, 1.5*thickness*)
x_5:	(3*width*/8, 5*width*/8)	x_6:	(4.0*thickness*, 6.0*thickness*)
x_7:	(1.5*thickness*, 3.0*thickness*)	x_8:	(0.000015A_c, 0.0004A_c)
x_9:	(0.000015A_c, 0.0004A_c)		

OBJECTIVE FUNCTIONS

The primary-objective function was the total cost per-unit length, which includes the costs of concrete, mild steel, prestressing steel and formwork and it was stated as:

$$f_1 = LC_f^o + V_cC_c + A_sC_s + A_{ps}C_{ps} \tag{13}$$

where C_f^o, C_c, C_s, and C_{ps} = unit cost of framework, concrete, mild steel and prestressed steel per length, volume, and areas respectively. In the paper, we assumed $C_f^o = 76.0\ ESC/m$, $C_c = 350.0\ ESC/m^2$, $C_s = 5{,}000\ ESC/m^2$, $C_p = 19{,}500\ ESC/m^2$.

The another objective function was designed as the minimization of the initial camber due to prestressing and own weight. The objective and all constraints described after are governed by the finite element equilibrium equation (8b), i.e.

$$f_2 = u = K^{-1}P \text{ or } exp[k(u_1 - \delta)] \tag{14}$$

The depth of the girder could be considered as the third objective function as:

$$f_3 = x_1 \tag{15}$$

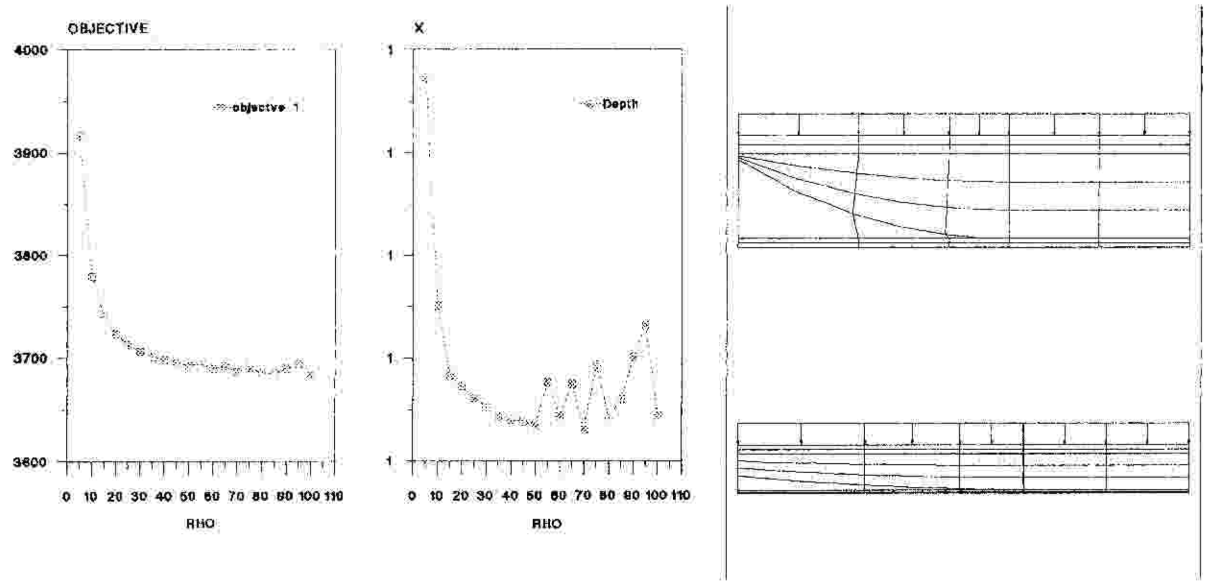


FIG 3.Parameter Study on ρ and Initial & Optimum Mesh

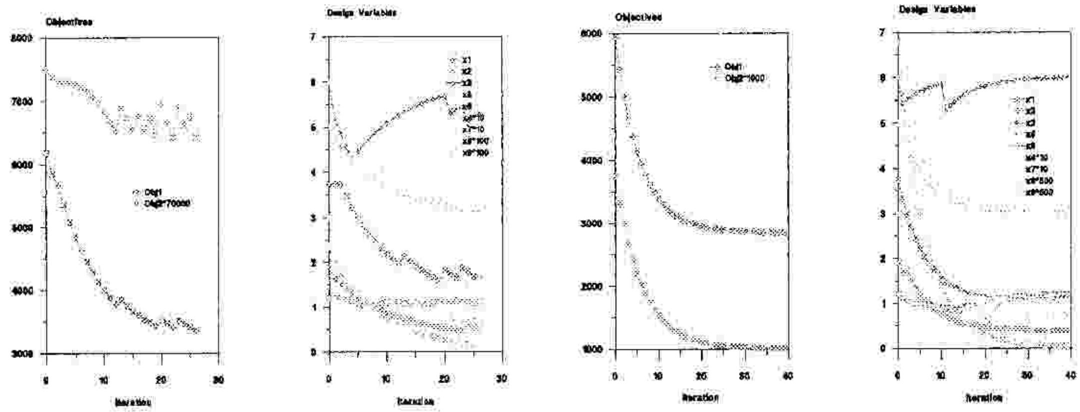
NUMERICAL RESULT

The parameter ρ in (4) is a very important parameter in the algorithms. In order to set the parameter, a single objective optimization was implemented which ρ was set from 5.0 to 100.0 . The result are shown in FIG.3. It is obviously that the higher ρ could archive bit lower objective function value. When the ρ is over 50.0 the results is not very stable. In the follows example ρ = 20.0 are assumed.

Six examples were implemented. It was concluded that:

1. All pareto solutions in the table were acceptable;
2. When the objective functions don't conflict, they converged quite smoothly, compared with conflicted objective functions exist, the process had quite variation(FIG.4);
3. In the design load cases, prestressed tendon keep quite longer straight line; In the optimization process, the depth of beam trends to some level, and the eccentricity at support trends to zero;
4. The thicknesses of top and bottom slabs convergence into their lower bounds; the thickness of web is quite sensitivity in the study;

5. The area of prestressed steel convergence into acceptable range, in the meantime, the mild steel trends to zero. The numerical result could be thought adopt the full prestressed concrete design theory, which ignore the contribution of mild steel;



Conflicting Objectives (f_1 vs f_2) Non-Conflicting Objectives (f_1 vs f_3)

FIG.4 Typical Multicriteria Optimization processes to Pareto Solutions

Table Pareto Solutions

	obj1×10^4	obj2×10^{-2}	x1×10^0	x2×10^0	x3×10^1	x4×10^0	x5×10	x6×10	x7×10^0	x8×10^{-3}	x9×10^{-2}	F/I
1	.3364	.0918	.1612	.5185	.5282	.1603	.3120	.1097	.3166	.7024	.1427	I
2	.3399	.0987	.1798	.4951	.5819	.1431	.3045	.1001	.3045	.7596	.0820	F
3	.3303	.1010	.1756	.5093	.5906	.1042	.3042	.0884	.3042	.5702	.1044	F
4	.3353	.0602	.1735	.4847	.5577	.1112	.3112	.0934	.3155	.6083	.1339	I
5	.3673	.0693	.1851	.6550	.4915	.2670	.3151	.1171	.3362	.7419	.4828	I
6	.3433	.0953	.1608	.5972	.5770	.2462	.3115	.1163	.3115	.7795	.1992	F

F/I: initial point in Feasible domain or Infeasible domain

Conclusions

The multicriteria optimization based on maximum entropy formulation approach presented in this paper demonstrates the potential of the advanced optimization strategy coupled with finite element techniques to solve a variety of prestressed and reinforcement concrete design problems in a quite efficient way. The major merits of the approach are: (1) very concrete theory foundation to determine the direction for a multicriteria problem to the non-unique Pareto Solution; (2) inclusion of all possible (even conflicting) objective functions for a given structural design problem; (3) satisfaction of all constraints (very fast to convergence into feasible designs); and (4) because a general FEM was employed, suitability of a variety of structural designs in the civil engineering, which had been widely used in the mechanical and aero-space engineering.

The advantage of the approach based on maximum entropy formulation over other approaches for transforming multicriteria problem into a single criterion problem lies in its automatically comprise among the possible conflict objective functions.

The multicriteria optimization approach enables the solution of optimization problems for which adequate allowable limits on some structural responses (e.g. initial camber) are not known .

This approach enables some insight into the sensitivity of various objectives functions and design variable variables to design problem.

More structural details should be investigated such as anchorage, prestressed concrete block and bridge bearing, which is more suitable a multi-level optimization algorithms.

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Notation

$\gamma_{c,s,ps}$	:	Partial safety factor for concrete, reinforcement and prestressing steel
f_c'	:	Compressive strength of concrete
$f_{cc,ct}$	:	Flexural compressive and tensile strength of concrete
$f'_{cc,ct}$	:	Initial flexural compressive and tensile strength of concrete
ρ_c	:	the density of concrete
τ_{Rd}	:	basic design shear strength of concrete
f_{ps}	:	Tensile strength of prestressing steel
f_{yk}	:	Characteristic tensile strength of prestressing steel
f_s	:	design strength of reinforcing steel
f_y	:	yield stress of reinforcing steel
w_g	:	own weight
P_{con}	:	Concentrated loan load
E, E_c, E_{ci}	:	Steel and Concrete Young's modulus
γ	:	Poisson's ratio of Concrete Material
φ	:	Safety factor
x	:	Design variables
f	:	objective functions
h, g	:	vectors of constraints
K	:	stiffness matrix
P	:	load vector
ω, thickness, width, span	:	constant

REFERENCES

[1] M.Z.Cohn, Theory and practice of structural optimization, Structural Optimization, **7**, 20-31, 1994

[2] L.Duckstein, Multiobjective in Structural Design: the Model Choice Problem, pp..459-481, New Directions in Optimum Structural Design, John Wiley & Sons, 1984

[3] S.J.Fenves, Status, Need and Opportunity in Computer Assisted Design and Analysis, Structural Engineering International, **2**, 43-47, 1991

[4] Haftka,R.T., Gürdal,Z., Elements of Structural Optimization, Kluwer Academic Publishers, 1992

[5] Jaynes,E.T., Information Theory and Statistical Mechanics, The Physical Review, 1957, **106**, pp.620-630 and **108**, pp.171-190

[6] Nash,S.G., Newton-type Minimization via the Lanczos Method, SIAM. J. Numer. Anal., 1984, **21**, pp. 770-788

[7] C.E.Shannon, A Mathematical Theory of Communication, The Bell System Technical Journal, Vol.XXVII, July,1948, No.3, pp.379-423

[8] Simões,L.M.C., and Templeman,A.B., Entropy-based synthesis of pretensioned cable net structures', Engineering Optimization 1989, **15**, 2, pp121-140