Reliability of Portal Frames with Interacting Stress Resultants[a]

Discussion by James J. Zimmerman,[2] Student Member, ASCE, J. Hugh Ellis,[3] and Ross B. Corotis,[4] Fellow, ASCE

The author is to be commended for illustrating techniques for including axial force–moment interaction in rigid-plastic structural reliability analysis and reliability-based design. Useful as well is the review of methods for solving the associated nonconvex mathematical programs.

The discussers' points for discussion are threefold. First, the author states that an equivalence exists between the mathematical programs given by (21) and (23), and in particular, that **z** in (23) represents a vector of random variables. No random variables exist in the original formulation [(21)], so the equivalence between the two programs is not obvious. The only random variables under consideration are the loads and member capacities (**L**,**X**), neither of which appear in (21). What is then the relationship between the vector of random variables in (23), **Z**, the vectors of decision variables in (21*a*), ($\boldsymbol{\Theta}_*,\boldsymbol{\delta}_*$), and the random vectors (**L**,**X**)?

Next, the global solution of the mathematical program given by (18)–(20) is the reliability index of the least reliable kinematically admissible mode. The optimal values of the decision variables define that mode. The limit state for this mode is a hyperplane in random variable space [(15)], forming a portion of the system limit state. That is, the least reliable kinematically admissible mode is also statically admissible over a range of the random variables. All other (local) optima identified by solution of this mathematical program are guaranteed to be kinematically admissible due to the constraint set, but it is not clear whether static admissibility is also guaranteed. Is it possible that some modes identified as local optima will not form a portion of the system limit state? Modes not on the system limit state are not physically realizable and typically not of interest.

Finally, the discussers (Ellis et al. 1991; Zimmerman et al. 1991) and others (Ma and Ang 1981; Arnbjerg-Nielsen and Ditlevsen 1990) have solved a nonconvex mathematical program to find the least reliable mode [(18)–(20)] by solving the associated mathematical program repeatedly using a set of starting points. A discussion of the advantages (computational or otherwise) of using the author's solution techniques as opposed to the multiple starting method would be most useful.

Appendix. References

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[a]December, 1990, Vol. 116, No. 12, by Luis Miguel da Cruz Simões (Paper 25366).

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Ma, H. F., and Ang, A. H-S. (1981). "Reliability analysis of redundant ductile structural systems." *Civil engineering studies*, Structural Research Series No. 494, University of Illinois, Urbana, Ill.
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Closure by Luis Miguel da Cruz Simões[5]

The author wishes to thank the discussers for their valuable comments. In response to the first query, vector **z** in the mathematical program (23) contains the vectors $\boldsymbol{\theta}_*$ and $\boldsymbol{\delta}_*$. Each random variable in vector **X** is associated with an element of the vector $\boldsymbol{\theta}_*$, which is obtained by adding all the activation parameters corresponding to the critical sections represented by the same plastic capacity in **X**. Similarly, the elements of vector $\boldsymbol{\delta}_*$ are sums of displacement rates associated with the random variables in the applied loading vector **L**. Vector **y** represents $\mathbf{u}_*$, which are mechanism activation parameters. Although both **z** and **y** are deterministic, the random variables are implicit functions of **z**: Local solutions of the mathematical program (23) can be used to obtain reduced random variables **X**′ and **L**′ given by (22). The evaluation of the corresponding random variables is straightforward

$$\mathbf{X} = \mathbf{X}'\boldsymbol{\sigma}_{\mathbf{X}} + \boldsymbol{\mu}_{\mathbf{X}} \quad (51a)$$

$$\mathbf{L} = \mathbf{L}'\boldsymbol{\sigma}_{\mathbf{L}} + \boldsymbol{\mu}_{\mathbf{L}} \quad (51b)$$

where $\boldsymbol{\mu}_{\mathbf{X}}$, $\boldsymbol{\sigma}_{\mathbf{X}}$, and $\boldsymbol{\mu}_{\mathbf{L}}$, $\boldsymbol{\sigma}_{\mathbf{L}}$ = the vectors of mean and standard deviation of the plastic resistances **X** and applied loading **L**, respectively. Next, deterministic plastic limit analysis consists of minimizing the ratio of internal to external work subject to kinematic admissibility. Traditionally, the nonlinearity of the objective function has been avoided by setting the external work to unity. The external work is then included in the constraint set as an equality constraint, and the objective function becomes the minimization of the internal work (a linear function). The fractional programs and corresponding concave quadratic minimization problems that find the stochastically most important mechanism must contain this normality constraint to avoid degeneracy, as it is indicated in the mathematical program [(13)–(19)] (Simões 1990).

With respect to the final point, optimization problems that arise in structural engineering are, in general, of the nonconvex type. Multiple optima may exist due to the following:

1. Nonconvex feasible region.
2. Nonconvex objective function.
3. Disjoint feasible sets.

These features are of great significant when convex optimization is carried out. By using this type of method only one local minimum is guaranteed, and when there are several, it depends on the initial design, which will be found at the end of the process. In some cases, such as the least-weight

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design of elastic structures and optimal shape design, a physical insight is available. It is usual to begin the optimization procedure from very dissimilar starting points to make distinct local minima easier to identify (multiple starting methods). By using this procedure, one just tries to find an improved solution with respect to the starting point, and it is assumed the nonconvex behavior is mild. Unfortunately, this is not the case of concave quadratic minimization, which possesses a local optimum at each vertex. To deal with such nonconvexities, either stochastic or deterministic (such as the one described in the paper) global optimization methods should be employed (Pardalos and Vavasis 1991). In the deterministic methods, the search for the global optimum is carried out by choosing the least objective function value among all those within the boundary of the feasible region. This procedure fits nicely in concave quadratic programming. Stochastic methods (multistart, clustering, multilevel) aim to obtain the global optimum of the function by probabilistic techniques. First, in the global phase the set of sampling points is defined in the domain. Then, a local phase is accomplished by working out with only a portion of the whole set of sampling points. They are usually expensive from a computational point of view. An alternative procedure (Levy and Gomes 1984) employs convex algorithms to locate the global optimum: After obtaining a local minimum the algorithm proceeds by finding only better minima. This tunneling method has not yet been tried out for failure mode identification. Moreover, reliability-based optimal design usually involves (all iterations considered) a large number of critical collapse mechanisms. For a six-story, three-bay frame, it is unlikely to list more than 200 important failure modes just by providing multiple starting points in a nonsystematic way.

Appendix. References

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Uplift Capacity of Z-Purlins[a]

Discussion by Robert W. Dannemann[2]

The biaxial bending concept for uplift capacity evaluation of Z-purlins, as proposed by the author, is a valuable contribution to the steel design practice. For this effort and for the simplicity of this proposal, the author deserve the recognition of the profession.

The discusser, based on the author's idea, thinks a similar procedure may be easily applicable to C-purlins. Instead of comparing with test results,

[a]April, 1991, Vol. 117, No. 4, by Roger A. LaBoube (Paper 25712).

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