Maria Benilde Faria de Oliveira e Costa

Programa, conteúdo e métodos de ensino da disciplina
Fundamentos de Física Moderna

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INDÍCE

1. Introdução

Iremos começar por enquadrar a disciplina de Fundamentos de Física Moderna no plano curricular. Esta disciplina é frequentada por alunos do 1º ano dos cursos de Física e Engenharia Física, sendo uma disciplina do 2º semestre. Anteriormente têm uma disciplina de Física Geral I onde revêem os conceitos de Mecânica Clássica. A preparação física e matemática dos alunos é suficiente, excepto no que respeita a temas como derivadas parciais e equação de propagação de ondas electromagnéticas.

A escolaridade é de uma aula teórica por semana com duração de duas horas e uma aula teórico-prática semanal também com duração de duas horas.

A Física é uma ciência experimental que tem as suas origens nos métodos experimentais aplicados à Mecânica por Galileo (séc. XVII). Até ao fim do século XIX enquadrava-se fundamentalmente nas áreas clássicas: Mecânica, Termodinâmica, Óptica e Electromagnetismo, e atingiu um auge na segunda metade do século XIX com as equações de Maxwell. Contudo, por volta dos fins do século XIX, foram descobertos vários fenómenos físicos que abriram novas ideias para o desenvolvimento da Física, como é o caso dos resultados negativos da experiência de Michelson-Morley (1887) para detectar o hipotético éter, a experiência de Hertz sobre o efeito fotoeléctrico (1887), a descoberta dos raios X (Roentgen, 1895), a radioactividade (Becquerel, 1896), as leis experimentais da radiação térmica emitida por um corpo negro (Stefan, 1879 -Boltzmann, 1884 e de Wien,1893).

A designação de física Moderna refere-se geralmente à Física que desabrochou nos fins do século XIX e que amadureceu fundamentalmente na primeira metade do século XX. Os fenómenos físicos não explicados pela Física Clássica deram origem, quase em simultâneo, a novas teorias: a teoria dos quanta (Max Planck, 1900), a teoria da relatividade (Einstein, 1905), a teoria do efeito fotoeléctrico (Einstein, 1905). Estas teorias, associadas à descoberta do núcleo atómico e à formulação do modelo do átomo com um núcleo central rodeado por electrões (Rutherford, 1911) levaram ao desenvolvimento de novas áreas da Física: Teoria da Relatividade, Mecânica Quântica, Física Nuclear, Física Atómica e Molecular, Física do Estado Sólido, Física das Particulas Elementares, Óptica Quântica, Astrofísica, etc.

A disciplina proposta está concebida com a preocupação não só de informar os alunos sobre os factos acima expostos como de os preparar para as disciplinas que frequentarão nos anos posteriores. O programa que será discutido adiante não aprofunda muito certos temas, visto os alunos serem apenas do primeiro ano. Será feita “uma ponte” entre a Física Clássica e a Moderna e as ferramentas necessárias para o estudo dos temas propostos serão fornecidas sempre que se verifique que os alunos ainda não as possuam.

Os alunos de Engenharia Física não terão mais nenhuma disciplina onde abordem a Teoria da Relatividade. Por esta razão dá-se um peso maior à Relatividade no programa da disciplina.

2. Estrutura da disciplina e metodologia

Como já dissemos a disciplina de Fundamentos de Física Moderna está enquadrada no 2º semestre do 1º ano dos cursos de Física e Engenharia Física.

Segundo a Ficha de Unidade Curricular, a escolaridade é de uma aula teórica por semana com duração de duas horas e uma aula teórico-prática semanal também com duração de duas horas, tendo 4,5 créditos ECTS.

Embora o semestre tenha uma duração oficial de 15 semanas, a ocorrência de feriados e tolerâncias de ponto fazem com que o número de aulas teóricas e de teórico-práticas seja em média de 14 aulas, cada uma.

As aulas teóricas serão expositivas e as deduções feitas no quadro negro por considerarmos que assim a interacção com os alunos é maior. O recurso a acetatos ou informação em “Powerpoint” será feito no caso de apresentação de figuras, gráficos ou tabelas. Este recurso a um ensino mais tradicional é o reflexo de considerarmos que embora a exposição da matéria seja mais lenta, os alunos apreendem-na melhor uma vez que a dedução do raciocínio feita passo a passo é mais interactiva e suscita mais interrogações aos alunos. Aliás, do contacto que temos tido com os alunos ao longo dos anos, concluímos que este tipo de ensino tem maior receptividade da sua parte. Por outro, a existência das novas ferramentas informáticas permite a utilização no ensino de técnicas de simulação computacional como é o caso dos diversos “applets” que a mais adiante nos referiremos.

As aulas teórico-práticas serão de discussão/realização de problemas e de realização/demonstração de trabalhos práticos. Discutir-se-ão exercícios sobre a matéria exposta nas aulas teóricas, que os alunos devem autonomamente resolver fora da sala de aulas. Tentar-se-á que cada exercício acabe por ser completamente resolvido no quadro, preferencialmente por um aluno, uma vez que a disciplina é do primeiro ano e os conceitos são quase sempre novos. Serão elaboradas folhas de exercícios, dos quais alguns serão os que os alunos resolvem autonomamente e se discutem nas aulas. Outros ficarão para os alunos resolverem em casa, consolidando a matéria apreendida nas aulas. Outros ainda terão a sua solução apresentada na página da disciplina na Internet. Alguns exercícios serão resolvidos nas aulas individualmente pelos alunos, o que contará para avaliação. Para a realização de trabalhos práticos, os alunos dividir-se-ão em grupos de 2 ou 3 elementos e ao longo do semestre realizarão 4 ou 5 experiências, consoante o número total de alunos inscritos na disciplina, seguindo um protocolo que lhes é fornecido. No final terão de elaborar um relatório. Os alunos poderão voltar ao laboratório, fora do período lectivo, após a aula em que fizeram a experiência, para a completar ou repetir, até à data indicada para entrega do relatório. Para avaliação, conta o modo como o aluno "está" no laboratório e o modo como foi elaborado o relatório. Uma das últimas aulas será reservada para discussão e demonstrações dos trabalhos práticos.

O Departamento de Física possui dois interferómetros de Michelson bem como um laboratório de Física Moderna com várias experiências que podem ser realizadas. Numa aula teórica ou teórico-prática, poderá ser montada a experiência que demonstra a existência do núcleo: Espalhamento de Rutherford, que por ser demorada propomos deixar a acumular contagens durante a aula e ir vendo os valores de tempos a tempos. Propomos também que o professor monte a experiência de Franck-Hertz e o interferómetro de Michelson para observação de franjas de interferência e que estes trabalhos sejam de demonstração, uma vez que o número de alunos é relativamente elevado.

A escolha dos trabalhos experimentais aqui apresentados, assim como a elaboração dos guiões propostos como protocolo para os alunos, foram realizadas com a colaboração de alguns colegas do Departamento de Física, a quem agradecemos vivamente.

3. Avaliação de conhecimentos

A avaliação de conhecimentos far-se-á por testes tradicionais, com uma parte teórica e outra de problemas. Apesar da brevidade do semestre propomos haver duas provas de frequência, envolvendo a primeira, a Parte I da matéria e a segunda frequência as restantes partes da matéria e uma prova de exame final, para todos os alunos cobrindo toda a matéria. Os testes terão um peso de 70% na nota final, sendo que os restantes 30% serão divididos entre o desempenho na resolução de problemas (os problemas que contam para a avaliação serão resolvidos nas aulas práticas) e na realização dos trabalhos práticos indicados. A nota final será a nota mais elevada obtida num dos conjuntos de provas, para um aluno que realize os dois. Na época de recurso haverá um exame semelhante ao exame final, mas que contará 100%, por neste caso poderem ser avaliados alunos que não fizeram avaliação contínua. Os alunos, de acordo com as disposições vigentes, podem ainda repetir exames para melhoria de classificação dentro de um prazo estabelecido. Nestes exames, recurso e especiais, haverá questões relacionadas directamente com os trabalhos práticos, uma vez que os alunos terão acesso aos protocolos das experiências, via página da disciplina, e aos laboratórios que estarão disponíveis para realização das mesmas ou apenas para observação dos aparelhos envolvidos nos trabalhos práticos.

4. Conteúdos

4.1- Programa da disciplina

Parte I- Introdução à Teoria da Relatividade Restrita

1- ***Revisão das leis da Mecânica Newtoniana***

1.1- Referênciais de inércia.

1.2- Transformação de coordenadas de Galileu.

1.3- Invariância das leis de Newton.

1.4- Princípio da relatividade de Galileu.

1.5- Electromagnetismo e relatividade Newtoniana.

2- *Os postulados de Einstein*

2.1- O espaço absoluto e o éter.

2.2- A experiência de Michelson-Morley.

2.3- Tentativa de preservar o conceito de éter: hipótese da contracção dos espaços; hipótese do arrastamento do éter. A aberração das estrelas. A experiência de Fizeau.

2.4- Os postulados de Einstein e a origem da Teoria da Relatividade.

3- *Cinemática Relativista*

3.1- Sincronismo e simultaneidade

3.2- Dedução das transformações de Lorentz para as coordenadas do espaço e do tempo de um acontecimento.

3.3- Consequências das transformações de Lorentz: invariância das equações do Electromagnetismo; relatividade do intervalo de tempo entre dois acontecimentos e do conceito de simultaneidade.

3.4- Medidas relativistas de comprimentos; a contracção dos espaços.

3.5- Medidas relativistas de tempos; a dilatação do tempo.

3.6- O paradoxo dos gémeos.

3.6- O conceito de espaço-tempo.

3.7- O espaço tempo de Minkowski como espaço euclideano a 4 dimensões.

3.8- Invariância do intervalo entre dois acontecimentos.

3.9- Diagramas espaço-tempo.

3.10- Adição de velocidades segundo a Teoria da Relatividade.

3.11- Dedução das transformações das velocidades.

3.12- Dedução das expressões para o efeito Doppler relativista.

4- *Dinâmica Relativista*

4.1- Necessidade de redefinir quantidade de movimento.

4.2- A equação fundamental da dinâmica relativista.

4.3- A massa relativista.

4.4- A relação entre massa e energia: a equação $E=mc^2$.

4.5- Energia e quantidade de movimento de um corpo.

4.6- Transformações de Lorentz para a energia e a quantidade de movimento de um corpo.

4.7- Transformações de Lorentz para as componentes de uma força.

5- ***Introdução à Teoria da Relatividade Geral***

5.1- O Princípio da Equivalência; a curvatura do espaço.

5.2- O deslocamento gravitacional para o vermelho. Comparação entre previsões relativísticas e testes no laboratório utilizando a espectroscopia de Mössbauer.

5.3- Buracos negros e raio de Schwarzchild.

Parte II- Introdução à Física Quântica

1- ***Radiação e radiadores***

1.1- Espectros de emissão e de absorção.

1.2- Espectros de radiação térmica.

1.3- Absorvibilidade e emissividade; o corpo negro ideal e o espelho ideal.

2- ***Leis do corpo negro***

2.1- Lei de Stefan-Boltzmann e de Wien.

2.2- Funções de distribuição espectral de Wien e de Planck.

2.3- A expressão de Rayleigh-Jeans.

2.4- Dedução da lei de Planck; os quanta de Planck.

2.5- Algumas aplicações das leis do corpo negro; o efeito estufa e estimativa da temperatura da Terra.

3- ***A teoria quântica da radiação electromagnética***

3.1- A natureza ondulatória da radiação electromagnética.

3.2- A natureza corpuscular da radiação electromagnética. O efeito fotoeléctrico, leis experimentais do efeito fotoeléctrico; a teoria de Einstein. O fotão.

3.3- O efeito de Compton para a radiação X. Comprimento de onda de Compton.

3.4- A produção e aniquilação de pares electrão–positrão. O positrónio.

3.5- Absorção de fotões.

3.6- Produção de raios-X e suas características. Espectros de raios-X característicos.

3.7- Electrões Auger.

3.8- Fluorescência de raios-X.

3.9- Difracção de raios-X por um cristal. Lei de Bragg

4- ***Ondas de matéria***

4.1- A dualidade onda-partícula para a radiação electromagnética e para a matéria.

4.2- A hipótese de de Broglie.

4.3- Velocidades de grupo e de fase de uma onda.

4.4- Experências de Davisson-Germer e de Thomson.

4.5- Leis da reflexão e refracção.

4.6- Princípio de Huygens-Fresnel. Experiência de Young.

4.7- Quantização da energia.

4.8- Princípio de incerteza de Heisenberg.

5- ***O modelo atómico de Bohr***

5.2- Níveis de energia de um átomo.

5.3- Transições entre os diversos níveis.

5.4- O espectro do Hidrogénio.

5.5- Experiência de Franck-Hertz.

Parte III- Introdução à Física Nuclear

1- ***Detecção de radiações ionizantes***

1.1- O contador proporcional. O contador Geiger e o contador de cintilação.

1.2- Efeitos das radiações ionizantes.

1.3- Dosimetria das radiações ionizantes.

2- ***A difusão de particulas alfa e a descoberta do núcleo***

2.1- A fórmula de Rutherford.

2.2- Propriedades nucleares.

2.3- A força nuclear.

3- *Estabilidade nuclear e radioactividade*

3.1- Leis do decaimento radioactivo; período de decaímento, constante de decaímento e vida média.

3.2- Actividade de uma amostra radioactiva.

3.3- Decaimentos alfa, beta e gama.

3.4- A quantidade de movimento do fotão e o recuo do núcleo na transição gama; o efeito de Mössbauer.

3.5- Radioactividade natural.

4.2- Sumários da aulas teóricas

<u>1ª lição</u>

Apresentação do programa da disciplina. Bibliografia. Métodos de avaliação. O que é a Física Moderna.

Revisão dos conceitos da Mecânica Clássica. Referenciais de inércia. Eventos e coordenadas.

Transformações de Galileu. Invariância das leis de Newton quando sujeitas a uma transformação de coordenadas de Galileu.

O Princípio da Relatividade de Galileu.

<u>2ª lição</u>

As ondas electromagnéticas e o éter. Propriedades do hipotético éter.

A experiência de Michelson-Morley.

O interferómetro de Michelson-Morley. Cálculo dos desvios esperados nas franjas de interferência na experiência de Michelson-Morley.

Tentativas de preservar o conceito de éter como referêncial preferencial. A hipótese da contracção dos espaços de Fitzgerald-Lorentz. A hipótese do arrastamento do éter.

<u>3ª lição</u>

Contradições à hipótese do arrastamento do éter: A aberração das estrelas e a experiência de Fizeau.

Os Postulados de Einstein: invariância das leis da Física em todos os referenciais inerciais e a constância da velocidade da luz no vácuo, como princípios da Teoria da Relatividade Restrita.

Sincronização de relógios.

4ª lição

Consequências dos postulados de Einstein: Derivação das transformações de Lorentz.

As transformações inversas.

Algumas consequência das transformações de Lorentz: Contracção dos espaços e dilatação dos tempos; A relatividade da simultaneidade.

Comprimento próprio e tempo próprio.

Invariância das equações do Electromagnetismo.

5ª lição

O paradoxo dos gémeos.

O conceito espaço-tempo. A não invariância do intervalo de tempo entre dois acontecimentos quando sujeitos a uma transformação de Lorentz.

Diagramas espaço-tempo. O espaço de Minkowski. Cones de luz de um acontecimento; linhas do universo. O diagrama espaço-tempo para o paradoxo dos gémeos.

A adição de velocidades em teoria da Relatividade Restrita. Dedução das transformações das velocidades. Casos particulares.

6ª lição

O efeito Doppler e Doppler Relativista.

Dinâmica Relativista. A necessidade de redefinir quantidade de movimento. Quantidade de movimento relativista. Equação fundamental da dinâmica relativista. Variação da massa com a velocidade.

Não proporcionalidade entre força e aceleração. Relação entre massa e energia de um corpo; a obtenção da equação $E=mc^2$. Energia de repouso.

Energia cinética relativista e sua validade no limite clássico.

Relação entre energia e quantidade de movimento.

7ª lição

Transformações de Lorentz para a energia total de um corpo e para as componentes da sua quantidade de movimento.

Transformações de Lorentz para as componentes de uma força.

Conservação da energia e da quantidade de movimento. Aplicação ao estudo do choque de duas partículas de massas iguais em dois referenciais distintos.

Introdução à teoria da Relatividade Geral. O Princípio da Equivalência de Einstein.

Massa de inércia e massa gravítica. O desvio gravitacional para o vermelho. Resultados obtidos em laboratório pela espectroscopia de Mössbauer. A experiência de Pound e Rebka. Previsões de Josephson. Buracos negros e raio de Schwarzchild.

A curvatura do espaço em teoria da Relatividade Geral.

8ª lição

Introdução à Física Quântica.

Radiação e radiadores. Espectros de emissão e de absorção. Espectros de radiação térmica. Absorvilidade e emissividade. O corpo negro ideal e o espelho ideal.

A radiação térmica. A teoria de troca de Prévost.

Radiação do corpo negro. Radiância espectral. Lei de Stefan-Boltzmann e lei do deslocamento de Wien.

Dedução da expressão de Rayleigh-Jeans para a radiação térmica do corpo negro e a chamada “catástrofe do ultravioleta”.

9ª lição

Teoria de Planck da radiação do corpo negro.

Dedução da expressão de Planck para o espectro do corpo negro e sua adequação aos resultados experimentais.

Obtenção da lei de Stefan e da lei de Wien a partir da lei de Planck.

Consequências do postulado de Planck.

Algumas aplicações das leis do corpo negro: o radiómetro, o efeito de estufa e a estimativa da temperatura da Terra.

10ª lição

Introdução à Teoria Quântica da Radiação Electromagnética. A natureza ondulatória e a natureza corpuscular da radiação electromagnética.

O efeito fotoeléctrico. A sua descoberta por Hertz. As três leis experimentais do efeito fotoeléctrico. A teoria quântica de Einstein sobre o efeito fotoeléctrico. O fotão. Função trabalho.

O efeito Compton. A experiência e análise dos resultados. Interpretação baseada na natureza corpuscular da luz. Comprimento de onda de Compton.

Produção e aniquilação de pares electrão-positrão. O positrónio.

Absorção de fotões.

11ª lição

Produção de raios-X. Radiação de *Bremstralung*. Linhas espectrais K_α, K_β …, L_α, L_β, …

Efeito Auger. Fluorescência de raios-X. Riscas K_α e K_β.

Difracção de raios-X por um cristal. Lei de Bragg.

Ondas de matéria. A dualidade onda-partícula para a radiação electromagnética e para partículas. A hipótese de de Broglie.

12ª lição

Velocidade de fase e velocidade de grupo de uma onda.

As experiências de Davisson-Germer e de Thomson sobre a difracção de electrões. Interpretação à luz da hipótese de de Broglie sobre a natureza ondulatória das partículas.

Partícula numa caixa a uma dimensão. Função de onda e analogia com uma onda estacionária numa corda. Pacote de onda.

Quantização da energia. Densidade de probabilidade $|\Psi|^2$.

Probabilidade e incerteza. Difracção por uma única fenda. Princípio de incerteza de Heisenberg. Incerteza na energia. Interferências em fenda dupla.

13ª lição

O modelo atómico de Bohr. Níveis de energia de um átomo. Transições entre os níveis de energia. O espectro do hidrgénio. Séries de Lyman, Balmer, Pashen e Brackett. A experiência de Franck-Hertz.

Introdução à Física Nuclear.

Detecção de radiações ionizantes. O contador Proporcional. O contador de Geiger.

O contador de cintilação. Efeito das radiações ionizantes. Taxa de exposição. Dose absorvida.

A experiência do Espalhamento de Rutherford.

A difusão de partículas alfa e a descoberta do núcleo.

14ª lição

A fórmula de Rutherford. Propriedades do núcleo. A força nuclear.

Estabilidade nuclear e radioactividade.

Leis do decaimento radioactivo; período de decaimento, constante de decaimento e vida média. Actividade de uma fonte radioactiva. Decaimentos alfa, beta e gama. O recuo do núcleo na transição gama; efeito de Mössbauer. Radioactividade natural.

4.3- Sumários das aulas teórico-práticas

1ª lição

Resolução de alguns problemas sobre as transformações de Galileu.

Electromagnetismo e relatividade Newtoniana. A não invariância da equação de propagação de uma Onda Electromagnética quando sujeita a uma transformação de Galileu.

Observação das franjas de interferência com um interferómetro de Michelson.

2ª lição

Resolução de problemas sobre a experiência de Michelson-Morley e sobre os postulados de Einstein.

Resolução de problemas sobre coordenadas de um acontecimento em referenciais de inércia distintos.

3ª lição

Problemas sobre a contracção dos espaços e dilatação do tempo.

Transformações de velocidades em relatividade.

4ª lição

O efeito de Doppler relativista. Dedução das equações que relacionam os ângulos e as frequências de emissão e recepção. O efeito Doppler longitudinal e transversal.

Resolução de exercícios de aplicação sobre o efeito de Doppler relativista e não relativista. Caso do som e da luz.

5ª lição

Quantidade de movimento e relação massa-energia; resolução de problemas.

Radiância e leis do corpo negro. Resolução de exercícios de aplicação.

6ª lição

Resolução de problemas sobre a interacção da radiação com a matéria: Efeito fotoeléctrico, efeito de Compton, produção e aniquilação de pares.

7ª lição

Absorção da radiação pela matéria. Produção de raios-X. Ondas e partículas. Resolução de problemas. Observação da demonstração da Experiência de Franck-Hertz.

8ª lição

Resolução de problemas sobre o Princípio de Incerteza e sobre o decaimento radioactivo. Demonstração da Experiência do Espalhamento de Rutherford.

9ª lição a 14ª lição

Realização de trabalhos práticos.

15ª lição

Discussão dos trabalhos práticos realizados pelos alunos nas aulas anteriores.

5. Aplicações informáticas disponíveis na Internet

5.1- Apontamentos e problemas de disciplinas afins de outras universidades

Na Internet estão disponíveis inúmeros apontamentos e exercícios, muitos deles resolvidos, sobre assuntos tratados nesta disciplina. Escolheram-se alguns endereços com essas aplicações, com a finalidade de dar um panorama sobre o tratamento destas matérias noutras universidades.

No endereço http://student.mit.edu/catalog/m8a.html#8.05 encontram-se apontamentos do MIT sobre a Teoria da Relatividade Restrita (nas lições de "Relativity") e de Introdução à Física Quântica e de Ondas e Partículas (nas lições de "Quantum Physics I"). Também do MIT se encontram apontamentos e muitos problemas resolvidos sobre Teoria da Relatividade Restrita no apontador sobre lições de Relatividade do endereço http://web.mit.edu/8.033/ .

Apontamentos da Universidade da Virgínia encontram-se em http://rockpile.phys. virginia.edu/252.html . Entre outros, encontram-se apontamentos sobre Relatividade Restrita, Ondas e Partículas e Teoria Quântica da Radiação Electromagnética. Tem, em particular, uma discussão muito boa sobre as Transformações de Lorentz. Há também muitos problemas (não resolvidos). Ainda da Universidade da Virgínia encontra-se no endereço http://galileo.phys.virginia.edu/classes/252/home.html um curso que engloba Relatividade Restrita, Introdução à Física Quântica, Ondas e Partículas, Átomos e Introdução à Física Nuclear. Este endereço tem também apontadores para muitos exercícios não resolvidos e pontos de exame.

Tutoriais na Web sobre Teoria da Relatividade encontram-se em http://math.ucr.edu/ home/baez/RelWWW/tutorial.html . Este endereço indica também outros portais populares. Um endereço mais geral é o "Relativity on the World Wide Web" http://math.ucr.edu/ home/baez/relativity.html .

http://www2.slac.stanford.edu/vvc/theory/relativity.html é um site com os conceitos básicos da Teoria da Relatividade Restrita.

Em "Hyperphysics" http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html encontra-se uma espécie de Enciclopédia da Física que cobre várias áreas, de entre as quais as escolhidas para esta disciplina. Como exemplos encontram-se os seguintes portais: http://hyperphysics.phy-astr.gsu.edu/hbase/rutsca.html#c1 que explica a experiência do Espalhamento de Rutherford e de partículas alfa por um núcleo e a própria fórmula de Rutherford. Tem várias referências de livros sobre o assunto e conceitos relacionados.

Do género do endereço anterior é http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/crosec.html#c1 que explica o conceito de secção eficaz relacionando-o também com a experiência do Espalhamento de Rutherford.

O endereço da Universidade de Toronto http://www.upscale.utoronto.ca/GeneralInterest/Key/relspec.htm tem apontamentos sobre a Teoria da Relatividade Restrita que cobrem os temas escolhidos no programa desta disciplina.

Em http://www.aip.org/history/einstein/voice1.htm temos a explicação do próprio Einstein sobre a Equivalência entre Energia e Massa, onde se pode ouvir a sua voz a explicar a fórmula $E=mc^2$.

Em http://www.motionmountain.net/text.html pode-se fazer o "download" de um livro de Física muito interessante que cobre algumas partes da matéria escolhida para esta disciplina, sobretudo a Relatividade Restrita.

No endereço http://www.mhhe.com/physsci/astronomy/applets/Blackbody/frame.html pode-se aceder a várias questões sobre a radiação do corpo negro e a luminosidade das estrelas. O endereço tem desde a explicação do que é um corpo negro até exercícios sobre radiação do corpo negro e as respectivas soluções.

A explicação de assuntos como o efeito fotoeléctrico, os quanta de Planck e a energia de um fotão pode ser encontrada em http://www.colorado.edu/physics/2000/quantumzone/photoelectric.html . Aí, através de um índice, também se encontra muita informação sobre raios-X. Ainda sobre a explicação do efeito fotoeléctrico, desde a sua descoberta por Hertz à teoria de Einstein, pode-se consultar em http://photoelectric-effect.search.ipupdater.com/.

Da universidade da Virgínia temos o endereço http://rockpile.phys.virginia.edu/252.html que nos fornece apontamentos sobre Física Moderna, cobrindo grande parte da matéria deste curso.

De um endereço on-line http://physics.nist.gov/cuu/Reference/unitconversions.html podem-se fazer conversões de unidades de várias grandezas físicas.

Datas importantes de descobertas e teorias que fizeram a Física Moderna podem encontrar-se em http://musr.physics.ubc.ca/~jess/p200/.

5.2- “Applets”

Pode-se visualizar a experiência de Michelson-Morley no endereço http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/MichelsonMorley/MichelsonMorley.html .

Em http://www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=70 pode-se simular a relação espaço-tempo em teoria da relatividade e concluir que não há simultaneidade absoluta. Neste endereço pode-se conseguir uma explicação detalhada deste conceito a partir dos postulados de Einstein. Uma simulação da dilatação do tempo pode ser encontrada em http://www.walter-fendt.de/ph11e/timedilation.htm . Ainda uma explicação e simulação sobre a dilatação do tempo pode ser encontrada em http://faraday.physics.utoronto.ca /PVB/Harrison/SpecRel/Flash/TimeDilation.html. Um bonito applet da universidade de Toronto sobre a contração relativista dos espaços encontra-se em http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/ContractInvisible.html. Em http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/LengthContract.html ainda se pode encontrar uma simulação e explicação sobre a contracção relativista dos espaços fazendo a experiência com muões provenientes do espaço e revendo brevemente o fenómeno da dilatação do tempo. Sobre o paradoxo dos gémeos encontra-se uma simulação e explicação em http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/TwinParadox.html. Várias simulações sobre as transformações de Lorentz podem ser encontradas em http://ankh-morpork.maths.qmul.ac.uk/~saha/teach/lorentz/. Ainda sobre a teoria da Relatividade Restrita se encontram simulações sobre vários temas em http://math.ucr.edu/home/baez/relativity.html.

Animações sobre o Efeito Doppler encontram-se em http://www.walter-fendt.de/ph11e/dopplereff.htm e http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/Doppler/DopplerEffect.html estando incluida neste último endereço uma explicação matemática da variação da frequência com a velocidade.

Em http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/home.html podem-se fazer animações sobre velocidades de grupo e de fase de uma onda.

Visualizações de figuras de interferência na difracção de electrões por dupla fenda podem ser feitas em http://www.upscale.utoronto.ca/PVB/Harrison/DoubleSlit/Flash/Histogram.html e http://www.upscale.utoronto.ca/PVB/Harrison/DoubleSlit/Flash/DoubleSlit.html. O applet http://www.walter-fendt.de/ph11e/interference.htm mostra a interferência de duas ondas circulares provenientes de fontes que oscilam com a mesma fase. Ainda em http://www.quantum-physics.polytechnique.fr/ se encontram animações sobre a dualidade onda-partícula em mecânica quântica simulando a experiência de Young com fotões ou partículas.

Em www.manole.com.br/fisicamoderna está disponível a simulação da experiência de difracção de electrões de Thomson.

Uma simulação bastante interactiva da experiência de difracção de raios-X encontra-se em http://www.eser.stonybrook.edu/projectjava/bragg/index.html, e onde se mostram os difractogramas obtidos numa experiência.

Para fazer uma simulação de espectros de emissão fazendo incidir um feixe paralelo de radiação num prisma consultar http://micro.magnet.fsu.edu/primer/java/scienceoptics/newton. Para ver os espectros de emissão e de absorção dos diversos elementos do Quadro Periódico consultar http://jersey.uoregon.edu/vlab/elements/Elements.html . Pode-se visualizar a distribuição espectral de um corpo negro no endereço http://jersey.uoregon.edu/vlab/prf/PRF_plugin.html. Também no endereço http://infrared.als.lbl.gov/calculators/bb2001.html se pode calcular a emissão do corpo negro em função do comprimento de onda, a partir do tamanho e temperatura do corpo. O applet http://webphysics.davidson.edu/alumni/MiLee/java/bb_mjl.htm visualiza os resultados previstos pela lei de Stefan-Boltzmann. Estabelece-se a temperatura do corpo negro e pode-se ver uma simulação do espectro a cores. Ainda em http://www.mhhe.com/physsci/astronomy/applets/Blackbody/applet_files/BlackBody.html temos a visualização do espectro do corpo negro em função da temperatura. Pode também visualizar-se a luminosidade estelar e seu tamanho em função do raio estelar.

Uma simulação de batimentos de uma onda pode-se encontrar em http://www.walter-fendt.de/ph11e/beats.htm, podendo-se variar as frequências das ondas envolvidas. Ainda sobre batimentos temos simulações com tambores (simulação sonora) e osciladores, mostrando as equações matemáticas das ondas em http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/Beats/Beats.html .

Em http://theory.uwinnipeg.ca/lts/java_phys.html podemos visualizar o modelo do átomo de Bohr com os níveis de energia e simular experiências sobre interferências de ondas que demonstram o princípio da dualidade onda-partícula. Pode-se ainda simular as órbitas do átomo de hidrogénio segundo a teoria de Bohr, usando o modelo de partícula ou o de onda, em http://www.walter-fendt.de/ph11e/bohrh.htm .

A explicação, montagem da experiência e visualização do efeito fotoeléctrico com vários elementos como cátodo pode ser feita em http://www.walter-fendt.de/ph11e/photoeffect.htm. Ainda sobre o efeito fotoeléctrico pode-se encontrar um applet em http://www.ifae.es/xec/phot2.html, mostrando o efeito da luz em vários metais.

Simulações sobre a interação da radiação com a matéria podendo escolher-se o tipo de interação: efeito fotoeléctrico, efeito de Compton ou produção de pares, podem ser encontradas em http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Nuclear/XRayInteract/XRayInteract.html.

Ainda sobre o efeito fotoeléctrico, pode-se visualizar em http://www.sc.ehu.es/sbweb/fisica a experiência mostrando a dependência da energia cinética dos electrões emitidos em relação à intensidade da luz incidente e á dependência do material do emissor. Este site é bastante interactivo, encontrando-se aí muitas outras simulações relativas a outros temas da Mecânica Quântica e ao Movimento Ondulatório.

A visualização da lei do decaimento radioactivo pode ser feita em http://www.walter-fendt.de/ph11e/lawdecay.htm,
http://www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=291 e
http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/Nuclear/Decay/NuclearDecay.html.

6. Bibliografia

Todos os livros aqui indicados são demasiado extensos para serem dados num curso semestral. Dividimos estes livros em duas secções; livros base e outros livros de consulta.

6.1- Livros de base

- Arthur Beiser, *"Concepts of Modern Physics"*, McGraw-Hill, 6th edition, 2003.

Bom livro que cobre praticamente todos os assuntos da disciplina, embora no que diz respeito à parte de Física Quântica, fá-lo com pormenor mais extenso do que é possível leccionar numa disciplina do primeiro ano. A parte da relatividade não é tão aprofundada como o que fazemos neste curso, mas aborda assuntos modernos. Este livro tem muitos exemplos de resolução de exercícios que ilustram os temas tratados.

- Paul A. Tippler e Ralph A. Llewellyn, *"Física Moderna"*, LTC, 3ª edição, 2006.

Livro em português bastante completo e moderno que aborda muitos temas (por exemplo da Teoria da Relatividade tratados no R. Resnick) em secções chamadas "Exploratórias" ou de "Leitura Suplementar", mas que fazem parte deste curso. É um livro muito esquemático, com "Sumário" em cada capítulo e "Notas" sobre acontecimentos, referidos no tempo, relacionados com os temas de cada capítulo.
Efeito de Mössbauer claramente tratado de maneira muito acessível, numa secção exploratória do capítulo de Física Nuclear.

- Robert Resnick, "*Introduction to Special relativity"*, John Willey Publisher, 1968.

Apesar de antigo consideramos o melhor livro de estudo na parte da Relatividade. Contém muitos exercícios que podem ajudar os alunos a consolidar a matéria. Dado que os alunos de engenharia Física e de alguns ramos da Física nunca mais abordarem a Teoria da relatividade, convém escolher um livro como este com uma abordagem com rigor apropriado para alunos do primeiro ano, nomeadamente no que diz respeito à dedução das transformações de Lorentz.

- Keneth S. Krane, *"Modern Physics"*, 2^{nd} edition, John Willey Publisher, 1996.

Livro actualizado, bastante completo para um curso do primeiro ano de Física ou de Engenharia Física, ou mesmo para outros cursos de Engenharia, embora aconselhado mais no aspecto da Física Quântica especialmente em Física Nuclear. Não aprofunda o suficiente a parte da Relatividade.

- Ronald Gautreau e William Savin, *"Modern Physics"*, Schaum's Outline Series- Mc Graw Hill, 2^{nd} edition, 1999.

Tem muitos exercícios resolvidos e não resolvidos. Livro aconselhado aos alunos para problemas nas aulas teórico-práticas, embora tenha também em cada capítulo uma introdução teórica bastante útil. Trata com suficiente pormenor a Teoria da Relatividade bem como a Física Quântica.

- Hugh D. Young e Roger A. Freedman, *"Física IV- Óptica e Física Moderna"*, 10ª edição, Pearson, Addison Wesley, 2004

Livro muito completo, que cobre tanto a parte da Teoria da Relatividade, embora com pouca profundidade, como a da Teoria Quântica. Contém muitos exercícios com vários graus de dificuldade. Para os alunos do primeiro ano, tem a vantagem de ser um livro em Português.

6.2- Livros de consulta

- Arthur Evett, *"Understanding the space-time concepts of Special Relativity"*, John Wiley and Sons, 1982.

Livro que explica detalhadamente a parte da cinemática relativista. Tem muitos diagramas explicativos e aconselha vários outros livros de leitura na mesma área.

- Serway, Moses e Moyer, "*Modern Physics"*, 3rd edition, Thomson, Brooks/Cole, 2005.

Livro bastante completo com muitos exercícios e exemplos de resolução de exercícios. No que respeita à Física Quântica, aborda os assuntos com pormenor mais extenso do que é possível leccionar nesta disciplina.

- Paul A. Tipler, "*Elementary Modern Physics"*, Worth Publishers, 1992

Excelente livro com uma extensiva abordagem de diversos temas de física moderna, embora por vezes de modo pouco aprofundado. Na parte da Teoria da Relatividade, por este tema não voltar a ser abordado por alunos de Engenharia Física e outros de algumas especialidades de Física, o livro não apresenta um nível de aprofundamento suficiente.

- Frank J. Blatt, *"Modern Physics"*, Mc Graw Hill, 1992.

É um bom livro de texto para estudo da disciplina sobretudo no que diz respeito à parte da Física Quântica.

- John J. Brehem e William J. Mullin, *"Introduction to the Structure of Matter- A Course in Modern Physics"*, John Willey Publisher, 1989.

Pouco extenso na parte da Teoria da Relatividade. Aprofunda com muito pormenor as partes da Física Quântica a um nível que pode ser tratado em cursos avançados.

- Jeremy I. Pfeffer e Shlomo Nir, *"Modern Physics- An Introductory Text"*, Imperial College Press, 2000.

Bom livro de consulta para a disciplina. Embora não aborde todos os temas escolhidos para a disciplina, fá-lo com profundidade naqueles que trata.
Num capítulo de Aplicações Seleccionadas, tem o efeito de Mössbauer muito bem tratado, de uma forma já um pouco avançada para este nível, e com aplicação ao desvio gravitacional e buracos negros.

- Michael Podesta, *"Understanding the Properties of Matter"*, University College, London, 1996.

Livro com uma abordagem muito interessante e intuitiva no que diz respeito à apresentação de conceitos fundamentais da Física Quântica, sobretudo em temas de Física do estado Sólido.

- Ray Skinner, *"Relativity for Scientists and Engineers"*, Dover Publications, Inc., New York, 1982.

Livro que embora não se adeque à parte da Teoria da Relatividade da disciplina que apresentamos, é aconselhavel para consulta.

- Lawrence S. Lerner, *"Modern Physics"*, Jones and Barlett Publishers, 1996.

Livro interessante mas demasiado conciso e com uma abordagem que não é suficiente para os alunos alvo desta disciplina.

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Anexo A : Folhas de exercícios

Apresentam-se nas páginas seguintes exemplos de folhas de problemas a ser usadas nas aulas teórico-práticas. A maioria dos problemas apresentados foram coligidos nos anos em que foram dadas as aulas práticas da disciplina (1985/86, 1986/87 e 1987/88).

Fundamentos de Física Moderna

Folha 1: Transformações de Galileu. Experiência de Michelson-Morley

1. Um passageiro de um comboio que se desloca à velocidade de 30 m/s passa por um homem que está de pé na plataforma da estação no instante $t = t' = 0$. Vinte segundos mais tarde, o homem na plataforma verifica que um pássaro, que voa à altura de 50 m, paralelamente à linha e no sentido do movimento do comboio, está a 800 de distância de si.

a) Calcular as coordenadas do pássaro no referencial do passageiro

b) Cinco segundos depois da primeira observação, o homem da plataforma verifica que o pássaro está agora a 850 m. Determinar a velocidade (constante) da ave em relação ao homem da plataforma e em relação ao passageiro do comboio.

2. Uma amostra de material radioactivo, em repouso no laboratório emite electrões. Verifica-se que um dos electrões sai com velocidade 0,6c e que outro sai com velocidade 0,7c, mas em sentido oposto. Obter, de acordo com a regra clássica de adição de velocidades, a velocidade de um dos electrões medido no referencial que acompanha o outro.

3. Num automóvel descapotável que se desloca à velocidade constante de 90 km/h viaja uma criança que, num determinado instante, atira verticalmente para cima uma bola, com velocidade de 6 m/s. Escrever a equação de movimento da bola para um observador no automóvel e para um observador parado na berma da estrada.

4. Uma pessoa viaja num barco em direcção a Este com uma velocidade de 5 m/s. No instante em que o barco passa pelo cais, uma criança atira do cais uma pedra em direcção a Norte e a pedra cai a 50 m do cais, 6 segundos depois. Determinar as coordenadas do ponto em que caiu a pedra, medidas pela pessoa que vai no barco.

5. Um rapaz que viaja num comboio, atira uma bola segundo a orientação do seu deslocamento e com uma velocidade de 20 km/h. O comboio desloca-se com uma velocidade de 80 km/h. Qual a velocidade da bola calculada por um observador em terra?

6. Um condutor numa plataforma de caminho de ferro sincroniza o seu relógio com o maquinista na parte da frente de um comboio circulando a 100 km/h. O comboio tem 300 m de comprimento. Dois minutos depois da dianteira do comboio passar a plataforma, um homem na traseira acende um cigarro. Quais as coordenadas deste acontecimento determinadas pelo condutor e pelo maquinista?

7. Uma bola de massa 1kg move-se na direcção Sul-Norte com velocidade de 3 m/s e colide elasticamente com uma bola igual que se encontra em repouso. Após o choque, o movimento é ainda segundo a mesma direcção.

a) Calcular o momento linear e a energia do sistema antes e depois da colisão.

b) Calcular o momento linear e a energia antes e depois do choque, medidos por um observador que se desloca de Sul para Norte à velocidade de 1,5 m/s.

c) Efectuar o cálculo pedido na alínea b) mas agora para um observador que se desloca para Este à velocidade de 2 m/s.

8. Uma massa *m*, ligada a uma mola elástica de constante *k*, move-se sem atrito sobre uma superfície horizontal. Mostrar que a equação do movimento para a massa *m* tem a mesma forma, quer para um observador em repouso em relação à mesa, quer para outro que se mova com velocidade constante na direcção paralela à mola.

9. Considerar uma colisão elástica unidimensional que ocorre ao longo do eixo *x*- *x'* do referencial *O*. Mostrar que, pelas transformações clássicas das equações, a energia cinética também se conserva quando determinada por um segundo observador *O'* que se mova com velocidade constante *u* ao longo do eixo *x*- *x'* de *O*.

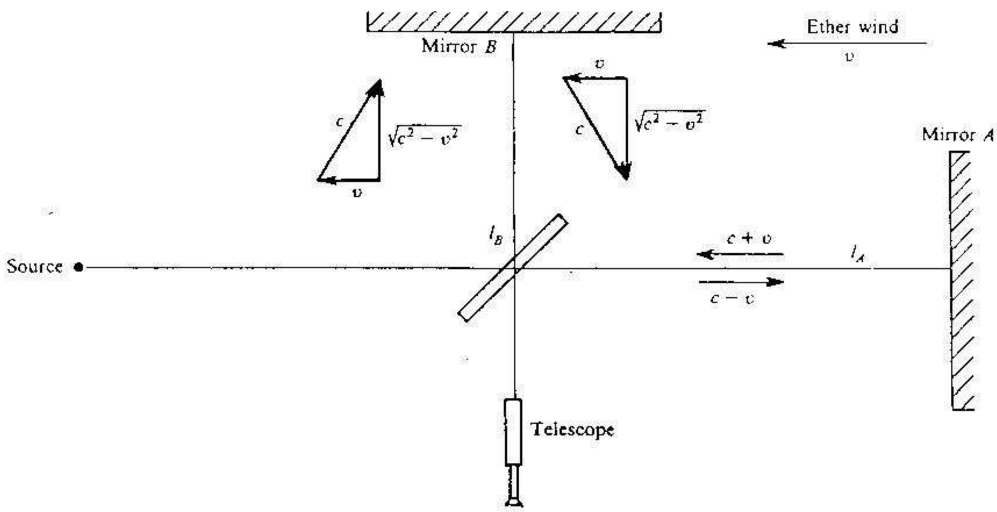
10. Mostrar que a equação de uma onda electromagnética

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$$

não é invariante sob as transformações de coordenadas de Galileu.

11. A figura representa, em esquema, o interferómetro de Michelson-Morley, com um dos braços paralelo ao "vento do éter". Mostrar que rodando o sistema de 90º se deveria observar um deslocamento na figura de interferência correspondente a um número de franjas dado por

$$\Delta N=\frac{v^2}{\lambda\, c^2}\,(l_A+l_B)$$



12. Na experiência original, Michelson e Morley usaram um interferómetro de braços iguais, com $l_A = l_B$ = 11m, e luz de sódio de comprimento de onda λ = 5900 Å. A sensibilidade do aparelho usado na experiência permitia detectar valores de ΔN até 0.005. Calcular o valor limite para a velocidade da Terra através do éter, consistente com os resultados da experiência.

Fundamentos de Física Moderna

Folha 2: Postulados de Einstein da Teoria da relatividade. Transformações de Lorentz e suas consequências.

1. Dois acontecimentos ocorrem a iguais distâncias de um observador. Suponha que ele adopta a seguinte definição de simultaneidade de acontecimentos equidistantes: " Os dois acontecimentos são simultâneos se os sinais de luz emitidos de cada acontecimento me atingirem ao mesmo tempo". Mostrar que, de acordo com esta definição, se o observador determinar que dois acontecimentos são simultâneos, então outro observador, que se mova em relação a ele, determina em geral que os acontecimentos não são simultâneos.

2. Uma barra move-se da esquerda para a direita. Quando a extremidade esquerda passa por uma máquina fotográfica, tira-se uma fotografia juntamente com uma régua estacionária. Depois de revelada a fotografia, vê-se que a extremidade esquerda coincide com o zero e a direita com 0.90 m, da régua. Se a barra se mover a 0,8*c* relativamente à máquina fotográfica, calcular *o verdadeiro* comprimento da barra.

3. Uma fonte luminosa, localizada a 30 km de um observador, emite um *"flash"* que é detectado pelo observador à uma da tarde. Qual o instante em que o *"flash"* foi emitido?

4. Verificar que a equação de uma onda electromagnética

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=0$$

é invariante sob as transformações de coordenadas de Lorentz..

5. Uma partícula que se move com velocidade 0,8*c* no laboratório, decai após ter percorrido 3m. Quanto tempo existiu essa partícula antes de decair?

6. Um *"flash"* é emitido no ponto x = 100 km, y = 10 km, z = 1 km e $t = 5 \times 10^{-4}$ s. Determinar as coordenadas *x'*, *y'*, *z'*, *t'* deste acontecimento para um observador *O'*, que se move relativamente a *O* com velocidade v = -0,8*c* î.

7. Uma partícula move-se relativamente a *O'* com velocidade constante *c*/2 no plano *x'y'*, fazendo a sua trajectória num ângulo de 60º com o eixo *x*'. Sabendo que *O'* se move relativamente a *O* com velocidade 0,6*c* na direcção *x*-*x'*, obter as equações do movimento da partícula no referencial *S*.

8. Uma barra com 1m de comprimento faz um ângulo de 30º com o eixo *x*-*x'* de *S'*. Calcular a velocidade do referencial *S'* em relação ao referencial *S* (segundo a direcção *x*-*x'*) para que a barra faça um ângulo de 45º relativamente ao eixo *x*-*x'* de *S*.

9. Um cubo tem um volume próprio de 1 dm^3. Determinar o volume desse cubo medido por um observador *O'*, que se move em relação ao cubo com velocidade 0,8*c*, segundo uma direcção paralela a uma aresta.

10. Numa colisão a alta energia de raios cósmicos na atmosfera, 120 km acima do nível do mar, foi criado um mesão que depois viajou verticalmente. No seu sistema próprio o mesão tem uma vida de $2,5 \times 10^{-6}$ s. Da terra a sua velocidade é v = 0,99*c*. Determinar:

a) o tempo de vida do mesão medido da Terra

b) a altitude a que se dá a desintegração.

11. Qual a velocidade de um foguetão para que um observador que nele viaje envelheça a uma taxa igual a metade da taxa de envelhecimento para um observador na Terra.

12. Dois observadores, *O* e *O'*, aproximam-se um do outro com velocidade de 0,6*c*. Se *O* registar que inicialmente a sua distância a *O'* é igual a 20m, calcular o tempo que decorrerá até que os dois observadores se encontrem.

13. Para um observador *O*, dois acontecimentos são simultâneos e ocorrem separados espacialmente de 600 km. Qual a diferença de tempo entre estes dois acontecimentos quando medida por *O'* que mede a sua separação espacial como sendo de 1200 km

14. Uma barra de 15 m está em repouso num sistema *S* e faz um ângulo de 60º com o eixo *x*-*x'*. Um observador *S'* move-se em relação a *S* com velocidade v = 0,8*c* segundo o sentido positivo do eixo comum *x*-*x'*, coincidindo as origens dos dois sistemas para t = t' = 0.

a) Determinar o comprimento da barra medido por *S'*.

b) Determinar as coordenadas dos dois extremos da barra medidas pelo observador *S'* quando um dos extremos tem as seguintes coordenadas no sistema *S*: x_1 = 2m; y_1 = 1m; z_1 = 0m; t_1 = 4s e t_2 = 4s.

c) Se uma lâmpada de iodo for colocada num dos extremos da barra tem uma vida média de 60h, qual a sua vida média medida pelo observador *S'*?

d) Se uma segunda lâmpada for ligada em *x'* = 10m e *t'* = 2 x 10^{-7} s em *O'*, qual é o intervalo de tempo entre os dois acontecimentos medido por *O*?

15. Duas naves espaciais com 100m de comprimento próprio cada, passam uma pela outra mas em sentidos opostos. Se um astronauta na frente de uma das naves mede um intervalo de tempo de 2,50 x 10-6 s para a segunda nave passar por ele, então:

a) Qual é a velocidade relativa das duas naves?

b) Qual o intervalo de tempo medido na primeira nave, para a frente da segunda nave passar dum extremo ao outro da primeira nave?

16. Um mesão desloca-se em relação à Terra com a velocidade v = 0,8c atravessando um tubo, fixo na Terra, com 100m de comprimento e sem ar.

a) Determinar o intervalo de tempo gasto pelo mesão a atravessar o tubo medido num referencial ligado ao mesão.

b) No instante em que o mesão entra no tubo emite um sinal luminoso. Calcular o tempo que o sinal demorou a atravessar o tubo, medido por um observador ligado à Terra e por outro ligado ao mesão.

17. A estrela Arcturus dista 36 anos-luz da Terra. (1 ano-luz = $9,463 \times 10^{15}$ m, distância que a luz percorre num ano).

a) Qual deve ser a velocidade de uma nave relativamente à Terra para que a distância medida pelo sistema de referência da nave seja apenas 1 ano-luz?

b) Qual o tempo que a nave demora da Terra à estrela, medido por um relógio na nave?

c) Aplicando a relatividade Galileana, qual a velocidade da nave se a viagem demorar um ano?

Fundamentos de Física Moderna

Folha 3: Transformações de velocidades. Efeito Doppler.

1. Um observador *S'* move-se em relação a um observador *S* segundo o eixo comum *x*-*x'* com velocidade constante de 0,6*c*. O observador *S'* dispara um projéctil de 20cm de comprimento (medido no sistema de repouso do projéctil) com uma velocidade de 0,2*c* segundo uma direcção que faz um ângulo de 60º com o eixo *x*-*x'*. Calcular:

a) a velocidade, em módulo e direcção, do projéctil em relação ao observador *S*.

b) o comprimento do projéctil medido por *S'*.

2. Da origem de coordenadas *O*, de um sistema de inércia *S*, parte no instante $t = 0$ uma nave A na direcção e sentido do eixo *x*-*x'* com velocidade v = 0,8*c*. No instante $t = 1$s parte de um ponto do eixo *x*-*x'* de coordenada 6×10^5 km uma segunda nave B também na direcção e sentido do mesmo eixo, mas com velocidade v = 0,2*c*.

a) Determinar a velocidade da nave A medida pelo piloto da nave B, depois de ambas terem partido.

b) Calcular a velocidade de um observador *O'*, relativamente a *O*, que determina que a partida das duas naves é simultânea.

c) Sabendo que as origens dos dois sistemas (o sistema *S* e o sistema em que se encontra *O'* cujo tempo próprio é *t'*) coincidem para $t = t' = 0$, determinar a posição *x'* da nave A em função de *t'*.

3. Num dado sistema inercial *S*, duas naves A e B são enviadas a partir de um dado ponto *O* com a mesma velocidade (em módulo) v = 0,8*c*, mas seguindo trajectórias perpendiculares.

a) Qual a velocidade de cada uma das naves relativamente à outra?

b) A nave A tem 100m de comprimento e quando a parte da frente desta nave passa por um observador em *S*, o piloto, colocado nesse mesmo ponto, envia um sinal luminoso em direcção à retaguarda. Que tempo leva o sinal luminosos a tingir a cauda da nave, medido

   i) pelo piloto

   ii) pelo observador *S*.

4. Um núcleo radioactivo, que se desloca em relação ao laboratório com velocidade constante de grandeza 0,5*c*, decai, emitindo um electrão com velocidade 0,9*c*, relativamente ao núcleo. Calcular a velocidade desse electrão no referencial do laboratório, considerando que, no referencial do núcleo:

a) o electrão é emitido na mesma direcção e sentido do movimento do núcleo

b) o electrão é emitido na direcção perpendicular à do movimento do núcleo.

5. Uma partícula move-se com velocidade de 0,8*c* fazendo um ângulo de 30º com o eixo *x-x'* no referencial *O*. Qual a velocidade da partícula determinada por um segundo observador, *O'*, movendo-se com velocidade de – 0,6*c* ao longo do mesmo eixo *x-x'*?

6. Fazer o estudo do efeito Doppler para o som nas seguintes situações:

a) Fonte em movimento e receptor parado

b) Fonte parada e receptor em movimento

c) Fonte em movimento deslocando-se com uma velocidade que faz um ângulo θ com a direcção de propagação da onda.

7. Mostrar que a expressão que traduz o efeito Doppler relativista

$$\nu = \nu_0 \frac{\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}\cos\theta}$$

Contém, quer o efeito Doppler longitudinal, quer o efeito transversal.

8. Obter a expressão do efeito Doppler para a luz, considerando apenas termos até *v/c*, quer para o caso em que a fonte (ou o receptor) se está a afastar com velocidade *v*, quer para a situação em que se está a aproximar.

9. Determinar qual a variação registada na frequência de $20\text{x}10^9$ Hz do radar da Polícia de Trânsito, ao detectar um automóvel que circula a 150 km/h.

10. A velocidade de afastamento de uma estrela é de 0,005*c*. Calcular a variação de comprimento de onda da risca D_2 (λ = 5890 Å) emitida pelos átomos de sódio.

11. O comprimento de onda mais elevado ainda visível para um determinado observador em repouso na Terra é λ = 6500 Å. Determinar a velocidade a que deverá mover-se uma nave, de modo que sinais de luz na região do verde (λ = 5000Å) emitidos a partir da nave já não sejam captados por este observador

12. Observa-se um desvio para o vermelho (*redshift*) de 100 Å na risca D_2 (λ = 5890 Å) do sódio proveniente de uma estrela distante. Determinar a velocidade a que se desloca a estrela.

13. Um satélite aproximando-se à velocidade de 10 000 km/h e à altitude de 200 km emite um sinal com a frequência de 1 GHz quando se encontra a 500 km da vertical da estação receptora.

a) Admitindo que a superfície da Terra é plana, calcular a frequência com que o sinal é recebido.

b) Calcular o tempo que o sinal demora a ir do satélite para a estação receptora, para um observador no satélite e para um observador na estação receptora.

14. Mostrar, usando o efeito de Doppler relativistico, que um fotão quando “cai” de uma altura H, a variação da sua frequência é dada por $\delta\upsilon/\upsilon = gH/c^2$, desde a velocidade “com que cai” seja pequena quando comparada com a da luz. Nesta queda há um desvio gravitacional “redshift” ou “blueshift”?

15. A Torre Eiffel tem uma altura de 300m. Qual é a fracção do desvio gravitacional devido a esta altura?

Fundamentos de Física Moderna

Folha 4: Variação da energia com a velocidade. Dinâmica relativista

1. A energia de uma partícula é o dobro da sua energia em repouso. Calcular a sua velocidade.

2. Calcular as razões das massas de repouso de um electrão e de um protão que, partindo do repouso, atravessam uma diferença de potencial de 10^7 V. Calcular as respectivas velocidades.

3. Uma partícula tem energia cinética de 65 MeV e momento linear de 335 MeV/c. Calcular a sua massa em repouso e a sua velocidade.

4. A vida média dos mesões μ em repouso é $2{,}3\times10^{-6}$ s e a massa de repouso é 207 m_e Uma medida de laboratório fornece-nos um valor para a vida média desses mesões de $6{,}9\times10^{-6}$ s. Considerando $m_e c^2$ = 0,511 MeV; $m_e = 9{,}109\times10^{-31}$ kg, determinar:

a) a velocidade dos mesões no laboratório

b) a massa efectiva dos mesões quando se movem a à velocidade determinada na alínea anterior

c) a sua energia cinética

d) A sua quantidade de movimento.

5. Uma partícula com velocidade de repouso m_0 que se move com velocidade 0,6*c* colide e fica unida a uma partícula idêntica inicialmente em repouso. Qual é a massa de repouso e velocidade da partícula composta.

6. Um electrão está a ser estudado por um observador ligado ao laboratório e por outro que se move à velocidade de 0,5*c* relativamente ao laboratório. A velocidade do electrão em relação ao laboratório é de 0,8*c*. Considerando $m_0 = 9{,}109\times10^{-31}$ kg; E_0 = 0,511 MeV; $q = 1{,}602\times10^{-19}$ C, calcular a energia cinética (em MeV) e a quantidade de movimento do electrão para cada observador.

7. Um mesão π^0 (m_0 = 264,2 m_{0e}) decai em apenas dois fotões e tem uma vida média de $2x10^{-16}$ s quando em repouso.

c) Se um destes mesões fosse produzido num núcleo de um átomo, que velocidade mínima deveria ter para sair do átomo durante uma vida média? (raio do átomo = 10^{-10} m).

d) Na situação da alínea anterior, e admitindo que o mesão e os dois fotões se propagam segundo o eixo *x-x´*, calcular a energia dos dois fotões medida por dois observadores A e B colocados no eixo *x-x'*; A está para a direita no eixo e B está para a esquerda.

8. Um objecto de massa m = 80 kg move-se com velocidade igual a 90% da velocidade da luz.

a) Calcular o valor da sua energia cinética.

b) Considerando que este objecto emite um fotão de comprimento de onda λ = 50 nm, segundo um ângulo igual a 0º relativamente a um observador em repouso, determinar o valor do comprimento de onda medido pelo observador.

Fundamentos de Física Moderna

Folha 5: Radiação e interacção da radiação com a matéria

1. Qual é a frequência, comprimento de onda e momento linear de um fotão cuja energia é igual à energia de repouso dum electrão (E_{0e} = 511 keV).

2. Calcular o comprimento de onda, a energia e o momento linear de um fotão de frequência 106 Hz.

3. Qual a massa efectiva de um fotão de λ = 5000 Å.

4. Uma estação de rádio opera a uma frequência de 103,7 MHz com uma potência de saída de 200 keV. Determinar a taxa de emissão dos quanta da estação.

5. Uma estação emissora de F. M. (considerar f = 100 MHz) com uma potência média de 10kW, está na Louzã a 20km de distância da Universidade de Coimbra. Quantos fotões são recebidos por m^2 na Universidade?

6. Um filamento de carbono, que pode ser considerado um radiador corpo negro, é aquecido sucessivamente a 500ºC, 1000ºC, 1500ºC e 2500ºC. Calcular os correspondentes comprimentos de onda para os quais a intensidade da radiação emitida é máxima a estas temperaturas.

7. Supondo que as superfícies estelares se comportam como corpos negros, calcular a temperatura da superfície do sol e a da estrela polar, supondo que o comprimento de onda para o qual a radiância espectral atinge o seu máximo é 5100Å e 3500Å , respectivamente.

8. A emissividade do tungsténio é aproximadamente igual a 0,35. Uma esfera deste material com 1cm de raio está suspensa num vasilhame de raio muito maior, estando este vasilhame a uma temperatura de 300K. Qual a potência requerida para manter a esfera a uma temperatura de 3000K (despreze a condução térmica ao longo do suporte).

9. Duas superfícies muito extensas e próximas uma da outra são mantidas à temperatura de 200K e 300K, respectivamente. Considere as superfícies como corpos negros ideais.

a) Qual a taxa de perda de calor da superfície mais quente?

Considere agora que é colocada uma folha de alumínio muito fina, de emissividade igual a 0,1, entre as duas superfícies. Considere que é atingido o estado estacionário, ficando ambas as faces da folha de alumínio à mesma temperatura.

b) Qual é a temperatura a que se encontra a folha de alumínio?

c) Qual é agora a taxa de perda de calor da superfície mais quente?

10. O potencial de corte para fotoelectrões emitidos de uma superfície por luz de λ = 4910Å é 0,71V. Qual o comprimento de onda da radiação incidente quando se encontra para este potencial um valor de 1,43V ?

11. Luz de comprimento de onda 2000Å incide numa placa de alumínio. No alumínio são necessários 4,2 eV para retirar um electrão.

a) Qual é a energia cinética do fotoelectrão emitido mais rápido? E a do fotoelectrão emitido mais lento?

b) Qual é o potencial de paragem?

c) Qual é o comprimento de onda limite para o alumínio?

d) Se a intensidade da luz incidente for de 2 W/m^2, qual é o número médio de fotões por unidade de tempo e por unidade de área que atinge a superfície?

12. Se a função trabalho do potássio for 2,21 eV quando iluminado por luz ultravioleta de λ = 2500Å, qual é a energia cinética máxima dos fotoelectrões emitidos?

13. Um electrão que sofre uma colisão frontal com um fotão de raios-X, tem um potencial de paragem de 70kV. Se o electrão estava inicialmente em repouso, quais são os comprimentos de onda dos fotões de raios-X inicial e difundido?

14. Determinar o comprimento de onda final de um fotão cuja energia inicial é de 12 MeV, que sofre uma colisão de Compton e é desviado de 90º por um protão. (Para protões: $m_o c^2$ = 938,3 MeV)

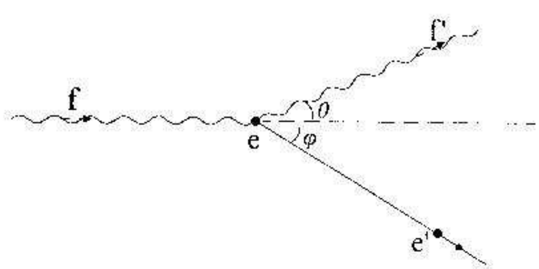
15. Numa colisão de Compton, detectam-se o fotão e o electrão difundidos. Determina-se que o electrão tem uma energia cinética de 75 keV e o fotão uma energia de 200 keV. Determinar:

a) o comprimento de onda inicial do fotão

b) os ângulos de difusão do electrão e do fotão.

16. Um fotão de energia $E = h\nu = hc/\lambda$ colide com um electrão estacionário. O fotão é emitido na direcção θ, com energia $E' = hc/\lambda'$, e o electrão é emitido na direcção φ. Mostrar que:

$$\lambda' - \lambda = \frac{h}{m_0\, c}(1 - \cos\theta)$$



17. Um fotão de λ = 0,0005Å produz um par electrão-positrão na vizinhança de um núcleo pesado.

a) Se as partículas tiverem as mesmas energias cinéticas, determinar essas energias.

b) Se o positrão tiver uma energia cinética cinco vezes maior que a energia cinética do electrão, determinar a energia cinética de cada partícula.

18. Um electrão com velocidade de 0,8c aniquila-se com um positrão em repouso, produzindo dois fotões. Um dos fotões emergentes é observado a mover-se na direcção perpendicular à do electrão inicial. Determinar a energia de cada fotão.

19. O coeficiente de absorção de um material é 0,061 mm^{-1}. Se a intensidade do feixe incidente for I_0, calcular a espessura do material necessária para reduzir a intensidade do feixe pata $I_0/3$.

20. Dois tipos de radiação X de comprimentos de onda λ_a e $\lambda_b > \lambda_a$, com a mesma intensidade, atravessam um material para o qual os coeficientes de absorção linear são $\mu_a = 0,3$ mm^{-1} e $\mu_b = 0,72$ mm^{-1}. Verifica-se que a radiação de comprimento de onda menor emergente do material tem intensidade dupla da outra radiação. Calcular a espessura do material.

21. Usando a conservação de energia e da quantidade de movimento, mostre que um electrão livre em repouso não pode sofrer efeito fotoeléctrico, isto é, não pode absorver um fotão. E se o electrão livre estiver em movimento?

Fundamentos de Física Moderna

Folha 6: Raios-X. Dualidade onda-partícula. Partícula numa caixa. Princípio de incerteza. Decaimento radioactivo.

1. Qual a diferença de potencial que deve ser aplicada a uma ampola de raios-X para que sejam emitidos raios-X com um comprimento de onda mínimo de 30 pm.

2. Electrões são acelerados em tubos de televisão através de uma diferença de potencial de 10kV. Calcular a frequência mais alta das ondas electromagnéticas emitidas quando estes electrões embatem no ecrã do tubo. Que tipo de ondas são estas?

3. O ângulo de Bragg mais pequeno para o KCl é 28,4º para raios-X de λ=0,30 nm. Calcular a distância entre os planos atómicos no KCL.

4. A distância entre planos atómicos adjacentes na calcite ($CaCO_3$) é de 0,300 nm. Calcular o ângulo de Bragg mais pequeno para raios-X de 0,030 nm.

5. Um fotão e uma partícula têm o mesmo comprimento de onda. Comparar os seus momentos lineares e a energia do fotão com a energia total da partícula. Qual a relação entre a energia do fotão e a energia cinética da partícula?

6. Um electrão parte do repouso numa região onde existe uma diferença de potencial de 100 V. Qual é o seu comprimento de onda de de Broglie?

7. Calcular o comprimento de onda de de Broglie de um grão de areia com 1 mg de massa empurrado pelo vento com uma velocidade de $2x10^8$ m/s.

8. Um electrão e um protão têm a mesma velocidade. Comparar os comprimentos de onda e velocidades de fase e de grupo das suas ondas de de Broglie.

9. A velocidade de fase das ondas do oceano é $\sqrt{g\,\lambda\,/\,2\,\pi}$, onde g é a aceleração da gravidade. Calcular a velocidade de grupo das ondas do oceano.

10. Calcular as velocidades de fase e de grupo de ondas de de Broglie de um electrão com energia cinética de 500 eV.

11. Qual o efeito de se aumentar a energia do electrão no ângulo de espalhamento, na experiência de Davisson-Germer?

12. Um feixe de electrões com energia de 50 eV incide num cristal e os electrões difractados são detectados num ângulo de 50º relativamente ao feixe incidente. Qual é o espaçamento dos planos atómicos do cristal?

13. Um electrão encontra-se numa caixa de largura 0,10 nm, que é a ordem de grandeza das dimensões atómicas. Determinar as suas energias permitidas.

14. Um protão numa caixa unidimensional tem energia de 400keV no seu primeiro estado excitado. Qual é a largura da caixa?

15. Uma medida determina a posição de um protão com uma precisão de $\pm$ 1,00 x10^{-11}m. Calcular a incerteza na posição do protão passados 1s. Considerar $v \ll c$.

16. Um átomo de hidrogénio tem 5,3x10^{-11} m de raio. Estimar a energia mínima que um electrão pode ter neste átomo.

17. A posição e o momento linear de um electrão com energia de 1,00 keV são determinadas simultaneamente. Se a posição tiver sido determinada com uma incerteza de 0,100 nm, qual é a percentagem de incerteza no momento linear?

18. A actividade de uma amostra de um isótopo radioactivo é de 115,0 Bq, imediatamente após a sua formação, na sequência de reacções nucleares num reactor nuclear. Decorridas 2h15min, verifica-se que a actividade desta amostra é 82,5 Bq. Calcular:

a) a constante de decaimento e o período de meia-vida deste isótopo.

b) O número de núcleos radioactivos existentes na atmosfera, no instante da sua formação.

19. Entre os produtos radioactivos que se escaparam num acidente nuclear encontrava-se o $^{131}_{53}I$, cujo período de meia-vida é 8,0 dias e o $^{137}_{55}Cs$, para o qual o $t_{1/2}$ = 30 anos. Foram libertados 10 vezes mais átomos de iodo que de césio. Quanto tempo demorará até se ter igual quantidade dos dois isótopos? Ao fim de quanto tempo serão as duas actividades iguais?

20. O carbono-14 decai por emissão β^- (emissão de electrões), sendo o seu período de meia-vida , $t_{1/2}$ = 5730 anos.

a) Escrever a reacção que traduz o decaimento do $^{14}_{6}C$.

b) Uma amostra de carvão produzido por queima de madeira recente tem cerca de 1 átomo de $^{14}_{6}C$ 10^{12} átomos de $^{12}_{6}C$.

Calcular a actividade (em becquerel, Bq, e em desintegrações /minuto), de uma amostra de 1 grama de carbono natural recente. Determinar a actividade desta amostra decorridos 200 anos, 600 anos e 10000 anos.

21. O isótopo potássio 40, $^{40}_{19}K$ é usado na datação geológica, tendo como base o seu decaimento para árgon 40, $^{40}_{18}Ar$, sendo de 1,25 x 109 anos, o seu período meia-vida. A análise feita a um fragmento de rocha mostrou que a razão entre o número de átomos de árgon e o número de átomos de potássio era igual a 10,3.

Supondo que todos os átomos de árgon presentes resultaram do decaimento de átomos de potássio, determinar a idade provável desta rocha.

22. Quando um núcleo no estado excitado emite fotões, recua no sentido oposto ao da emissão do fotão.

a) O núcleo $^{57}_{27}Co$ decai por captura electrónica para o $^{57}_{26}Fe$, o qual emite depois um fotão de 14,4 keV ao atingir o estado fundamental. A massa do átomo $^{57}_{26}Fe$ é 9,5x10^{-26} kg. Qual é a redução da energia do fotão devido ao recuo do núcleo?

b) Há certos cristais aos quais os átomos estão ligados, de tal modo que o cristal como um todo recua quando um fotão γ é emitido, em vez do átomo isolado. Este fenómeno é conhecido como efeito de Mössbauer. Qual a redução da energia do fotão nesta situação se o núcleo $^{57}_{26}Fe$ fizer parte de 1,0 g de um tal cristal?

c) A emissão sem recuo considerada na alínea anterior, significa que é possível construir uma fonte monoenergética (emitindo fotões monocromáticos). Uma fonte como esta foi usada na experiência de Pound e Rebka. Qual é a frequência original e a variação de frequência de um fotão de energia 14,4 keV, depois de "cair" 20m?

Anexo B : Folhas de Trabalhos Práticos

Apresentam-se nas páginas seguintes folhas de trabalhos práticos (guiões e manual com instruções do equipamento a ser usado) para os alunos lerem e se poderem preparar para a execução dos mesmos ao longo do semestre. A maioria dos guiões apresentados foram coligidos por colegas do Departamento de Física.

Espectroscopia de Mössbauer e o Paradoxo dos Gémeos

Guião

1. O efeito de Mössbauer

O efeito de Mössbauer envolve a absorção ressonante de fotões emitidos sem recuo dos núcleos. Como as larguras das riscas das transições são muito pequenas comparadas com as suas energias, o processo de absorção ressonante é muito sensível a pequenas variações na energia dos fotões. A resolução deste efeito é tipicamente da ordem de 1 parte em 10^{12}, tornando-o uma ferramenta poderosa na espectroscopia nuclear e física da matéria condensada.

2. Absorção ressonante

Quando um fotão é emitido numa transição entre dois níveis de energia, a frequência do fotão tem uma largura natural da linha que resulta do tempo de vida finito do estado excitado. Se este fotão incidir num sistema similar no estado fundamental, é provável que seja reabsorvido, levando o sistema ao estado excitado. É este o significado de absorção ressonante de fotões.

Contudo, numa transição, a energia não é toda transferida para o fotão. Como a quantidade de movimento se conserva, alguma da energia é perdida do recuo do emissor. Da mesma maneira, quando o fotão é absorvido, o absorvente recua. As distribuições das energias de emissão e de absorção são assim separadas pelo dobro da energia de recuo. A probabilidade da absorção ressonante é proporcional à sobreposição destas distribuições.

No caso das transições atómicas, as energias de recuo são pequenas comparadas com as larguras naturais das linhas. Há então uma grande probabilidade de absorções ressonantes nas transições atómicas. No caso das transições nucleares, as energias dos fotões são muito maiores do que as energias de recuo e do que as larguras naturais das linhas. Por isso há pouca ou nenhuma sobreposição das transições de energia. Resulta daí que a absorção ressonante não é muito provável para transições nucleares.

3. Emissão sem recuo

Quando se determina a energia de um núcleo que faz parte de um sólido a interacção com toda a estrutura da rede do sólido deve também ser considerada. Aumentando a massa efectiva do sistema que recua, a energia de recuo diminui. O núcleo transmite energia para a rede por excitação vibracional de estados através da criação de fonões. Se não for produzido nenhum fonão (a mecânica quântica permite-o), então toda a energia da transição vai para o fotão emitido. É isto que significa a emissão sem recuo.

A fracção da emissão a zero-fonões, dada pelo factor de Debye-Waller, é função da temperatura de Debye, da energias dos raios gama e das temperaturas do emissor e do absorvente. Devido à quantificação da excitação vibracional, há alguma (não pequena) probabilidade de que uma dada transição não transfira energia para a rede: emissão sem recuo.

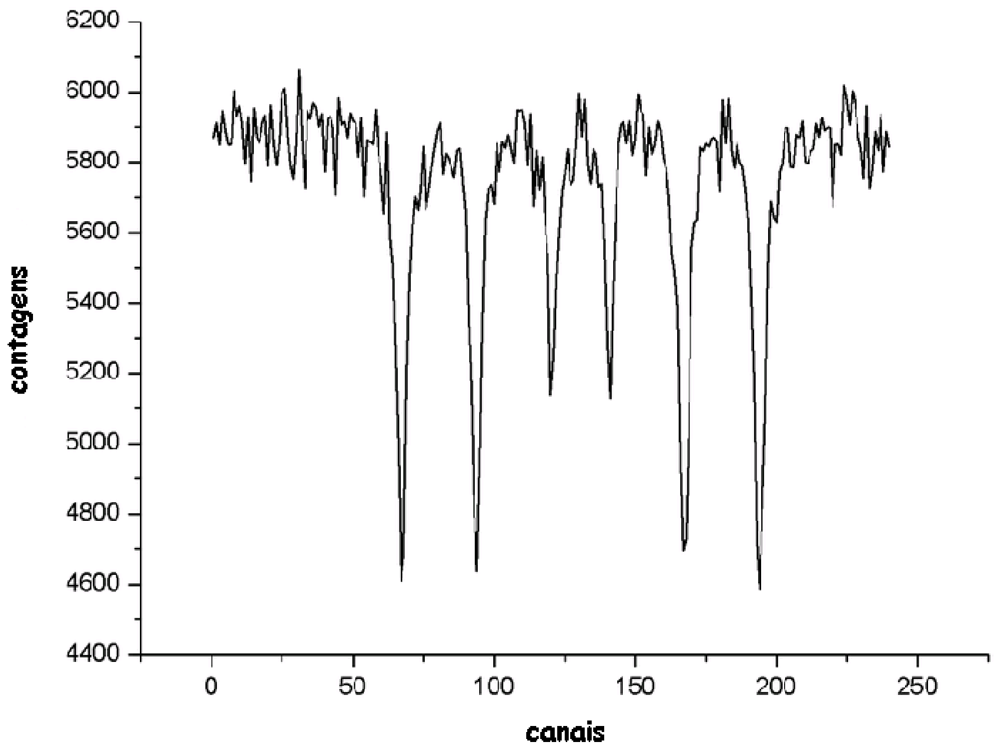
4. Desvio Doppler

Quando um emissor se move em relação a um absorvente, a energia dos fotões sofre um desvio de Doppler. Para velocidades não relativistas, $\Delta E = h\Delta\nu = \beta h\nu_0$ onde $\beta = v/c$ e ν_0 é a frequência de um fotão emitido a partir de um átomo em repouso. Este é o Desvio Doppler de primeira ordem e é válido para $v << c$. De acordo com esta relação, a variação na energia do fotão é proporcional à velocidade relativa.

Variando a velocidade relativa do emissor, é possível varrer as energias do fotão numa gama de valores. Na espectrometria de Mössbauer, a taxa de absorção é medida em função da velocidade da fonte. Usando a fórmula de Doppler pode converter-se velocidade em desvio de energia. A análise dos picos de absorção observados revela detalhes sobre níveis de energia nuclear.

5. Interacções observáveis

Uma vez que os níveis de energia dos núcleos dependem de alguns factores externos, é possível obter informação sobre a estrutura atómica e molecular de materiais a partir de espectros de Mössbauer. Dependendo da natureza das interacções, pode haver uma separação ou desvio dos níveis nucleares. No espectro de Mössbauer três interacções possíveis são observadas, a que não nos referiremos aqui.



Um espectro de absorção de Mössbauer do ^{57}Fe

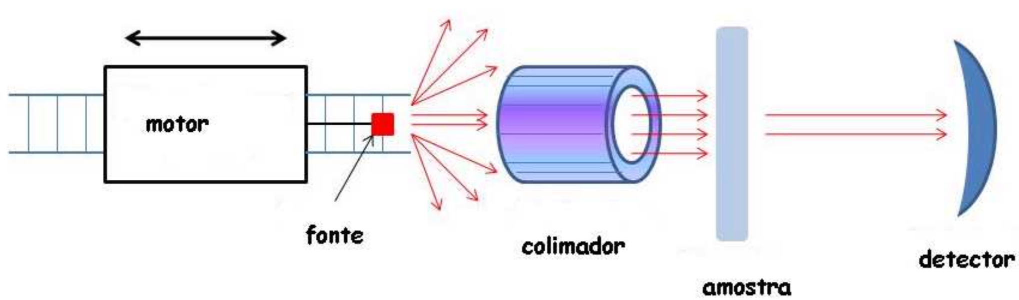
6. Método

Na espectroscopia de Mössbauer de transmissão, uma amostra sólida é exposta a um feixe de radiação gama, e um detector mede a intensidade do feixe transmitido através da amostra. Os átomos na fonte emissora de raios gama devem ser do mesmo isótopo que os átomos na amostra absorvente.

Se os núcleos emissores e absorventes estiverem em ambientes químicos idênticos, as energias de transição nucleares serão exactamente iguais e a absorção ressonante será observada com ambos os materiais em repouso. A diferença nos ambientes químicos, faz com que os níveis de energia nucleares se desloquem. Apesar desses desvios de energia serem muito pequenos (às vezes menores que um micro-electrãovolt), a extremamente estreita largura espectral dos raios gama para alguns nuclidos faz com que pequenos desvios na energia correspondam a largas variações na absorvância. Para conseguir ressonância entre núcleos emissores e absorventes, é necessário modificar ligeiramente a energia dos raios gama, e na prática isto faz-se sempre com o efeito Doppler.

A fonte é acelerada numa zona de velocidades usando um motor linear para produzir o efeito Doppler e varrer a energia dos raios gama numa dada faixa. Uma gama típica de velocidades para o ^{57}Fe é, por exemplo, ±10 mm/s (1mm/s = 48.075 neV).

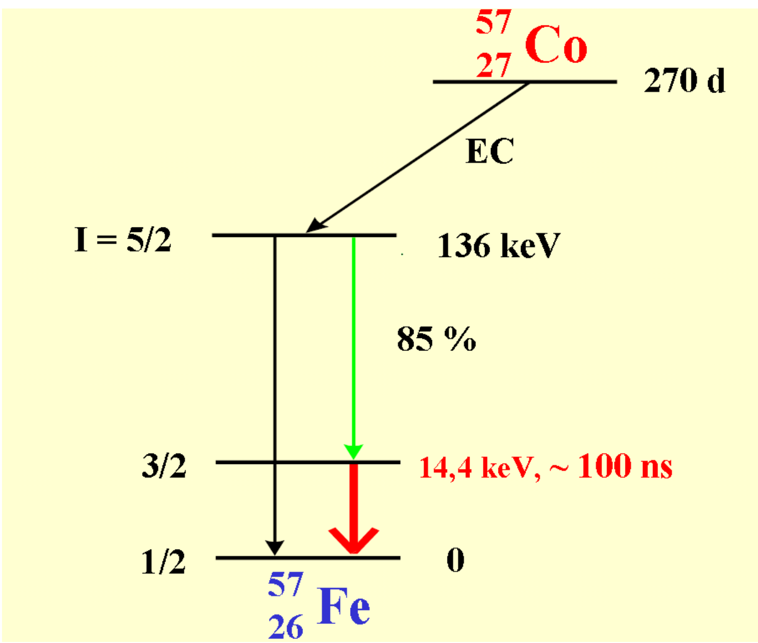
No espectro resultante, a intensidade dos raios gama é representada em função da velocidade da fonte. Para velocidades correspondentes aos níveis de energia ressonantes da amostra, uma parte dos raios gama é absorvida, o que resulta numa queda da intensidade medida e num pico para baixo, no espectro. O número, posição e intensidade dos picos, fornecem informação sobre o ambiente químico dos núcleos absorventes e podem ser usados para caracterizar a amostra.

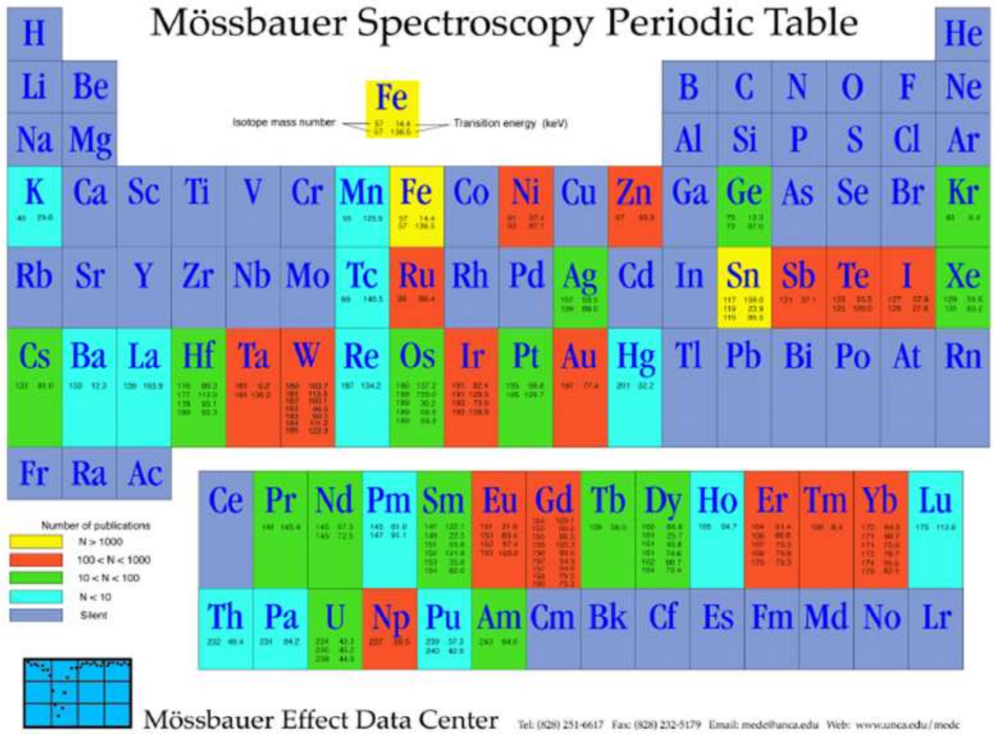


Esquema de um espectrómetro em geometria de transmissão

7. Escolha da fonte adequada

A espectroscopia de Mössbauer é limitada pela necessidade de uma fonte de raios gama adequada. Habitualmente, esta consiste num parente radioactivo que decai para o isótopo desejado. Por exemplo, a fonte de ^{57}Fe consiste em ^{57}Co, que decai por captura electrónica para um estado excitado do ^{57}Fe, o qual decai para o estado fundamental emitido o desejado raio gama. Idealmente o isótopo parente terá uma meia-vida suficientemente longa para ser útil, mas também terá de ter uma boa taxa de decaimento para se obter uma boa intensidade de radiação. A energia do raio gama deverá ser relativamente baixa para que o recuo não seja grande porque isto faz com que a relação sinal ruído seja má e sejam necessários longos tempos de medida.





8. Desvio de Doppler de segunda ordem

Quando um núcleo num cristal decai de um estado excitado para o seu estado fundamental por emissão sem recuo, o núcleo perde energia e a sua massa é reduzida de $\delta M = E/c^2$. A quantidade de movimento fica inalterada. Contudo, a energia cinética do átomo aumenta com a emissão do raio gama. Há uma diminuição da energia do fotão emitido da mesma quantidade. Essa diminuição é igual a

$$\delta E = \frac{1}{2}\frac{< v^2 >_T}{c^2} E$$

em que $<v^2>$ é a velocidade média quadrática do núcleo na rede, $< >_T$ denota a média térmica, c é a velocidade da luz e E a energia transferida.

Assumindo que a energia cinética é metade da energia média do átomo, de acordo com o modelo do sólido de Einstein, em que

$$< E > = \frac{3\hbar w_E}{\exp(\frac{\hbar w_E}{K T}) - 1}$$

Em que w_E é a frequência característica do sólido. Então,

$$\frac{1}{2} M < v^2 >_T = \frac{1}{2} < E >$$

onde M é a massa do átomo Mössbauer.

Assim, o desvio de temperatura relativista é dado por

$$\frac{\delta E}{E} = \frac{1}{2} \frac{< E >}{M c^2}$$

No limite clássico,

$$\frac{\partial}{\partial T} (\frac{\delta E}{E}) = c_p / 2 c^2$$

em que c_p é o calor específico.

Pound e Rebka em 1960 mostraram que o desvio de temperatura relativista ou desvio de Doppler de segunda ordem no ^{57}Fe existe. Boyle em 1960, mostrou que este efeito existe no ^{119}Sn. Eles verificaram a equação anterior experimentalmente. Simultaneamente, a partir de outro ponto de vista, Josephson, também em 1960, previu este efeito.

As vibrações dos átomos à temperatura ambiente são da ordem de 10^{13} Hz. Logo, um núcleo de ^{57}Fe no seu estado excitado oscila antes de decair com a velocidade média quadrática $\approx \frac{K T}{m}$ durante este período.

De acordo com a teoria especial da relatividade, um relógio movendo-se com o núcleo é mais lento relativamente a outro em repouso no laboratório pela fracção

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}$$

O centro da energia de uma linha de emissão de Mössbauer à temperatura T tem um desvio negativo da ordem de

$$< (\frac{v}{c})^2 > E = \frac{K T}{M c^2} E$$

O centro da energia de absorção é desviado de modo idêntico. Como já referido, este efeito foi descoberto na experiência do desvio gravitacional para o vermelho de Pound e Rebka. Eles tiveram especial atenção em manter a fonte e o absorvente à mesma temperatura numa fracção de 1º de modo que a diferença de temperatura não mascarasse o desvio gravitacional. O efeito da temperatura dá uma evidência clara do “paradoxo dos gémeos” da teoria da relatividade especial e resolve a dúvida de se as acelerações envolvidas nas viagens negam a dilatação do tempo sofrida pelos relógios em movimento.

9. Realização da experiência: Coeficiente de temperatura das linhas Mössbauer e o paradoxo dos gémeos

O efeito de temperatura é um desvio na frequência de ressonância de uma linha de absorção devido ao efeito de Doppler de segunda ordem que se relaciona com o atraso de um relógio atómico numa amostra aquecida, devido ao movimento térmico relativo ao relógio de referência no laboratório. De acordo com a teoria da relatividade, esse efeito é da ordem de 10^{-13} como referido anteriormente, pequeno, mas acessível à espectroscopia de Mössbauer. A estratégia da experiência é a seguinte:

a) operar o espectrómetro em muito alta dispersão e calibrá-lo medindo a separação entre os dois picos centrais do ^{57}Fe

b) medir o desvio fraccional na posição $\Delta E/E$ do pico único do aço inox, entre a temperatura ambiente T e $T + \Delta T$

c) comparar o resultado com a teoria de Josephson.

Material a utilizar

1- folha de ferro metálica
2- folha de aço inox
3- forno
4- controlador de temperatura e termopar

Procedimento

1- Reduzir a velocidade do motor de modo a que as duas componentes centrais do espectro do ^{57}Fe estejam separadas, digamos, de 400 canais, i.e, aumentar a dispersão do espectrómetro para que o pequeno efeito de temperatura seja capaz de provocar um desvio de vários canais nos centróides das linhas.

2- Calibrar a escala de velocidades com a folha de ferro metálica nesta velocidade.

3- Colocar a folha de aço inox dentro do forno. Adquirir um espectro à temperatura ambiente, com estatística suficiente para que as posições dos picos sejam determinadas com incerteza inferior a 11 canais.

4- Ajustar a temperatura do controlador de temperatura para 120ºC, ligar o forno e deixar estabilizar. Adquirir um espectro a esta temperatura com a mesma estatística.

5- Repetir as aquisições a baixa e alta temperatura várias vezes para diminuir e determinar o erro.

6- Determinar o coeficiente de temperatura a partir dos dados, i.e, a fracção variacional nas energias dos picos por grau centígrado, e comparar os resultados com as previsões teóricas.

Determinação da velocidade da luz pelo método de Foucault

Manual de instruções da "Pasco" (incluindo guião)

Instruction Manual and Experiment Guide for the PASCO scientific Model OS-9261A, 62 and 63A

SPEED OF LIGHT APPARATUS

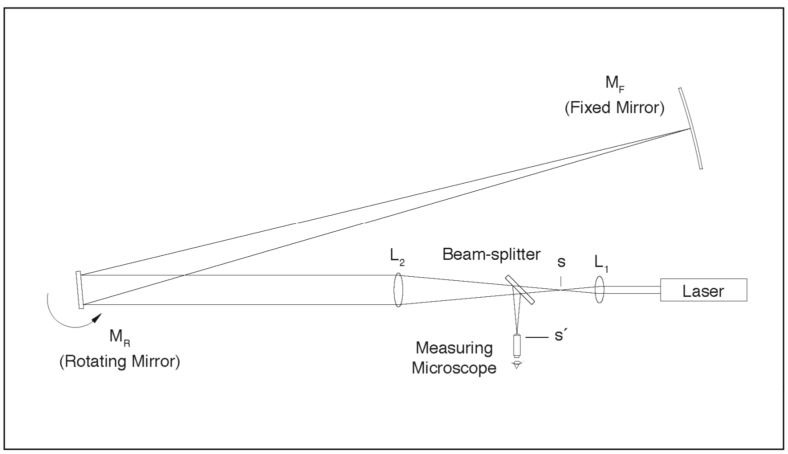


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Table of Contents

Introduction

The velocity of light in free space is one of the most important and intriguing constants of nature. Whether the light comes from a laser on a desk top or from a star that is hurtling away at fantastic speeds, if you measure the velocity of the light, you measure the same constant value. In more precise terminology, the velocity of light is independent of the relative velocities of the light source and the observer.

Furthermore, as Einstein first presented in his *Special Theory of Relativity*, the speed of light is critically important in some surprising ways. In particular:

1. The velocity of light establishes an upper limit to the velocity that may be imparted to any object.
2. Objects moving near the velocity of light follow a set of physical laws drastically different, not only from Newton's Laws, but from the basic assumptions of human intuition.

With this in mind, it's not surprising that a great deal of time and effort has been invested in measuring the speed of light. Some of the most accurate measurements were made by Albert Michelson between 1926 and 1929 using methods very similar to those you will be using with the PASCO Speed of Light Apparatus. Michelson measured the velocity of light in air to be 2.99712 x 10^8 m/sec. From this result he deduced the velocity in free space to be 2.99796 x 10^8 m/sec.

But Michelson was by no means the first to concern himself with this measurement. His work was built on a history of ever-improving methodology.

Measuring the Velocity of Light: History

Galileo

Through much of history, those few who thought to speculate on the velocity of light considered it to be infinite. One of the first to question this assumption was the great Italian physicist Galileo, who suggested a method for actually measuring the speed of light.

The method was simple. Two people, call them A and B, take covered lanterns to the tops of hills that are separated by a distance of about a mile. First A uncovers her lantern. As soon as B sees A's light, she uncovers her own lantern. By measuring the time from when A uncovers her lantern until A sees B's light, then dividing this time by twice the distance between the hill tops, the speed of light can be determined.

However, the speed of light being what it is, and human reaction times being what they are, Galileo was able to determine only that the speed of light was far greater than could be measured using his procedure. Although Galileo was unable to provide even an approximate value for the speed of light, his experiment set the stage for later attempts. It also introduced an important point: to measure great velocities accurately, the measurements must be made over a long distance.

Römer

The first successful measurement of the velocity of light was provided by the Danish astronomer Olaf Römer in 1675. Römer based his measurement on observations of the eclipses of one of the moons of Jupiter. As this moon orbits Jupiter, there is a period of time when Jupiter lies between it and the Earth, and blocks it from view. Römer noticed that the duration of these eclipses was shorter when the Earth was moving toward Jupiter than when the Earth was moving away. He correctly interpreted this phenomena as resulting from the finite speed of light.

Geometrically the moon is always behind Jupiter for the same period of time during each eclipse. Suppose, however, that the Earth is moving away from Jupiter. An astronomer on Earth catches his last glimpse of the moon, not at the instant the moon moves behind Jupiter, but only after the last bit of unblocked light from the moon reaches his eyes. There is a similar delay as the moon moves out from behind Jupiter but, since the Earth has moved farther away, the light must now travel a longer distance to reach the astronomer. The astronomer therefore sees an eclipse that lasts longer than the actual geometrical eclipse. Similarly, when the Earth is moving toward Jupiter, the astronomer sees an eclipse that lasts a shorter interval of time.

From observations of these eclipses over many years, Römer calculated the speed of light to be 2.1×10^8 m/sec. This value is approximately 1/3 too slow due to an inaccurate knowledge at that time of the distances involved. Nevertheless, Römer's method provided clear evidence that the velocity of light was not infinite, and gave a reasonable estimate of its true value—not bad for 1675.

Fizeau

The French scientist Fizeau, in 1849, developed an ingenious method for measuring the speed of light over terrestrial distances. He used a rapidly revolving cogwheel in front of a light source to deliver the light to a distant mirror in discrete pulses. The mirror reflected these pulses back toward the cogwheel. Depending on the position of the cogwheel when a pulse returned, it would either block the pulse of light or pass it through to an observer.

Fizeau measured the rates of cogwheel rotation that allowed observation of the returning pulses for carefully measured distances between the cogwheel and the mirror. Using this method, Fizeau measured the speed of light to be 3.15×10^8 m/sec. This is within a few percent of the currently accepted value.

Foucault

Foucault improved Fizeau's method, using a rotating mirror instead of a rotating cogwheel. (Since this is the method you will use in this experiment, the details will be discussed in considerable detail in the next section.) As mentioned, Michelson used Foucault's method to produce some remarkably accurate measurements of the velocity of light. The best of these measurements gave a velocity of 2.99774×10^8 m/sec. This may be compared to the presently accepted value of 2.99792458×10^8 m/sec.

The Foucault Method

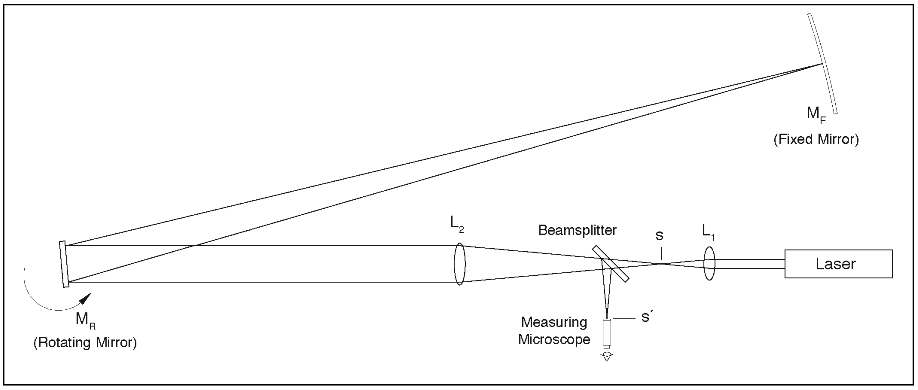


Figure 1: Diagram of the Foucault Method

A Qualitative Description

In this experiment, you will use a method for measuring the speed of light that is basically the same as that developed by Foucault in 1862. A diagram of the experimental setup is shown in Figure 1, above.

With all the equipment properly aligned and with the rotating mirror stationary, the optical path is as follows. The parallel beam of light from the laser is focused to a point image at point **s** by lens $\mathbf{L_1}$. Lens $\mathbf{L_2}$ is positioned so that the image point at s is reflected from the rotating mirror $\mathbf{M_R}$, and is focused onto the fixed, spherical mirror $\mathbf{M_F}$. $\mathbf{M_F}$ reflects the light back along the same path to again focus the image at point **s**.

In order that the reflected point image can be viewed through the microscope, a beam splitter is placed in the optical path, so a reflected image of the returning light is also formed at point **s´**.

Now, suppose $\mathbf{M_R}$ is rotated slightly so that the reflected beam strikes $\mathbf{M_F}$ at a different point. Because of the spherical shape of $\mathbf{M_F}$, the beam will still be reflected directly back toward $\mathbf{M_R}$. The return image of the source point will still be formed at points s and **s´**. The only significant difference in rotating $\mathbf{M_R}$ by a slight amount is that the point of reflection on $\mathbf{M_F}$ changes.

Now imagine that $\mathbf{M_R}$ is rotating continuously at a very high speed. In this case, the return image of the source point will no longer be formed at points **s** and **s´**. This is because, with $\mathbf{M_R}$ rotating, a light pulse that travels from $\mathbf{M_R}$ to $\mathbf{M_F}$ and back finds $\mathbf{M_R}$ at a different angle when it returns than when it was first reflected. As will be shown in the following derivation, by measuring the displacement of the image point caused by the rotation of $\mathbf{M_R}$, the velocity of light can be determined.

A Quantitative Description

In order to use the Foucault method to measure the speed of light, it's necessary to determine a precise relationship between the speed of light and the displacement of the image point. Of course, other variables of the experimental setup also affect the displacement. These include:

- the rate of rotation of $\mathbf{M_R}$
- the distance between $\mathbf{M_R}$ and $\mathbf{M_F}$
- the magnification of $\mathbf{L_2}$, which depends on the focal length of $\mathbf{L_2}$ and also on the distances between $\mathbf{L_2}$, $\mathbf{L_1}$, and $\mathbf{M_F}$.

Each of these variables will show up in the final expression that we derive for the speed of light.

To begin the derivation, consider a beam of light leaving the laser. It follows the path described in the qualitative description above. That is, first the beam is focused to a point at **s**, then reflected from $\mathbf{M_R}$ to $\mathbf{M_F}$, and back to $\mathbf{M_R}$. The beam then returns through the beamsplitter, and is refocused to a point at point **s´**, where it can be viewed through the microscope. This beam of light is reflected from a particular point on $\mathbf{M_F}$. As the first step in the derivation, we must determine how the point of reflection on $\mathbf{M_F}$ relates to the rotational angle of $\mathbf{M_R}$.

Figure 2a shows the path of the beam of light, from the laser to $\mathbf{M_F}$, when $\mathbf{M_R}$ is at an angle $\boldsymbol{\theta}$. In this case, the angle of incidence of the light path as it strikes $\mathbf{M_R}$ is also $\boldsymbol{\theta}$ and, since the angle of incidence equals the angle of reflection, the angle between the incident and reflected rays is just $\mathbf{2\theta}$. As shown in the diagram, the pulse of light strikes $\mathbf{M_F}$ at a point that we have labeled **S**.

Figure 2b shows the path of the pulse of light if it leaves the laser at a slightly later time, when $\mathbf{M_R}$ is at an angle $\boldsymbol{\theta_1 = \theta + \Delta\theta}$. The angle of incidence is now equal to $\boldsymbol{\theta_1 = \theta + \Delta\theta}$, so that the angle between the incident and reflected rays is just $\boldsymbol{2\theta_1 = 2(\theta + \Delta\theta)}$. This time we label the point where the pulse strikes $\mathbf{M_F}$ as $\mathbf{S_1}$. If we define **D** as the distance between $\mathbf{M_F}$ and $\mathbf{M_R}$, then the distance between **S** and $\mathbf{S_1}$ can be calculated:

$$\mathbf{S_1 - S = D(2\theta_1 - 2\theta) = D[2(\theta + \Delta\theta) - 2\theta] = 2D\Delta\theta \ (EQ1)}$$

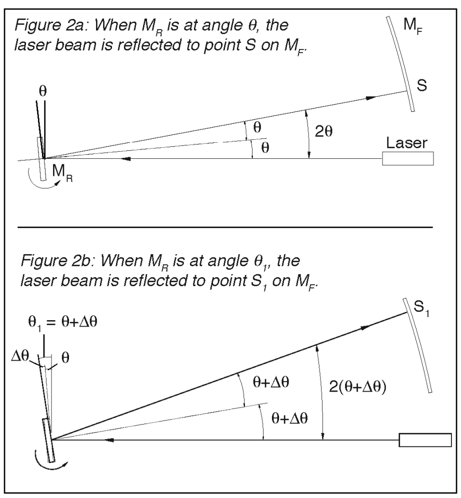


Figure 2a: When M_R is at angle θ, the laser beam is reflected to point S on M_F.

Figure 2b: When M_R is at angle θ_1, the laser beam is reflected to point S_1 on M_F.

Figure 2 a,b: The Reflection Point on M_F

In the next step in the derivation, it is helpful to think of a single, very quick pulse of light leaving the laser. Suppose $\mathbf{M_R}$ is rotating, and this pulse of light strikes $\mathbf{M_R}$ when it is at angle $\boldsymbol{\theta}$, as in Figure 2a. The pulse will then be reflected to point **S** on $\mathbf{M_F}$. However, by the time the pulse returns to $\mathbf{M_R}$, $\mathbf{M_R}$ will have rotated to a new angle, say angle $\boldsymbol{\theta_1}$. If $\mathbf{M_R}$ had not been rotating, but had remained stationary, this returning pulse of light would be refocused at point **s**. Clearly, since $\mathbf{M_R}$ is now in a different position, the light pulse will be refocused at a different point. We must now determine where that new point will be.

The situation is very much like that shown in Figure 2b, with one important difference: the beam of light that is returning to $\mathbf{M_R}$ is coming from point **S** on $\mathbf{M_F}$, instead of from point $\mathbf{S_1}$. To make the situation simpler, it is convenient to remove the confusion of the rotating mirror and the beam splitter by looking at the virtual images of the beam path, as shown in Figure 3.

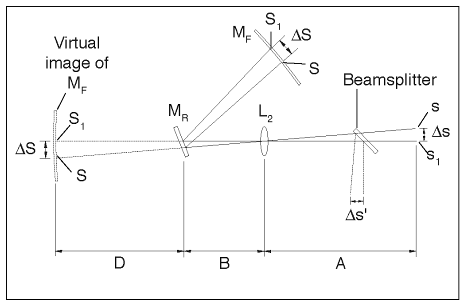


Figure 3: Analyzing the Virtual Images

The critical geometry of the virtual images is the same as for the reflected images. Looking at the virtual images, the problem becomes a simple application of thin lens optics. With $\mathbf{M_R}$ at angle $\boldsymbol{\theta_1}$, point $\mathbf{S_1}$ is on the focal axis of lens $\mathbf{L_2}$. Point **S** is in the focal plane of lens $\mathbf{L_2}$, but it is a distance $\mathbf{\Delta S = S_1 - S}$ away from the focal axis. From thin lens theory, we know that an object of height $\mathbf{\Delta S}$ in the focal plane of $\mathbf{L_2}$ will be focused in the plane of point **s** with a height of $\mathbf{(-i/o)\Delta S}$. Here **i** and **o** are the distances of the lens from the image and object, respectively, and the minus sign corresponds to the inversion of the image. As shown in Figure 3, reflection from the beam splitter forms a similar image of the same height.

Therefore, ignoring the minus sign since we aren't concerned that the image is inverted, we can write an expression for the displacement (**Δs´**) of the image point:

$$\Delta s' = \Delta s = (i/o)\Delta S = \frac{A}{D+B}\Delta S \qquad \textbf{(EQ2)}$$

Combining equations 1 and 2, and noting that **$\Delta S = S_1 - S$,** the displacement of the image point relates to the initial and secondary positions of $\mathbf{M_R}$ by the formula:

$$\Delta s' = \frac{2DA\,\Delta\theta}{D+B} \qquad \textbf{(EQ3)}$$

The angle Δθ depends on the rotational velocity of $\mathbf{M_R}$ and on the time it takes the light pulse to travel back and forth between the mirrors $\mathbf{M_R}$ and $\mathbf{M_F}$, a distance of **2D**. The equation for this relationship is:

$$\Delta\theta = \frac{2D\omega}{c} \qquad \textbf{(EQ4)}$$

where **c** is the speed of light and **ω** is the rotational velocity of the mirror in radians per second. (**2D/c** is the time it takes the light pulse to travel from $\mathbf{M_R}$ to $\mathbf{M_F}$ and back.)

Using equation 4 to replace **Δθ** in equation 3 gives:

$$\Delta s' = \frac{4AD^2\omega}{c(D+B)} \qquad \textbf{(EQ5)}$$

Equation 5 can be rearranged to provide our final equation for the speed of light:

$$c = \frac{4AD^2\omega}{(D+B)\,\Delta s'} \qquad \textbf{(EQ6)}$$

where:

c = the speed of light

w = the rotational velocity of the rotating mirror (M_R)

A = the distance between lens L_2 and lens L_1, minus the focal length of L_1

B = the distance between lens L_2 and the rotating mirror (M_R)

D = the distance between the rotating mirror (M_R) and the fixed mirror (M_F)

Δs´ = the displacement of the image point, as viewed through the microscope. (**$\Delta s' = s_1 - s$**; where **s** is the position of the image point when the rotating mirror (**M_R**) is stationary, and **s_1** is the position of the image point when the rotating mirror is rotating with angular velocity **ω**.)

Equation 6 was derived on the assumption that the image point is the result of a single, short pulse of light from the laser. But, looking back at equations 1-4, the displacement of the image point depends only on the *difference* in the angular position of M_R in the time it takes for the light to travel between the mirrors. The displacement does not depend on the specific mirror angles for any given pulse.

If we think of the continuous laser beam as a series of infinitely small pulses, the image due to each pulse will be displaced by the same amount. All these images displaced by the same amount will, of course, result in a single image. By measuring the displacement of this image, the rate of rotation of M_R, and the relevant distances between components, the speed of light can be measured.

The Equipment

What You Need to Measure the Speed of Light

In order to measure the speed of light as described in this manual, you will need all the items listed below (see Figure 4). If you have an OS-9261 Complete Speed of Light Apparatus, everything is included. If you have the OS-9262 Basic Speed of Light Apparatus or the OS-9263A High Speed Rotating Mirror, you will need additional components, as listed, to make the measurement.

The OS-9261 Complete Speed of Light Apparatus includes:

- OS-9262 Basic Speed of Light Apparatus, which includes:
  - –OS-9263A High Speed Rotating Mirror Assembly
  - –Fixed Mirror
  - –Measuring Microscope
- SE-9367 0.5 mW He-Ne Laser
- OS-9103 One-Meter Optics Bench
- OS-9172 Laser Alignment Bench
- OS-9142 Optics Bench Couplers
- OS-9133 Lens (48 mm FL)
- OS-9135 Lens (252 mm FL)
- OS-9109 Calibrated Polarizers (2)
- OS-9107 Component Holders (3)
- OS-8514 Laser Adapter Kit
- Alignment Jigs (2) – Part Number 648-02230

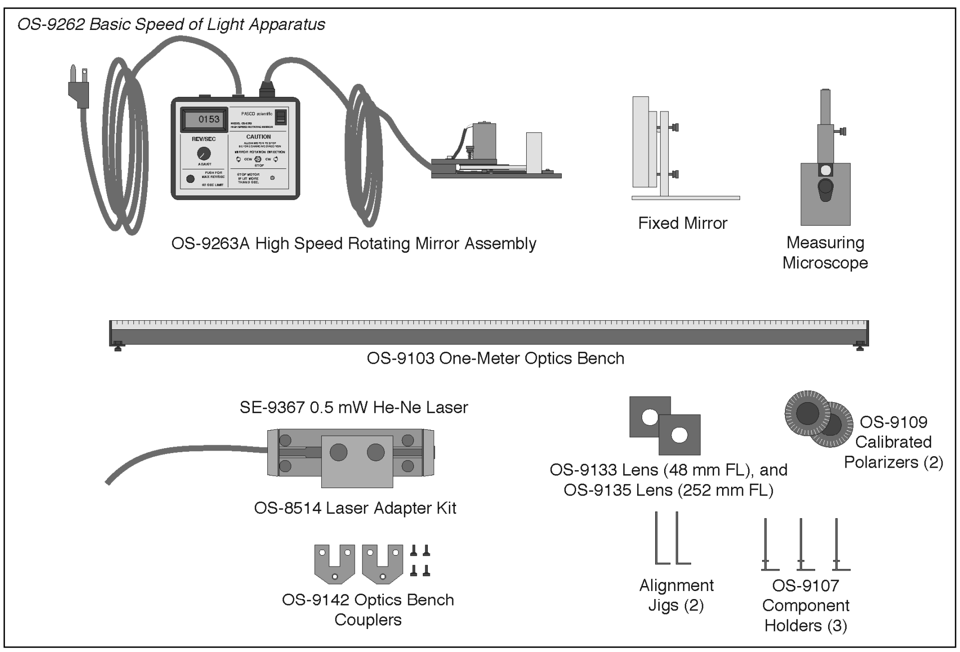


Figure 4: Equipment Included with the OS-9261A Complete Speed of Light Apparatus

About the Equipment

1. **High Speed Rotating Mirror Assembly**

The High Speed Rotating Mirror comes with its own power supply and digital display. The mirror is flat to within 1/4 wavelength. It's supported by high speed ball bearings, mounted in a protective housing, and driven by a DC motor with a drive belt. A plastic lock-screw lets you hold the mirror in place during the alignment procedure.

An optical detector and the digital display provide measurements of mirror rotation to within 0.1% or 1 rev/sec. The display and the controls for mirror rotation are on the front panel of the power supply. Rotation is reversible and the rate is continuously variable from 100 to 1,000 rev/sec. In addition, holding down the MAX REV/SEC button will bring the rotation speed quickly to its maximum value at approximately 1,500 rev/sec.

➤ **CAUTION:** Before turning on the motor for the rotating mirror, carefully read the cautionary notices in the section of this manual entitled "Making the Measurement".

2. **Measuring Microscope**

The 90X microscope is mounted on a micrometer stage for precise measurements of the displacement of the image point. Measurements are most easily made by visually centering the image point on the microscope cross-hairs before and after the displacement. By noting the change in the micrometer setting, the displacement can be resolved to within 0.005 mm.

To focus the cross-hairs, slide the eyepiece up or down in the microscope. To focus the microscope, loosen the lock-screw on the side of the mounting tube and slide the microscope up or down within the tube.

With the lock-screw loosened, the microscope can also be removed from the mounting tube. This can be helpful when you are trying to locate the image point. A piece of tissue paper placed over the tube provides a screen that allows you to view the point without focusing the microscope.

In addition to the microscope and micrometer, the micrometer stage also contains the beamsplitter. The lever on the side of the stage is used to adjust the angle of the beamsplitter. When the lever points directly down, the beamsplitter is at a forty-five degree angle.

3. **Fixed Mirror**

The Fixed Mirror is a spherical mirror with a radius of curvature of 13.5 meters. It is mounted to a stand and has separate x and y alignment screws.

4. **OS-9103 Optics Bench**

The 1.0 meter long Optics Bench provides a flat, level surface for aligning the optical components. The bench is equipped with a one meter scale, four leveling screws, and a magnetic top surface. The "fence", a raised edge on the back of the bench, provides a guide for aligning components along the optical axis.

5. **SE-9367 Laser with the OS-9172 Alignment Bench**

The 0.5 mW, TEM_{00} mode, random polarization laser has an output wavelength of 632.8 nm. The Alignment Bench attaches to the Optics Bench for precise, stable positioning of the laser.

6. **Alignment Jigs (2)**

These jigs mount magnetically to the Optics Bench. Each has a 2 mm diameter hole that is used to align the laser beam.

7. **Optical Components**

The use of the lenses and polarizers is described in the *Setup and Alignment* section of the manual.

Setup and Alignment

The following alignment procedure is tailored for those using the OS-9261A Complete Speed of Light Apparatus. For those using only some of the components in the complete system, the general procedure is the same, though the details depend on the optical components used.

> ➤ **IMPORTANT:** Proper alignment is critical, not only for getting good results, but for getting any results at all. Please follow this alignment procedure carefully. *Allow yourself about three hours to do it properly the first time*. Once you have set up the equipment a few times, you may find that the alignment summary at the end of this section is a helpful guide.

For reference as you set up the equipment, Figure 5 shows the approximate positioning of the components with respect to the metric scale on the side of the Optics Bench. The exact placement of each component depends on the position of the Fixed Mirror (**M_F**) and must be determined by following the steps of the alignment procedure described below.

All component holders, the Measuring Microscope, and the Rotating Mirror Assembly should be mounted flush against the "fence" of the Optics Bench (Figure 6). This will insure that all components are mounted at right angles to the beam axis.

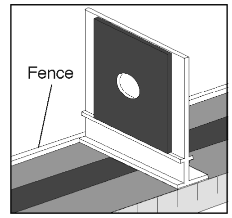


Figure 6: Placing Components Flush Against the Fence for Proper Alignment

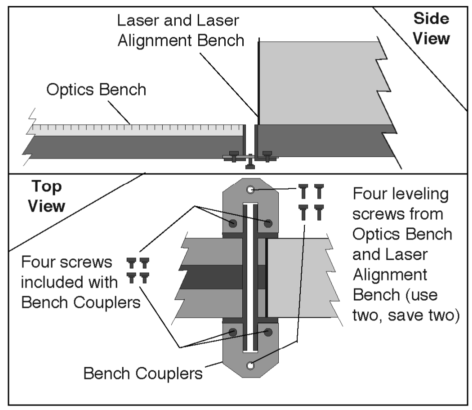


Figure 7: Coupling the Optics Bench and the Laser Alignment Bench

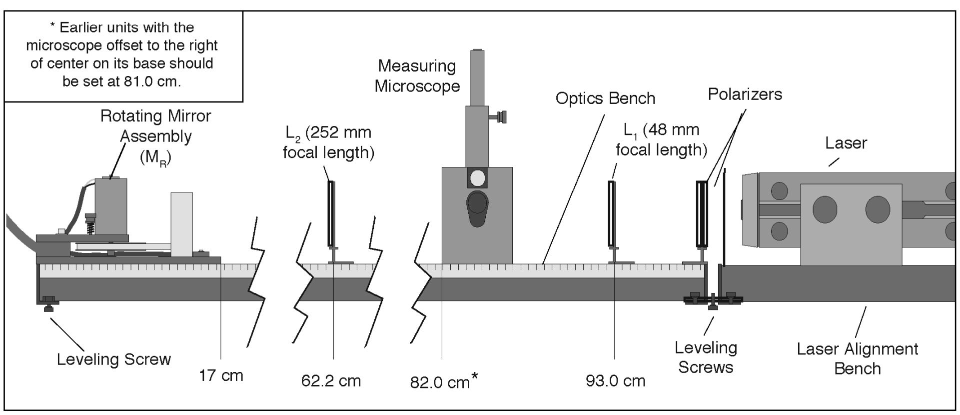


Figure 5: Equipment Alignment

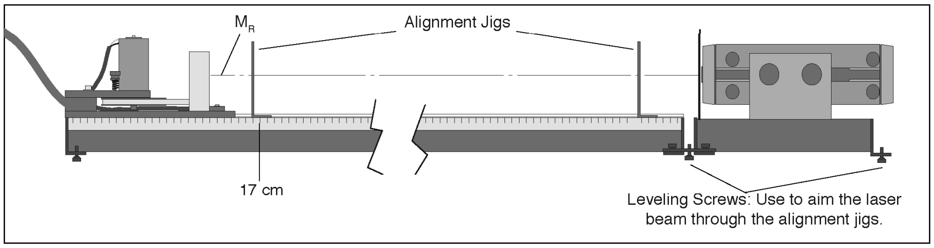


Figure 8: Using the Alignment Jigs to Align the Laser

To Set up and Align the Equipment:

1. Place the Optics Bench on a flat, level surface.
2. Place the Laser, mounted on the Laser Alignment Bench, end-to-end with the Optics Bench, at the end corresponding to the 1-meter mark of the metric scale.
3. Use the Bench Couplers and the provided screws to connect the Optics Bench and the Laser Alignment Bench. Details are shown in Figure 7. Do not yet tighten the screws holding the Bench Couplers.
   ➤ Note that the leveling screws must be removed from the Optics Bench and from the Laser Alignment Bench to attach the Bench Couplers. Two of the removed leveling screws are then inserted into the threaded holes in the Bench Couplers and are used for leveling.
4. Mount the Rotating Mirror Assembly on the opposite end of the bench. Be sure the base of the assembly is flush against the fence of the Optics Bench and align the front edge of the base with the 17 cm mark on the metric scale of the Optics Bench (see Figure 8).
5. The laser must be aligned so the beam strikes the center of the Rotating Mirror (M_R). Two alignment jigs are provided for this purpose. Place one jig at each end of the Optics Bench as shown in Figure 8, with the edges flush against the fence of the bench. When properly placed, the holes in the jigs define a straight line that is parallel to the axis of the Optics Bench.
6. Turn on the Laser.

> ➤ **CAUTION:** Do not look into the laser beam, either directly or as it reflects from either mirror. Also, when arranging the equipment, be sure the beam path does not traverse an area where someone might inadvertently look into the beam.

7. Adjust the position of the front of the laser so the beam passes directly through the hole in the first jig. (Use the two front leveling screws to adjust the height. Adjust the position of the laser on the Laser Alignment Bench to adjust the lateral position.) Then adjust the height and position of the rear of the laser so the beam passes directly through the hole in the second jig.

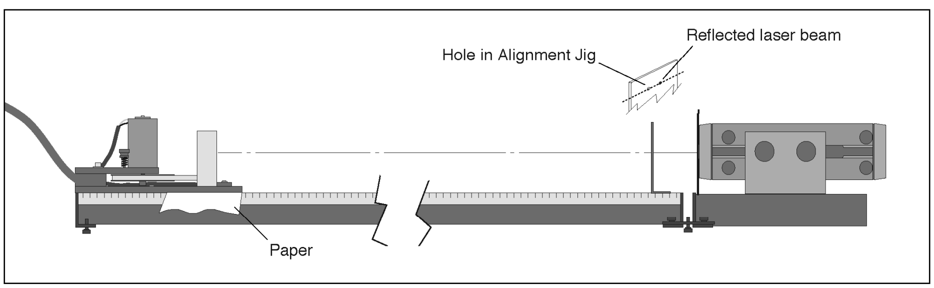


Figure 9: Aligning the Rotating Mirror (M_R)

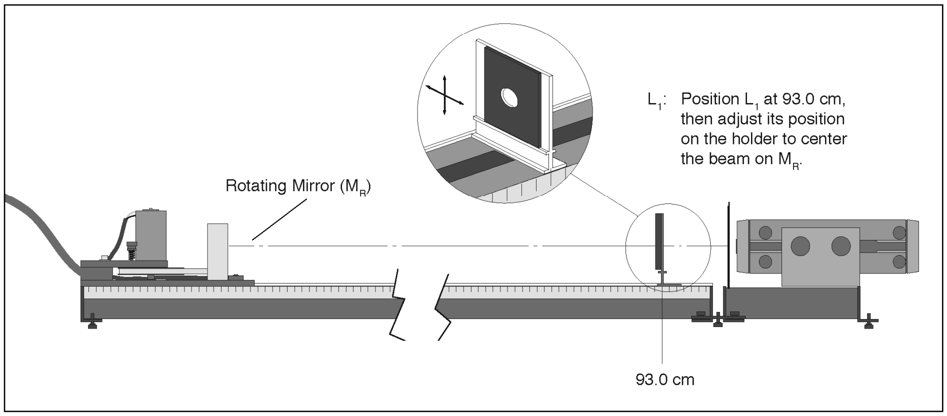


Figure 10: Positioning and Aligning L_1

8. To fix the laser in position with respect to the Optics Bench, tighten the screws on the Bench Couplers. Then recheck the alignment of the laser.

9. Align the Rotating Mirror. $\mathbf{M_R}$ must be aligned so that its axis of rotation is vertical and also perpendicular to the laser beam. To accomplish this, remove the second alignment jig and then rotate $\mathbf{M_R}$ so that the laser beam reflects back toward the hole in the first alignment jig (Figure 9). Be sure to use the reflective side of the mirror. It helps to tighten the lockscrew on the rotating mirror assembly just enough so $\mathbf{M_R}$ holds its position as you adjust its rotation.

   If needed, use pieces of paper to shim between the Rotating Mirror Assembly and the Optics Bench so that the laser beam is reflected back through the hole in the first jig.

10. Remove the first alignment jig.

11. Mount the 48 mm focal length lens ($\mathbf{L_1}$) on the Optics Bench so that the center line of the Component Holder is aligned with the 93.0 cm mark on the metric scale of the bench. Without moving the Component Holder, slide $\mathbf{L_1}$ as needed on the holder to center the beam on $\mathbf{M_R}$ (see Figure 10). Notice that L_1 has spread the beam at the position of $\mathbf{M_R}$.

12. Mount the 252 mm focal length lens ($\mathbf{L_2}$) on the Optics Bench so the center line of the Component Holder aligns with the 62.2 cm mark on the metric scale of the bench. As for $\mathbf{L_1}$ in step 11, adjust the position of $\mathbf{L_2}$ on the Component Holder so that the beam is again centered on $\mathbf{M_R}$.

13. Place the Measuring Microscope on the Optics Bench so that the left edge of the mounting stage is aligned with the 82.0 cm mark on the bench (see Figure 5). The lever that adjusts the tilt of the beam splitter should be on the same side as the metric scale of the Optics Bench. Position this lever so it points directly down.

> ➤ **CAUTION:** Do not look through the microscope until the polarizers have been placed between the laser and the beam splitter in step 19.

   The beamsplitter will slightly alter the position of the laser beam. Readjust $\mathbf{L_2}$ on the Component Holder so the beam is again centered on $\mathbf{M_R}$.

14. Place the Fixed Mirror ($\mathbf{M_F}$) from 2 to 15 meters from M_R, as shown in Figure 11. The angle between the axis of the Optics Bench and a line from $\mathbf{M_R}$ to $\mathbf{M_F}$ should be approximately 12 degrees. (If it is greater than 20-degrees, the reflected beam will be blocked by the Rotating Mirror enclosure.) Also be sure that $\mathbf{M_F}$ is not on the same side of the optical bench as the micrometer knob, so you will be able to make the measurements without blocking the beam.

> ➤ **NOTE:** Best results are obtained when $\mathbf{M_F}$ is 10 to 15 meters from $\mathbf{M_R}$. See *Notes on Accuracy* near the end of the manual.

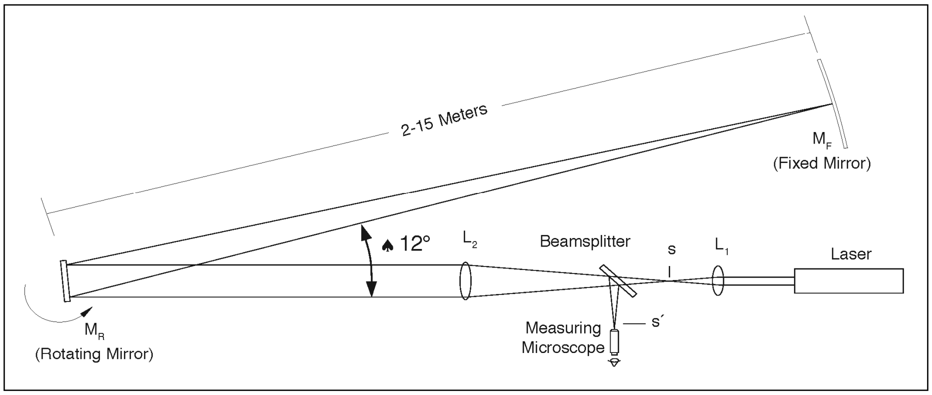


Figure 11: Positioning the Fixed Mirror (M_F)

15. Position $\mathbf{M_R}$ so the laser beam is reflected toward $\mathbf{M_F}$. Place a piece of paper in the beam path and "walk" the beam toward $\mathbf{M_F}$, adjusting the rotation of $\mathbf{M_R}$ as needed.
16. Adjust the position of $\mathbf{M_F}$ so the beam strikes it approximately in the center. Again, a piece of paper in the beam path will make the beam easier to see.
17. With a piece of paper still against the surface of $\mathbf{M_F}$, slide $\mathbf{L_2}$ back and forth along the Optics Bench to focus the beam to the smallest possible point on $\mathbf{M_F}$.
18. Adjust the two alignment screws on the back of $\mathbf{M_F}$ so the beam is reflected directly back to the center of $\mathbf{M_R}$. This step is best performed with two people: one adjusting $\mathbf{M_F}$, and one watching the beam position at $\mathbf{M_R}$.
19. Place the polarizers (attached to either side of a single Component Holder) between the laser and $\mathbf{L_1}$. Begin with the polarizers at right angles to each other, than rotate one until the image in the microscope is bright enough to view comfortably.

If you can't find the point image there are several things you can try:

- Vary the tilt of the beamsplitter slightly (no more than a few degrees) and turn the micrometer knob to vary the transverse position of the microscope until the image comes into view.
- Loosen the lock-screw on the microscope. As shown in Figure 13, remove the microscope and place a piece of tissue paper over the tube to locate the beam. Adjust the beamsplitter angle and the micrometer knob to center the point image in the tube of the microscope.
- Slide the Measuring Microscope a centimeter or so in either direction along the axis of the Optics Bench. Be sure that the Microscope stays flush against the fence of the Optics Bench. If this doesn't work, recheck the alignment, beginning with step 1.

20. Bring the cross-hairs of the microscope into focus by sliding the microscope eyepiece up and down.
21. Focus the microscope by loosening the lock-screw and sliding the scope up and down. If the apparatus is properly aligned, you will see the point image through the microscope. Focus until the image is as sharp as possible.

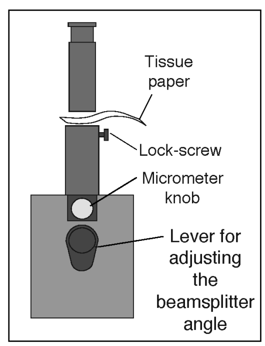


Figure 13: Looking for the Beam Image

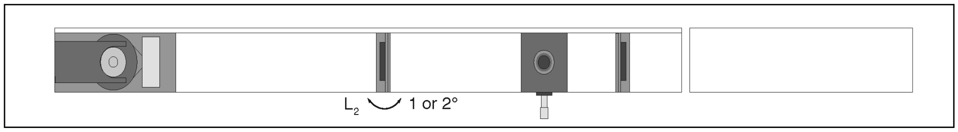


Figure 12: Turning L_2 Slightly Askew to Clean Up the Image

➤ **IMPORTANT:** In addition to the point image, you may also see some extraneous beam images resulting, for example, from reflection of the laser beam from $\mathbf{L_1}$. To be sure you are observing the right image point, place a piece of paper between $\mathbf{M_R}$ and $\mathbf{M_F}$ while you watch the image in the microscope. If the point does not disappear, it is not the correct image.

Cleaning Up the Image

22. In addition to the point image, you may also see interference fringes through the microscope (as well as the extraneous beam images mentioned above). These fringes cause no difficulty as long as the point image is clearly visible. However, the fringes and extraneous beam images can sometimes be removed without losing the point image. This is accomplished by turning L2 slightly askew, so it is no longer quite at a right angle to the beam axis (see Figure 12).

Alignment Summary

(see Figure 14 for approximate component placement)

This summary is for those who are familiar with the equipment and the experiment, and just need a quick reminder of the steps in the alignment procedure. If you have not successfully aligned the apparatus before, we recommend that you take the time to go through the detailed alignment procedure in the preceding section.

1. Align the laser so the laser beam strikes the center of $\mathbf{M_R}$ (use the alignment jigs).
2. Adjust the rotational axis of $\mathbf{M_R}$ so it is perpendicular to the beam (i.e. as $\mathbf{M_R}$ rotates, there must be a position at which it reflects the laser beam directly back into the laser aperture).
3. Insert $\mathbf{L_1}$ to focus the laser beam to a point. Adjust $\mathbf{L_1}$ so the beam is still centered on $\mathbf{M_R}$.
4. Insert $\mathbf{L_2}$ and adjust it so the beam is still centered on $\mathbf{M_R}$.
5. Place the Measuring Microscope in position and, again, be sure that the beam is still centered on $\mathbf{M_R}$.

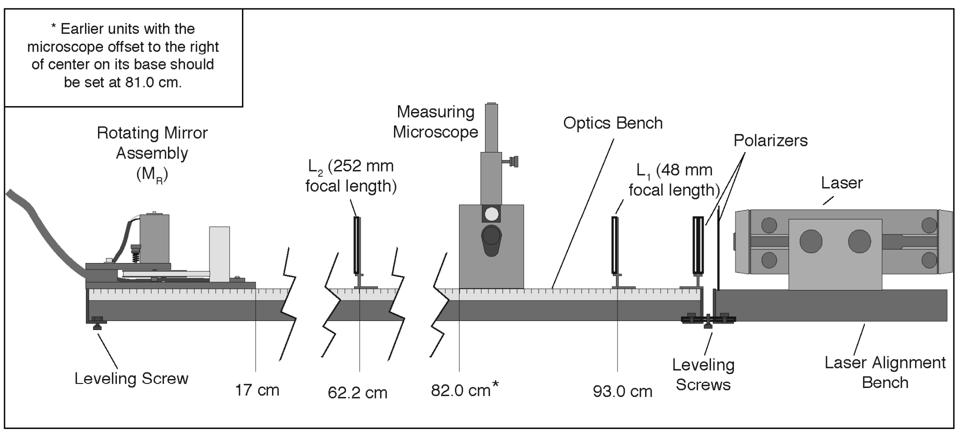


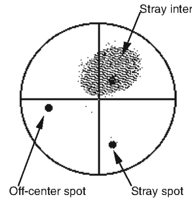
Figure 14: Equipment Alignment

> ➤ **CAUTION:** Do not look through the microscope until the polarizers have been placed between the laser and the beamsplitter.

6. Position $\mathbf{M_F}$ at the chosen distance from $\mathbf{M_R}$ (2 - 15 meters), so the reflected image from $\mathbf{M_R}$ strikes the center of $\mathbf{M_F}$.
7. Adjust the position of $\mathbf{L_2}$ to focus the beam to a point on $\mathbf{M_F}$.
8. Adjust $\mathbf{M_F}$ so the beam is reflected directly back onto $\mathbf{M_R}$.
9. Insert the polarizers between the laser and the beam splitter.
10. Focus the microscope on the image point.
11. Remove polarizers.

Alignment Hints

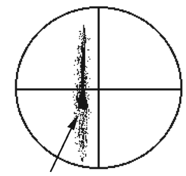
Once you have the microscope focused, it may still be difficult to obtain a good spot. There may be several other lights visible in the microscope besides the spot reflected from the fixed mirror.



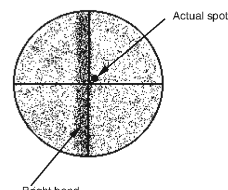
The most common of these are stray interference patterns. These are caused by multiple reflections from the surfaces of the lenses, and may be ignored. If necessary, you may be able to eliminate them by angling the lenses 1 – 2°.

Stray Spots are most often caused by reflections off the window of the rotating mirror housing. To determine which spot is the one you must measure, block the beam path between the rotating mirror and the fixed mirror. The relevant spot will disappear.

If the spot you need to measure is significantly off-center, you can move it by adjusting the angle of the beamsplitter.



Another common problem is a spot that is "stretched" with no easily discernible maxima. Check first to make sure that this is the spot you need by blocking the beam path between the moving and fixed mirrors. If it is, then twist L_2 slightly until the image coalesces into a single spot.



Once the mirror begins to rotate, it is safe to look into the microscope without the polarizers. You will notice that your carefully aligned pattern has changed: now the entire field is covered with a random interference pattern, and there is a bright band down the center of the field. Ignore the interference pattern; there's nothing you can do about it anyway. The band is the image of the laser when, once each rotation, the mirror reflects it into the microscope beamsplitter. This is also unavoidable.

Your actual spot will probably be just to one side of the bright band. You can check for it by blocking and unblocking the beam path between the rotating mirror and fixed mirror and watching to see what disappears.

If you aligned everything ***perfectly***, the spot will be hidden by the bright band; in this case, make sure that you have a spot when the rotating mirror is fixed and is reflecting the laser to the fixed mirror. If you do have the correct spot under stationary conditions, then misalign the fixed mirror *very slightly* (0.004° or less) around the horizontal axis. This will bring the actual spot out from under the bright band.

Making the Measurement

The speed of light measurement is made by rotating the mirror at high speeds and using the microscope and micrometer to measure the corresponding deflection of the image point. By rotating the mirror first in one direction, then in the opposite direction, the total beam deflection is doubled, thereby doubling the accuracy of the measurement.

> ➤ **Important—to Protect the Rotating Mirror Assembly:**
>
> – Before turning on the motor, be sure the lock-screw for the rotating mirror is completely loosened, so the mirror rotates freely by hand.
>
> – Whenever the speed of the motor is accelerated, the red LED on the front panel of the motor control box will light up. As the speed stabilizes, this light should go off. If it does not, turn off the motor. Something is interfering with the motor rotation. Check to be sure the lock-screw for $\mathbf{M_R}$ is fully loosened.
>
> – Never run the motor with the MAX REV/SEC button pushed for more than one minute at a time, and always allow about a minute between runs for the motor to cool off.

1. With the apparatus aligned and the beam image in sharp focus (see the previous section), set the direction switch on the rotating mirror power supply to CW, and turn on the motor. If the image was not in sharp focus, adjust the microscope. You should also turn $\mathbf{L_2}$ slightly askew (about 1 - 2°) to improve the image. To get the best image you may need to adjust the microscope and $\mathbf{L_2}$ several times. Let the motor warm up at about 600 revolutions/sec for at least 3 minutes.
2. Slowly increase the speed of rotation. Notice how the beam deflection increases.
3. Use the ADJUST knob to bring the rotational speed up to about 1,000 revolutions/sec. Then push the MAX REV/SEC button and hold it down. When the rotation speed stabilizes, rotate the micrometer knob on the microscope to align the center of the beam image with the cross hair in the microscope that is perpendicular to the direction of deflection. Record the speed at which the motor is rotating, turn off the motor, and record the micrometer reading.

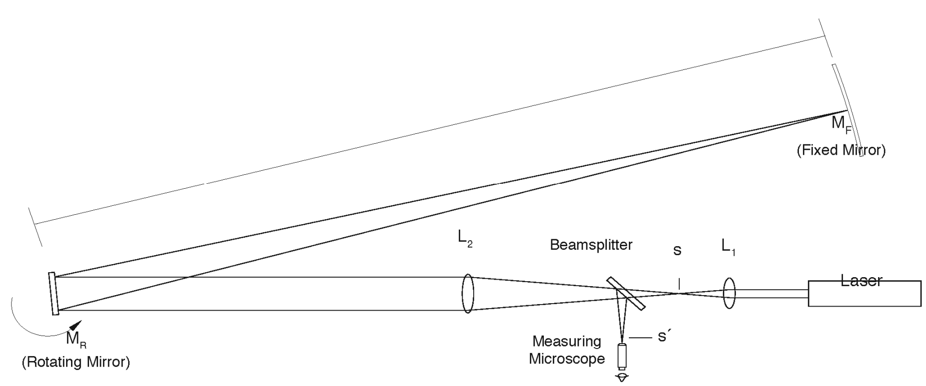


Figure 15: Diagram of the Foucault Method

➤ **NOTE:**

When reversing the direction of movement of the micrometer carriage, there will always be some movement of the micrometer knob before the carriage responds. Though this source of error is small, it can be eliminated. Just adjust the initial position of the micrometer stage so that you always turn the micrometer knob in the same direction as you adjust it.

Reverse the direction of the mirror rotation by switching the direction switch on the power supply to CCW. Allow the mirror to come to a complete stop before reversing the direction. Then repeat your measurement as in step 3.

➤ **NOTES:**

- When the mirror is rotated at 1,000 rev/sec or more, the image point will widen in the direction of displacement. Position the microscope cross-hair in the center of the resulting image.
- The micrometer on the Measuring Microscope is graduated in increments of 0.01 mm for the beam deflections.

5. The following equation was derived earlier in the manual:

$$c = \frac{4AD^2\omega}{(D+B)\,\Delta s'}$$

When adjusted to fit the parameters just measured, it becomes:

$$c = \frac{8\pi AD^2(Rev/sec_{cw} - Rev/sec_{ccw})}{(D+B)(s'_{cw} - s'_{ccw})}$$

Use this equation, along with the diagram in Figure 15, to calculate c, the speed of light. (To measure A, measure the distance between $\mathbf{L_1}$ and $\mathbf{L_2}$, then subtract the focal length of $\mathbf{L_1}$, 48 mm.)

➤ **NOTES:**

This equation is the same as the original equation in step 5, but with two differences:

The rotational velocity is expressed in rad/s.

The CCW rotational velocity is expressed as a negative number, reflecting the direction of rotation.

$$c = \frac{4AD^2(\omega_{cw} - \omega_{ccw})}{(D+B)(s'_{cw} - s'_{ccw})}$$

Radiação térmica

Manual de instruções da "Pasco" (incluindo guião)

Includes Teacher's Notes and Typical Experiment Results

Instruction Manual and Experiment Guide for the PASCO scientific Model TD-8553/8554A/8555

012-04695D
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THERMAL RADIATION SYSTEM

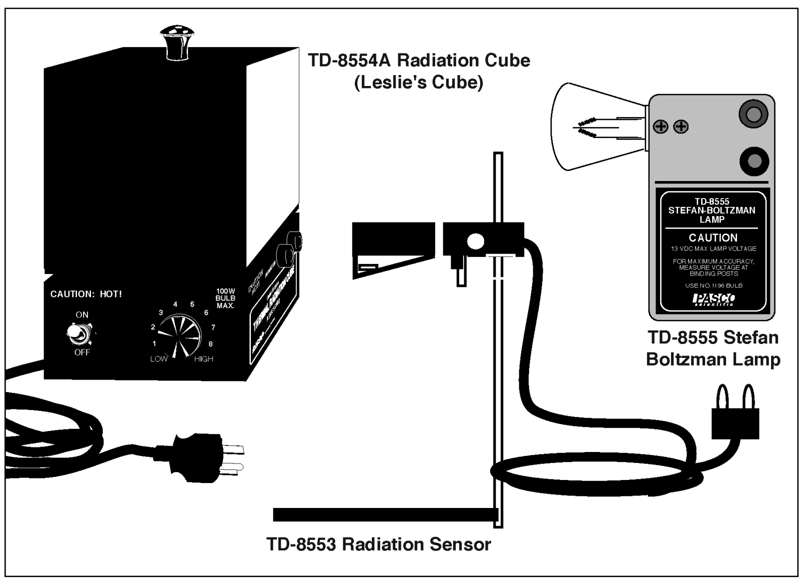

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Table of Contents

Introduction

The PASCO Thermal Radiation System includes three items: the TD-8553 Radiation Sensor, the TD-8554A Radiation Cube (Leslie's Cube), and the TD-8555 Stefan-Boltzmann Lamp. This manual contains operating instructions for each of these items plus instructions and worksheets for the following four experiments:

① Introduction to Thermal Radiation,

② Inverse Square Law,

③ Stefan-Boltzmann Law* (at high temperatures),

④ Stefan-Boltzmann Law* (at low temperatures).

* The Stefan-Boltzmann law states that the radiant energy per unit area is proportional to the fourth power of the temperature of the radiating surface.

In addition to the equipment in the radiation system, several standard laboratory items, such as power supplies and meters are needed for most experiments. Check the experiment section of this manual for information on required equipment.

If you don't have all the items of the radiation system, read through the operating instructions for the equipment you do have, then check the experiment section to determine which of the experiments you can perform. (A radiation sensor is required for all the experiments.)

Radiation Sensor

The PASCO TD-8553 Radiation Sensor (Figure 1) measures the relative intensities of incident thermal radiation. The sensing element, a miniature thermopile, produces a voltage proportional to the intensity of the radiation. The spectral response of the thermopile is essentially flat in the infrared region (from 0.5 to 40 μm), and the voltages produced range from the microvolt range up to around 100 millivolts. (A good millivolt meter is sufficient for all the experiments described in this manual. See the current PASCO catalog for recommended meters.)

The Sensor can be hand held or mounted on its stand for more accurate positioning. A spring-clip shutter is opened and closed by sliding the shutter ring forward or back. During experiments, the shutter should be closed when measurements are not actively being taken. This helps reduce temperature shifts in the thermopile reference junction which can cause the sensor response to drift.

> ➤ **NOTE:** When opening and closing the shutter, it is possible you may inadvertently change the sensor position. Therefore, for experiments in which the sensor position is critical, such as Experiment 3, two small sheets of opaque insulating foam have been provided. Place this heat shield in front of the sensor when measurements are not actively being taken.

The two posts extending from the front end of the Sensor protect the thermopile and also provide a reference for positioning the sensor a repeatable distance from a radiation source.

Specifications

Temperature Range: -65 to 85 °C.
Maximum Incident Power: 0.1 Watts/cm^2.
Spectral Response: .6 to 30µm.
Signal Output: Linear from 10^{-6} to 10^{-1} Watts/cm^2.

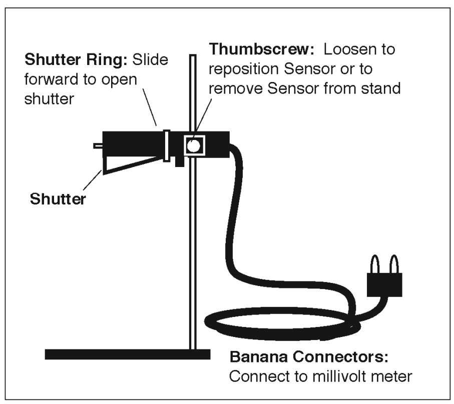


Figure 1 Radiation Sensor

Thermal Radiation Cube (Leslie's Cube)

The TD-8554A Radiation Cube (Figure 2) provides four different radiating surfaces that can be heated from room temperature to approximately 120 °C. The cube is heated by a 100 watt light bulb. Just plug in the power cord, flip the toggle switch to "ON", then turn the knob clockwise to vary the power.

Measure the cube temperature by plugging your ohmmeter into the banana plug connectors labeled THERMISTOR. The thermistor is embedded in one corner of the cube. Measure the resistance, then use Table 1, below, to translate the resistance reading into a temperature measurement. An abbreviated version of this table is printed on the base of the Radiation Cube.

➤ **NOTE:** For best results, a digital ohmmeter should be used. (See the current PASCO catalog for recommended meters.)

➤ **IMPORTANT:** When replacing the light bulb, use a 100-Watt bulb. Bulbs of higher power could damage the cube.

CAUTION: Cube may be HOT!

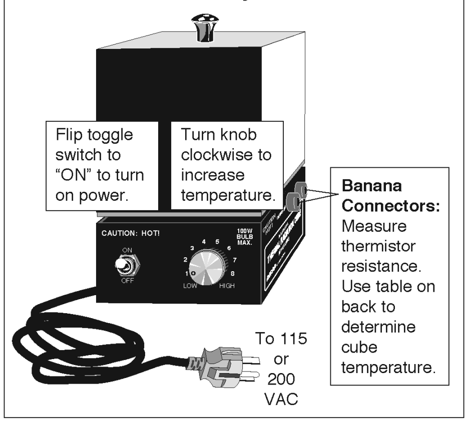


Figure 2 Radiation Cube (Leslie's Cube)

Table 1

Resistance versus Temperature for the Thermal Radiation Cube

Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

Stefan-Boltzmann Lamp

IMPORTANT: The voltage into the lamp should **NEVER exceed 13 V**. Higher voltages will burn out the filament.

The TD-8555 Stefan-Boltzmann Lamp (Figure 3) is a high temperature source of thermal radiation. The lamp can be used for high temperature investigations of the Stefan-Boltzmann Law. The high temperature simplifies the analysis because the fourth power of the ambient temperature is negligibly small compared to the fourth power of the high temperature of the lamp filament (see Experiments 3 and 4). When properly oriented, the filament also provides a good approximation to a point source of thermal radiation. It therefore works well for investigations into the inverse square law.

By adjusting the power into the lamp (13 Volts max, 2 A min, 3 A max), filament temperatures up to approximately 3,000 °C can be obtained. The filament temperature is determined by carefully measuring the voltage and current into the lamp. The voltage divided by the current gives the resistance of the filament.

Equipment Recommended

AC/DC LV Power Supply (SF-9584) or equivalent capable of 13 V @ 3 A max

$$T = \frac{R - R_{ref}}{\alpha R_{ref}} + T_{ref}$$

For small temperature changes, the temperature of the tungsten filament can be calculated using **a**, the temperature coefficient of resistivity for the filament:

where,

T = Temperature

R = Resistance at temperature T

T_{ref} = Reference temperature (usually room temp.)

R_{ref} = Resistance at temperature T_{ref}

α = Temperature coefficient of resistivity for the filament ($\alpha = 4.5 \times 10^{-3}\ K^{-1}$ for tungsten)

For large temperature differences, however, **a** is not constant and the above equation is not accurate.

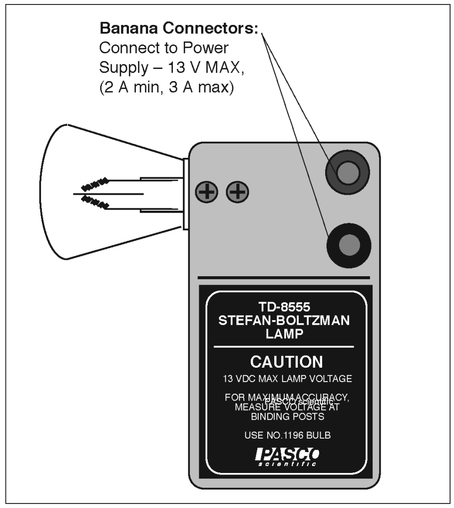


Figure 3 Stefan-Boltzmann Lamp

REPLACEMENT BULB: GE Lamp No. 1196, available at most auto parts stores.
➤ **NOTE:** When replacing the bulb, the leads should be soldered to minimize resistance.

For large temperature differences, therefore, determine the temperature of the tungsten filament as follows:

① Accurately measure the resistance (R_{ref}) of the tungsten filament at room temperature (about 300 °K). Accuracy is important here. A small error in R_{ref} will result in a large error in your result for the filament temperature.

② When the filament is hot, measure the voltage and current into the filament and divide the voltage by the current to measure the resistance (R_T).

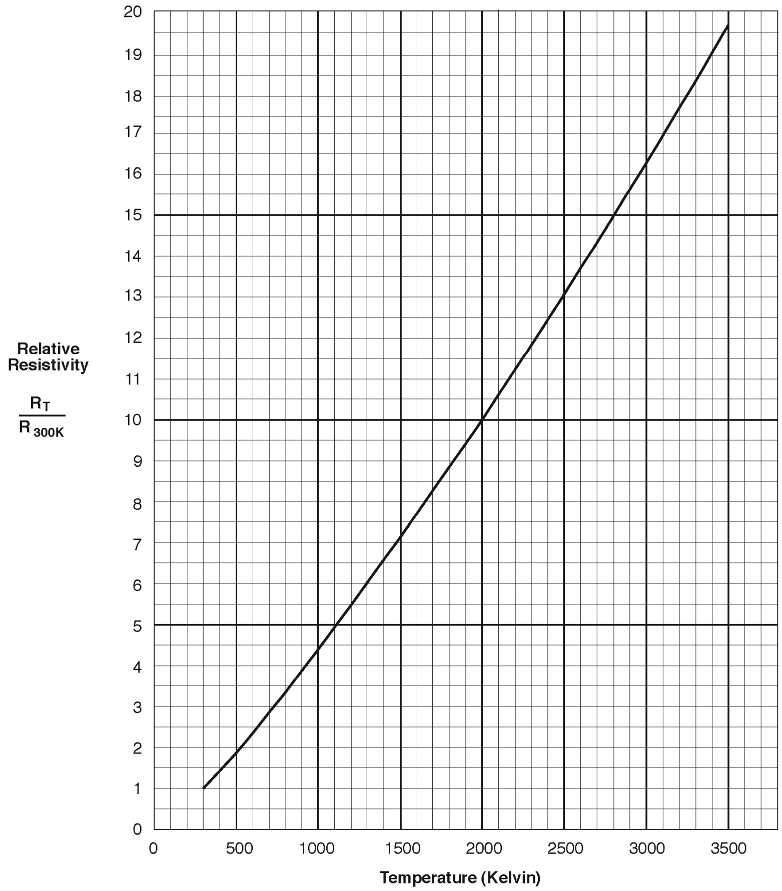
③ Divide R_T by R_{ref} to obtain the relative resistance (R_T/R_{ref}).

④ Using your measured value for the relative resistivity of the filament at temperature T, use Table 2 on the following page, or the associated graph, to determine the temperature of the filament.

Table 2 Temperature and Resistivity for Tungsten

R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

Temperature versus Resistivity for Tungsten



Experiment 1: Introduction to Thermal Radiation

EQUIPMENT NEEDED:

— Radiation Sensor, Thermal Radiation Cube
— Millivoltmeter
— Window glass
— Ohmmeter.

➤ **NOTES:**

① If lab time is short, it's helpful to preheat the cube at a setting of 5.0 for 20 minutes before the laboratory period begins. (A very quick method is to preheat the cube at full power for 45 minutes, then use a small fan to reduce the temperature quickly as you lower the power input. Just be sure that equilibrium is attained with the fan off.)

② Part 1 and 2 of this experiment can be performed simultaneously. Make the measurements in Part 2 while waiting for the Radiation Cube to reach thermal equilibrium at each of the settings in Part 1.

③ When using the Radiation Sensor, always shield it from the hot object except for the few seconds it takes to actually make the measurement. This prevents heating of the thermopile which will change the reference temperature and alter the reading.

Radiation Rates from Different Surfaces

Part 1

① Connect the Ohmmeter and Millivoltmeter as shown in Figure 1.1.

② Turn on the Thermal Radiation Cube and set the power switch to “HIGH”. Keep an eye on the ohmmeter reading. When it gets down to about 40 kΩ, reset the power switch to 5.0. (If the cube is preheated, just set the switch to 5.0.)

③ When the cube reaches thermal equilibrium—the ohmmeter reading will fluctuate around a relatively fixed value—use the Radiation Sensor to measure the radiation emitted from each of the four surfaces of the cube. Place the Sensor so that the posts on its end are in contact with the cube surface (this ensures that the distance of the measurement is the same for all surfaces). Record your measurements in the appropriate table on the following page. Also measure and record the resistance of the thermistor. Use the table on the base of the cube to determine the corresponding temperature.

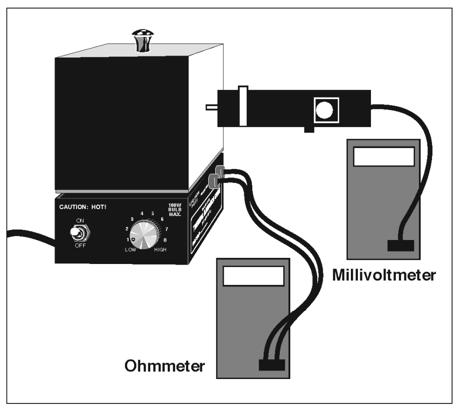


Figure 1.1 Equipment Setup

④ Increase the power switch setting, first to 6.5, then to 8.0, then to “HIGH”. At each setting, wait for the cube to reach thermal equilibrium, then repeat the measurements of step 1 and record your results in the appropriate table.

Part 2

Use the Radiation Sensor to examine the relative magnitudes of the radiation emitted from various objects around the room. On a separate sheet of paper, make a table summarizing your observations. Make measurements that will help you to answer the questions listed below.

Absorption and Transmission of Thermal Radiation

① Place the Sensor approximately 5 cm from the black surface of the Radiation Cube and record the reading. Place a piece of window glass between the Sensor and the bulb. Does window glass effectively block thermal radiation?

② Remove the lid from the Radiation Cube (or use the Stefan-Boltzmann Lamp) and repeat the measurements of step 1, but using the bare bulb instead of the black surface. Repeat with other materials.

Radiation Rates from Different Surfaces

Data and Calculations

Power Setting 5.0

Therm. Res. ________ Ω
Temperature ________ °C

Surface	**Sensor Reading (mV)**
Black	
White	
Polished Aluminum	
Dull Aluminum	

Power Setting 6.5

Therm. Res. ________ Ω
Temperature ________ °C

Surface	**Sensor Reading (mV)**
Black	
White	
Polished Aluminum	
Dull Aluminum	

Power Setting 8.0

Therm. Res. ________ Ω
Temperature ________ °C

Surface	**Sensor Reading (mV)**
Black	
White	
Polished Aluminum	
Dull Aluminum	

Power Setting 10.0

Therm. Res. ________ Ω
Temperature ________ °C

Surface	**Sensor Reading (mV)**
Black	
White	
Polished Aluminum	
Dull Aluminum	

Questions (Part 1)

① List the surfaces of the Radiation Cube in order of the amount of radiation emitted. Is the order independent of temperature?

② It is a general rule that good absorbers of radiation are also good emitters. Are your measurements consistent with this rule? Explain.

Questions (Part 2)

① Do different objects, at approximately the same temperature, emit different amounts of radiation?

② Can you find materials in your room that block thermal radiation? Can you find materials that don't block thermal radiation? (For example, do your clothes effectively block the thermal radiation emitted from your body?)

Absorption and Transmission of Thermal Radiation

Questions

① What do your results suggest about the phenomenon of heat loss through windows?

② What do your results suggest about the Greenhouse Effect?

Experiment 2: Inverse Square Law

EQUIPMENT NEEDED:

— Radiation Sensor
— Stefan-Boltzmann Lamp, Millivoltmeter
— Power Supply (12 VDC; 3 A), meter stick.

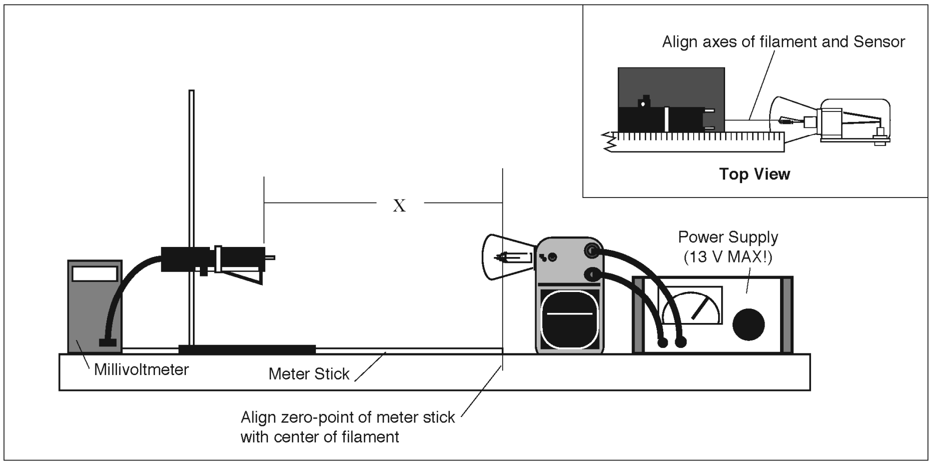


Figure 2.1 Equipment Setup

① Set up the equipment as shown in Figure 2.1.

a. Tape a meter stick to the table.

b. Place the Stefan-Boltzmann Lamp at one end of the meter stick as shown. The zero-point of the meter stick should align with the center of the lamp filament.

c. Adjust the height of the Radiation Sensor so it is at the same level as the filament of the Stefan-Boltzmann Lamp.

d. Align the lamp and sensor so that, as you slide the Sensor along the meter stick, the axis of the lamp aligns as closely as possible with the axis of the Sensor.

e. Connect the Sensor to the millivoltmeter and the lamp to the power supply as indicated in the figure.

② With the lamp OFF, slide the sensor along the meter stick. Record the reading of the millivolt-meter at 10 cm intervals. Record your values in Table 2.1 on the following page. Average these values to determine the ambient level of thermal radiation. You will need to subtract this average ambient value from your measurements with the lamp on, in order to determine the contribution from the lamp alone.

③ Turn on the power supply to illuminate the lamp. Set the voltage to approximately 10 V.

➤ **IMPORTANT:** Do not let the voltage to the lamp exceed 13 V.

④ Adjust the distance between the Sensor and the lamp to each of the settings listed in Table 2.2. At each setting, record the reading on the millivoltmeter.

➤ **IMPORTANT:** Make each reading quickly. Between readings, move the Sensor away from the lamp, or place the reflective heat shield between the lamp and the Sensor, so that the temperature of the Sensor stays relatively constant.

X (cm)	Ambient Radiation Level (mV)
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	
Average Ambient Radiation Level =	

Table 2.1
Ambient Radiation Level

X (cm)	Rad (mV)	$1/X^2$ (cm^{-2})	Rad - Ambient (mV)
2.5			
3.0			
3.5			
4.0			
4.5			
5.0			
6.0			
7.0			
8.0			
9.0			
10.0			
12.0			
14.0			
16.0			
18.0			
20.0			
25.0			
30.0			
35.0			
40.0			
45.0			
50.0			
60.0			
70.0			
80.0			
90.0			
100.0			

Table 2.2
Radiation Level versus Distance

Calculations

① For each value of X, calculate $1/X^2$. Enter your results in Table 2.2.

② Subtract the Average Ambient Radiation Level from each of your Rad measurements in Table 2.2. Enter your results in the table.

③ On a separate sheet of paper, make a graph of Radiation Level versus Distance from Source, using columns one and four from Table 2.2. Let the radiation level be the dependent (y) axis.

④ If your graph from part 3 is not linear, make a graph of Radiation Level versus $1/X^2$, using columns three and four from table 2.2.

Questions

① Which of the two graphs is more linear? Is it linear over the entire range of measurements?

② The inverse square law states that the radiant energy per unit area emitted by a point source of radiation decreases as the square of the distance from the source to the point of detection. Does your data support this assertion?

③ Is the Stefan-Boltzmann Lamp truly a point source of radiation? If not, how might this affect your results? Do you see such an effect in the data you have taken?

Experiment 3: Stefan-Boltzmann Law (high temperature)

EQUIPMENT NEEDED:

— Radiation Sensor
— Ohmmeter
— Voltmeter (0-12 V)
— Ohmmeter
— Stefan-Boltzmann Lamp
— Ammeter (0-3 A)
— Millivoltmeter
— Thermometer.

Introduction

The Stefan-Boltzmann Law relates R, the power per unit area radiated by an object, to T, the absolute temperature of the object. The equation is:

$$R = \sigma T^4; \quad \left(\sigma = 5.6703 \quad x\ 10^{-8} \frac{W}{m^2 K^4}\right)$$

In this experiment, you will make relative measurements of the power per unit area emitted from a hot object, namely the Stefan-Boltzmann Lamp, at various temperatures. From your data you will be able to test whether the radiated power is really proportional to the fourth power of the temperature.

Most of the thermal energy emitted by the lamp comes from the filament of the lamp. The filament temperature can be determined using the procedure given on pages 3 and 4 of this manual.

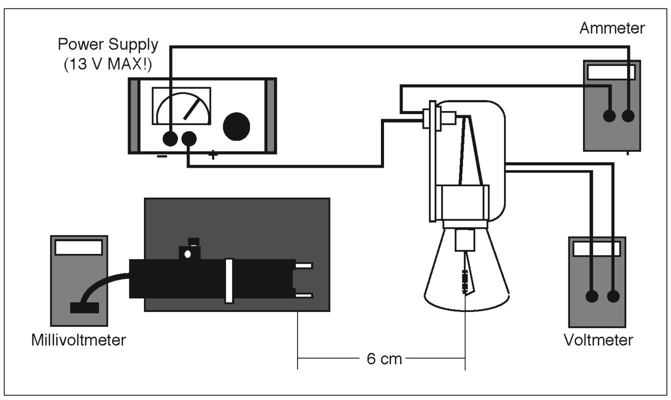


Figure 3.1 Equipment Setup

Procedure

➤ **IMPORTANT:** The voltage into the lamp should NEVER exceed 13 V. Higher voltages will burn out the filament.

① **BEFORE TURNING ON THE LAMP**, measure T_{ref}, the room temperature in degrees Kelvin, (K=°C + 273) and R_{ref}, the resistance of the filament of the Stefan-Boltzmann Lamp at room temperature. Enter your results in the spaces on the following page.

② Set up the equipment as shown in Figure 3.1. The voltmeter should be connected directly to the binding posts of the Stefan-Boltzmann Lamp. The Sensor should be at the same height as the filament, with the front face of the Sensor approximately 6 cm away from the filament. The entrance angle of the thermopile should include no close objects other than the lamp.

③ Turn on the power supply. Set the voltage, V, to each of the settings listed in Table 3.1 on the following page. At each voltage setting, record I, the ammeter reading, and Rad, the reading on the millivoltmeter.

➤ **IMPORTANT:** Make each Sensor reading quickly. Between readings, place both sheets of insulating foam between the lamp and the Sensor, with the silvered surface facing the lamp, so that the temperature of the Sensor stays relatively constant.

Data and Calculations

① Calculate R, the resistance of the filament at each of the voltage settings used (R = V/I). Enter your results in Table 3.1.

② Use the procedure on pages 3 and 4 of this manual to determine T, the temperature of the lamp filament at each voltage setting. Enter your results in the table.

③*Calculate T^4 for each value of T and enter your results in the table.

④*On a separate sheet of paper, construct a graph of Rad versus T^4. Use Rad as your dependent variable (y-axis).

*In place of calculations ① and , some may prefer to perform a power regression on Rad versus T to determine their relationship, or graph on log-log paper and find the slope.

Questions

① What is the relationship between Rad and T? Does this relationship hold over the entire range of measurements?

② The Stefan-Boltzmann Law is perfectly true only for ideal, black body radiation. A black body is any object that absorbs all the radiation that strikes it. Is the filament of the lamp a true black body?

③ What sources of thermal radiation, other than the lamp filament, might have influenced your measurements? What affect would you expect these sources to have on your results?

$\alpha = 4.5 \times 10^{-3}\ K^{-1}$

T_{ref} (room temperature) = _______ K (K = °C + 273)

R_{ref} (filament resistance at T_{ref}) = ________Ω

Table 3.1

Data			Calculations		
V (Volts)	I (Amps)	Rad (mV)	R (Ohms)	T (K)	T^4 (K^4)
1.00					
2.00					
3.00					
4.00					
5.00					
6.00					
7.00					
8.00					
9.00					
10.00					
11.00					
12.00					

Experiment 4: Stefan-Boltzmann Law (low temperature)

EQUIPMENT NEEDED:

— Radiation Sensor
— Millivoltmeter
— Thermal Radiation Cube
— Ohmmeter.

Introduction

In experiment 3, you investigated the Stefan-Boltzmann Law ($R_{rad} = sT^4$) for the high temperatures attained by an incandescent filament. At those high temperatures (approximately 1,000 to 3,000 K), the ambient temperature is small enough that it can be neglected in the analysis. In this experiment you will investigate the Stefan-Boltzmann relationship at much lower temperatures using the Thermal Radiation Cube. At these lower temperatures, the ambient temperature can not be ignored.

If the detector in the Radiation Sensor were operating at absolute zero temperature, it would produce a voltage directly proportional to the intensity of the radiation that strikes it. However, the detector is not at absolute zero temperature so it is also radiating thermal energy. According to the Stefan-Boltzmann law, it radiates at a rate, $R_{det} = sT_{det}^4$. The voltage produced by the sensor is proportional to the radiation striking the detector minus the radiation leaving it. Mathematically, the sensor voltage is proportional to $R_{net} = R_{rad} - R_{det} = s(T^4 - T_{det}^4)$. As long as you are careful to shield the Radiation Sensor from the Radiation Cube when measurements are not being taken, T_{det} will be very close to room temperature (T_{rm}).

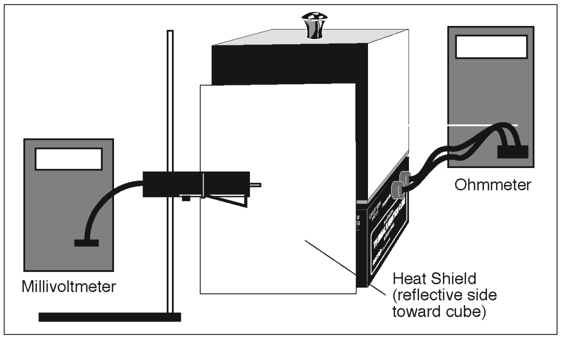


Figure 4.1 Equipment Setup

Procedure

① Set up the equipment as shown in Figure 4.1. The Radiation Sensor should be pointed directly at the center of one of the better radiating surfaces of the cube (the black or white surface). The face of the Sensor should be parallel with the surface of the cube and about 3 to 4 cm away.

② With the Thermal Radiation Cube off, measure R_{rm}, the resistance of the thermistor at room temperature. Enter this data in the space on the following page.

③ Shield the sensor from the cube using the reflecting heat shield, with the reflective side of the shield facing the cube.

④ Turn on the Radiation Cube and set the power switch to 10.

⑤ When the thermistor resistance indicates that the temperature is about 12 C° above room temperature, turn the power down so the temperature is changing slowly. Read and record R, the ohmmeter reading, and Rad, the millivoltmeter reading. The readings should be taken as nearly simultaneously as possible while briefly removing the heat shield. Record these values in Table 4.1.

> ➤ **IMPORTANT:** Make each reading quickly, removing the heat shield only as long as it takes to make the measurement. Take care that the position of the sensor with respect to the cube is the same for all measurements.

⑥ Replace the heat shield, and turn the cube power to 10. When the temperature has risen an additional 12-15 C°, repeat the measurements of step 5. Repeat this procedure at about 12-15° intervals until the maximum temperature of the cube is reached.

Data and Calculations

Room Temperature: R_{rm} = ________ Ω

T_{rm} = ____ °C = _____ K

Table 4.1

Data			Calculations		
R (ý)	Rad (mV)	T_c (°C)	T_k (K)	T_k^4 (K^4)	$T_k^4 - T_{rm}^4$ (K^4)

① Using the table on the base of the Thermal Radiation Cube, determine T_c, the temperature in degrees Centigrade corresponding to each of your thermistor resistance measurements. For each value of T_c, determine T_k, the corresponding value in degrees Kelvin (K = °C + 273). Enter both sets of values in Table 4.1, above. In the same manner, determine the room temperature, T_{rm}.

② CalculateT_k^4 for each value of T_k and record the values in the table.

③ Calculate $T_k^4 - T_{rm}^4$ for each value of T_k and record your results in the table.

④ On separate sheet of paper, construct a graph of Rad versus $T_k^4 - T_{rm}^4$. Use Rad as the dependent variable (y-axis).

Questions

① What does your graph indicate about the Stefan-Boltzmann law at low temperatures?

② Is your graph a straight line? Discuss any deviations that exist.

Díodos emissores de luz (LED) e a constante de Planck

Guião

OBJECTIVO

Os objectivos desta experiência são:

· Determinar o comprimento de onda da luz emitida por um LED
· Determinar a constante de Planck

INTRODUÇÃO

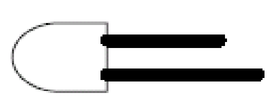
Os díodos emissores de luz ou LED (de *Light Emitting Diode*) emitem luz quando são percorridos por uma corrente eléctrica. Esta emissão de luz ocorre quando electrões transitam entre estados de diferentes energias ao passarem na junção entre os dois tipos (n e p) do material semicondutor de que é feito o díodo. A diferença de energia entre estes estados é uma propriedade do material semicondutor. Num díodo, a passagem de corrente só é significativa quando o díodo é polarizado no sentido directo (corrente eléctrica convencional do lado p para n) e, nestas condições, ocorre a emissão de luz. Na polarização directa é aplicada uma diferença de potencial V e, para que um electrão atravesse a junção semicondutora, é necessário realizar um certo trabalho W. Este trabalho é convertido, em grande parte, na energia dos fotões emitidos. No entanto, há pequenas perdas de energia, devidas ao efeito de Joule e processos que ocorrem no interior da junção, que têm um valor praticamente constante para LEDs dum mesmo tipo quando **atravessados por uma mesma corrente eléctrica**. Nestas condições,

$$W = E_f + k \ ,$$

onde E_f é a energia do fotão emitido e k uma constante que representa outras perdas de energia.

A luz emitida por um LED é praticamente monocromática. É possível fabricar LEDs que emitem luz de diferentes cores, alterando a composição química do material semicondutor. Os LEDs mais comuns são feitos de ligas de gálio, arsénio e alumínio. Alterando a proporção de gálio e alumínio é possível fabricar LEDs que emitem várias cores na gama do visível e do infravermelho.

Os LEDs comerciais são fornecidos com o material semicondutor encapsulado (plástico) e com dois terminais, sendo o mais longo o positivo (lado p).



Para determinar o comprimento de onda, λ, da luz emitida por um LED podemos usar uma rede de difracção. Os ângulos θ_n para os quais ocorrem os máximos de intensidade difractada por uma rede com espaçamento entre linhas *d*, são dados pela equação:

$d \sin\theta_n = n \lambda$ *n*: um número inteiro

MATERIAL

- Régua graduada
- Folhas de papel branco A3
- Conjunto de LEDs do mesmo tipo montados num suporte
- Pilha de 9 V
- Fios e garras (crocodilos) para ligações
- Potenciómetro (470 Ω)
- Resistência (220 Ω)
- Resistência (11,4 kΩ)
- Dois multímetros
- Rede de difracção (1000 linhas/mm) montada num suporte

DADOS

Carga elementar: $e = 1{,}602 \times 10^{-19}$ C
Velocidade da luz no vácuo: $c = 2{,}998 \times 10^{8}$ m s^{-1}
Constante de Boltzmann: $k_B = 1{,}381 \times 10^{-23}$ J K^{-1}

PRECAUÇÕES

1- A corrente eléctrica que atravessa o LED não deverá exceder cerca de 50 mA, pois este pode danificar-se. Para protecção do LED, a resistência de 220 Ω **deve estar sempre ligada em série com o LED.**

2-Tenha em atenção as escalas na utilização segura dos multímetros como amperímetros ou como voltímetros. A manipulação incorrecta do multímetro poderá queimar o seu fusível interno, comprometendo o seu trabalho.

INFORMAÇÃO

1 - O potenciómetro tem 3 terminais e permite variar a tensão entre o terminal central e uma das extremidades, desde 0 até ao valor máximo fornecido pela pilha.

2 - São fornecidos LEDs com as seguintes características:

Tabela 1:

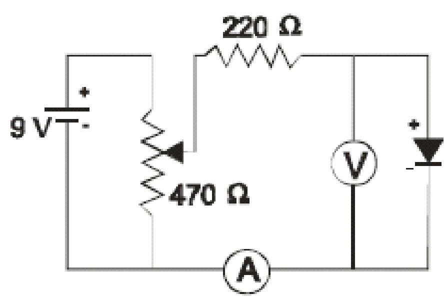
LED	**λ / nm**
Azul	470
Verde	??
Vermelho	630
Infravermelho	950

MEDIDAS E ANÁLISE

A) COMPORTAMENTO ELÉCTRICO DOS LEDS:

Pretende-se determinar a curva característica do LED verde, ou seja, a relação entre a corrente, *I*, que o atravessa e a tensão, *V*, aplicada aos seus terminais.

1 - Monte um circuito que permita alimentar o LED verde com uma tensão variável em polarização directa, de acordo com a Fig.1. A resistência de 220 Ω assegura que a corrente no LED não excede os 50 mA, respeitando os limites de segurança.



2 - Encontre a curva característica para o LED verde: apresente uma tabela com os valores medidos de tensão *V* e corrente *I*, com a indicação das respectivas unidades; faça um gráfico de ln *I* em função de *V* (não se esqueça de converter a corrente de mA para A).

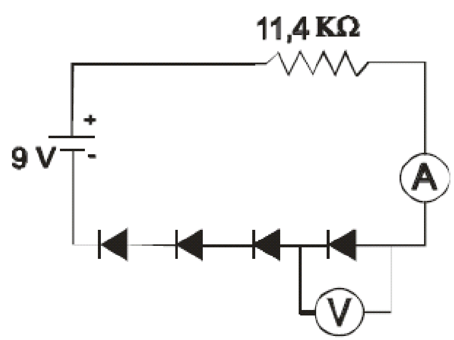
3 - A expressão *teórica* que relaciona a corrente com a tensão é:

$$I = I_0 \left(e^{\frac{eV}{\eta k_B T}} - 1 \right) \approx I_0 \, e^{\frac{eV}{\eta k_B T}}$$

em que I_0 e η são constantes características de cada LED, *e* é a carga do electrão, k_B é a constante de Boltzmann e *T* é a temperatura ambiente (em kelvin). A expressão aproximada é válida para $V > 2$V.
O gráfico que obteve está de acordo com o que espera da expressão teórica? Justifique.

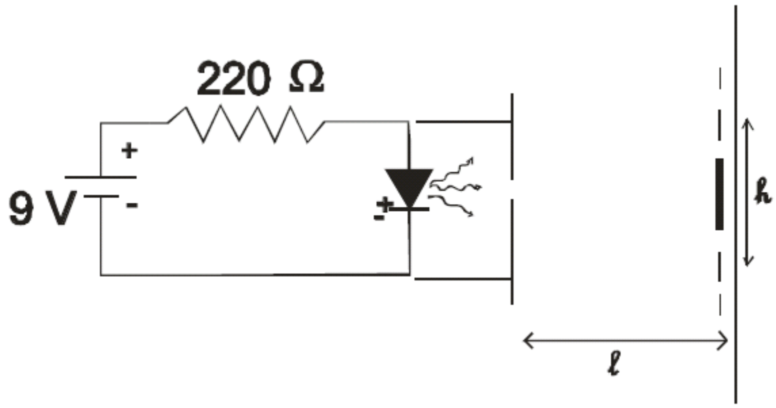
Faça um gráfico de ln I em função de V só com os pontos para os quais a expressão aproximada é válida. Determine o valor das constantes I_0 e η para o LED verde (considere $T = 293$ K).

4 – Ligue os LEDs em série e também em série com a resistência de 11,4 kΩ e aplique ao circuito a tensão de 9 V (pode usar os crocodilos para fazer as ligações entre os LEDs), de acordo com a Fig2.
Registe a corrente que percorre o circuito. Registe numa tabela o valor da tensão, *V*, nos terminais de cada LED, bem como o valor do comprimento de onda (tabela 1) e frequência da radiação que cada um emite (o valor para o LED verde é determinado na parte B).



B) DETERMINAÇÃO DO COMPRIMENTO DE ONDA DO LED VERDE:

5 – Ligue o LED verde à pilha de 9V em série com a resistência de 220 Ω. Para determinar o comprimento de onda do LED verde coloque o tubo preto com a rede de difracção a envolver o LED e verifique visualmente que ocorrem máximos de intensidade de um lado e do outro da direcção frontal. Utilize as folhas A3 para projectar o espectro de difracção. Meça a distância *h* entre os dois máximos de primeira ordem e a distância *l* entre a rede de difracção e o alvo, indicando as respectivas incertezas.



Determine, a partir de h e l , o ângulo θ_1 correspondente à difracção de primeira ordem. Determine o comprimento de onda emitido pelo LED verde. Apresente o resultado com a respectiva incerteza.

C) DETERMINAÇÃO DA CONSTANTE DE PLANCK, h :

6 - Utilizando os dados da tabela obtida em 4 (parte A), completada com a informação obtida em B, represente graficamente a tensão V em função da frequência ν da luz emitida pelos LED's. Note que de

$$W = E_f + k$$

Vem

$$eV = h\nu + k$$

ou

$$V = \frac{h}{e}\nu + k$$

7 - Determine, a partir do gráfico, o valor da constante de Planck e da respectiva incerteza. Comente o resultado. (O valor tabelado desta constante obtida por métodos muito precisos é $h = 6{,}626 \times 10^{-34}$ J s.)

Efeito fotoeléctrico e determinação da constante de Planck

Manual de instruções da "Pasco" (incluindo guião)

Includes
Teacher's Notes
and
Typical
Experiment Results

**Instruction Manual
and Experiment Guide
for the PASCO scientific
Model AP-9368 and AP-9369**

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h/e Apparatus
and
h/e Apparatus Accessory Kit

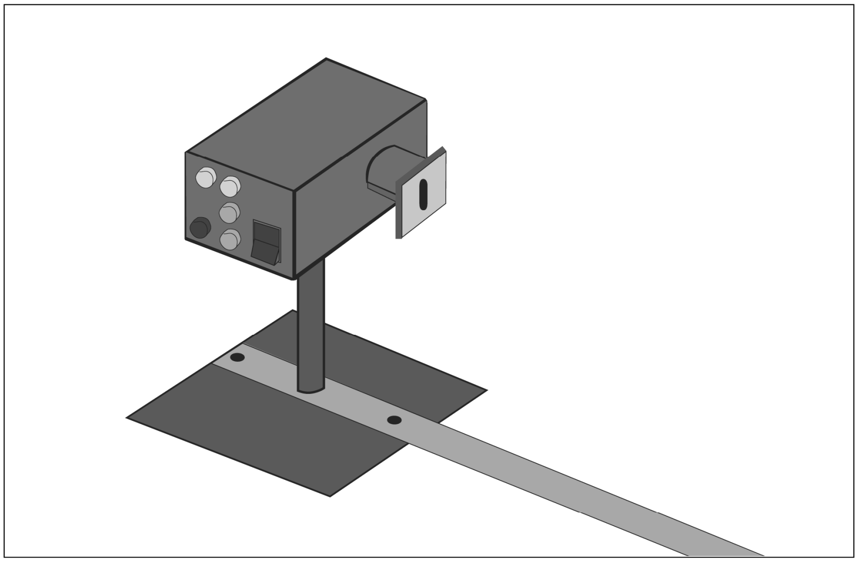

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Table of Contents

Introduction

The emission and absorption of light was an early subject for investigation by German physicist Max Planck. As Planck attempted to formulate a theory to explain the spectral distribution of emitted light based on a classical wave model, he ran into considerable difficulty. Classical theory (Rayleigh-Jeans Law) predicted that the amount of light emitted from a black body would increase dramatically as the wavelength decreased, whereas experiment showed that it approached zero. This discrepancy became known as the ultraviolet catastrophe.

Experimental data for the radiation of light by a hot, glowing body showed that the maximum intensity of emitted light also departed dramatically from the classically predicted values (Wien's Law). In order to reconcile theory with laboratory results, Planck was forced to develop a new model for light called the quantum model. In this model, light is emitted in small, discrete bundles or quanta.

The relationship between the classical and quantum theories for the emission of light can be investigated using the PASCO scientific h/e Apparatus. Using the Apparatus in combination with the PASCO Mercury Vapor Light Source (Model OS-9286) allows an accurate determination of the h/e ratio and thus a determination of h, Planck's constant.

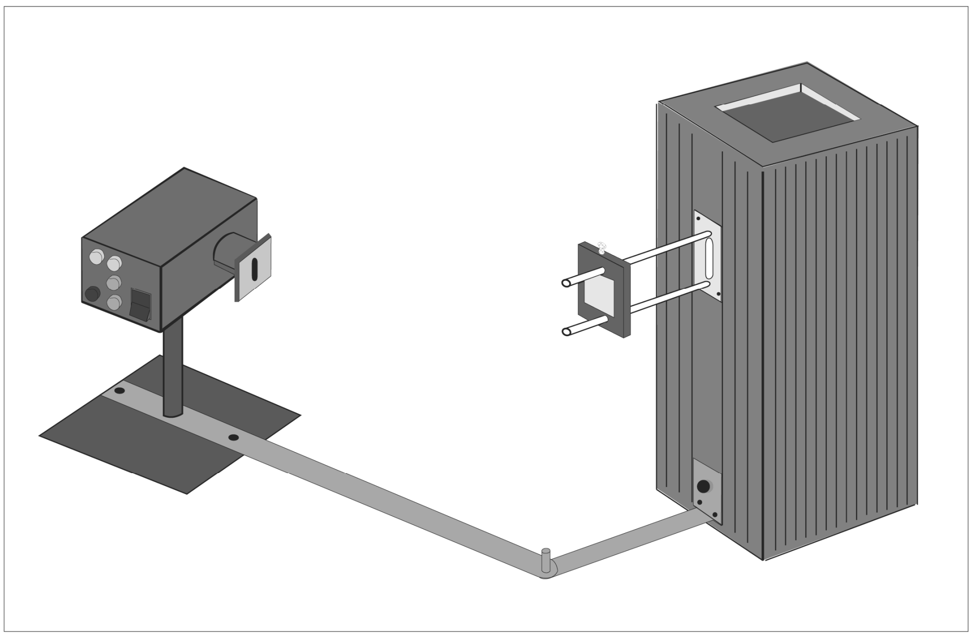


Figure 1. The h/e Apparatus Shown With the Accessory Kit and Mercury Vapor Light Source

Background Theory

Planck's Quantum Theory

By the late 1800's many physicists thought they had explained all the main principles of the universe and discovered all the natural laws. But as scientists continued working, inconsistencies that couldn't easily be explained began showing up in some areas of study.

In 1901 Planck published his law of radiation. In it he stated that an oscillator, or any similar physical system, has a discrete set of possible energy values or levels; energies between these values never occur.

Planck went on to state that the emission and absorption of radiation is associated with transitions or jumps between two energy levels. The energy lost or gained by the oscillator is emitted or absorbed as a quantum of radiant energy, the magnitude of which is expressed by the equation:

$$E = h\nu$$

where E equals the radiant energy, ν is the frequency of the radiation, and h is a fundamental constant of nature. The constant, h, became known as Planck's constant.

Planck's constant was found to have significance beyond relating the frequency and energy of light, and became a cornerstone of the quantum mechanical view of the subatomic world. In 1918, Planck was awarded a Nobel prize for introducing the quantum theory of light.

The Photoelectric Effect

In photoelectric emission, light strikes a material, causing electrons to be emitted. The classical wave model predicted that as the intensity of incident light was increased, the amplitude and thus the energy of the wave would increase. This would then cause more energetic photoelectrons to be emitted. The new quantum model, however, predicted that higher frequency light would produce higher energy photoelectrons, independent of intensity, while increased intensity would only increase the number of electrons emitted (or photoelectric current). In the early 1900s several investigators found that the kinetic energy of the photoelectrons was dependent on the wavelength, or frequency, and independent of intensity, while the magnitude of the photoelectric current, or number of electrons was dependent on the intensity as predicted by the quantum model. Einstein applied Planck's theory and explained the photoelectric effect in terms of the quantum model using his famous equation for which he received the Nobel prize in 1921:

$$E = h\nu = KE_{max} + W_O$$

where KE_{max} is the maximum kinetic energy of the emitted photoelectrons, and W_O is the energy needed to remove them from the surface of the material (the work function). E is the energy supplied by the quantum of light known as a photon.

The h/e Experiment

A light photon with energy $h\nu$ is incident upon an electron in the cathode of a vacuum tube. The electron uses a minimum W_O of its energy to escape the cathode, leaving it with a maximum energy of KE_{max} in the form of kinetic energy. Normally the emitted electrons reach the anode of the tube, and can be measured as a photoelectric current. However, by applying a reverse potential V between the anode and the cathode, the photoelectric current can be stopped. KE_{max} can be determined by measuring the minimum reverse potential needed to stop the photoelectrons and reduce the photoelectric current to zero.* Relating kinetic energy to stopping potential gives the equation:

$$KE_{max} = Ve$$

Therefore, using Einstein's equation,

$$h\nu = Ve + W_O$$

When solved for V, the equation becomes:

$$V = (h/e)\,\nu - (W_O/e)$$

If we plot V vs ν for different frequencies of light, the graph will look like Figure 2. The V intercept is equal to $-W_O/e$ and the slope is h/e. Coupling our experimental determination of the ratio h/e with the accepted value for e, 1.602 x 10^{-19} coulombs, we can determine Planck's constant, h.

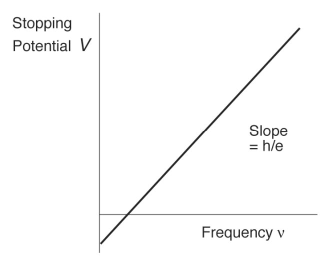


Figure 2. The graph of V vs. ν

> ***NOTE:** In experiments with the PASCO h/e Apparatus the stopping potential is measured directly, rather than by monitoring the photoelectric current. See the *Theory of Operation* in the Technical Information section of the manual for details.

Equipment and Setup

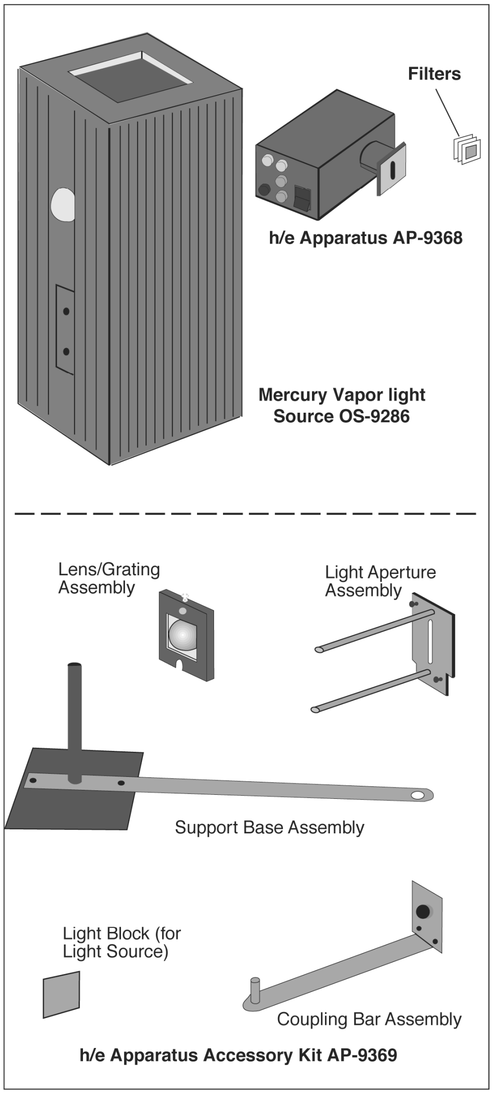


Figure 3. h/e Equipment Identification

* These items may be purchased separately from PASCO scientific, or together as an AP-9370 h/e System.

Equipment Required:

– Digital voltmeter (SE-9589)
– h/e Apparatus, (AP-9368*)
– h/e Apparatus Accessory Kit, (AP-9369*)
– Mercury Vapor Light Source, (OS- 9286*)

Installing the Batteries

The h/e Apparatus requires two 9-volt batteries (supplied but not installed). The battery compartment is accessed by loosening the thumbscrew on the rear end panel, and removing the cover plate.

➤ **NOTE:** The h/e Apparatus can also be powered using a ±9 V dual power supply. Just remove the batteries and connect +9 V to the "+6 V MIN" battery test terminal and -9 V to the "-6 V MIN" battery test terminal.

Battery Voltage Check

Although the h/e Apparatus draws only a small amount of current and batteries normally last a long time, it's a good idea to check the output voltage before each use. Battery test points are located on the side panel of the Apparatus near the ON/OFF switch. Batteries functioning below the recommended minimum operating level of 6 volts may cause erroneous results in your experiments.

To check the batteries, use a voltmeter to measure between the OUTPUT ground terminal and each BATTERY TEST terminal (-6V MIN and +6V MIN). If either battery tests below its minimum rating, it should be replaced before running experiments.

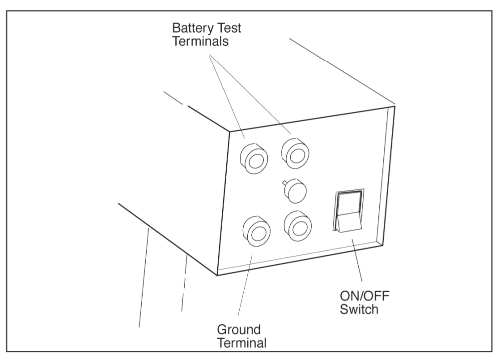


Figure 4. Battery Test Points

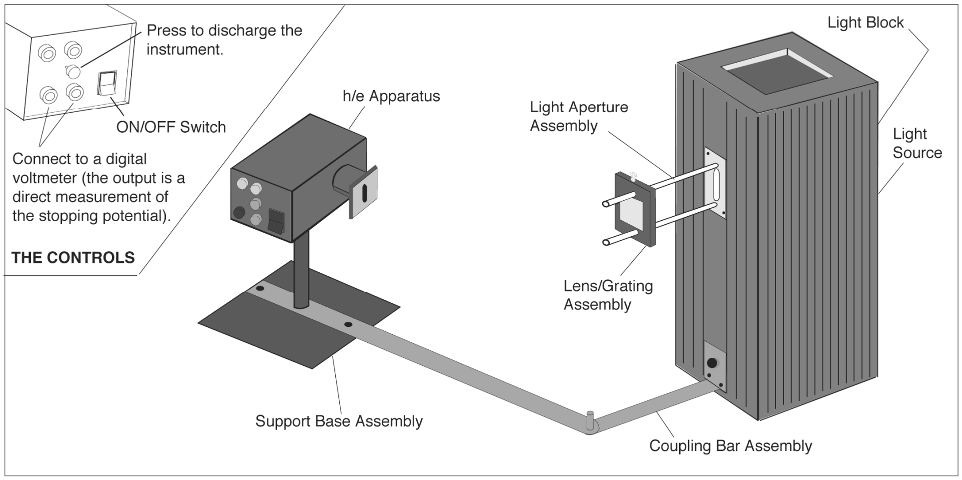


Figure 5. Equipment Setup Using a Mercury Vapor Light Source and the h/e Apparatus

Equipment Setup

The standard setup for h/e experiments is shown in Figure 5. Details for setting up the apparatus are described below.

1. The Light Source design allows simultaneous connection of two Light Aperture assemblies: one on the front and one on the back. If you are using only one Light Aperture and h/e Apparatus, install the Light Block (supplied with the Accessory Kit) in the mounting groove closest to the body of the housing on the back of the Light Source (see Figure 6).
2. Slide the Light Aperture Assembly into the center mounting groove on the front of the Light Source. Secure it in place by finger-tightening the two thumbscrews against the front of the Light Source housing.
3. The Lens/Grating Assembly mounts on the support bars of the Light Aperture Assembly (Figure 7). Loosen the thumbscrew, slip it over the bars, and finger-tighten the thumbscrew to hold it securely.

> ➤ **NOTE:** The grating is blazed to produce the brightest spectrum on one side only. During your experiment, you may need to turn the Lens/Grating Assembly around in order to have the brightest spectrum on a convenient side of your lab table.

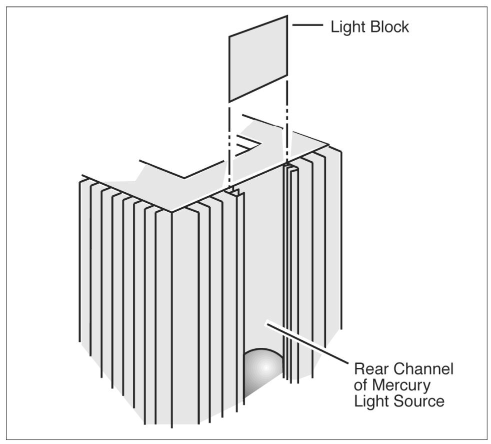


Figure 6. Installing the Light Block

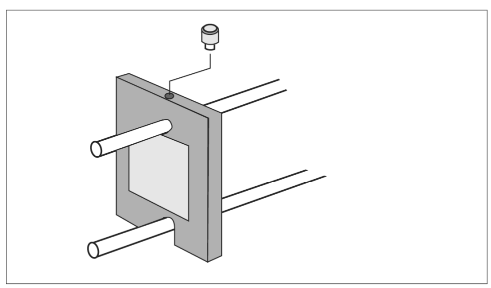


Figure 7. Lens/Grating Mounting Detail

4. Turn on the Light Source and allow it to warm up for five minutes. Check the alignment of the Light Source and the Aperture by looking at the light shining on the back of the Lens/Grating assembly. If necessary, adjust the back plate of the Light Aperture Assembly by loosening the two retaining screws (Figure 8) and sliding the aperture plate left or right until the light shines directly on the center of the Lens/Grating Assembly.

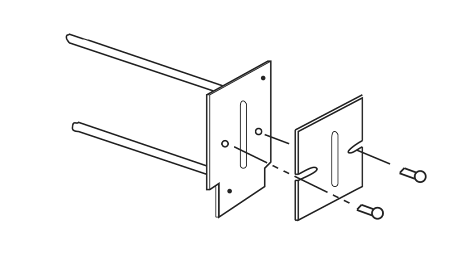


Figure 8. Light Aperture Adjustment

5. Insert the Coupling Bar assembly into the lower mounting groove of the Light Source (Figure 5). Secure in place by tightening the thumbscrew against the front of the Light Source housing.
6. Remove the screw from the end of the Support Base rod. Insert the screw through the hole in the Support Base plate and attach the rod to the Support Base plate by tightening the screw (use Phillips drive screwdriver).
7. Place the h/e Apparatus onto the Support Base Assembly.
8. Place the Support Base assembly over the pin on the end of the Coupling Bar assembly.
9. Connect a digital voltmeter (DVM) to the OUTPUT terminals of the h/e Apparatus. Select the 2V or 20V range on the meter.
10. Set the h/e Apparatus directly in front of the Mercury Vapor Light Source. By sliding the Lens/Grating assembly back and forth on its support rods, focus the light onto the white reflective mask of the h/e Apparatus (Figure 9).

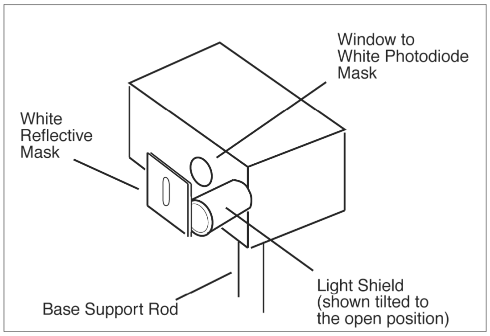
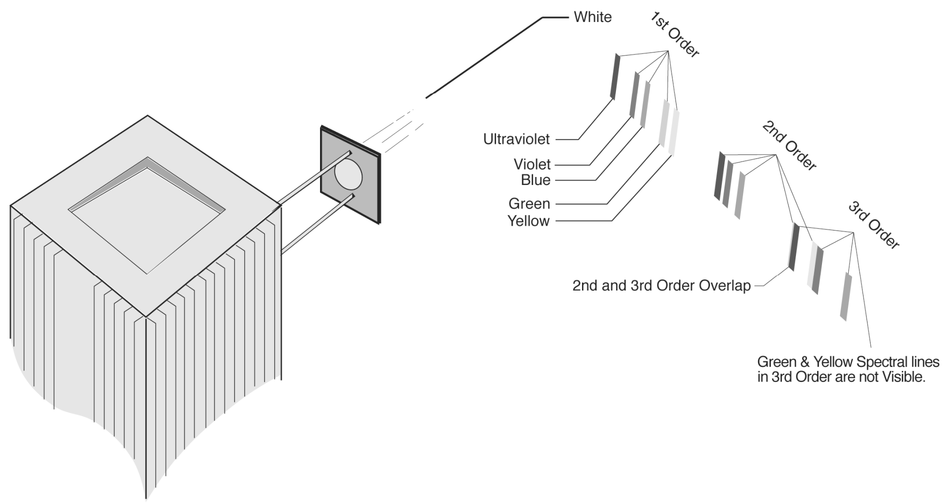


Figure 9. h/e Light Shield

11. Roll the light shield of the Apparatus out of the way to reveal the white photodiode mask inside the Apparatus. Rotate the h/e Apparatus until the image of the aperture is centered on the window in the photodiode mask. Then tighten the thumbscrew on the base support rod to hold the Apparatus in place.
12. As in step 9, slide the Lens/Grating assembly back and forth on its support rods, until you achieve the sharpest possible image of the aperture on the window in the photodiode mask. Tighten the thumbscrew on the Lens/Grating assembly and replace the light shield.
13. Turn the power switch ON. Rotate the h/e Apparatus about the pin of the Coupling Bar Assembly until one of the colored maxima in the first order shines directly on the slot in the white reflective mask. Rotate the h/e Apparatus on its support base so that the same spectral maxima that falls on the opening in the White Reflective Mask also falls on the window in the photodiode mask.

➤ **NOTE:** The white reflective mask on the h/e apparatus is made of a special fluorescent material. This allows you to see the ultraviolet line as a blue line, and it also makes the violet line appear more blue. You can see the actual colors of the light if you hold a piece of white non-fluorescent material in front of the mask. (The palm of your hand works in a pinch, although it fluoresces enough that the UV line will still be visible.)

When making measurements it is important that only one color falls on the photodiode window. There must be no overlap from adjacent spectral maxima.



All values except wavelength for yellow line are from *Handbook of Chemistry and Physics, 46th ed*. The wavelength of the yellow was determined experimentally using a 600 line/mm grating.

NOTE: The yellow line is actually a doublet with wavelengths of 578 and 580mm.

Color	Frequency (Hz)	Wavelength (nm)
Yellow	5.18672E+14	578
Green	5.48996E+14	546.074
Blue	6.87858E+14	435.835
Violet	7.40858E+14	404.656
Ultraviolet	8.20264E+14	365.483

Figure 10. The Three Orders of Light Gradients

14. Press the "PUSH TO ZERO" button on the side panel of the h/e Apparatus to discharge any accumulated potential in the unit's electronics. This will assure the Apparatus records only the potential of the light you are measuring. Note that the output voltage will drift with the absence of light on the photodiode.

15. Read the output voltage on your digital voltmeter. It is a direct measurement of the stopping potential for the photoelectrons. (See *Theory of Operation* in the Technical Information section of the manual for an explanation of the measurement.)

➤ **NOTE:** For some apparatus, the stopping potential will temporarily read high and then drop down to the actual stopping potential voltage.

Using the Filters

The (AP-9368) h/e Apparatus includes three filters: one Green and one Yellow, plus a Variable Transmission Filter. The filter frames have magnetic strips and mount to the outside of the White Reflective Mask of the h/e Apparatus.

Use the green and yellow filters when you're using the green and yellow spectral lines. These filters limit higher frequencies of light from entering the h/e Apparatus. This prevents ambient room light from interfering with the lower energy yellow and green light and masking the true results. It also blocks the higher frequency ultraviolet light from the higher order spectra which may overlap with lower orders of yellow and green.

The Variable Transmission Filter consists of computer-generated patterns of dots and lines that vary the intensity (not the frequency) of the incident light. The relative transmission percentages are 100%, 80%, 60%, 40%, and 20%.

Experiment 1: The Wave Model of light vs. the Quantum Model

According to the photon theory of light, the maximum kinetic energy, KE_{max}, of photoelectrons depends only on the frequency of the incident light, and is independent of the intensity. Thus the higher the frequency of the light, the greater its energy.

In contrast, the classical wave model of light predicted that KE_{max} would depend on light intensity. In other words, the brighter the light, the greater its energy.

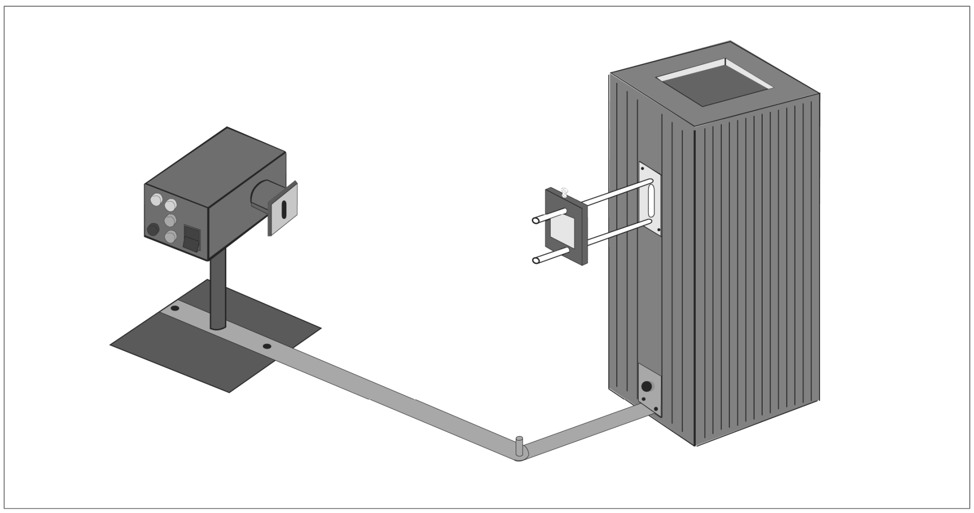
This lab investigates both of these assertions. Part A selects two spectral lines from a mercury light source and investigates the maximum energy of the photoelectrons as a function of the intensity. Part B selects different spectral lines and investigates the maximum energy of the photoelectrons as a function of the frequency of the light.

Setup

Set up the equipment as shown in the diagram below. Focus the light from the Mercury Vapor Light Source onto the slot in the white reflective mask on the h/e Apparatus. Tilt the Light Shield of the Apparatus out of the way to reveal the white photodiode mask inside the Apparatus. Slide the Lens/Grating assembly forward and back on its support rods until you achieve the sharpest image of the aperture centered on the hole in the photodiode mask. Secure the Lens/Grating by tightening the thumbscrew.

Align the system by rotating the h/e Apparatus on its support base so that the same color light that falls on the opening of the light screen falls on the window in the photodiode mask, with no overlap of color from other spectral lines. Return the Light Shield to its closed position.

Check the polarity of the leads from your digital voltmeter (DVM), and connect them to the OUTPUT terminals of the same polarity on the h/e Apparatus.



Experiment 1. Equipment Setup

Procedure

Part A

1. Adjust the h/e Apparatus so that only one of the spectral colors falls upon the opening of the mask of the photodiode. If you select the green or yellow spectral line, place the corresponding colored filter over the White Reflective Mask on the h/e Apparatus

2. Place the Variable Transmission Filter in front of the White Reflective Mask (and over the colored filter, if one is used) so that the light passes through the section marked 100% and reaches the photodiode. Record the DVM voltage reading in the table below.

   Press the instrument discharge button, release it, and observe approximately how much time is required to return to the recorded voltage.

3. Move the Variable Transmission Filter so that the next section is directly in front of the incoming light. Record the new DVM reading, and approximate time to recharge after the discharge button has been pressed and released.

   Repeat Step 3 until you have tested all five sections of the filter.

   Repeat the procedure using a second color from the spectrum.

Color #1 ________ (name)	%Transmission	Stopping Potential	Approx. Charge Time
	100		
	80		
	60		
	40		
	20		
Color #2 ________ (name)	**%Transmission**	**Stopping Potential**	**Approx. Charge Time**
	100		
	80		
	60		
	40		
	20		

Part B

1. You can easily see five colors in the mercury light spectrum. Adjust the h/e Apparatus so that only one of the yellow colored bands falls upon the opening of the mask of the photodiode. Place the yellow colored filter over the White Reflective Mask on the h/e Apparatus.
2. Record the DVM voltage reading (stopping potential) in the table below.
3. Repeat the process for each color in the spectrum. Be sure to use the green filter when measuring the green spectrum.

Analysis

1. Describe the effect that passing different amounts of the same colored light through the Variable Transmission Filter has on the stopping potential and thus the maximum energy of the photoelectrons, as well as the charging time after pressing the discharge button.
2. Describe the effect that different colors of light had on the stopping potential and thus the maximum energy of the photoelectrons.
3. Defend whether this experiment supports a wave or a quantum model of light based on your lab results.

   Explain why there is a slight drop in the measured stopping potential as the light intensity is decreased.

> ➤ **NOTE:** While the impedance of the zero gain amplifier is very high (♠10^{13} Ω), it is not infinite and some charge leaks off. Thus charging the apparatus is analogous to filling a bath tub with different water flow rates while the drain is partly open.

Light Color	Stopping Potential
Yellow	
Green	
Blue	
Violet	
Ultraviolet	

Experiment 2: The Relationship between Energy, Wavelength, and Frequency

According to the quantum model of light, the energy of light is directly proportional to its frequency. Thus, the higher the frequency, the more energy it has. With careful experimentation, the constant of proportionality, Planck's constant, can be determined.

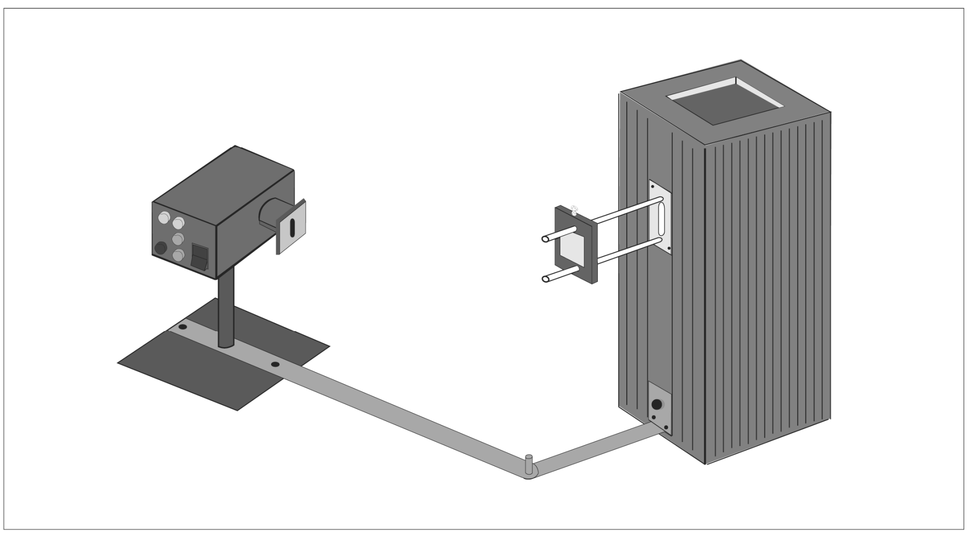
In this lab you will select different spectral lines from mercury and investigate the maximum energy of the photoelectrons as a function of the wavelength and frequency of the light.

Setup

Set up the equipment as shown in the diagram below. Focus the light from the Mercury Vapor Light Source onto the slot in the white reflective mask on the h/e Apparatus. Tilt the Light Shield of the Apparatus out of the way to reveal the white photodiode mask inside the Apparatus. Slide the Lens/Grating assembly forward and back on its support rods until you achieve the sharpest image of the aperture centered on the hole in the photodiode mask. Secure the Lens/Grating by tightening the thumbscrew.

Align the system by rotating the h/e Apparatus on its support base so that the same color light that falls on the opening of the light screen falls on the window in the photodiode mask with no overlap of color from other spectral bands. Return the Light Shield to its closed position.

Check the polarity of the leads from your digital voltmeter (DVM), and connect them to the OUTPUT terminals of the same polarity on the h/e Apparatus.



Experiment 2. Equipment Setup

Procedure

1. You can see five colors in two orders of the mercury light spectrum. Adjust the h/e Apparatus carefully so that only one color from the first order (the brightest order) falls on the opening of the mask of the photodiode.
2. For each color in the first order, measure the stopping potential with the DVM and record that measurement in the table below. Use the yellow and green colored filters on the Reflective Mask of the h/e Apparatus when you measure the yellow and green spectral lines.
3. Move to the second order and repeat the process. Record your results in the table below.

Analysis

Determine the wavelength and frequency of each spectral line. Plot a graph of the stopping potential vs. frequency.

Determine the slope and y-intercept. Interpret the results in terms of the *h/e* ratio and the W_o/e ratio. Calculate *h* and W_o.

In your discussion, report your values and discuss your results with an interpretation based on a quantum model for light.

First Order Color	**Wavelength nm**	**Frequency $x10^{14}$ Hz**	**Stopping Potential volts**
Yellow			
Green			
Blue			
Violet			
Ultraviolet			
Second Order Color	**Wavelength nm**	**Frequency $x10^{14}$ Hz**	**Stopping Potential volts**
Yellow			
Green			
Blue			
Violet			
Ultraviolet			

Technical Information

Theory of Operation

In experiments with the h/e Apparatus, monochromatic light falls on the cathode plate of a vacuum photodiode tube that has a low work function, W_0. Photoelectrons ejected from the cathode collect on the anode.

The photodiode tube and its associated electronics have a small capacitance which becomes charged by the photoelectric current. When the potential on this capacitance reaches the stopping potential of the photoelectrons, the current decreases to zero, and the anode-to-cathode voltage stabilizes. This final voltage between the anode and cathode is therefore the stopping potential of the photoelectrons.

To let you measure the stopping potential, the anode is connected to a built-in amplifier with an ultrahigh input impedance ($> 10^{13}\ \Omega$), and the output from this amplifier is connected to the output jacks on the front panel of the apparatus. This high impedance, unity gain (Vout/Vin = 1) amplifier lets you measure the stopping potential with a digital voltmeter.

Due to the ultra high input impedance, once the capacitor has been charged from the photodiode current it takes a long time to discharge this potential through some leakage. Therefore a shorting switch labeled “PUSH TO Zero” enables the user to quickly bleed off the charge. However, the op-amp output will not stay at 0 volts after the switch is released since the op-amp input is floating.

Due to variances in the assembly process, each apparatus has a slightly different capacitance. When the zero switch is released, the internal capacitance along with the user's body capacitance coupled through the switch is enough to make the output volatge jump and/or oscillate. Once photoelectrons charge the anode the input voltage will stabilize.

Difracção de electrões

Guião

Objectivos:

ii) Verificar que electrões com energias da ordem de 1-10 keV são difractados por um filme de grafite, exibindo o seu carácter ondulatório;

ii) verificar a relação de de Broglie, $\lambda = h/p$, onde λ é o comprimento de onda do feixe de electrões, p é a sua quantidade de movimento e h é a constante de Planck.

Material: ampola de difracção de electrões, fontes de alimentação, craveira

Introdução:

No interior do tubo de difracção existe um filme de grafite constituído por micro-cristais cuja orientação no espaço é aleatória. O tubo contém também um cátodo que emite electrões e grelhas de aceleração, através das quais é aplicado um campo eléctrico. Os electrões acelerados por uma diferença de potencial V (que varia num intervalo de cerca de 1 a 10 kV) incidem na grafite, atravessam-na e são depois projectados sobre um écran fluorescente .

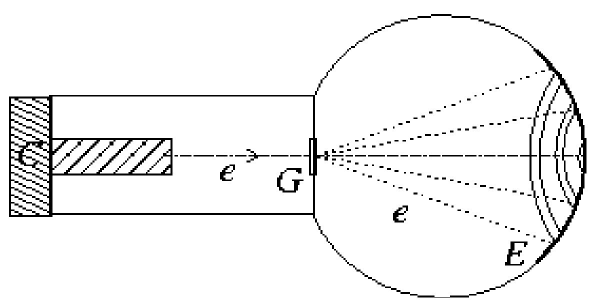


Figura 1: Representação esquemática do tubo de difracção de electrões. **G** representa a amostra de grafite, ***e*** o feixe de electrões e ***E*** o écran fluorescente.

Tratando-se de partículas, esperar-se-ia a formação de uma mancha no ecrã com a forma do filme de grafite. Em seu lugar, porém, observa-se um padrão de interferência, com anéis circulares correspondentes a máximos e mínimos, tal como é típico de uma onda! Os electrões comportam-se afinal como ondas que se difractam nos planos de átomos de carbono que constituem a grafite. O seu comprimento de onda λ pode então ser determinado se conhecermos a estrutura da grafite e se medirmos o diâmetro dos anéis que se formam no ecrã. Por outro lado, a quantidade de movimento p dos electrões que colidem com a grafite pode ser determinada a partir da diferença de potencial eléctrico V a que são sujeitos, através da relação $eV = p^2/2m$. Assim, para cada valor da tensão de aceleração V podemos determinar o comprimento de onda λ e a quantidade de movimento p dos electrões; repetindo as medidas para um conjunto de valores V obtemos um conjunto de pares de valores (p, λ) e podemos verificar a validade da relação de de Broglie:

$$\lambda = h/p \; .$$

Note-se que, tal como em qualquer figura de difracção, há uma fracção do feixe que atravessa o alvo sem sofrer desvio e produz uma mancha no centro do tubo. Se diminuirmos a tensão de aceleração, o diâmetro dos anéis aumenta e o diâmetro da mancha central aumenta também. Focando a imagem torna-se evidente que essa mancha central, associada a electrões que não sofreram desvio, é de facto a uma imagem da grafite, tal como esperaríamos para um feixe de partículas. É mesmo possível reconhecer a forma dos parafusos de suporte do filme de grafite!

Como podemos determinar o comprimento de onda λ dos electrões?
A amostra de grafite é composta por cristais que têm um arranjo regular de átomos de carbono (ver Fig.2). Os planos desses átomos, separados por uma distância d , actuam como uma rede de difracção tal como está esquematizado na Fig.3. A condição de interferência construtiva dos feixes emergentes é dada pela lei de Bragg:

$$n\,\lambda = 2\,d \sin\theta \quad \text{com } n = 1,2,3, \ldots$$

onde θ é o ângulo entre o feixe de electrões e os planos da rede.

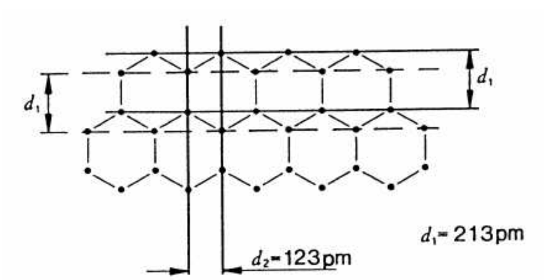


Figura 2:Planos de átomos na grafite associados aos primeiros dois aneis de interferência.

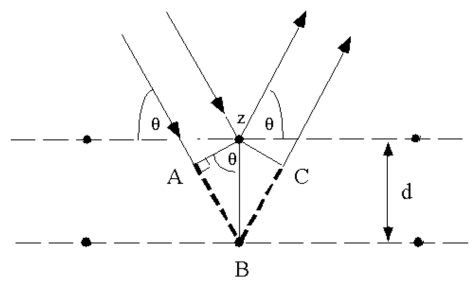


Figura 3: Os feixes difractados pelos dois planos de átomos estão
em fase e interferem construtivamente se a distancia ABC= 2 d sin θ for igual a um número inteiro de comprimentos de onda, isto é se $n\,\lambda = 2\,d \sin\theta$.

A amostra de grafite é policristalina estando os diferentes cristais orientados de forma aleatória. O ângulo de Bragg θ pode ser determinado a partir do ângulo de desvio α do feixe (ver Figuras 4 e 6) sendo $\alpha = 2\,\theta$.

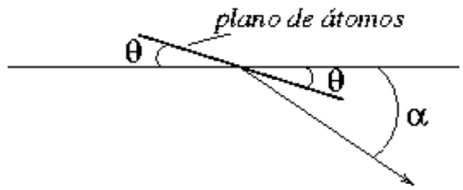


Figura 4: O ângulo de Bragg é o ângulo entre o feixe de electrões e um plano átomos, em condições de interferência construtiva; o ângulo de desvio a correspondente a um máximo de interferência é o dobro do ângulo de Bragg, $\alpha = 2\ \theta$.

Da fig. 6 temos

$\sin(2\alpha) = r/R = 2r/D$,

onde D=127 mm é a distância nominal entre a amostra de grafite e a parede do tubo onde se encontra o ecrã fluorescente.
Para ângulos pequenos,

$\sin(2\alpha) \approx 2 \sin\alpha = 2 \sin(2\ \theta) \approx 4 \sin\theta$.

Assim

$n\ \lambda = d\ r/D$.
A grafite tem uma estrutura hexagonal na qual é possível encontrar várias distâncias interplanares d_i (ver Fig. 5). Os dois valores de d_i mais elevados são d_1 = 0,213 nm e d_2 = 0,123 nm, representados na Fig.2. Para cada ordem *n* de difracção e para cada valor de λ (determinado pelo valor de V) haverá diferentes anéis para diferentes d_i. Os dois anéis de difracção com raio menor (e mais facilmente mensuráveis) correspondem à difracção de primeira ordem (n=1) dos planos da grafite com valor de *d* mais elevado, d_1 e d_2 . Conhecidos os valores de d_1, d_2 e D, se medirmos os raios r_1 e r_2 dos dois primeiros anéis de interferência obtemos duas medidas independentes para cada λ :

$\lambda = d_1\ r_1/D$ e $\lambda = d_2\ r_2/D$.

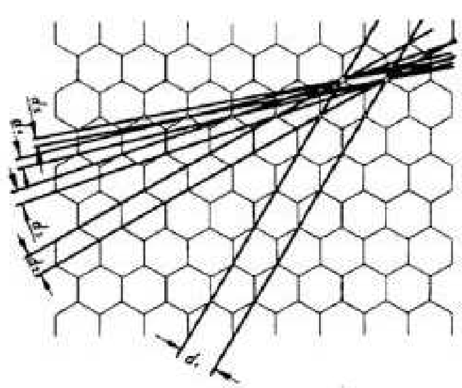


Figura 5: Distâncias d_i entre planos de átomos na grafite.

Sistema Experimental:

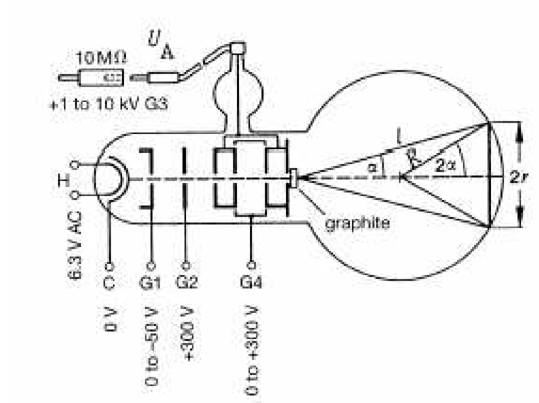


Figura 6: Representação esquemática do sistema de emissão, focagem, aceleração e projecção num alvo do feixe de electrões na ampola.

O sistema de emissão e aceleração dos electrões no tubo está esquematizado na Figura 6. O cátodo K é aquecido através da resistência H emitindo electrões. Os electrões são então acelerados por um campo eléctrico criado pelo sistema de grelhas G1-G4. O cilindro de Wehlnet G1 vai concentrar o feixe de electrões na direcção do eixo (eixo óptico). Este feixe é então acelerado pela grelha G2, a qual está a um potencial positivo, e seguidamente pela grelha G3 que está a um potencial ainda mais elevado. Finalmente G4 é usado para focar o feixe, funcionando como uma lente electrostática. Na sua montagem G4 está fixo e é igual a 250 V. O feixe de electrões vai então incidir na amostra que está fixa no eixo do tubo. A amostra consiste numa rede de cobre na qual se encontra um filme de grafite policristalina. Quando os electrões incidem na amostra são difractados com diferentes ângulos, projectando uma imagem no ecrã fluorescente.

A corrente no ânodo não deve exceder significativamente 1 mA. A limitação de corrente pode ser obtida usando uma resistência de cerca de 10 MW ou através de um circuito limitador de corrente incluído na própria fonte.

A intensidade e contraste dos anéis pode ser controlada com a tensão G1.

A mancha luminosa central (correspondente a electrões que atravessaram o cristal sem sofrer desvio) é intensa e pode danificar a camada fluorescente do tubo. Para o evitar reduza a intensidade da luz após cada leitura.

Execução:

1. Ajuste a tensão de aceleração dos electrões para V= 8 kV ($V=U_A$, na figura 6). **V não deve exceder 8 kV**. Ajuste a tensão G1 de forma a que a imagem de difracção seja nítida.

2. Meça os diâmetros 2r1 e 2r2 dos dois anéis de difracção de menor raio com o auxílio de uma craveira. Estime as incertezas σ_{2r1} e σ_{2r2} associadas a cada medida e anote esses valores.

3. Repita o procedimento anterior para um conjunto de valores de V inferiores a 8 kV. Note que espera que $\lambda = h/p = h\ (2\ m\ e\ V)^{-1/2}$, ou seja espera que $\lambda \propto V^{-1/2}$. Uma vez que λ não varia linearmente com V, a escolha de intervalos regulares de V não é a mais adequada. Uma vez que os anéis são visíveis num intervalo de V entre ≈2 kV e 8 kV pode estimar os valores mínimos e máximos esperados para λ e escolher os valores de V adequados para obter cerca de 10 pontos a intervalos aproximadamente regulares de λ. Por ex. V= 8, 6.5, 4.5, 3.9, 3.3, 2.9 , 2.6, 2.3, 2.0, 1.8 em kV.

4. Construa uma tabela com os valores medidos de V, σV , 2r1, σ_{2r1}, 2r2, e σ_{2r2} do tipo

V (KV)	σV (KV)	2r1 (mm)	σ_{2r1} (mm)	2r2 (mm)	σ_{2r2} (mm)

Análise:

1. A partir da tabela dos dados originais construa uma nova tabela com valores calculados para as grandezas $\lambda 1$= d1 r1/D , $\sigma\lambda 1$, $\lambda 2$= d2 r2/D , $\sigma\lambda 2$, λ , $\sigma\lambda$,
1/p = $(2\ m\ e\ V)^{-1/2}$, σ1/p ; do tipo (não se esqueça de indicar as unidades!):

V	$\lambda 1$= d1 r1/D	$\sigma\lambda 1$	$\lambda 2$= d2 2/D	$\sigma\lambda 2$	λ	$\sigma\lambda$	1/p = $(2\ m\ e\ V)^{-1/2}$	σ1/p

Use os valores D=127 mm , d1 = 0,213 nm e d2 = 0,123 nm referidos no texto. Note que para cada valor de V obtém dois valores independentes para λ ($\lambda 1$= d1 r1/D e $\lambda 2$ =d2 r2/D) com incertezas $\sigma\lambda 1$ e $\sigma\lambda 2$ diferentes. Assim, os valores $\lambda 1$ e $\lambda 2$ devem ser ponderados com as respectivas incertezas no cálculo do valor médio, λ , i.e.

$$\lambda = \frac{\dfrac{\lambda_1}{\sigma_{\lambda_1}^2} + \dfrac{\lambda_2}{\sigma_{\lambda_2}^2}}{\dfrac{1}{\sigma_{\lambda_1}^2} + \dfrac{1}{\sigma_{\lambda_2}^2}}$$

As incertezas $\sigma\lambda_1$. $\sigma\lambda_2$, $\sigma\lambda$ e σ_1/p são calculadas usando a fórmula de propagação de erros (ver notas sobre análise de dados). Assim, por ex. ,

$$\sigma_{\lambda i} = d_i \ \sigma_{2\theta i} /(2D) \qquad \text{e} \qquad \sigma_{\lambda} = \sqrt{\frac{1}{\frac{1}{\sigma_{\lambda_1}^2} + \frac{1}{\sigma_{\lambda_2}^2}}}$$

Verifique as expressões anteriores e deduza a expressão de σ1/p.

2. Represente graficamente λ em função de 1/p. Utilize barras de erro e faça o ajuste a uma recta utilizando o método dos mínimos desvios quadrados. Interprete o valor do declive e compare com o valor esperado. (link para programa de ajuste a uma recta http://mars.fis.uc.pt/~jpinto/simulacoes/fit/linear_fit.html).

Discussão e Conclusões:

1. Discuta os resultados obtidos e conclua se estes permitem verificar a relação de de Broglie.

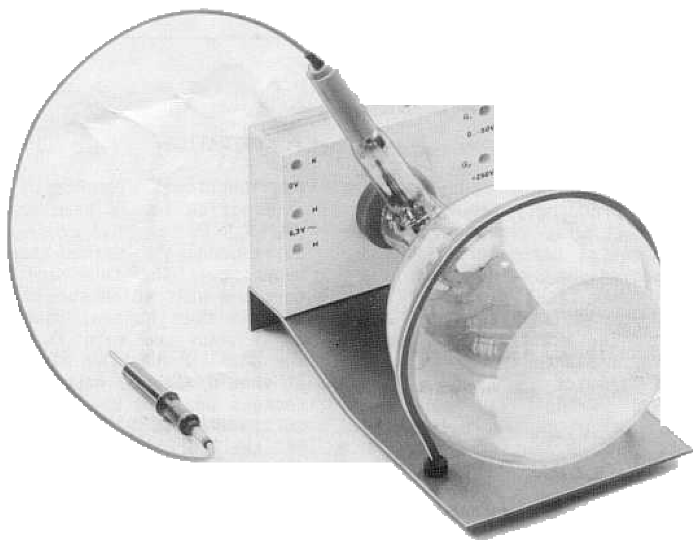
2. Compare a ordem de grandeza dos valores de λ obtidos com a ordem de grandeza de d_1 e d_2. Se se tratasse de um feixe de neutrões com uma energia da mesma ordem de grandeza (1-10 KeV) esperaria observar a difracção do feixe de neutrões na amostra de grafite? Justifique. Estime a ordem de grandeza da energia de um feixe de neutrões para que este seja difractado nos planos dos átomos da grafite.

Difracção de Electrões

Manual da "Phywe"

PHYWE
Operating Instructions

Electron Diffraction Tube with mounting 06721.00



CONTENTS

1. PURPOSE

With the electron diffraction tube, the corpuscular and wave characteristics of electrons can be demonstrated and studied. In comparison with other experiments in the quantum physics of electrons, the method of electron diffraction on a crystal grid proves particularly advantageous because:

- the diffracted image can be made directly visible with the help of a fluorescent screen and only one simple to operate, compact test instrument is required.

The electron diffraction tube enables the de-Broglie principle to be proved experimentally and with considerable accuracy, as a basis for wave particle dualism applicable to electrons.

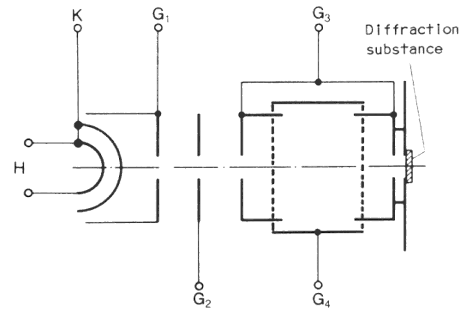


Fig. 2

2. DESCRIPTION AND MODE OF OPERATION

The electron diffraction tube has a mounting to which the necessary power supply units and measuring equipment can be connected.

In the electron diffraction tube, an electron beam is produced which can be controlled, as it is dependent on external, experimental parameters. The electron diffraction tube beam emission system is shown schematically in Fig. 2. Thermionic cathode K is heated by a heating coil H. The electrons emitted from the cathode are accelerated in an electric field created by the grid system G_1 to G_4. Wehnelt cylinder G_1 deflects a narrow electron bundle in the direction of the symmetrical axis (optical axis). The electrons from the Wehnelt cylinder are easily accelerated forward by Grid G_2, which has a positive potential. Then the electrons flying through the opening at Grid G_2 are accelerated very vigorously because of the strong positive potential at the anode, G_3. Finally, Grid G_4 is used to focus the electron beam, providing, with the anode, an electron optic lens system. The accelerated particles land on a film attached vertically to the optical axis. This film consists of a copper mesh on which there is a layer of polycrystalline graphite. As the particles penetrate this film, they are deflected from their path to varying degrees and enter the spherical section of the electron tube, see also Fig. 5.

Where the electrons reach the inner wall of the glass sphere, they encounter a fluorescent layer. In areas where individual electrons land in clusters on the fluorescent layer, they produce a clearly visible luminescence, due to the fluorescent rays.

3. OPERATION

The functional elements of the electron diffraction tube's beam emission system (H, K, G_1 to G_4) connect with the correspondingly marked sockets on the mounting. The tube and the mounting form one unit which should not be split. If this does happen, then care must be taken when the tube is re-fitted that the small connecting pins are not bent. It should also be noted that one of the sockets on the tube mounting is sealed so that the tube can only be plugged in in the correct way for the contact arrangement.

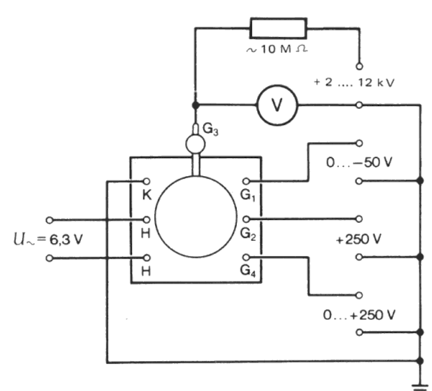


Fig. 3

Fig. 3 shows the electrical wiring for the electron diffraction tube, taking in a meter for the anode voltage. An AC current is sufficient for the heating. To power the other functions, well-filtered DC voltages are always required. The individual voltage ranges and values are also reproduced in Fig. 3. All voltages (apart from the anode voltage) can be taken from a universal power supply 11725.93.

The anode current should not far exceed 1 mA. Therefore, a limiter resistor of some 10 MΩ (e.g. 10 MΩ resistor with plug and socket, Order no. 07160.00) should generally be wired in front of the anode. If a HV unit with short-circuit current < 1 mA is used to provide the anode voltage (e.g. HV power supply 25 kV, Order no. 11730.93), then a limiter resistor is not required. The HV cable coupled firmly to the anode has dialectric strength, so that no special conductor lead is required.

If HV power supply 11730.93 is used, no separate earthing of the cathode is needed, as the negative pole on the unit has a zero potential.

For all the experiments described in Paras. 4 and 5, the following initial setting, based on the cathode, should be selected for the grid voltages (any further adjustment then depends on the aim of each experiment):

- Wehnelt cylinder (Grid G_1) -25 V
- Grid G_2: +250 V
- Anode (Grid G_3): +10 kV
- Grid G_4: approx. +250 V

4. DIFFRACTION OF THE ELECTRONS ON GRAPHITE

The graphite layer on the diffraction film is made up of many micro-crystals which are spatially randomly oriented. Each crystal shows the typical structure for graphite. The two largest grid-level distances are:

$$d_1 = 213 \text{ pm}$$

and

$$d_2 = 123 \text{ pm}$$

The electrons deflected at the graphite have a de-Broglie wave length dependent on the anode voltage U_a, as follows:

$$\lambda = \sqrt{\frac{1500 \text{ kV}}{U_a}} \text{ pm},$$

Where U_a is the anode voltage (acceleration voltage), or the potential to earth at Grid G_3. Since the anode voltage should not exceed 12 kV, the smallest de-Broglie wavelength that can be achieved with the electron diffraction tube is:

$$\lambda_{min} = 11.2 \text{ pm}$$

This value is clearly smaller than the grid constants for the structure of graphite. Thus, there may be interference on the diffraction rings of the individual wave packages.

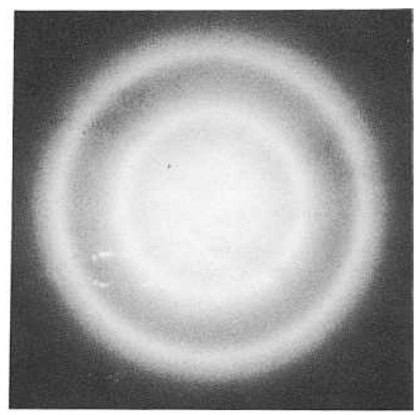


Fig. 4

In the experiment, the additional electric power supply equipment is first of all adjusted to the initial setting. The negative voltage at the Wehnelt cylinder (Grid G_1) is reduced until the strongest possible diffraction rings have formed.

Fig. 4 shows the diffraction image in an experimental example. Both rings seen in the image comply with the Bragg definition

$$2\, d_{1,2} \sin \theta_{1,2} = \lambda = \sqrt{\frac{1500 \text{ kV}}{U_a}} \text{ pm},$$

Where $\theta_{1,2}$ represents the angles of diffraction of the electron beam, dependent on anode voltage U_a (see Fig. 5).

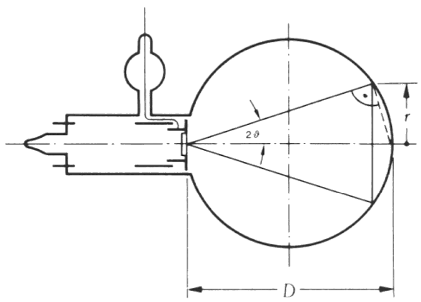


Fig.

If the relationship between the angles of diffraction θ_1 and θ_2 and the anode voltage U_a is to be studied, then the latter must be increased step by step (say from U_a = 2 kV up to U_a = 12 kV), and, at the same time, the voltage at the Wehnelt cylinder must be adjusted, so that the diffraction image is as clear as possible (when increasing the Wehnelt voltage, the anode voltage previously set will always drop slightly, because there is increasing load on the HV supply unit).

Angles of diffraction $\theta_1 + \theta_2$ can be determined from the definitive equation:

$$\theta_{1,2} = \frac{1}{4} \arcsin \frac{2r_{1,2}}{D}$$

using the appropriate diffraction ring diameters 2 r_1 and 2 r_2. The ring diameters can best be measured with a slide rule, noting that the rule should be aligned with the maximum intensity of the rings. (Relatively accurate results can be obtained if the diameter is measured from the edges of the rings in each case and the position of maximum intensity calculated by finding mean values.)

D is the greatest distance of the graphite layer, the diffraction substance, from the wall of the glass sphere. The nominal dimension to be used in the calculation is:

D = 127 mm

The graphs in Fig. 6 show the relationship found between angles of diffraction θ_1 and θ_2 and the anode voltage, U_a in a measuring example.

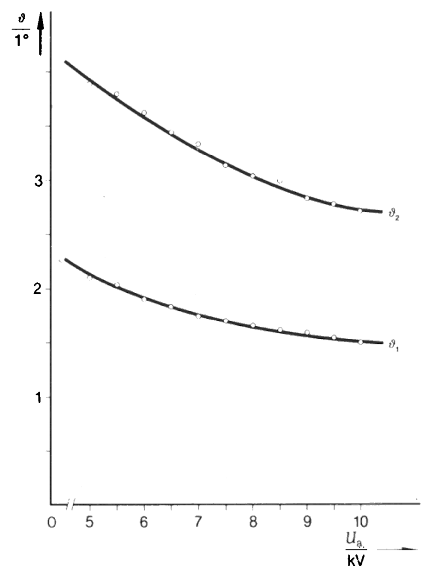


Fig. 6

The values of $\lambda/2 \sin \theta_{1,2}$ are, according to Fig. 7, mostly identical to the values of the grid-level distances d_1 and d_2.

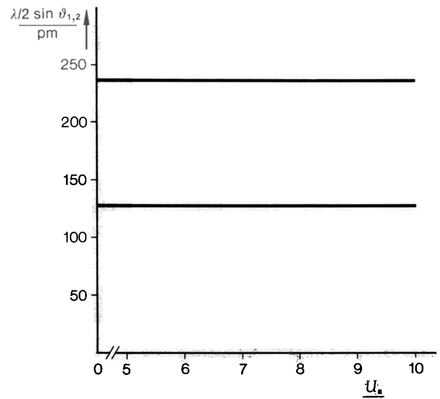


Fig. 7

5. ELECTRON OPTIC REPRESENTATION OF THE CARRIER MESH

If the electron bundle that hits the sample is expanded sufficiently, the copper mesh that carries the graphite layer necessary for diffraction can be shown on the screen.

The connected electrical power supply equipment is adjusted to the initial setting. The voltage at the Wehnelt cylinder (Grid G_1) is reduced until the diameter of the patch of light is large enough for the shadow of the carrier mesh to be seen.

The mesh structure seen on the screen shows that objects can be represented geometrically with the help of electrons in an optical context.

6. FAULT LEVELS

With an anode voltage of 12 kV, the speed of the electrons in relation to the speed of light is some 22%. It is therefore acceptable to regard the electrons as non-relativistic.

For the de-Broglie wavelength, this approximation relates to a fault level of max. 1.2%.

Systematic or equipment-related deviations in the experiment results as compared with theoretical values arise from the inevitable inaccuracy of D (distance from graphite layer to glass sphere). In accordance with the tolerances set for D, a fault level of up to 2.5% can be expected.

7. ADDITIONAL INFORMATION

As well as the two strong diffraction rings, other less intense rings of larger diameter can be seen. If the room is in complete darkness, these rings can also be seen clearly.

To demonstrate the diffraction phenomenon in a large room or lecture theatre, it is advisable to use TV transmission equipment.

8. PROTECTION FROM RAYS

When using the electron diffraction tube, X-rays are produced. As defined by the regulations with regard to protection against damage by ionising rays, 13th October, 1976 (Regulations on Radiation Protection) and the regulations with regard to protection against damage by X-rays of 1st March, 1973 (Regulations on X-rays), the electron diffraction tube emits hazardous rays. Clause 5 (2) of the regulations on X-rays states how the instrument should be handled. Since, under operating conditions (maximum operating voltage: U_a = 12 kV), the localised emission of X-rays at a distance of 5 cm from the surface of the instrument is less than 36 pA/kg, the electron diffraction tube can be used in the lecture room without hesitation and without any special measures to protect against radiation. The legislation demands no design permit. The instrument can be used without special permission from or notification to authorities.

It should, however, be pointed out in this connection, that, if the maximum operating voltage of 12 kV is exceeded, not only could the instrument be damaged, but the radiation emission becomes unacceptably high.

9. EXPERIMENT LITERATURE

Experiments in Physics
Wave Particle Dualism
Order No. 16052.41

University Laboratory
Experiments - Physics
Part 2, Order No. 16500.41

10. TECHNICAL DATA

Diffraction substance:	Graphite
Carrier Mesh:	Copper
Nominal distance from diffraction substance to inner wall of sphere:	127 mm ± 3 mm
Diameter of fluorescent screen	10 cm
Heating voltage:	U AC = 6.3 V 300 mA
Wehnelt voltage (filtered):	0 to -50 V
Acceleration voltage (filtered):	+250 V
Anode voltage (filtered):	+2 to +12 kV
Anode current	< 1 mA
Focussing voltage (filtered):	0 to +250 V

11. EQUIPMENT LIST

The most important instruments used in connection with the electron diffraction tube are given below with their order nos. The complete equipment lists required for various experiments can be found in the experiment literature quoted.

Order No.	Equipment
11151.00	Electrostatic voltmeter 26 kV
11725.93	Power supply, universal
11730.93	HV power supply 25 kV

PHYWE

PHYWE AG
P.O.Box 30 44
D-3400 Göttingen/F.R.G. · Phone (05 51) 604-493
Telex 96 808 · Telegrams PHYWE Goettingen

Druck-Nr.: P 0685024

Determinação dos espectros de emissão de H e Hg

Guião

1. Inicie o programa Scienceworkshop seguindo as indicações na página 11 do manual da Pasco.

2. O sensor de rotação determina a posição angular a partir das rotações do pino do sensor. O pino tem um raio cerca de 60 vezes inferior ao da placa giratória e por isso os desvios angulares na placa são cerca de 60 vezes inferiores aos medidos pelo sensor. Para corrigir os valores medidos pelo sensor use o programa ScienceWorkshop tal como está indicado no topo da página 17 do manual. Em vez de 60 use o valor 59,8 que foi determinado como é sugerido na página 7.

3. Use o programa Science workshop para amplificar o sinal do sensor de luz. Para isso
carregue no símbolo de sensor de luz e mude a sensibilidade de "low" para "medium".

4. Ligue a lâmpada de sódio. Reduza as fendas ao mínimo e foque a imagem das fendas. Abra uma janela gráfica que represente a intensidade em função do desvio angular corrigido pelo factor 59,8. Registe os dados, que devem incluir a posição angular da linha de 1ª ordem à esquerda e à direita da linha central. Uma vez obtido o espectro, registe a posição θesq e θdir das linhas de primeira ordem com o auxílio do cursor em forma de cruz.

5. Determine o desvio angular $\theta=|\theta esq - \theta dir|/2$ correspondente à difracção de 1ª ordem da linha amarela do sódio. Use estes resultados e o conhecimento do comprimento de onda λ = 589,3 nm da linha amarela do sódio para determinar o espaçamento d das linhas de rede de difracção, tal como indicado na página 13 do manual.

6. Substitua a lâmpada de sódio por uma lâmpada de mercúrio e registe o espectro, incluindo linhas de 1ª ordem à esquerda e à direita da linha central. Analise o espectro e verifique se é possível melhorá-lo reduzindo a luz parasita, ajustando as fendas, a focagem, ou a amplificação do sinal. Determine o valor de $\theta=|\theta esq - \theta dir|/2$ para cada linha. Use o valor d obtido anteriormente e a relação $\lambda = d \sin \theta$ para determinar os comprimentos de onda das linhas de mercúrio. Registe os dados e os resultados numa tabela.

7. Substitua a lâmpada de mercúrio pela lâmpada de hidrogénio. Note que a luz da lâmpada de hidrogénio é muito menos intensa do que a de mercúrio ou sódio. Terá que ter mais cuidado com luz parasita e terá que escolher a amplificação máxima no sensor de luz. Seleccione fendas mais largas, verifique a focagem e faça vários registos com diferentes larguras de fendas até decidir qual o melhor compromisso de intensidade versus resolução.

8. Determine os comprimentos de onda das linhas do hidrogénio. Registe os dados e os resultados numa tabela.

9. As linhas do hidrogénio na região do visível pertencem à série de Balmer, isto é a transições cujo estado final tem número quântico principal n_f=2. Use os comprimentos de onda obtidos para determinar

i) o número quântico n_i do estado inicial, para cada linha.

ii) o valor da constante de Rydberg.

10. Não se esqueça que qualquer resultado tem sempre uma incerteza associada. As tabelas e resultados finais devem sempre incluir as incertezas nas medidas e nas grandezas calculadas a partir das medidas.

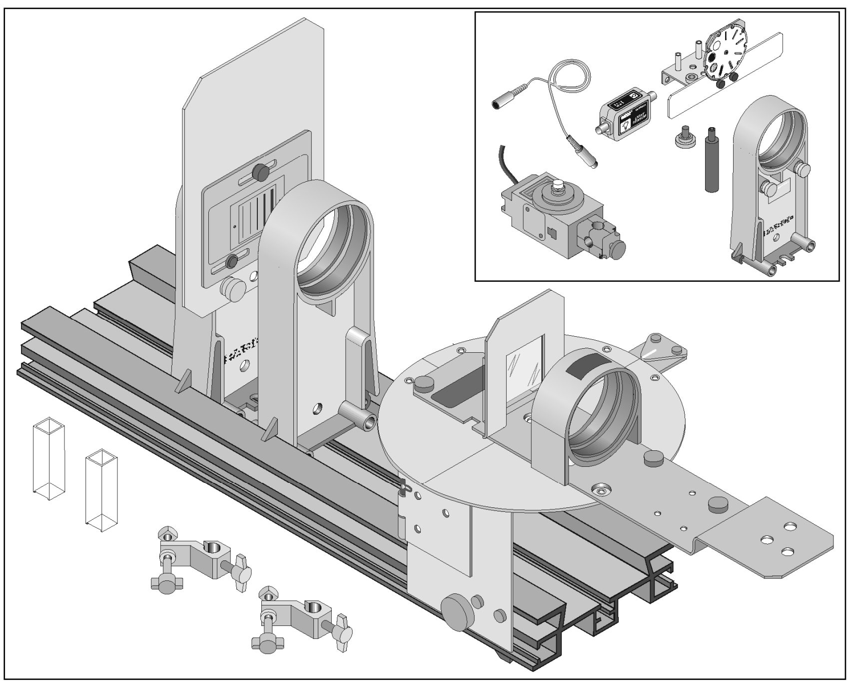
Determinação dos espectros de emissão de H e Hg

Manual da "Pasco"

Instruction Manual and Experiment Guide for the PASCO scientific Model OS-8537 and OS-8539

012-06575A
3/98

EDUCATIONAL SPECTROPHOTOMETER ACCESSORY KIT AND EDUCATIONAL SPECTROPHOTOMETER SYSTEM



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Quick Start

The following pages give an overview of the Spectrophotometer equipment setup.

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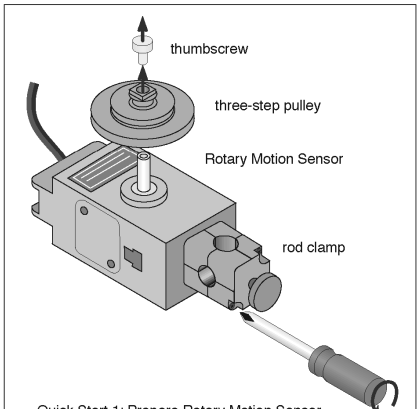
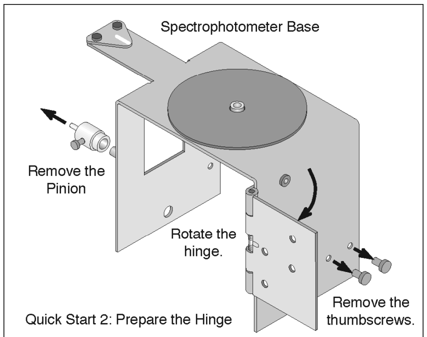


Quick Start 1: Prepare Rotary Motion Sensor

Step Two: Prepare the Spectrophotometer Base by removng the two small thumbscrews and Pinion and by rotating the hinge away from the Base.



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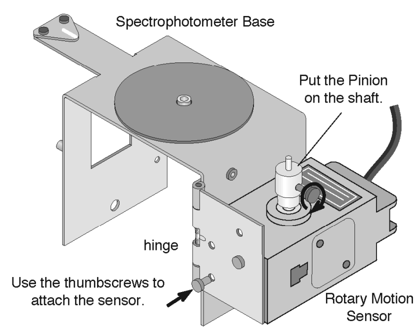

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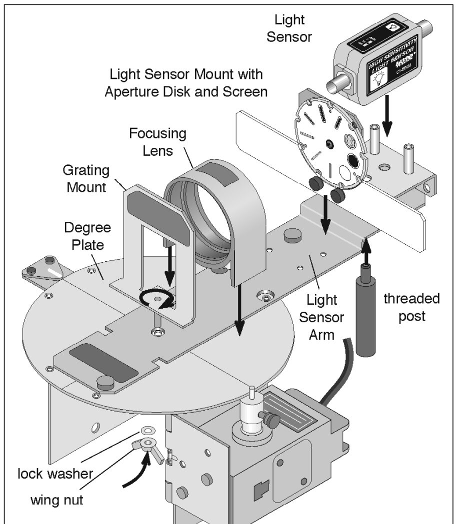

Quick Start 4: Degree Plate & Light Sensor Arm, Grating Mount, Focusing Lens, Light Sensor Mount, and Light Sensor

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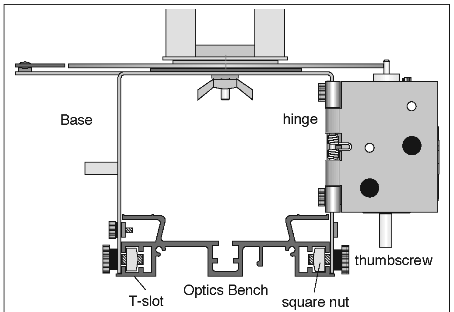

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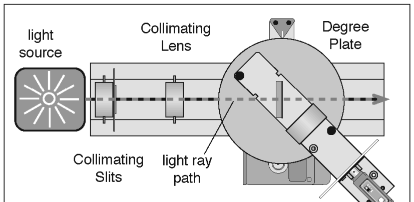

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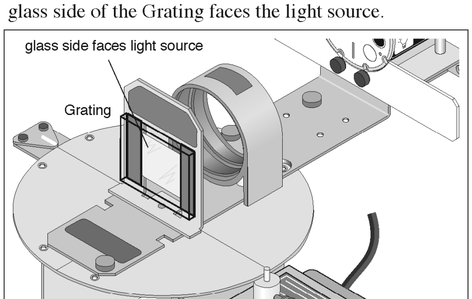

Quick Start 7: Attach the Grating

CAUTION: Avoid touching the Grating surface.

Step Eight: Set up the experiment in the *ScienceWorkshop* program.

1. Select the Light Sensor for Analog Channel A.
2. Select the Rotary Motion Sensor for Digital Channels 1 and 2.
3. Set the Rotary Motion Sensor to high resolution (1440 Divisions/Rotation).
4. Create a calculation for "Actual Angular Position" based on the Angular Position data from the Rotary Motion Sensor *and* the ratio of the radius of the Degree Plate to the radius of the small post on the Pinion (typically, a 60 to 1 ratio).
5. Select a Graph display. Set the *vertical* axis to Light Intensity and the *horizontal* axis to your calculation of "Actual Angular Position".
6. Set the sampling rate to 20 Hz (20 measurements per second).

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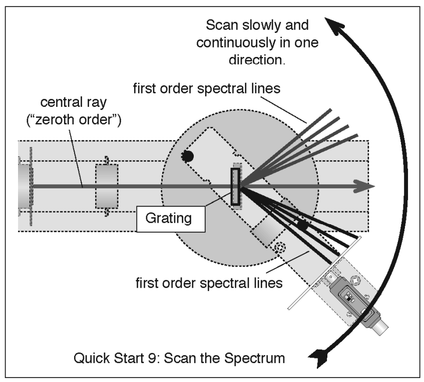
1. Mask or hood the light source if necessary.
2. Move the Light Sensor Arm so the Light Sensor is beyond the edge of the first order spectral pattern.
3. Start recording data. Slowly and continuously scan the spectrum. Scan the first order spectrum on one side of the central ray, through the central ray, and through the first order spectrum on the other side.



Quick Start 9: Scan the Spectrum

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Introduction

About This Manual

This manual describes the PASCO OS-8537 Educational Spectrophotometer Accessory Kit and the PASCO OS-8539 Educational Spectrophotometer System. The OS-8537 Accessory Kit is designed to be mounted on the Optics Bench of the OS-8515 Basic Optics System.

Components of the Kit

The OS-8537 Educational Spectrophotometer Accessory Kit (see Figure 1a) includes the following items:

- Spectrophotometer Base
- Grating Mount
- Focusing Lens
- Collimating Slits
- Rod Stand Mounting Brackets (2)
- Degree Plate with Light Sensor Arm
- Grating (~600 lines per mm)
- Collimating Lens
- Cuvettes (2)

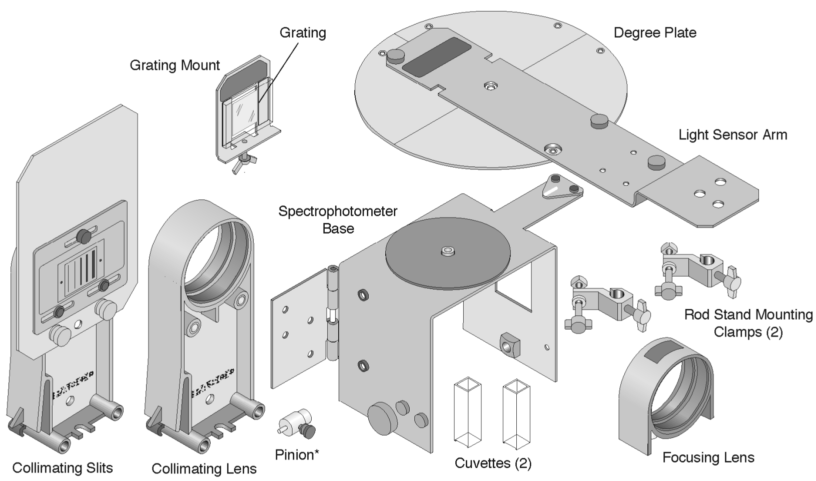

Figure 1a: Educational Spectrophotometer Accessory Kit

(*The pinion shown in Figure 1a can be mounted on a post on the Spectrophotometer Base when not in use.)

Recommended Equipment for use with the Spectrophotometer Accessory Kit:

- Basic Optics System (OS-8515)
- Aperture Bracket (OS-8534)
- High Sensitivity Light Sensor (CI-6604)
- Rotary Motion Sensor (CI-6538)

Components of the System

The OS-8539 Educational Spectrophotometer System includes the items in the Spectrophotometer Accessory Kit plus the following:

Optics Bench (60 cm)
Rotary Motion Sensor (CI-6538)
High Sensitivity Light Sensor (CI-6604)
Aperture Bracket (OS-8534)

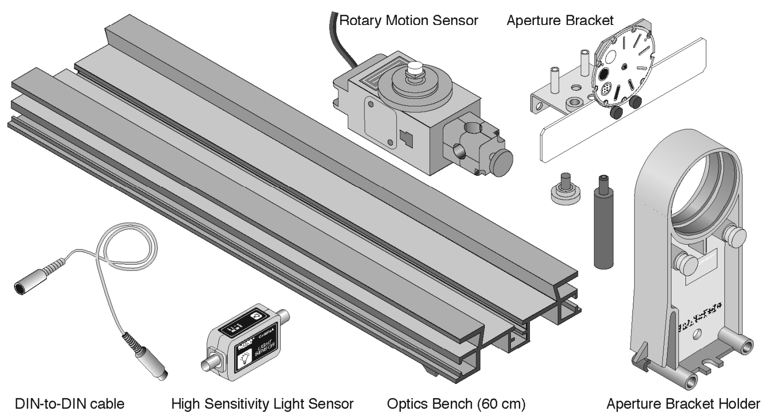

Figure 1b: Additional Components of the Spectrophotometer System

Recommended Equipment for use with both the Kit *and* the System:

Light Source (such as OS-9286 Mercury Light)
Large Rod Stand (ME-8735) (2)
Rod, 45 cm (ME-8736) (2)

Description

The Spectrophotometer allows you to view and measure the spectral pattern (spectrum) produced by a light source. The Collimating Slits and Collimating Lens produce a narrow beam of parallel light rays. The Grating disperses the beam of light into a spectrum with different colors at different angles but with all of the light of a given color in a parallel beam. The Focusing Lens focuses these parallel beams of color into spectral lines (see Fig. 2). The narrow slit on the Aperture Disk (part of the Aperture Bracket) allows light of a single color to enter the High Sensitivity Light Sensor. The High Sensitivity Light Sensor (included with the Spectrophotometer System) measures the intensity of the light while the Rotary Motion Sensor (included with the Spectrophotometer System) measures the angle to which the light is diffracted by the Grating.

You can find the wavelength of each color of light using the measured angle and the Grating spacing "d".

$$m\lambda = d \sin\theta,\ m = 0, 1, 2\ldots$$

where d is the distance between the rulings on the Grating, m is the order of the particular principal maximum, θ is the angle of the diffracted light, and λ is the wavelength.

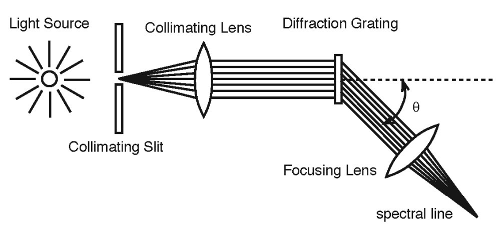

Figure 2: Grating Spectrometer

The Grating disperses the beam of light into a first order spectrum and higher order spectra. The higher order spectra are broader and less bright than the first order spectra, and may overlap.

The Grating is *blazed*, so one side of the spectrum is much brighter than the other.

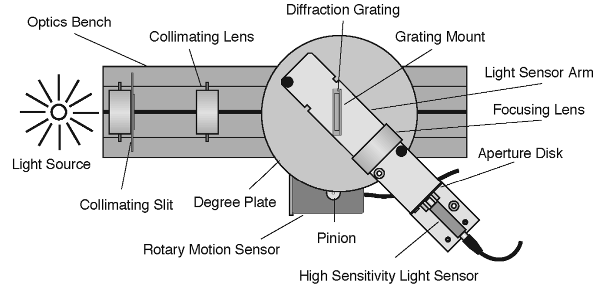

Figure 3: Spectrophotometer System (Top View)

Set Up

This part of the manual describes how to set up the Spectrophotometer System (see Fig. 3).

Mounting the Rotary Motion Sensor

This describes how to mount the Rotary Motion Sensor to the hinge on the side of the Spectrophotometer Base.

The top of the Spectrophotometer Base has a short threaded post for centering the circular Degree Plate and for holding the Grating Mount. It also has a magnetic pad for holding the Degree Plate, and a triangular shaped index marker. One side of the base has a post upon which the Pinion can be stored when it is not in use. The other side has a spring-loaded hinge and two small thumbscrews for mounting the Rotary Motion Sensor (included in the Spectrophotometer System). On both sides of the base are thumbscrews and square nuts used for mounting the Spectrophotometer Base on the Optics Bench (see Fig. 4).

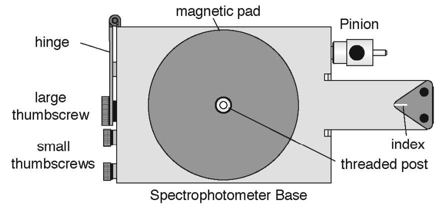

Figure 4: Spectrophotometer Base (Top View)

The Rotary Motion Sensor has a three step pulley attached to its shaft with a small thumbscrew. The sensor also has a rod clamp attached at end end.

First, remove the small thumbscrew and three step pulley from the Rotary Motion Sensor shaft. Then, remove the rod clamp from the Rotary Motion Sensor (see Fig. 5).

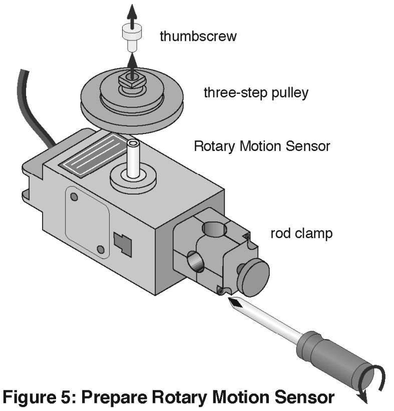

Figure 5: Prepare Rotary Motion Sensor

Remove the two small thumbscrews from the threaded storage holes on the side of the Spectrophotometer Base and set them aside for the moment. Remove the Pinion from the storage post on the opposite side of the Spectrophotometer Base and set the Pinion aside for a moment (see Fig. 6).

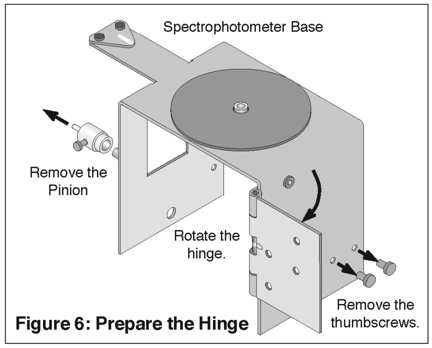

Figure 6: Prepare the Hinge

Rotate the hinge away from the side of the base until the hinge is almost perpendicular to the base. Use the two small thumbscrews to fasten the Rotary Motion Sensor to the *lower* set of holes on the inside of the hinge.

Place the Pinion all the way onto the Rotary Motion Sensor shaft and tighten the Pinion on the shaft by turning the small thumbscrew on the side of the Pinion (see Fig. 7).

Connect the Rotary Motion Sensor plugs into the *ScienceWorkshop* interface.

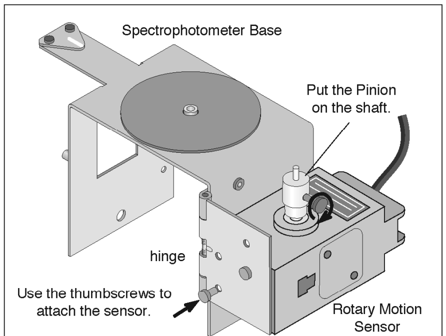

Figure 7: Attach the Sensor and Pinion

Mounting the Degree Plate and Light Sensor Arm

The Degree Plate and Light Sensor Arm are shipped as a unit. The Light Sensor Arm is attached to the circular Degree Plate with two small thumbscrews. The hole in the center of the Degree Plate fits over the short threaded post on the top of the Spectrophotometer Base.

Hold the Rotary Motion Sensor slightly away from the base so the small diameter post on top of the Pinion is not in the way of the edge of the Degree Plate. Position the hole in the plate over the short threaded post on the top of the base. Place the Degree Plate onto the Spectrophotometer Base. Let the small diameter post on the top of the Pinion rest against the *edge* of the Degree Plate (see Fig. 8).

More Information About the Degree Plate

The ratio between the radius of the Degree Plate and the radius of the small post on the top of the Pinion is designed to be 60 to 1. In other words, the Pinion rotates 60 times for one rotation of the Degree Plate.

This assumed ratio of 60 to 1 is included in a calculation for the actual angular displacement of the Degree Plate as it turns during the measurement of a spectrum (see "Using the *ScienceWorkshop* Program" in the Procedure section).

Using the exact ratio of the Degree Plate to the small Pinion post can slightly improve the accuracy of measurement. To determine the *exact* ratio of the Degree Plate to the small Pinion post, do the following to calibrate the Degree Plate:

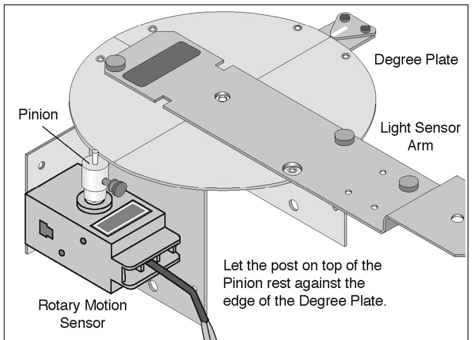

Figure 8: Degree Plate onto Base

1. Remove the Light Sensor Arm from the Degree Plate by unscrewing the two small thumbscrews. (You can store the thumbscrews in the empty threaded holes on the Light Sensor Arm.) Turn the Degree Plate so the zero degree mark is *exactly* aligned with the index mark on the arm that extends from the Spectrophotometer Base.
2. Start the *ScienceWorkshop* program. (See the *ScienceWorkshop* User's Guide for more information.)
3. In the *ScienceWorkshop* program, select the Rotary Motion Sensor to be connected to Digital Channels 1 and 2. Set the resolution of the Rotary Motion Sensor to 1440 Divisions/Rotation.
4. In the program, use the built-in Calculator to create a calculation. Base the calculation on the "Angular Position" measurement from the Rotary Motion Sensor. In the calculation, divide "Angular Position" by 2π, where π is approximately 3.1416… Label your calculation "Ratio of Radii" (Fig. 9).
5. In the program, select a Graph display and set it to show the calculation "Ratio of Radii" on its vertical axis.
6. Start recording data. Turn the Degree Plate in one direction slowly and continuously for exactly one complete rotation. Stop recording data.

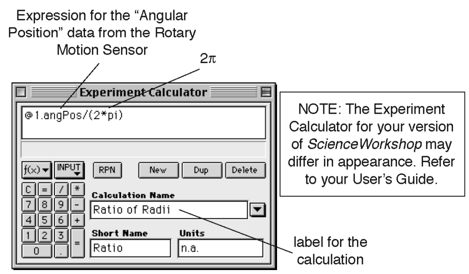

Figure 9: Calculation for Ratio of Radii

7. In the Graph display, use the built-in Statistics tools to find the maximum value of the "Ratio of Radii" (i.e., the maximum of y - see Fig. 10). This value is the exact ratio of the Degree Plate radius to the small Pinion post radius. (Note: This value should be close to 60.) Record the actual ratio.

   Ratio of Radii = _______

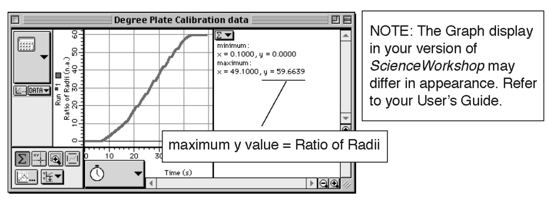

Figure 10: Sample Degree Plate Calibration Data

8. Replace the Light Sensor Arm on the Degree Plate and fasten it to the plate with the two small thumbscrews.

Mounting the Aperture Bracket Light Sensor Mount and Light Sensor

The Aperture Bracket has two main parts: the Light Sensor Mount and the Holder. (The Holder and thumbscrew are not used and can be put aside.) An Aperture Disk and Aperture Screen are attached to the front of the Light Sensor Mount.

The Light Sensor Mount has two holes: one hole in the center of the mount and a second hole at the rear of the mount. The end of the Light Sensor Arm has three holes: two holes along the centerline of the arm and one hole to the side of the centerline. Line up the center hole of the Light Sensor Mount with the centerline hole in the Light Sensor Arm that is closest to the Degree Plate.

Line up the High Sensitivity Light Sensor so the threaded hole in its base is above the center hole of the Light Sensor Mount. Use the threaded post (included with the Aperture Bracket) to fasten the High Sensitivity Light Sensor and Light Sensor Mount to the Light Sensor Arm (Fig. 11).

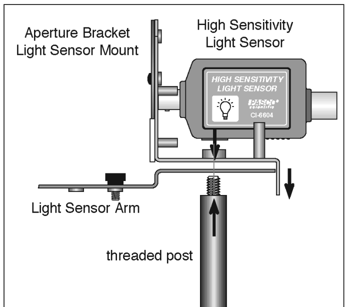

Figure 11: Sensor and Aperture Bracket onto Light Sensor Arm

Rotate the Aperture Disk on the front of the Light Sensor Mount so that the narrowest slit is in line with the opening to the sensor.

Connect one end of the DIN-to-DIN cable that is included with the High Sensitivity Light Sensor to the DIN connector on the end of the sensor.

Mounting the Grating Mount

The threaded rod on the bottom of the Grating Mount has a lock washer and a wing nut. Remove the lock washer and wing nut from the threaded rod, and screw the threaded rod into the short threaded post in the center of the Degree Plate. Do *not* screw the Grating Mount all the way down onto the Light Sensor Arm. The Grating Mount must be slightly above the Light Sensor Arm so the Degree Plate can move.

Use the Light Sensor Arm to rotate the Degree Plate until the zero line on the Degree Plate is aligned with the index mark on the Spectrophotometer Base. Turn the Grating Mount so it is aligned with the zero line on the Degree Plate and the label side of the Grating Mount faces *away* from the sensor on the Light Sensor Arm.

Put the lock washer and wing nut back onto the threaded rod and tighten the wing nut until the Grating Mount remains in place when the Degree Plate is rotated in either direction (Fig. 12).

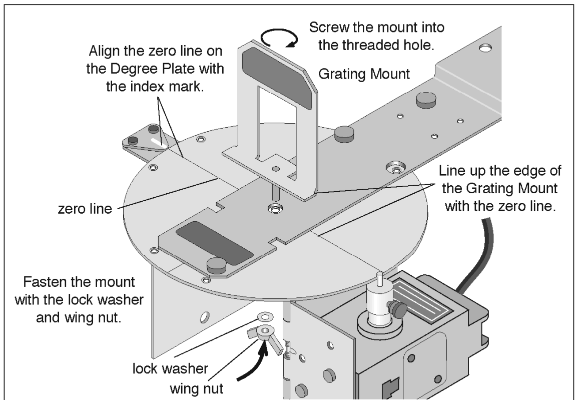

Figure 12: Grating Mount onto Rotating Table

Mounting the Spectrophotometer Base on the Optics Bench

You can mount the components of the Spectrophotometer on the 1.2 meter Optics Bench that is part of the OS-8515 Basic Optics System, or the 0.6 meter (60 cm) Optics Bench that is included with the OS-8539 Spectrophotometer System. The Spectrophotometer Base mounts into the T-slots on the sides of the Optics Bench and the Collimating Slits and Collimating Lens snap into the center of the Optics Bench.

To mount the Spectrophotometer Base on the Optics Bench, loosen the thumbscrew on each side of the Base so the square nuts can slip into the T-slots on the sides of the Optics Bench. Insert the square nuts into the slots and slide the Base along the bench until it is about 20 cm from the end (Fig 13). Tighten the thumbscrews to hold the Base firmly on the bench.

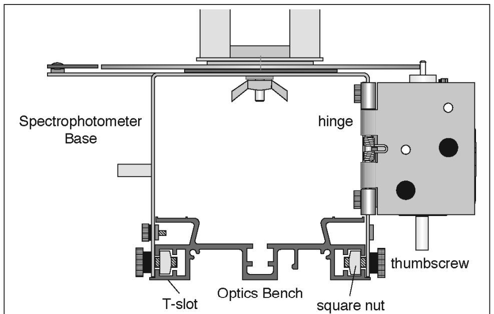

Figure 13: Base Onto Optics Bench (End View)

Rod Stand Mounting Clamps

You can adjust the height or angle of the Optics Bench using the Rod Stand Mounting Clamps and two rods and bases (not included). Each clamp has a thumbscrew, a spacer (washer), and a square nut at one end, and a second thumbscrew at the other end. The thumbscrew with the square nut holds the clamp onto the bench, and the other thumbscrew holds the clamp onto a rod.

Fixed Angle: To mount the clamp on the bench so it is parallel to the bench (at a fixed angle), put the clamp on the bench with the spacer on the opposite side of the clamp from the square nut. Tighten the thumbscrew (Fig. 14).

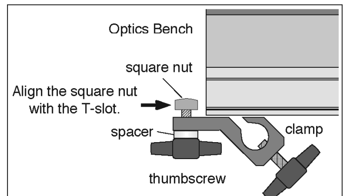

Figure 14: Mounting Clamp onto Optics Bench - Fixed Angle

Adjustable Angle: To mount the clamp on the bench so it can be adjusted to any angle, move the spacer so it is next to the square nut. Put the clamp on the bench with the square nut in the T-slot and the spacer between the bench and the clamp. Adjust the clamp to the angle you need and then tighten the thumbscrew (Fig 15).

Mounting and Adjusting the Collimating Slits and Lens

The Collimating Slits consist of two parts: the Collimating Slits Plate and the Collimating Slits Holder. The slits are in a piece of metal that is attached to a moveable slide. The slide is held on the Collimating Slits Plate with a small thumbscrew and two permanently attached socket screws. The Collimating Slits Plate is attached to the Collimating Slits Holder with two brass thumbscrews. Loosen the small thumbcrew in the Collimating Slits Plate in order to move the slits slide back and forth. The narrow slot in the Collimating Slits Plate lets light through one slit at a time. The top half of the Collimating Slits Plate helps to prevent extra light from being measured.

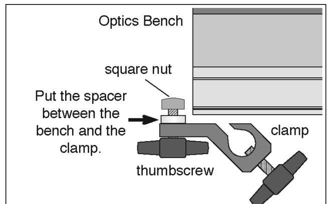

Figure 15: Mounting Clamp onto Optics Bench - Adjustable Angle

Snap the Collimating Slits Holder into the Optics Bench near the end of the bench that is closest to the light source you are using. To move the holder, grasp the holder at the base and push in on the locking clip on one side of the holder (Fig. 16). Slide the holder along the center section of the bench while squeezing the clip. Release the clip to lock the holder firmly in position.

Snap the Collimating Lens Holder into the Optics Bench between the Collimating Slits Holder and the Spectrophotometer Base. Position the Collimating Lens Holder about 10 cm from the Collimating Slits Holder.

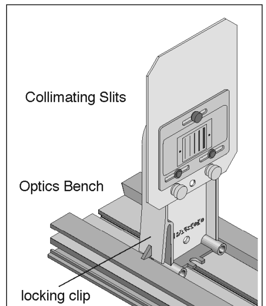

Figure 16: Locking Clip

Positioning the Collimating Slits and Lens

The focal length of the Collimating Lens is *about* 10 cm so the lens should be positioned about 10 cm from the slits. Use the following procedure to position the lens more precisely.

Set up a light source so that light from the source passes through one of the slits on the Collimating Slits and then through the Collimating Lens. Rotate the Light Sensor Arm so the Aperture Bracket and Light Sensor are out of the way and the beam of light can shine onto a distant vertical surface such as a wall (Fig. 17).

Adjust the distance between the Collimating Slits and the Collimating Lens so that the beam of light is neither converging nor diverging (i.e., light rays are parallel). The beam of light should stay about the same width all the way to the distant vertical surface. Hold a piece of paper in the light beam's path at various distances along

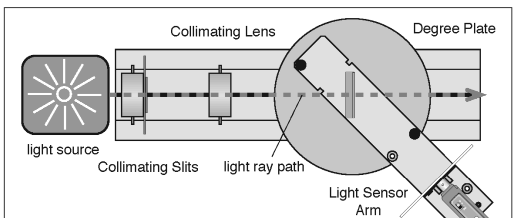

Figure 17: Collimation Setup

the beam's path. Check to see that the light beam's width is about the same at each point along its path. Note that the light may not be in focus during this process.

If the light beam is not perfectly vertical, loosen the small thumbscrew that holds the Collimating Slits slide and adjust the slide, or loosen the brass thumbscrews that hold the Collimating Slits Plate onto the holder and adjust the plate until the collimated beam is vertical. Remember to tighten the thumbscrews after you make the adjustment.

The distances between the light source and the Collimating Slits and between the Collimating Slits and the rest of the Spectrophotometer are not critical. However, the closer the light source, the brighter the spectrum.

Mounting the Grating

Caution! Handle the Grating carefully. Avoid touching the Grating or the glass plate to which the Grating is attached *except by the edges* of the glass plate.

The Grating is mounted on one side of a rectangular glass plate. There are two magnetic pads on the same side of the glass plate as the Grating. These pads hold the Grating in place on the Grating Mount.

Once the light from the source is collimated, attach the Grating to the Grating Mount so that the glass side of the Grating faces the light source (Fig. 18).

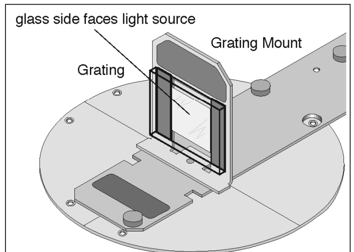

Figure 18: Grating Onto Mount

Mounting and Positioning the Focusing Lens

The Degree Plate has markings on either side of the Light Sensor Arm that indicate the appoximate position in which to place the Focusing Lens. The Focusing Lens has two small magnets in its base that hold it in place on top of the Light Sensor Arm. Place the Focusing Lens on the Light Sensor Arm between the Grating Mount and the High Sensitivity Light Sensor.

The Focusing Lens focal length is *about* 10 cm so the lens should be positioned on the Light Sensor Arm about 10 cm from the front of the Aperture Disk (Fig. 19).

Set up the light source, Collimating Slits, and Collimating Lens so a beam of light shines through the Grating and Focusing Lens. Darken the room so you can see the spectral lines more clearly. Move the Light Sensor Arm so the central ray of light (called the "zeroth order") is centered on the slit at the bottom of the Aperture Disk. You should be able to see the first order spectral lines on the Aperture Screen on either side of the central ray of light. Adjust the position of the Focusing Lens until the spectral lines are sharply focused.

Note: Because the Grating is strongly blazed, the spectral lines on one side of the central ray ("zeroth order") will be less bright than the spectral lines on the other side.

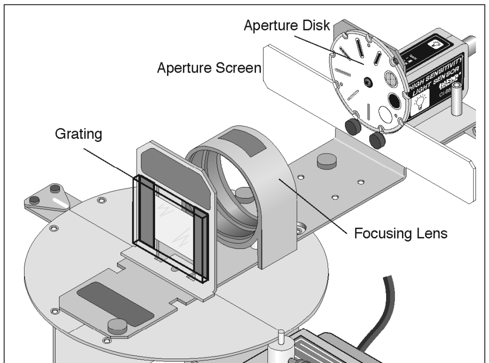

Figure 19: Position Focusing Lens

Procedures

Grounding

1. If you use an AC powered spectral light source, plug the power cord for the light source into a different outlet than the outlet used for your *ScienceWorkshop* interface and computer.
2. For a very dim light source you may need to use the 100 GAIN setting on the High Sensitivity Light Sensor and the 100x Sensitivity setting in the *ScienceWorkshop* program. If so, connect a wire from the Spectrophotometer Base to an earth ground in order to reduce electrical "noise". You can use one of the small thumbscrews from the Light Sensor Arm to connect a ground wire (not included) to the threaded hole on the side of the Spectrophotometer Base opposite to the Rotary Motion Sensor.

Turning the Degree Plate

The spring in the hinge on the Spectrophotometer Base is strong enough to keep the Pinion in contact with the edge of the Degree Plate as you move the Light Sensor Arm to turn the plate. If the small diameter post on the Pinion slips rather than turns, make sure that the thumbscrews that hold the Rotary Motion Sensor onto the hinge are tight. You may need to loosen the screws and then push the Rotary Motion Sensor so it is as close to the Base as possible before re-tightening the screws.

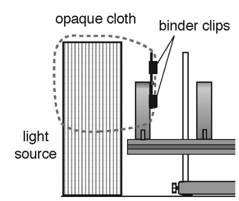

Figure 20: Mask the Light Source

Masking the Light Source or the Spectrophotometer

When using a bright spectral light source such as the PASCO Mercury Vapor Light Source or the Low Pressure Sodium Light Source, cover the source with an opaque cloth hood to block out ambient light. Cover the light source opening with a mask that has a 0.5 to 1.0 cm wide rectangular slot in it to reduce ghost images. Use clothespins or binder clips to attach the edge of the cloth hood to the plate on the Collimating Slits (Fig. 20).

When measuring solar spectra, put an opaque cloth hood over the entire Spectrophotometer so that the only opening to the sunlight is the Collimating Slit. Use clothespins or binder clips to attach the edge of the cloth hood to the plate on the Collimating Slits. Use one hand to hold up the center of the cloth hood (Fig. 21).

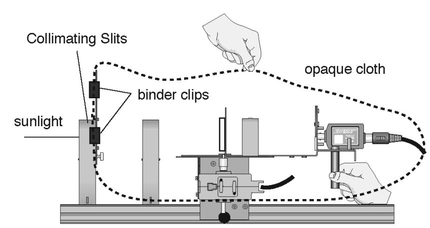

Figure 21: Mask the Spectrophotometer

Using the *ScienceWorkshop* Program

Note: See the *ScienceWorkshop* User's Guide for detailed information about using the *ScienceWorkshop* program.

1. Start *ScienceWorkshop*. Select the Light Sensor to be connected to Analog Channel A and select the Rotary Motion Sensor to be connected to Digital Channels 1 and 2.
2. In *ScienceWorkshop*, set the Rotary Motion Sensor so it can record 1440 divisions per rotation.
3. In the program, select a Graph display and set it to show "Light Intensity (% max)" on its vertical axis.
4. Use the Experiment Calculator in *ScienceWorkshop* to create a calculation of the actual angular position of the Degree Plate. If the small post on the top of the Pinion is in contact with the edge of the Degree Plate, the Angular Position of the Rotary Motion Sensor must be divided by the ratio of the radius of the Degree Plate

and the radius of the small post on the Pinion*. The ratio is approximately 60 to 1. If the larger diameter section at the bottom of the Pinion is used, the Angular Position of the Rotary Motion Sensor must be divided by 15* (Fig. 22).

5. Change the Graph display to show your calculation of Actual Angular Position on its horizontal axis.

(*See "Calibrating the Degree Plate" earlier in the Set Up section for more information on measuring the ratio of the radius of the Degree Plate and the radius of the Pinion.)

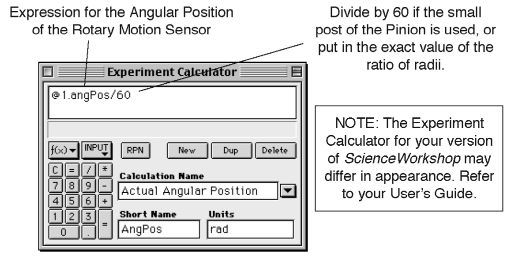

Figure 22: Create a Calculation

General Information About the Light Sensor

The High Sensivitiy Light Sensor has a GAIN (amplification) select switch on the top with three settings (1, 10 and 100). When you measure a spectrum, start with the lowest GAIN setting to measure the brightest lines. Then switch to the next setting and re-scan the spectrum to measure the dimmer lines. Then re-scan again at the highest GAIN setting to measure the dimmest lines.

It is also possible to amplify the signal from the Light Sensor using the *ScienceWorkshop* program. The normal Sensitivity setting is "Low (1x)". The other settings are "Medium (10x)" and "High (100x)".

In general, you will record better data if you increase the GAIN setting on the Light Sensor before you increase the Sensitivity setting in the *ScienceWorkshop* program.

General Information About Slit Widths

There are five slits on the Collimating Slits slide and six slits on the Aperture Disk. You can select wider slits in order to increase the amount of light that passes through the Grating and into the Light Sensor, but this will make a wider spectral pattern and decrease the accuracy of your measurements.

Scanning a Spectrum

To scan a spectrum, use the threaded post under the Light Sensor to move the Light Sensor Arm so the Light Sensor is beyond the far end of the first order spectral lines, but not in front of any of the spectral lines in the second order.

In the *ScienceWorkshop* program, begin recording data. Then, scan the spectrum continuously but slowly in one direction by pushing on the threaded post to rotate the Degree Plate. Scan all the way through the first order spectral lines on one side of the central ray ("zeroth order"), through the central ray, and all the way through the first order spectral lines on the other side of the central ray (Fig. 23).

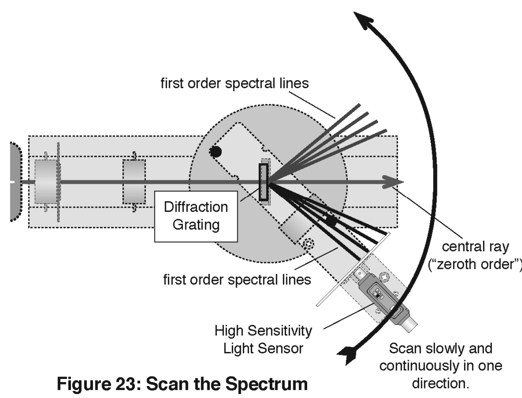

Figure 23: Scan the Spectrum

The angle θ of a particular line in the spectral pattern is one-half of the difference of the angle between the chosen spectral line in the first order on one side of the central ray and the matching spectral line in the first order on the

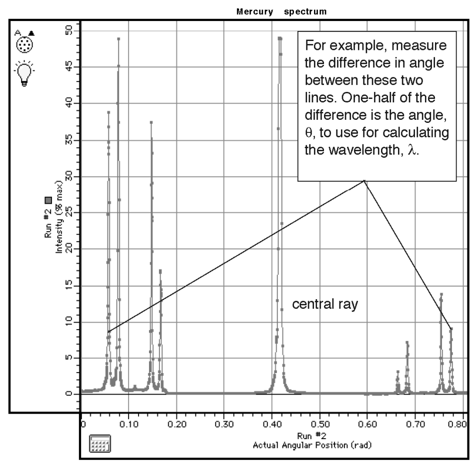

Figure 24: Measure the Angles for Both First Order Spectral Patterns.

other side of the central ray. Use the built-in analysis tools in the *ScienceWorkshop* Graph display to find the angle between the two matching spectral lines. The angle, θ, is one-half of the angle between the two lines. Use $m\lambda = d \sin\theta$ (and let the order, m, be 1) to calculate the wavelength of the chosen spectral line.

If a dim spectral line only appears on one side of the central ray ("zeroth order"), calculate where the central ray is using a brighter spectral line that is visible on both sides of the central ray. Then determine the angle from the central ray to the dim line to find the angle, θ, for that spectral line (Fig. 24).

Calibrating the Grating

The number of grating lines on the Grating is approximately 600 lines per millimeter. This translates to a Grating line spacing of d = 1666 nm ($1.666\ldots x10^{-6}$ m).

To find the *exact* Grating line spacing, use a sodium lamp to calibrate the Grating. The average wavelength, λ, of the yellow sodium doublet is 589.3 nm. Solving $m\lambda = d \sin\theta$ for the line spacing, d, assuming m = 1 (for the first order) gives

$$d = \lambda / \sin\theta$$

where λ is the known wavelength of the sodium doublet. To determine the angle, θ, find the difference in angle between the two first order yellow lines and divide by two.

Using the calibrated Grating line spacing, d, the wavelength of any spectral line in any other spectrum can be more accurately determined using $m\lambda = d \sin\theta$ where the order, m, is 0,1,2,…

Angular Resolution

When the Rotary Motion Sensor is fastened to the *lower* set of holes on the hinge on the Spectrophotometer Base (as described earlier), the small diameter post of the Pinion will make contact with the edge of the Degree Plate. The shaft of the Rotary Motion Sensor will rotate approximately 60 times for every rotation of the Degree Plate (or 360°). One rotation of the Rotary Motion Sensor shaft represents approximately 6° of angular displacement of the Degree Plate. The maximum resolution of the Rotary Motion Sensor is 1440 divisions per rotation. Since one rotation is 6˚, the angular resolution is 6˚ divided by 1440, or one quarter of a minute (0.25') or fifteen seconds of arc. This translates to a wavelength resolution of 2 nm, given a grating line spacing of approximately 1666 nm for the Diffraction Grating included with the Spectrophotometer.

If the Rotary Motion Sensor is fastened to the *upper* set of holes on the hinge, the larger diameter part of the Pinion will make contact with the edge of the Degree Plate. The shaft of the sensor will rotate 15 times for every rotation of the Degree Plate. The maximum angular resolution is one minute or sixty seconds of arc. This translates to a wavelength resolution of 8 nm, given a grating line spacing of approximately 1666 nm.

Cuvettes

The Educational Spectrophotometer Accessory Kit includes two plastic cuvettes. These flat sided containers can hold about five milliliters of liquid. You can use the Spectrophotometer and a cuvette to measure a substance's characteristic pattern of absorption and transmission.

To set up the Spectrophotometer and cuvette for measuring an absorption spectrum, do the following:

1. Unscrew the threaded post that holds the Light Sensor and Light Sensor Mount onto the Light Sensor Arm.
2. Move the Light Sensor back so its threaded hole is lined up with the rear centerline hole on the Light Sensor Mount. Use the threaded post to reattach the Light Sensor and Light Sensor Mount to the Light Sensor Arm.
3. Place the empty cuvette in front of the opening to the Light Sensor between the sensor and the backside of the Aperture Disk. Make sure that the cuvette is turned so that the smooth sides are in line with the Light Sensor (Fig. 25).
4. Scan the spectrum from an incandescent light source (such as a bulb powered by a regulated DC power supply).
5. Fill the cuvette with the liquid to be tested and then rescan the spectrum of the incandescent light that is transmitted through the liquid.

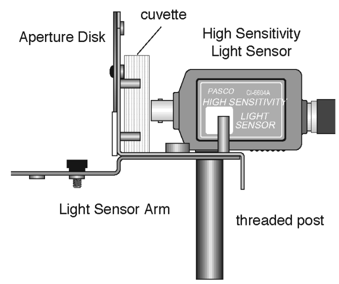

Figure 25: Cuvette with Light Sensor

Other Information

The material on the front of the Aperture Disk may "fluoresce" under ultraviolet (UV) light. It may produce a faint violet color when UV light shines on it. This faint violet color might be seen on the Aperture Disk when you use a mercury light source, for example. The violet line of color appears to be a part of the spectral pattern when the light falls on the disk, but disappears when the light falls only on the Aperture Screen. The High Sensitivity Light Sensor cannot measure ultraviolet light.

The *ScienceWorkshop* program interprets the starting position of the Rotary Motion Sensor as the "zero" angular (or linear) position. For some measurements it may be important to put a mark or stop on the edge of the Degree Plate for reference so you can begin each trial of measurement from the same position. The Degree Plate has several threaded holes near its outer edge. You can put one of the small thumbscrews that are stored on the Light Sensor Arm into one of these threaded holes to use as a reference point for beginning or ending a scan.

Activity 1: Emission (Bright Line) Spectrum

EQUIPMENT NEEDED

– Spectrophotometer System (OS-8539)
or
– Spectrophotometer Kit (OS-8537)
– Rotary Motion Sensor (CI-6538)
– Aperture Bracket (OS-8534)
– High Sensitivity Light Sensor (CI-6604)
– Basic Optics Bench (part of OS-8515)
and
– Mercury Vapor Light Source (OS-9286)
– Rod, 45 cm (ME-8736) (2)
– Large Rod Stand (ME-8735) (2)

Introduction

The purpose of this activity is to determine the wavelengths of the colors in the spectrum of a mercury vapor light.

Theory

An incandescent source such as a hot solid metal filament produces a continuous spectrum of wavelengths. Light produced by an electric discharge in a rarefied gas of a single element contains a limited number of discrete wavelengths - an emission or "bright line" spectrum. The pattern of colors in an emission spectrum is characteristic of the element. The individual colors appear in the shape of "bright lines" because the light that is separated into the spectrum usually passes through a narrow slit illuminated by the light source.

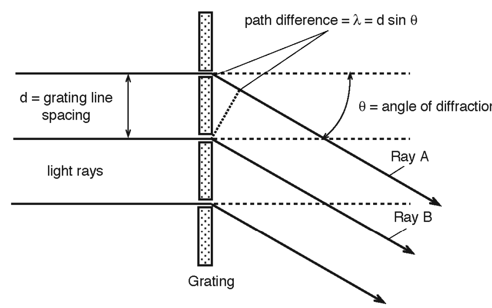

Figure 1.1: Ray diagram for first order diffraction pattern

A grating is a piece of transparent material on which has been ruled a large number of equally spaced parallel lines. The distance between the lines is called the grating line spacing, d.

Light that strikes the transparent material is diffracted by the parallel lines. The diffracted light passes through the grating at all angles relative to the original light path. If diffracted light rays from adjacent lines on the grating interfere and are *in phase*, an image of the light source can be formed. Light rays from adjacent lines will be in phase if the rays differ in path length by an integral number of wavelengths of the light. The first place that an image can be formed is where the path length between two adjacent light rays differs by one wavelength, λ. However, the difference in path length for two adjacent light rays also depends on the grating line spacing, d, and the angle, θ, at which the two light rays were diffracted by the grating.

The relationship between the wavelength of the light, λ, the grating line spacing, d, and diffraction angle, θ, is as follows:

$$\lambda = d \sin\theta$$

In the diagram (Fig. 1.1), the path length for Ray A is one wavelength longer than the path length of Ray B.

Procedure

In this activity, the High Sensitivity Light Sensor measures the relative intensity of colors of light in an emission spectrum produced by light from a mercury vapor light source passing through a grating. The Rotary Motion Sensor measures the angle, θ, of each band or "bright line" of color.

The *ScienceWorkshop* program records and displays the light intensity and the angle. You can use the program's built-in data analysis tools to find the angle for each color, and then you can determine the wavelength, λ, of each color.

Equipment Setup

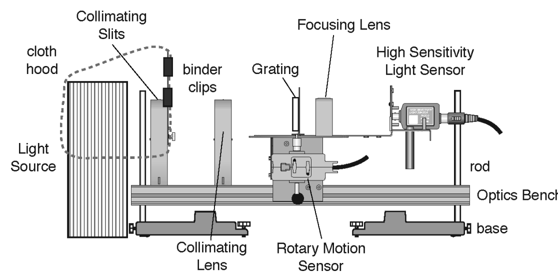

Figure 1.2: Equipment Setup

1. Set up the Spectrophotometer next to a mercury vapor light source as shown (Fig. 1.2). If needed, use the Rod Stand Mounting Clamps, two rods, and two bases to raise the Spectrophotometer to the same level as the opening to the light source. (Refer to the Introduction for more information.)
2. If the light source has a large opening, mask the opening so it transmits a narrow (0.5 to 1.0 cm) beam to the Collimating Slits. Put a cloth hood over the light source and attach the edge of the hood to the plate on the Collimating Slits.
3. Turn on the light source. Once it is warmed up, adjust the light source, Collimating Slits, Collimating Lens, and Focusing Lens so clear images of the central ray and the first order spectral lines appear on the Aperture Disk and Aperture Screen in front of the High Sensitivity Light Sensor. Turn the Aperture Disk so the smallest slit on the disk is in line with the central ray.
4. Connect the *ScienceWorkshop* interface to the computer, turn on the interface. Start *Science Workshop*.
5. Connect the High Sensitivity Light Sensor cable to Analog Channel A. Connect the Rotary Motion Sensor cable to Digital Channels 1 and 2.

Experiment Setup

Select the Sensors, Set the Sample Rate, and Create a Calculation

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on selecting sensors, changing the sample rate, and creating a calculation.

1. In the *ScienceWorkshop* program, select the Rotary Motion Sensor and connect it to Digital Channels 1 and 2 and select the Light Sensor and connect it to Analog Channel A.
2. In the program, set up the Rotary Motion Sensor for high resolution (1440 Divisions per Rotation) and set the sample rate to 20 Hz, or 20 measurements per second.

3. In the *ScienceWorkshop* program, use the Calculator to create a calculation of Actual Angular Position based on the Angular Position measurement made by the Rotary Motion Sensor and the ratio of the radius of the Spectrophotometer's Degree Plate to the radius of the small post on the Pinion (Fig. 1.3). (Refer to the Set Up section for more information.)

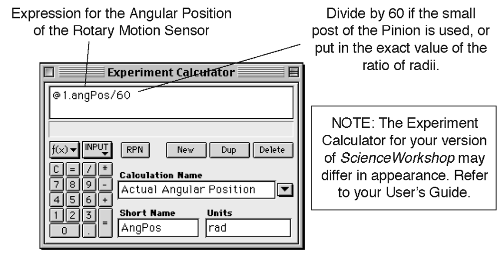

Figure 1.3: Create a Calculation

Select the Display

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on displays.

1. Select a Graph display.
2. Set the axes of the Graph display so *Light Intensity* is on the *vertical* axis and *Actual Angular Position* is on the *horizontal* axis.

Prepare to Record Data

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on *monitoring* and *recording* data.

1. Darken the room. Examine the spectrum closely. Determine which of the two first order spectral patterns is brightest. In the Data Table, list the colors you see in order starting with the color that appears farthest from the central ray.
2. Use the Light Sensor Arm on the Spectrophotometer to turn the Degree Plate until the light sensor is beyond the last line in the brightest first order spectral pattern.

Record Data

1. Set the GAIN select switch on top of the High Sensitivity Light Sensor to 1.
2. Start recording data.
3. Push on the threaded post under the light sensor to slowly and continuously scan the spectrum in one direction. Scan all the way through the first order spectral lines on one side of the central ray, through the central ray itself, and all the way through the first order spectral lines on the other side of the central ray (Fig. 1.4).
4. Stop recording data.
5. Set the GAIN select switch on top of the light sensor to 10. Put the light sensor back at its starting point. Repeat the data collection procedure.

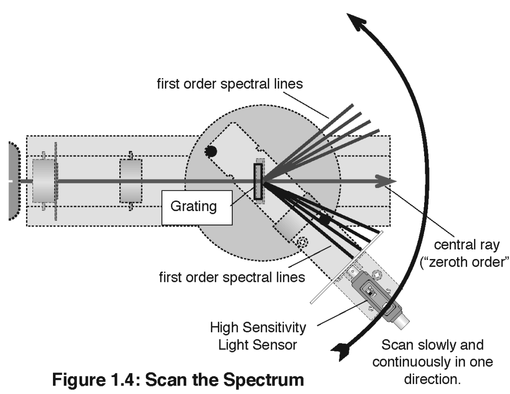

Figure 1.4: Scan the Spectrum

6. Set the GAIN select switch on top of the light sensor to 100 and repeat the data collection procedure.

Analyze the Data

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on using *ScienceWorkshop* for data analysis.

1. Use the Graph display to examine the plot of Light Intensity versus Actual Angular Position for the first run of data (GAIN select switch = 1).
2. Use the built-in analysis tools to determine the angle of the first line in the spectral pattern, and the angle of the matching line in the first order spectral pattern on the other side of the central ray.
3. Determine the difference in angle between the two lines and use one-half of the difference as the angle, θ, to determine the wavelength, λ, of that color. (If you did not calibrate the Diffraction Grating, assume d = 1666 nm.)
4. Repeat the process for the other colors in the first order spectral pattern.
5. Examine the plot of Light Intensity versus Actual Angular Position for your other two runs of data. Look for other lines in the spectral pattern that may be too dim to record when the sensor was set to GAIN = 1.

Data Table

Record your data here:

Color	θ_1	θ_2	$\Delta\theta$	$\theta=\Delta\theta/2$	$\lambda = d\sin\theta$

Conclusion

1. Compare your values for the wavelengths of color in the mercury vapor light spectrum to the accepted values for wavelengths.

Extensions

Repeat the process for a different gaseous element, such as hydrogen or helium.

Activity 2: Absorption (Dark Line) Spectrum

EQUIPMENT NEEDED

– Spectrophotometer System (OS-8539)
or
– Spectrophotometer Kit (OS-8537)
– Rotary Motion Sensor (CI-6538)
– Aperture Bracket (OS-8534)
– High Sensitivity Light Sensor (CI-6604)
– Basic Optics Bench (part of OS-8515)
and
– Incandescent Light Source, DC, regulated
– Rod, 45 cm (ME-8736) (2)
Large Rod Stand (ME-8735) (2)
Colored Liquid Sample (~5 mL)

Introduction

The purpose of this activity is to determine the wavelengths of colors absorbed by a liquid sample.

Theory

One of the most important applications of spectrophotometers is to identify substances by their absorption spectra. For example, it is possible to identify tiny amounts of sodium dissolved in a complicated liquid (such as beer) because sodium has a unique absorption spectrum.

An incandescent source such as a hot solid metal filament produces a continuous spectrum of wavelengths. A substance placed in the path of light from a continuous spectrum source will absorb certain colors from the continuous specrum. The individual colors that are absorbed appear as gaps or "dark lines" in the otherwise continuous spectrum (Fig. 2.1).

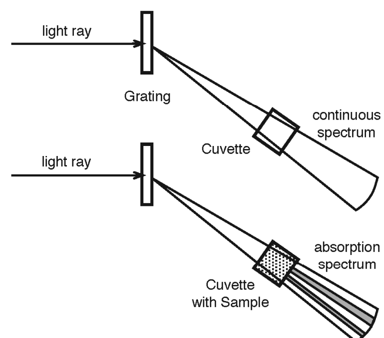

Figure 2.1: Continuous Spectrum and Absorption Spectrum

Procedure

In this activity, the High Sensitivity Light Sensor measures the relative intensity of colors of light in an continuous spectrum produced by an incandescent light. Then, the sensor measures the relative intensity of colors of light in an absorption spectrum produced when light from the incandescent source passes through a liquid sample. The Rotary Motion Sensor measures the angle, θ, of each part of the continuous spectrum and then the absorption spectrum.

The *ScienceWorkshop* program records and displays the light intensity and the angle. You can use the program's built-in data analysis tools to find the angle for each "gap" or dark line in the absorption spectrum, and then you can determine the wavelength, λ.

Equipment Setup

1. Set up the Spectrophotometer next to a DC powered incandescent light source as shown. Move the High Sensitivity Light Sensor to the second position on the Light Sensor Arm so there is room for a cuvette between the back of the Aperture Disk and the opening to the sensor. (Refer to the Set Up section for more information.)

2. Put an empty cuvette in front of the High Sensitivity Light Sensor between the sensor and the back of the Aperture Disk. Make sure that the smooth sides of the cuvette are in line with the opening to the sensor (Fig. 2.2).

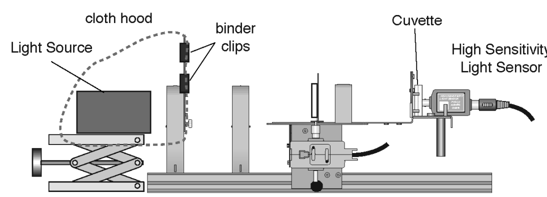

Figure 2.2: Equipment Setup for Absorption Spectrum

3. If the light source has a large opening, mask the opening so it transmits a narrow (0.5 to 1.0 cm) beam to the Collimating Slits. Adjust the Collimating Slits slide so the number 2 slit is in line with the light source. Put a cloth hood over the light source and attach the edge of the hood to the plate on the Collimating Slits.
4. Turn on the light source. Once it is warmed up, adjust the light source, Collimating Slits, Collimating Lens, and Focusing Lens so clear images of the central ray and the first order spectral pattern appear on the Aperture Disk and Aperture Screen. Turn the Aperture Disk so the second smallest slit on the disk is in line with the central ray.
5. Connect the *ScienceWorkshop* interface to the computer, turn on the interface. Start *Science Workshop*.
6. Connect the High Sensitivity Light Sensor cable to Analog Channel A. Connect the Rotary Motion Sensor cable to Digital Channels 1 and 2.

Experiment Setup

Select the Sensors, Set the Sample Rate, and Create a Calculation

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on selecting sensors, changing the sample rate and sensitivity, and creating a calculation.

1. In the *ScienceWorkshop* program, select the Rotary Motion Sensor and connect it to Digital Channels 1 and 2 and select the Light Sensor and connect it to Analog Channel A.
2. In the program, set up the Rotary Motion Sensor for high resolution (1440 Divisions per Rotation) and set the sample rate to 20 Hz, or 20 measurements per second.
3. Set the Sensitivity for the High Sensitivity Light Sensor to 10x.
4. Use the Calculator to create a calculation of Actual Angular Position based on the Angular Position measurement made by the Rotary Motion Sensor and the ratio of the radius of the Spectrophotometer's Degree Plate to the radius of the small post on the Pinion. (Refer to the Introduction for more information.)

Select the Display

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on displays.

1. Select a Graph display.
2. Set the axes of the Graph display so *Light Intensity* is on the *vertical* axis and *Actual Angular Position* is on the *horizontal* axis.

Prepare to Record Data

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on *monitoring* and *recording* data.

1. Darken the room. Examine the spectrum closely. Determine which of the two first order spectral patterns is brightest.
2. Use the Light Sensor Arm on the Spectrophotometer to turn the Degree Plate until the light sensor is beyond the last color in the brightest first order spectral pattern.

Record Data - Empty Cuvette

1. Set the GAIN select switch on top of the High Sensitivity Light Sensor to 10.
2. Start recording data.
3. Push on the threaded post under the light sensor to slowly and continuously scan the spectrum in one direction. Scan all the way through the first order spectral pattern on one side of the central ray, through the central ray itself, and all the way through the first order spectral pattern on the other side of the central ray.
4. Stop recording data.

Record Data - Cuvette with Liquid Sample

1. Remove the cuvette and fill it three-quarters full with the liquid sample you are testing. Cap the cuvette and replace it in front of the sensor.
2. Start recording data.
3. Push on the threaded post under the light sensor to slowly and continuously scan the spectrum in one direction. Scan all the way through the first order spectral pattern on one side of the central ray, through the central ray itself, and all the way through the first order spectral pattern on the other side of the central ray.
4. Stop recording data.

Analyze the Data

- Refer to the User's Guide for *your* version of *ScienceWorkshop* for detailed information on using *ScienceWorkshop* for data analysis.

1. Use the Graph display to compare the plot of Light Intensity versus Actual Angular Position for the first run of data (empty cuvette) to the plot of Light Intensity versus Actual Angular Position for the second run of data (cuvette plus liquid sample).
2. Use the built-in analysis tools of the program to find the angle of the first gap or "dark line" in the absorption spectrum of the liquid sample. Find the angle of the corresponding gap or line in the first order on the other side of the central ray.
3. Determine the difference between the angles and use one-half of the difference as the angle, θ, to determine the wavelength, λ, of that gap or dark line. (If you did not calibrate the Diffraction Grating, assume d = 1666 nm.)

$$\lambda = d \sin \theta$$

4. Repeat the process for the other gaps (if any) in the first order spectral pattern.

Data Table

Record your data here:

"Dark Line"	θ_1	θ_2	$\Delta\theta$	$\theta = \Delta\theta/2$	$\lambda = d \sin\theta$

Questions

1. What color corresponds to the wavelength for each "dark line" in your absorption spectrum?

2. How does the color or colors that are absorbed out of the continuous spectrum compare to the naked eye color of your liquid sample?

Extensions

Repeat the process for a different liquid sample, such as chlorophyll extracted from a spinach leaf.

Anexo C : Instruções para a elaboração de um relatório

Esta forma de apresentar um relatório não deve ser entendida como única, mas sim como um guia geral de elaboração de um relatório e deverá ser a seguida nesta disciplina.

A redacção do relatório deve permitir a qualquer pessoa, não sendo da área, entender a finalidade, o objectivo do trabalho, o que foi feito e como foi executado, de maneira a entender os resultados obtidos e as conclusões decorrentes. O leitor deve poder reproduzir a experiência para verificação dessas conclusões. O relatório, deve em consequência, seguir algumas regras básicas que permitam a sua elaboração com clareza.

A estrutura do relatório a apresentar, deverá ser a seguinte:

- **Cabeçalho:** Título, nomes dos autores e data da realização da experiência
- **Sumário:** Descrição muito sucinta do trabalho. Deve indicar os objectivos que se pretendem alcançar, o método seguido para os atingir, e os resultados obtidos. Não deve ocupar mais do que 4 ou 5 linhas.
- **Introdução:** Desenvolvimento simples da teoria subjacente ao trabalho. Deve indicar as equações e os princípios físicos em que se baseia a experiência, mas não é necessário deduzir as equações apresentadas. Não deve ser simplesmente copiada do protocolo.
- **Método Experimental:** Descrição do método e material utilizado. Sempre que possível incluir um esquema da montagem utilizada, com uma legenda elucidativa.
- **Resultados:** Devem ser apresentados os dados obtidos, os cálculos efectuados (com incertezas) e o resultado final obtido. Devem ser incluídas tabelas e figuras, todas com legenda, com os resultados obtidos
- **Discussão e conclusões:** Esta é a parte mais importante do relatório. É aqui que se devem apresentar as vossas próprias conclusões acerca do trabalho realizado e dos objectivos, alcançados ou não, bem como a discussão do método; comentando os resultados com valores conhecidos (se for possível), discutindo métodos alternativos e respondendo a questões pertinentes que tenham surgido durante a experiência.

- **Bibliografia:** Devem apresentar toda a bibliografia que consultaram. Não é aceitável utilizarem só o protocolo. Os endereços da Internet não devem ser genéricos (e.g., www.google.com).

Notas: As legendas das figuras colocam-se imediatamente abaixo delas e as legendas das tabela por cima delas.

O relatório não deve exceder duas páginas (excepto quando o número de figuras ou tabelas o exigir).

Anexo D : Notas sobre Análise de Dados

Notas sobre análise de dados

J. Pinto da Cunha
Departamento de Física, Universidade de Coimbra
Setembro de 2006

- FÓRMULA DE PROPAGAÇÃO DE ERROS

Seja f uma função (bem comportada) de N parâmetros variáveis, $f(a_1, a_2, \ldots, a_N)$. Como é sabido,

$$df = \sum_{k=1}^{N} \frac{\partial f}{\partial a_k} da_k , \qquad k = 1, \ldots, N \tag{1}$$

sendo N o n^o de variáveis e da_k uma variação infinitesimal de a_k. Isto é, quando as variáveis vão de $a_k \to a_k + da_k$, a função varia de $f \to f + df$. Assim, na prática, pequenas variações, δ, nos valores dos parâmetros $\{a_k\}$, quando $a_k \to a_k + \delta a_k$, fazem com que $f \to f + \delta f$, sendo

$$\delta f \approx \sum_{k=1}^{N} \frac{\partial f}{\partial a_k} \delta a_k \tag{2}$$

Para além disso,

$$\delta^2 f \approx \left(\sum_{k=1}^{N} \frac{\partial f}{\partial a_k} \delta a_k \right) \left(\sum_{j=1}^{N} \frac{\partial f}{\partial a_j} \delta a_j \right) = \sum_{kj}^{N} \frac{\partial f}{\partial a_k} \frac{\partial f}{\partial a_j} \delta a_k \delta a_j \tag{3}$$

Se os valores de $\{a_k\}$ forem o resultado de medidas, directas ou indirectas, esses valores estarão afectados de flutuações estatísticas, devidas a múltiplas causas, por hipótese pequenas, e que se traduzem numa incerteza de medida, que habitualmente se designa por σ_{a_k}. Isto é, de facto, $a_k \in [a_k - \sigma_{a_k}, a_k + \sigma_{a_k}]$; se repetirmos a medição, por força dessas flutuações, não obteremos sempre o mesmo valor para a_k mas quantidades que se situam em geral no intervalo de incerteza. Sendo, por hipótese, $\{a_k\}$ variáveis independentes (haverá sempre um conjunto de variáveis independentes), e sendo as flutuações inerentemente furtuitas, então estas não vão certamente todas no mesmo sentido em cada momento, sendo por isso zero em média cada um dos produtos cruzados $\delta a_k \delta a_j$, com $k \neq j$.[1]

Por conseguinte, não estando as variáveis por nenhuma forma correlacionadas, então a incerteza (ou erro) em f, que designaremos por σ_f, deve ser dada por

$$\sigma_f^2 = \sum_{k=1}^{N} \left(\frac{\partial f}{\partial a_k} \right)^2 \sigma_{a_k}^2 \tag{4}$$

onde σ_{a_k} é a incerteza na variável a_k, com $k = 1, \ldots, N$ e N o n^o de variáveis.

Exemplo:

"Para obter o volume de um cilindro mediu-se o raio da base e a altura, tendo-se obtido $r = (50 \pm 1)$ *cm e* $h = (200 \pm 1)$ *cm, respectivamente. Qual é a incerteza na determinação do volume,V ?"*

Sendo $V = V(r, h) = \pi r^2 h$, da eq. 4 vem $\sigma_V = \sqrt{\left(\frac{\partial V}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial V}{\partial h}\right)^2 \sigma_h^2}$. Por conseguinte, tem-se $V = \pi r^2 h \pm \sqrt{(2\pi r h)^2 \sigma_r^2 + (\pi r^2)^2 \sigma_h^2}$, o que neste caso dá $V = 1571 \pm 63\ \ell$.

[1] Pois umas vezes δa_k e δa_j têm sinais opostos, em outras não; considerando a soma de todas as combinações de valores obtemos certamente zero. Porém, se as variáveis estiverem de algum modo correlacionadas, então é necessário considerar essa correlação na incerteza de f, havendo que considerar explicitamente esses produtos cruzados.

- **Ajuste de uma função pelo método dos desvios mínimos quadrados**

Seja um conjunto de pares de valores obtidos em medidas, $\{(x, y)\}$,que é bem descrito pela função $f(x; a_1, a_2, \ldots)$, onde $\{a_k\}$ são parâmetros, cujos valores pretendemos determinar a partir das medidas. Fazem-se, por hipótese, M medidas, $\{(x_i, y_i \pm \sigma_i)\}$, com $i = 1, \ldots, M$, sendo σ_i a incerteza de cada uma das medidas (supomos que os valores de x_i não têm incerteza ou que ela pode ser desprezada em relação à de y_i em cada ponto)[2].

O problema de saber qual é o melhor conjunto de valores dos parâmetros da função por forma a que ela descreva o melhor possível um conjunto de medidas dá normalmente pelo nome de *ajuste* ou *fit*(em inglês). O método mais comum consiste em considerar que a melhor estimativa dos parâmetros é aquela que minimiza a discrepância entre os valores previstos pela função, $f(x_i; a_1, a_2, \ldots)$, e os respectivos valores medidos, $y_i \pm \sigma_i$. No método dos desvios mínimos quadrados essa discrepância é formulada em termos da função χ^2,

$$\chi^2(a_1, a_2, \ldots) = \sum_i^M \frac{(f(x_i; a_1, a_2, \ldots a_N) - y_i)^2}{\sigma_i^2}$$

Os melhores valores dos parâmetros são aqueles que minimizam a função χ^2. Onde for o mínimo a função χ^2 tem derivada nula. Por isso, na prática trata-se de resolver o sistema de N equações

$$\begin{cases} \frac{\partial \chi^2}{\partial a_1} = 0 \\ \frac{\partial \chi^2}{\partial a_2} = 0 \\ \vdots \end{cases} \tag{5}$$

Este método é aplicável qualquer que seja a função. Pode porém ser complicado resolver algebricamente o sistemae de equações, sendo então preferível resolvê-lo numericamente para obter uma solução aproximada.

x	f	σ_f
x_1	f_1	σ_1
x_2	f_2	σ_2
$\vdots$	$\vdots$	$\vdots$

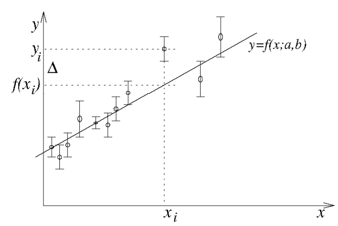

figura 5

Entende-se que o acordo entre as previsões da função e as medidas é bom se, tomando os valores de $a_1, a_2, \ldots, a_N$ calculados no ajuste anterior a diferença $(f - y)$ for em média igual à imprecisão da medida, σ. De tal facto vai resultar que o valor mínimo de χ^2 seja

$$\chi_0^2 = M \tag{6}$$

Isto é, o ajuste é bom se o valor de χ^2 que se obtém com os parâmetros que resultam do ajuste (χ_0^2) for próximo do número total de pontos medidos (M). Esta não é porém condição suficiente, devendo sempre analisar-se criticamente o acordo entre a previsão dada pela função ajustada e os valores medidos.

Devemos também avaliar o grau de incerteza com que é determinado cada um dos parâmetros. A incerteza na determinação do valor de a_k por ajuste aos dados decorre da incerteza desses mesmos

[2]Não analisaremos aqui o caso mais geral em que que se têm em conta as incertezas quer nas abcissas quer nas ordenadas.

dados. A uma variação de a_k igual à sua incerteza, $a_k \to a_k + \sigma_{a_k}$, deve corresponder em média a variação de χ^2 correspondente à incerteza dos dados. A variação de σ_i nos dados faz com que $\chi^2 = \sum_i \frac{(f-y)_i^2}{\sigma_i^2}$ varie em média de uma unidade. A correspondente variação do parâmetro a_k que daí decorre é uma medida da incerteza com que ele é determinado pelo ajuste. Isto é,

$$\chi^2(a_k + \sigma_{a_k}) = \chi_0^2 + 1 \tag{7}$$

Esta é a definição mais geral da incerteza dos parâmetros de um ajuste. Calculado, o mínimo de χ^2, obtidos os valores para os quais ocorre o mínimo, varia-se o valor de a_k até observar um incremento de uma unidade no χ^2; nesse ponto estamos em $a_k \pm \sigma_{a_k}$.

A expansão de χ^2 em série de Taylor em torno do mínimo fica

$$\chi^2(a_1, a_2, \ldots) = \chi_0^2 + \sum_k \left.\frac{\partial \chi^2}{\partial a_k^2}\right|_0 \delta a_k + \frac{1}{2}\sum_{kj} \left.\frac{\partial \chi^2}{\partial a_k \partial a_j}\right|_0 \delta a_k \delta a_j \; \cdots \tag{8}$$

onde $\left.\frac{\partial \chi^2}{\partial a_k^2}\right|_0 = 0$. Portanto, em face do que acima se disse,

$$1 - \frac{1}{2}\sum_{kj} \left.\frac{\partial \chi^2}{\partial a_k \partial a_j}\right|_0 \sigma_{a_k}\sigma_{a_j} \tag{9}$$

Fazendo $\alpha_{kj} = \frac{1}{2}\frac{\partial \chi^2}{\partial a_k \partial a_j}$, e $V_{kj} = \sigma_{a_k}\sigma_{a_j}$ podemos escrever a igualdade anterior na forma matricial, como o produto de duas matrizes, $\sum_{kj} \alpha_{kj} V_{kj} = 1$. Isto é, $\alpha\, V = 1$, ou $V = \alpha^{-1}$,

$$V_{kj} = \alpha_{kj}^{-1} = 2\left(\frac{\partial \chi^2}{\partial a_k \partial a_j}\right)^{-1} \tag{10}$$

A matriz V é conhecida como matriz de covariância, sendo assim chamada porque os seus elementos têm todas as variâncias e co-variâncias dos parâmetros do ajuste, i.e., explicitamente,

$$\begin{pmatrix} \sigma_{a_1}^2 & \sigma_{a_1,a_2} & \cdots \\ \sigma_{a_2,a_1} & \sigma_{a_2}^2 & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} = 2\begin{pmatrix} \partial_{a_1}^2\chi^2 & \partial_{a_1 a_2}^2\chi^2 & \cdots \\ \partial_{a_2 a_1}^2\chi^2 & \partial_{a_2}^2\chi^2 & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix}^{-1} \tag{11}$$

- **Ajuste a uma recta, $y(x) = a + bx$, sendo diferentes os erros em cada medida**

Se a função representar uma recta, $y = f(x; a, b) = a + bx$, o ajuste tem solução analítica simples. Nesse caso, $\chi^2(a, b) = \sum_i \frac{(a+bx_i-y_i)^2}{\sigma_i^2}$, sendo os parâmetros a e b dados pelo sistema de equações (ver eqs. 5)

$$\left\{\begin{array}{lll} \partial_a\chi^2 = 0\,, & \sum_i \frac{2}{\sigma_i^2}(a + bx_i - y_i) = 0\,, & a\sum_i \frac{1}{\sigma_i^2} + b\sum_i \frac{x_i}{\sigma_i^2} - \sum_i \frac{y_i}{\sigma_i^2} = 0 \\ \partial_b\chi^2 = 0\,, & \sum_i \frac{2x_i}{\sigma_i^2}(a + bx_i - y_i) = 0\,, & a\sum_i \frac{x_i}{\sigma_i^2} + b\sum_i \frac{x_i^2}{\sigma_i^2} - \sum_i \frac{x_i y_i}{\sigma_i^2} = 0 \end{array}\right. \tag{12}$$

cuja solução é[3]

$$\begin{aligned} a &= \frac{1}{\gamma}\left[\sum_i \frac{x_i^2}{\sigma_i^2}\sum_i \frac{y_i}{\sigma_i^2} - \sum_i \frac{x_i}{\sigma_i^2}\sum_i \frac{x_i y_i}{\sigma_i^2}\right] \\ b &= \frac{1}{\gamma}\left[\sum_i \frac{x_i y_i}{\sigma_i^2}\sum_i \frac{1}{\sigma_i^2} - \sum_i \frac{x_i}{\sigma_i^2}\sum_i \frac{y_i}{\sigma_i^2}\right] \\ \gamma &= \sum_i \frac{x_i^2}{\sigma_i^2}\sum_i \frac{1}{\sigma_i^2} - \left(\sum_i \frac{x_i}{\sigma_i^2}\right)^2 \end{aligned}$$

[3] As somas são sobre todos os pontos medidos. A melhor forma de usar estas expressões é proventura começar por calcular os vários somatórios, anotar o seu valor, e só depois calcular os parâmetros.

onde $\{(x_i, y_i)\}$ representa o conjunto das medidas e σ_i é o erro na medida de y_i.

A incerteza na determinação de cada um dos parâmetros, $a \pm \sigma_a$ e $b \pm \sigma_b$, é dada pela matriz de covariância,

$$\begin{pmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ba} & \sigma_b^2 \end{pmatrix} = 2 \begin{pmatrix} \partial_a^2 \chi^2 & \partial_{ab}^2 \chi^2 \\ \partial_{ba}^2 \chi^2 & \partial_b^2 \chi^2 \end{pmatrix}^{-1} = \begin{pmatrix} \sum_i \frac{1}{\sigma_i^2} & \sum_i \frac{x_i}{\sigma_i^2} \\ \sum_i \frac{x_i}{\sigma_i^2} & \sum_i \frac{x_i^2}{\sigma_i^2} \end{pmatrix}^{-1} = \frac{1}{\gamma} \begin{pmatrix} \sum_i \frac{x_i^2}{\sigma_i^2} & -\sum_i \frac{x_i}{\sigma_i^2} \\ -\sum_i \frac{x_i}{\sigma_i^2} & \sum_i \frac{1}{\sigma_i^2} \end{pmatrix}$$

Assim,

$$\sigma_a^2 = \frac{1}{\gamma} \sum_i \frac{x_i^2}{\sigma_i^2} \, ; \qquad \sigma_b^2 = \frac{1}{\gamma} \sum_i \frac{1}{\sigma_i^2} \, ; \qquad \sigma_{ab} = -\frac{1}{\gamma} \sum_i \frac{xi}{\sigma_i^2} \tag{13}$$

A boa prática tem mostrado que é conveniente organizar as medidas numa tabela do tipo

x	y	σ	$\frac{1}{\sigma^2}$	$\frac{x}{\sigma^2}$	$\frac{y}{\sigma^2}$	$\frac{x^2}{\sigma^2}$	$\frac{xy}{\sigma^2}$
x_1	y_1	σ_1	$\frac{1}{\sigma_1^2}$	$\frac{x_1}{\sigma_1^2}$	$\frac{y_1}{\sigma_1^2}$	$\frac{x_1^2}{\sigma_1^2}$	$\frac{x_1 y_1}{\sigma_1^2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\sum x$	$\sum y$	$\sum \sigma$	$\sum \frac{1}{\sigma^2}$	$\sum \frac{x}{\sigma^2}$	$\sum \frac{y}{\sigma^2}$	$\sum \frac{x^2}{\sigma^2}$	$\sum \frac{xy}{\sigma^2}$

- **Ajuste a uma recta, $y(x) = a + bx$, sendo constantes os erros de medida**

  Por vezes os valores medidos têm todos a mesma precisão. Nesse caso, $\sigma_i = \sigma =$ constante, ficando as expressões anteriores mais simples,

$$\begin{aligned} a &= \frac{1}{\Theta} \left(\sum_i x_i^2 \sum_j y_j - \sum_i x_i \sum_j x_j y_j \right) \\ b &= \frac{1}{\Theta} \left(N \sum_j x_j y_j - \sum_i x_i \sum_j y_j \right) \\ \sigma_a^2 &= \frac{\sigma^2}{\Theta} \sum_i x_i^2 \\ \sigma_b^2 &= \frac{\sigma^2}{\Theta} N \, , \qquad \text{com} \\ \Theta &= N \sum_i x_i^2 - \left(\sum_i x_i \right)^2 \end{aligned}$$

onde $\{(x_i, y_i)\}$ representa o conjunto das medidas e σ (constante) é o erro nas medidas y_i.

Anexo E : Trabalhos práticos de demonstração

Apresentam-se nas páginas seguintes folhas de trabalhos práticos (guiões e manual com instruções do equipamento a ser usado) para os alunos poderem seguir as demonstrações que se farão nas aulas. A maioria dos guiões apresentados foram coligidos por colegas do Departamento de Física

Observação de franjas de interferência no interferómetro de Michelson

Manual da “Pasco” (incluindo guião)

Includes Teacher's Notes and Typical Experiment Results

Instruction Manual and Experiment Guide for the PASCO scientific Models OS-9255A thru OS-9258A

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PRECISION INTERFEROMETER

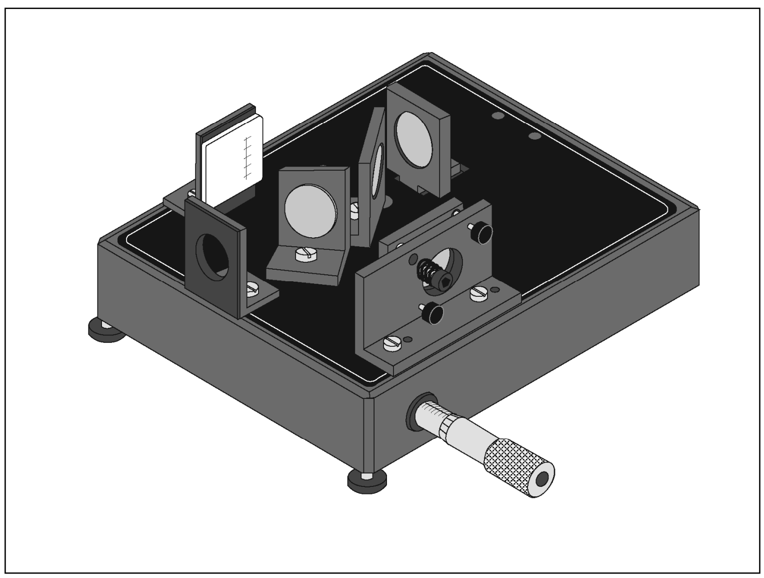

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Table of Contents

Equipment

The OS-9255A Precision Interferometer includes the following equipment:

- 5 kg Base with built-in micrometer
- Adjustable Mirror
- Movable Mirror
- Beam Splitter
- Compensator Plate
- (2) Component Holder
- Viewing Screen
- Lens, 18 mm Focal Length
- Diffuser
- Fitted Storage Case

Additional Equipment Required –

- Laser (OS-9171)
- Laser Bench (OS-9172)

➤ **NOTE:** The preceding equipment includes everything needed for basic Michelson interferometry. You can produce clear fringes and make precise measurements of the wavelength of your source. However, to perform the experiments in this manual, you will need additional components, such as the OS-9256A Interferometer Accessories or a comparable set of your own components.The Precision Interferometer is available as a complete system. Please refer to your current PASCO catalog for details.

Additional Equipment Recommended –

The OS-9256A Interferometer Accessories includes:

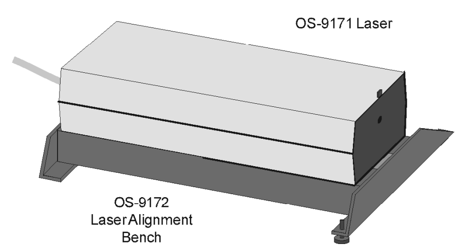
- Rotating Pointer
- Vacuum Cell
- Component Holder
- Lens, 18 mm Focal Length
- Lens, 48 mm Focal Length
- Glass Plate
- (2) Polarizer
- Vacuum Pump with Gauge

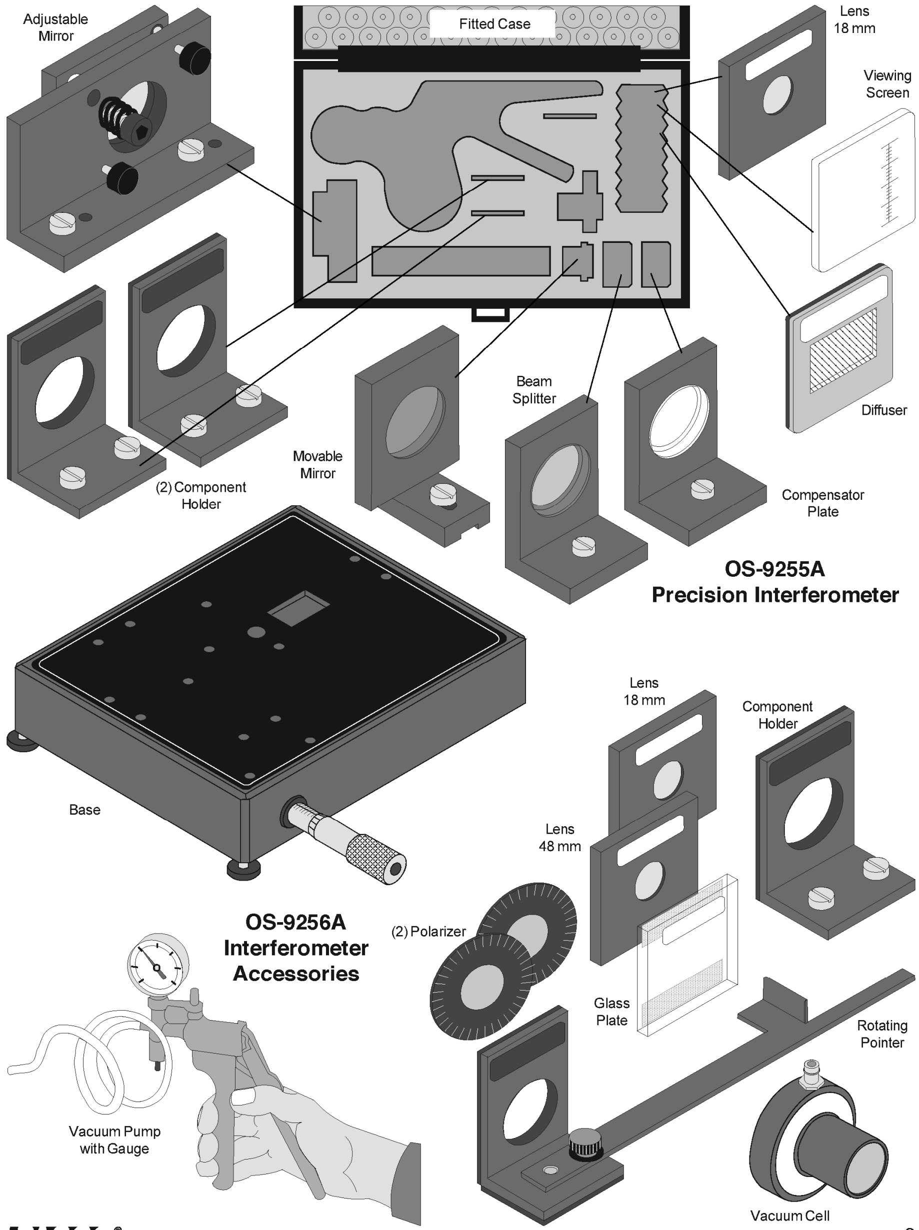
➤ **NOTE:** The OS-9255A Fitted Case also provides storage for these accessory components.

About Your Light Source

We strongly recommend a laser for most introductory applications. A spectral light source can be used (see the Appendix), but that really comprises an experiment in and of itself for beginning students. A laser source is easy to use and produces bright, sharp fringes.

The OS-9171 Laser and OS-9172 Laser Alignment Bench are available from PASCO. However, any low power laser that operates in the visible range will work well. If you want to demonstrate the importance of polarization in interferometry, a non-polarized laser should be used. For easy alignment, the beam should be approximately 4 cm above the level of the bench top.




Adjustable Mirror
Fitted Case
Lens 18 mm
Viewing Screen
(2) Component Holder
Movable Mirror
Beam Splitter
Diffuser
Compensator Plate
OS-9255A Precision Interferometer
Base
Lens 18 mm
Component Holder
Lens 48 mm
OS-9256A Interferometer Accessories
(2) Polarizer
Glass Plate
Rotating Pointer
Vacuum Pump with Gauge
Vacuum Cell

Theory of Operation

Interference Theory

A beam of light can be modeled as a wave of oscillating electric and magnetic fields. When two or more beams of light meet in space, these fields add according to the principle of superposition. That is, at each point in space, the electric and magnetic fields are determined as the vector sum of the fields of the separate beams.

If each beam of light originates from a separate source, there is generally no fixed relationship between the electromagnetic oscillations in the beams. At any instant in time there will be points in space where the fields add to produce a maximum field strength. However, the oscillations of visible light are far faster than the human eye can apprehend. Since there is no fixed relationship between the oscillations, a point at which there is a maximum at one instant may have a minimum at the next instant. The human eye averages these results and perceives a uniform intensity of light.

If the beams of light originate from the same source, there is generally some degree of correlation between the frequency and phase of the oscillations. At one point in space the light from the beams may be continually in phase. In this case, the combined field will always be a maximum and a bright spot will be seen. At another point the light from the beams may be continually out of phase and a minima, or dark spot, will be seen.

Thomas Young was one of the first to design a method for producing such an interference pattern. He allowed a single, narrow beam of light to fall on two narrow, closely spaced slits. Opposite the slits he placed a viewing screen. Where the light from the two slits struck the screen, a regular pattern of dark and bright bands appeared. When first performed, Young's experiment offered important evidence for the wave nature of light.

Young's slits can be used as a simple interferometer. If the spacing between the slits is known, the spacing of the maxima and minima can be used to determine the wavelength of the light. Conversely, if the wavelength of the light is known, the spacing of the slits could be determined from the interference patterns.

The Michelson Interferometer

In 1881, 78 years after Young introduced his two-slit experiment, A.A. Michelson designed and built an interferometer using a similar principle. Originally Michelson designed his interferometer as a means to test for the existence of the ether, a hypothesized medium in which light propagated. Due in part to his efforts, the ether is no longer considered a viable hypothesis. But beyond this, Michelson's interferometer has become a widely used instrument for measuring the wavelength of light, for using the wavelength of a known light source to measure extremely small distances, and for investigating optical media.

Figure 1 shows a diagram of a Michelson interferometer. The beam of light from the laser strikes the beam-splitter, which reflects 50% of the incident light and transmits the other 50%. The incident beam is therefore split into two beams; one beam is transmitted toward the movable mirror (M_1), the other is reflected toward the fixed mirror (M_2). Both mirrors reflect the light directly back toward the beam-splitter. Half the light from M_1 is reflected from the beam-splitter to the viewing screen and half the light from M_2 is transmitted through the beam-splitter to the viewing screen.

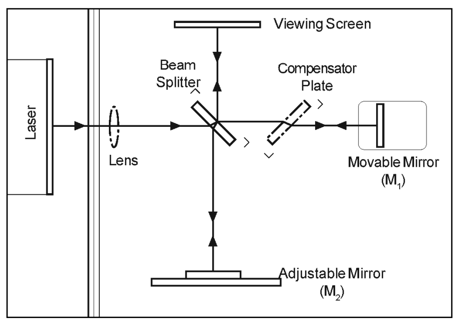


Figure 1. Michelson Interferometer

In this way the original beam of light is split, and portions of the resulting beams are brought back together. Since the beams are from the same source, their phases are highly correlated. When a lens is placed between the laser source and the beam-splitter, the light ray spreads out, and an interference pattern of dark and bright rings, or fringes, is seen on the viewing screen (Figure 2).

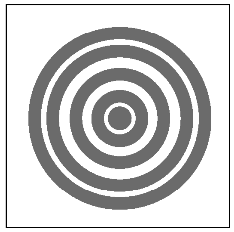


Figure 2. Fringes

Since the two interfering beams of light were split from the same initial beam, they were initially in phase. Their relative phase when they meet at any point on the viewing screen, therefore, depends on the difference in the length of their optical paths in reaching that point.

By moving M_1, the path length of one of the beams can be varied. Since the beam traverses the path between M_1 and the beam-splitter twice, moving M_1 1/4 wavelength nearer the beam-splitter will reduce the optical path of that beam by 1/2 wavelength. The interference pattern will change; the radii of the maxima will be reduced so they now occupy the position of the former minima. If M_1 is moved an additional 1/4 wavelength closer to the beam-splitter, the radii of the maxima will again be reduced so maxima and minima trade positions, but this new arrangement will be indistinguishable from the original pattern.

By slowly moving the mirror a measured distance $\mathbf{d_m}$, and counting **m**, the number of times the fringe pattern is restored to its original state, the wavelength of the light (λ) can be calculated as:

$$\lambda = \frac{2d_m}{m}$$

If the wavelength of the light is known, the same procedure can be used to measure $\mathbf{d_m}$.

➤ **NOTE: Using the Compensator**

In Figure 1, notice that one beam passes through the glass of the beam-splitter only once, while the other beam passes through it three times. If a highly coherent and monochromatic light source is used, such as a laser, this is no problem. With other light sources this is a problem.

The difference in the effective path length of the separated beams is increased, thereby decreasing the coherence of the beams at the viewing screen. This will obscure the interference pattern.

A compensator is identical to the beam-splitter, but without the reflective coating. By inserting it in the beam path, as shown in Figure 1, both beams pass through the same thickness of glass, eliminating this problem.

The Twyman-Green Interferometer

The Twyman-Green Interferometer is a variation of the Michelson Interferometer that is used to test optical components. A lens can be tested by placing it in the beam path, so that only one of the interfering beams passes through the test lens (see Figure 3). Any irregularities in the lens can be detected in the resulting interference pattern. In particular, spherical aberration, coma, and astigmatism show up as specific variations in the fringe pattern.

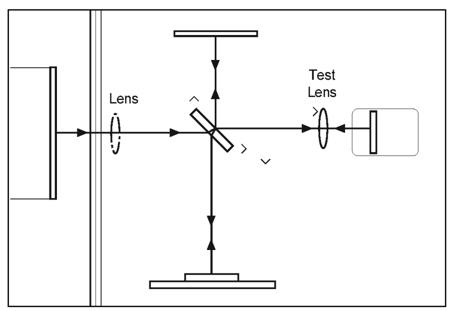


Figure 3. Twyman-Green Interferometer

The Fabry-Perot Interferometer

In the Fabry-Perot Interferometer, two partial mirrors are aligned parallel to one another, forming a reflective cavity. Figure 4 shows two rays of light entering such a cavity and reflecting back and forth inside. At each reflection, part of the beam is transmitted, splitting each incident ray into a series of rays. Since the transmitted rays are all split from a single incident ray, they have a constant phase relationship (assuming a sufficiently coherent light source is used).

The phase relationship between the transmitted rays depends on the angle at which each ray enters the cavity and on the distance between the two mirrors. The result is a circular fringe pattern, similar to the Michelson pattern, but with fringes that are thinner, brighter, and more widely spaced. The sharpness of the Fabry-Perot fringes makes it a valuable tool in high-resolution spectrometry.

As with the Michelson Interferometer, as the movable mirror is moved toward or away from the fixed mirror, the fringe pattern shifts. When the mirror movement is equal to 1/2 of the wavelength of the light source, the new fringe pattern is identical to the original.

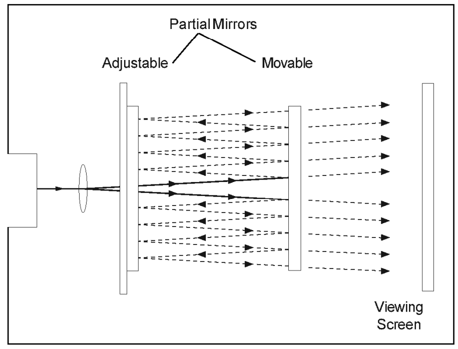


Figure 4. Fabry-Perot Interferometer

Setup and Operation

Laser Alignment

- If you are using a PASCO Laser and Laser Alignment Bench, the setup and alignment procedure is as follows.
- If you are using a different laser, the alignment procedure is similar. Adjust your laser so that the beam is approximately 4 cm above the table top. Then align the beam as in steps 4 and 5, below.
- If you are using a spectral light source instead of a laser, see *Suggestions for Additional Experiments*, near the end of the manual.

To set up and align your PASCO Laser:

1. Set the interferometer base on a lab table with the micrometer knob pointing toward you.
2. Position the laser alignment bench to the left of the base approximately perpendicular to the interferometer base and place the laser on the bench.
3. Secure the movable mirror in the recessed hole in the interferometer base.
4. Turn the laser on. Using the leveling screws on the laser bench, adjust its height until the laser beam is approximately parallel with the top of the interferometer base and strikes the movable mirror in the center. (To check that the beam is parallel with the base, place a piece of paper in the beam path, with the edge of the paper flush against the base. Mark the height of the beam on the paper. Using the piece of paper, check that the beam height is the same at both ends of the bench.)
5. Adjust the X-Y position of the laser until the beam is reflected from the movable mirror right back into the laser aperture. This is most easily done by gently sliding the rear end of the laser transverse to the axis of the alignment bench, as shown in Figure 5.

You are now ready to set up the interferometer in any of its three modes of operation.

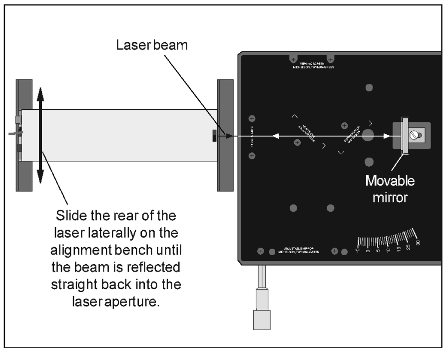


Figure 5. Aligning the Laser

> ➤ **NOTE:**
> For ease of installation the placement of the individual components in the various modes is indicated on the label.

Michelson Mode

1. Align the laser and interferometer base as previously described. The laser beam should be approximately parallel with the top of the base, should strike the center of the movable mirror, and should be reflected directly back into the laser aperture.
2. Mount the adjustable mirror on the interferometer base. Position one component holder in front of the laser. Place the other component holder opposite the adjustable mirror and attach the viewing screen to its magnetic backing. See Figure 6.
3. Position the beam-splitter at a 45 degree angle to the laser beam, within the crop marks, so that the beam is reflected to the fixed mirror. Adjust the angle of the beam-splitter as needed so that the reflected beam hits the fixed mirror near its center.
4. There should now be two sets of bright dots on the viewing screen; one set comes from the fixed mirror and the other comes from the movable mirror. Each set of dots should include a bright dot with two or more dots of lesser brightness (due to multiple reflections). Adjust the angle of the beam-splitter again until the two sets of dots are as close together as possible, then tighten the thumbscrew to secure the beam-splitter.

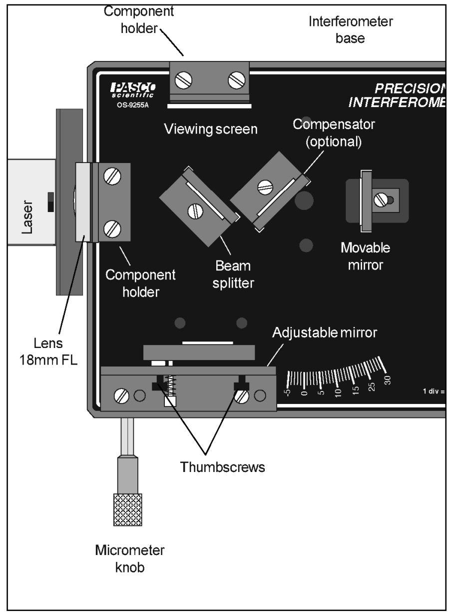


Figure 6. Michelson Mode Setup

5. Using the thumbscrews on the back of the adjustable mirror, adjust the mirror's tilt until the two sets of dots on the viewing screen coincide.
6. The compensator is not needed for producing interference fringes when using a laser light source. However, if you wish to use the compensator, it mounts perpendicular to the beam-splitter, as shown.
7. Attach the 18 mm FL lens to the magnetic backing of the component holder in front of the laser, as shown, and adjust its position until the diverging beam is centered on the beam-splitter. You should now see circular fringes on the viewing screen. If not, carefully adjust the tilt of the adjustable mirror until the fringes appear.
8. If you have trouble obtaining fringes, see *Trouble-Shooting* at the end of this section.

Twyman-Green Mode

1. Set up the interferometer in the Michelson mode, as described above.
2. Remove the pointer from the rotational componet holder. (It is recommended to store the pointer, washer and thumbscrew in the storage case.) Place the component holder between the beam-splitter and the movable mirror (see Figure 7). It attaches magnetically. Mount a second 18 mm FL lens (L_2) on its magnetic backing and position it .
3. Remove the original lens (L_1) from in front of the laser.

   Observe the two sets of dots on the viewing screen—one set from the movable mirror and one set from the adjustable mirror. Adjust the position of L_2 until both sets of dots are the same size.
4. Adjust the tilt of the adjustable mirror until the two sets of dots coincide.
5. Replace lens L_1 in front of the laser. Move the viewing screen so it's at least 12 inches from the edge of the interferometer base. Fringes should appear in the bright disk of the viewing screen. Fine adjustments of L_1 may be necessary to find the fringes. A piece of white paper or cardboard can be used in place of the viewing screen. A 48 mm FL convex lens may also be used to magnify the projected image of the fringes.

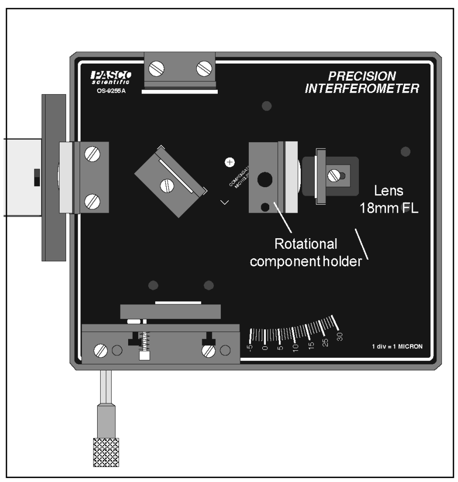


Figure 7. Twyman-Green Mode Setup

Fabry-Perot Mode

1. Align the laser and interferometer base as described in *Laser Alignment* at the beginning of this section. The laser beam should be approximately parallel with the top of the base, should strike the center of the movable mirror, and should be reflected directly back into the laser aperture.
2. Mount the adjustable mirror where indicated on the interferometer base and one component holder in front of the movable mirror. See Figure 8.
3. Place the other component holder behind the movable mirror and attach the viewing screen to its magnetic backing. You should see several images of the laser beam on the viewing screen.
4. Using the thumbscrews, adjust the tilt of the adjustable mirror until there is only one bright dot on the screen.
5. Now mount the 18 mm FL lens on the front component holder. A clear sharp interference pattern should be visible on the viewing screen. If you use light with two component wavelengths, instead of a laser, two sets of fringes can be distinguished on the viewing screen.

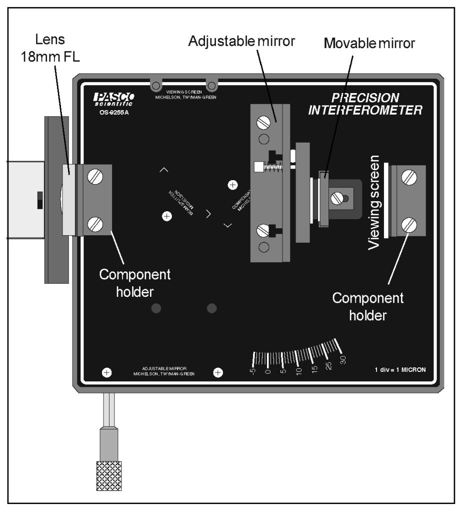


Figure 8. Fabry-Perot Mode Setup

Tips on Using the Interferometer

Accurate Fringe-Counting

The following techniques can help you make accurate measurements.

1. It's not necessary that your interference pattern be perfectly symmetrical or sharp. As long as you can clearly distinguish the maxima and minima, you can make accurate measurements.
2. It's easy to lose track when counting fringes. The following technique can help.

   Center the interference pattern on the viewing screen using the thumbscrews on the back of the fixed mirror. Select a reference line on the millimeter scale and line it up with the boundary between a maxima and a minima (see Figure 9). Move the micrometer dial until the boundary between the next maximum and minimum reaches the same position as the original boundary. (The fringe pattern should look the same as in the original position.) One fringe has gone by.

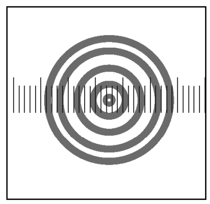


Figure 9. Counting Fringes

3. When turning the micrometer dial to count fringes, always turn it one complete revolution before you start counting, then continue turning it in the same direction while counting. This will almost entirely eliminate errors due to backlash in the micrometer movement.

   Backlash is a slight slippage that always occurs when you reverse the direction of motion in a mechanical instrument. (Turning the micrometer dial clockwise moves the movable mirror toward the right. Turning the dial counter-clockwise moves the mirror toward the left.) The PASCO micrometer is designed to minimize backlash. However, by using the technique described above, you can practically eliminate all effects of backlash in your measurements.
4. Always take several readings and average them for greater accuracy.
5. The slip ring at the base of the micrometer knob adjusts the tension in the dial. Before making a measurement, be sure the tension is adjusted to give you the best possible control over the mirror movement.

Calibrating the Micrometer

For even more accurate measurements of the mirror movement, you can use a laser to calibrate the micrometer. To do this, set up the interferometer in Michelson or Fabry-Perot mode. Turn the micrometer knob as you count off at least 20 fringes. Carefully note the change in the micrometer reading, and record this value as d'. The actual mirror movement, d, is equal to $N\lambda/2$, where λ is the known wavelength of the light (0.6328 μm for a standard helium-neon laser) and N is the number of fringes that were counted. In future measurements, multiply your micrometer readings by d/d' for a more accurate measurement.

> ➤ **NOTE:** You can also adjust the micrometer calibration mechanically. The process is not difficult, but for most accurate results, the above procedure is still recommended. See the *Maintenance* section at the end of the manual for the mechanical calibration procedure.

Demonstrations

The PASCO interferometer is not designed for large demonstrations. However, for small demonstrations, you can use the 48 mm focal length lens (included in the Interferometer Accessories) to magnify the fringe pattern and project it onto a wall or screen. It is helpful to have a powerful laser for large projections.

Using the Diffuser

It's sometimes more convenient to view the interference pattern through the diffuser rather than on the viewing screen. Just place the diffuser where you would normally place the viewing screen, and look through it toward the interferometer.

Sources of Experimental Error

Backlash— Although PASCO's carefully designed mirror movement reduces backlash considerably, every mechanical system is susceptible to backlash. However, the effects of backlash can be practically eliminated by using proper technique when counting fringes (see item 3 under Accurate Fringe-Counting, on the previous page).

Mirror Travel— The amount of mirror movement per dial turn of the micrometer is constant to within 1.5%. Most of this error occurs at the extreme ends of the mirror's total possible movement. For very accurate measurements, see *Calibrating the Micrometer*, above, and remember that the mirrors are flat to within 1/4 wavelength across their surface.

Troubleshooting

If you have trouble producing a clear set of interference fringes, consider the following possible sources of difficulty:

1. **Warm up your Laser—** Many lasers vary in intensity and/or polarization as they warm up. To eliminate any possible fringe or intensity variations, allow the laser to warm up prior to setting up an experiment. (The PASCO laser should warm up in about 1 hour.)
2. **Check your Mirrors—** The beam-splitter and movable mirror are carefully mounted in their brackets to remain perpendicular to the interferometer base when set up. If the brackets are bent slightly out of alignment, the resulting fringe patterns will be distorted somewhat. If they are significantly out of alignment, it may be impossible to obtain fringes.
3. **Background Fringes—** Reflections from the front and back surfaces of the mirrors and beam-splitter often cause minor interference patterns in the background of the main fringe pattern. These background patterns normally do not move when the mirror is moved, and have no impact on measurements made using the main interference pattern.
4. **Convection Currents—** If the fringe pattern appears to wave or vibrate, check for air currents. Even a slight breeze can effect the fringes.
5. **Vibration—** Under normal conditions, the interferometer base and mirror mounts are stable enough to provide a vibration free setup. However, if the experiment table is vibrating sufficiently, it will effect the interference pattern.

➤ **IMPORTANT:** If the movable mirror doesn't move when you turn the micrometer dial, see *Micrometer Spacer Replacement* in the *Maintenance* section at the end of this manual.

Component Specifications

Interferometer Mirrors— 3.175 cm in diameter; 0.635 $\pm$ 0.012 cm thick; flat to 1/4 wavelength on both sides; coated on one side for 80% reflectance and 20% transmission.

Beam-Splitter— 3.175 cm in diameter; 0.635 $\pm$ 0.012 cm thick; flat to 1/4 wavelength on both sides; coated on one side for 50% reflectance and 50% transmission.

Compensator— Identical to the beam-splitter, but uncoated.

Movable Mirror— movement is controlled by the micrometer that is built-into the interferometer base; turning the dial clockwise moves the mirror toward the right (looking from the micrometer side); 25 microns per micrometer dial revolution (±1% near center of movement); movement through full distance of travel is linear to within 1.5%.

➤ **IMPORTANT:** Avoid touching all mirror surfaces. Minute scratches and dirt can impair the clarity of interference images. See the *Maintenance* section at the end of this manual for cleaning instructions.

Experiência de Franck-Hertz

Manuais da “Leybold”

Atomic and Nuclear Physics

Atomic shell
Franck-Hertz experiment

LEYBOLD Physics Leaflets **P6.2.4.1**

Franck-Hertz experiment with mercury

Recording with the oscilloscope, the XY-recorder and point by point

Objects of the experiment

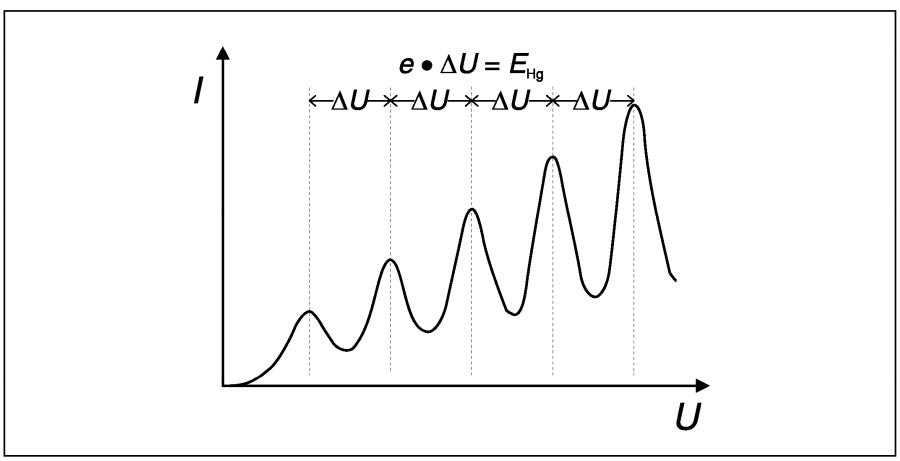
- To record a Franck-Hertz curve for mercury.
- To measure the discontinuous energy emission of free electrons for inelastic collision.
- To interpret the measurement results as representing discrete energy absorption by mercury atoms.

Principles

In 1914, *James Franck* and *Gustav Hertz* reported an energy loss occurring in distinct “steps” for electrons passing through mercury vapor, and a corresponding emission at the ultraviolet line (λ = 254 nm) of mercury. Just a few months later, *Niels Bohr* recognized this as evidence confirming his model of the atom. The Franck-Hertz experiment is thus a classic experiment for confirming quantum theory.

In a previously evacuated glass tube, mercury atoms are held at a vapor pressure of about 15 hPa, which is kept constant by temperature control. This experiment investigates the energy loss of free electrons due to inelastic scattering, and thus due to collision excitation of mercury atoms.

The electron current flowing to the collector as a function of the acceleration voltage in the Franck-Hertz experiment with mercury (schematic representation)



1105-Sel

Apparatus

1 Franck-Hertz tube, Hg	555 85
1 Socket for Franck-Hertz tube, Hg with multi-pin plug	555 861
1 Electric oven, 220 V	555 81
1 Franck-Hertz supply unit	555 88
1 Temperature sensor, NiCr-Ni	666 193

Recommended for optimizing the Franck-Hertz curve:

1 Two-channel oscilloscope 303	575 211
2 Screened cables BNC/4 mm	575 24

Recommended for recording the Franck-Hertz curve:

1 XY-Yt recorder SR 720	575 663
Connecting leads	

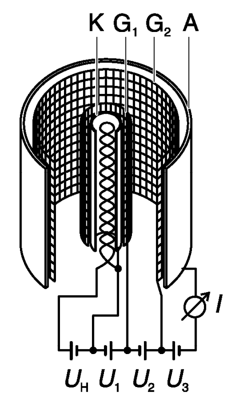


Fig. 1: Schematic diagram of the mercury Franck-Hertz tube

The glass tube contains a cylindrically symmetrical system of four electrodes (see Fig. 1). The cathode K is surrounded by a grid-type control electrode G_1 at a distance of a few tenths of a millimeter, an acceleration grid G_2 at a somewhat greater distance and finally the collector electrode A outermost. The cathode is heated indirectly, in order to prevent a potential differential along K.

Electrons are emitted by the hot electrode and form a charge cloud. These electrons are attracted by the driving potential U_1 between the cathode and grid G_1. The emission current is practically independent of the acceleration voltage U_2 between grids G_1 and G_2, if we ignore the inevitable punch-through. A braking voltage U_3 is present between grid G_2 and the collector A. Only electrons with sufficient kinetic energy can reach the collector electrode and contribute to the collector current.

In this experiment, the acceleration voltage U_2 is increased from 0 to 30 V while the driving potential U_1 and the braking voltage U_3 are held constant, and the corresponding collector current I_A is measured. This current initially increases, much as in a conventional tetrode, but reaches a maximum when the kinetic energy of the electrons closely in front of grid G_2 is just sufficient to transfer the energy required to excite the mercury atoms (E_{Hg} = 4.9 eV) through collisions. The collector current drops off dramatically, as after collision the electrons can no longer overcome the braking voltage U_3.

As the acceleration voltage U_2 increases, the electrons attain the energy level required for exciting the mercury atoms at ever greater distances from grid G_2. After collision, they are accelerated once more and, when the acceleration voltage is sufficient, again absorb so much energy from the electrical field that they can excite a mercury atom. The result is a second maximum, and at greater voltages U_2 further maxima of the collector currents I_A.

Preliminary remark

The complete Franck-Hertz curve can be recorded manually.

For a quick overview, e. g. for optimizing the experiment parameters, we recommend using a two-channel oscilloscope. However, note that at a frequency of the acceleration voltage U_2 such as is required for producing a stationary oscilloscope pattern, capacitances of the Franck-Hertz tube and the holder become significant. The current required to reverse the charge of the electrode causes a slight shift and distortion of the Franck-Hertz curve.
An XY-recorder is recommended for recording the Franck-Hertz curve.

a) Manual measurement:

– Set the operating-mode switch to MAN. and slowly increase U_2 by hand from 0 V to 30 V.
– Read voltage U_2 and current I_A from the display; use the selector switch to toggle between the two quantities for each voltage.

b) Representation on the oscilloscope:

– Connect output sockets U_2/10 to channel II (0.5 V/DIV) and output sockets U_A to channel I (2 V/DIV) of the oscilloscope. Operate the oscilloscope in XY-mode.
– Set the operating-mode switch on the Franck-Hertz supply unit to "Sawtooth".
– Set the Y-position so that the top section of the curve is displayed completely.

c) Recording with the XY-recorder:

– Connect output sockets U_2/10 to input X (0.2 V/cm CAL) and output sockets U_A to input Y (1 V/cm CAL) of the XY-recorder.
– Set the operating-mode switch on the Franck-Hertz supply unit to RESET.

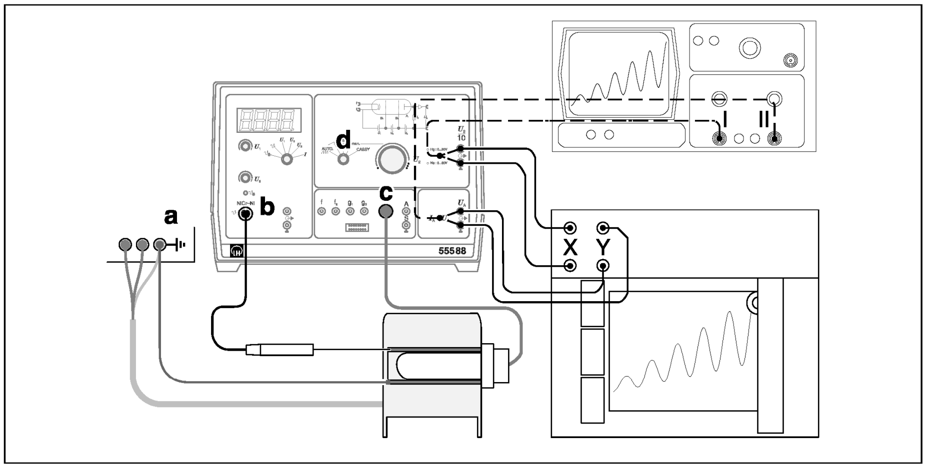


Fig. 2: Experiment setup for the Franck-Hertz experiment with mercury

- Adjust the zero-point of the recorder in the X and Y direction and mark this point by briefly lowering the recorder pen onto the paper.
- To record the curve, set operating-mode switch to "Ramp" and lower the recorder pen.
- When you have completed recording, raise the pen and switch to RESET.

Setup

Fig. 2 shows the experiment setup.

First:

- Make sure the Franck-Hertz supply unit is switched off.
- Connect the heating oven via the 4-mm safety sockets **(a)** on the rear of the supply unit.
- Additionally, connect the copper lead of the copper sleeve with 4-mm plug to the green-yellow safety socket (to screen the Franck-Hertz tube from interference fields).
- Insert the DIN plug of the temperature sensor in socket **(b)** of the supply unit and the DIN plug of the Franck-Hertz tube in socket **(c)**.

Heating:

Note:
If the thermal contact of the temperature sensor is poor, the measured oven temperature will be too low, resulting in overheating of the tube.

- Insert the temperature sensor in the corresponding blind hole of the heating oven as far as it will go and slide the Franck-Hertz tube with copper sleeve into the oven.
- Turn the operating-mode switch **(d)** to RESET and switch on the supply unit (after a few seconds, the LED indicator for mercury (Hg) changes from green to red).
- Check the default setting ϑ_S = 180 °C and wait until the operating temperature is reached (LED indicator changes from red to green; the temperature ϑ first reaches a maximum, and then declines to the final value).

If the indicator in the display flashes:

- There is a mistake in the setup for temperature measurement (see the Instruction Sheet).

Optimizing the Franck-Hertz curve:

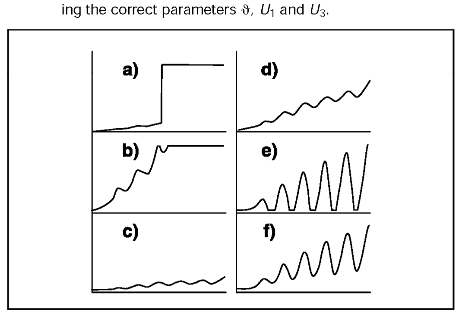
- Set the driving potential U_1 = 1.5 V and the braking voltage U_3 = 1.5 V and record the Franck-Hertz curve (see preliminary remark).

a) Optimizing ϑ

If the Franck-Hertz curve rises abruptly (see Fig. 3 a) and you can see a gas discharge in the Franck-Hertz tube through the insertion opening of the oven (blue glow):

- Immediately turn the operating-mode switch to RESET and wait until the setup reaches the operating temperature.
- If necessary, raise the set value ϑ_S using the screwdriver potentiometer (e.g. by 5 °C) and wait a few minutes until the system settles into the new thermal equilibrium.

Fig. 3: Overview for optimizing the Franck-Hertz curves by selecting the correct parameters ϑ, U_1 and U_3.



b) Optimizing U_1:

A higher driving potential U_1 results in a greater electron emission current.

If the Franck-Hertz curve rises too steeply, i.e. the overdrive limit of the current measuring amplifier is reached at values below U_2 = 30 V and the top of the Franck-Hertz curve is cut off (Fig. 3b):

– Reduce U_1 until the curve steepness corresponds to that shown in Fig. 3d.

If the Franck-Hertz curve is too flat, i.e. the collector current I_A remains below 5 nA in all areas (see Fig. 3c):

– Increase U_1 (max. 4.8 V) until the curve steepness corresponds to that shown in Fig. 3d.

If the Franck-Hertz curve is flat even after increasing U_1:

– Reduce the set value ϑ_s for the oven temperature using the screwdriver potentiometer.

c) Optimizing U_3:

A greater braking voltage U_3 causes better-defined maxima and minima of the Franck-Hertz curve; at the same time, however, the total collector current is reduced.

If the maxima and minima of the Franck-Hertz curve are insufficiently defined (see Fig. 3d):

– Alternately increase first the braking voltage U_3 (maximum 4.5 V) and then the driving potential U_1 until you obtain the curve form shown in Fig. 3f.

If the minima of the Franck-Hertz curve are cut off at the bottom (see Fig. 3e):

– Alternately reduce first the braking voltage U_3 (maximum 4.5 V) and then the driving potential U_1 until you obtain the curve form shown in Fig. 3f.

Carrying out the experiment

– Record the Franck-Hertz curve (see preliminary remark).
– To better demonstrate the first maxima, you can increase the sensitivity of the Y-input and repeat the recording process.

Measuring example and evaluation

U_1 = 1.58 V

U_3 = 3.95 V

ϑ_s = 180 °C

In Fig. 4, the average of the intervals between the successive maxima gives us the value

ΔU_2 = 5.1 V.

This corresponds to an energy transfer of

ΔE = 5.1 eV

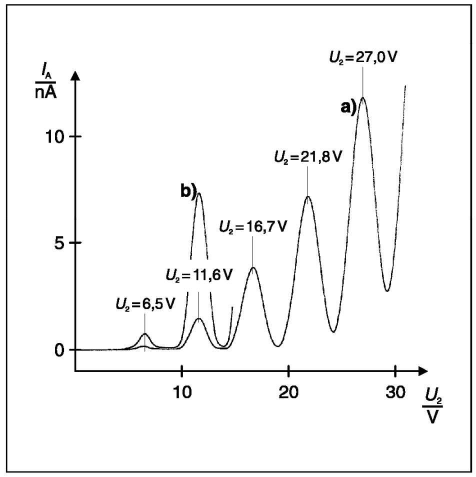


Fig. 4: a) Franck-Hertz curve of mercury (recorded with XY-recorder)
b) Curve section, with ordinate enlarged five times

We can compare this value with the literature value

E_{Hg} = 4.9 eV

for the transition energy of the mercury atoms from the ground state $^1S_0$ to the first $^3P_1$ state.

The kinetic energy of the electrons at grid G_2 can be calculated as

$E_{kin} = (U_1 + U_2)$

On the basis of this, we would expect the first maximum of the collector current at $U_1 + U_2$ = 4.9 V. In fact, the first maximum is not registered until $U_1 + U_2$ = 8.1 V. The difference between the two values is the effective contact potential between cathode K and grid G_2.

Supplementary information

A number of factors influence the effective contact potential. The most important of these deserve mention here.

The actual contact potential is caused by the different work of emission of the cathode and grid materials. The emission properties of the mixed-oxide cathode and the gas charge resp. the mercury coating of the grid play an important role here.

The electrons emitted by the hot cathode have an initial velocity which depends on the temperature of the cathode.

Name Partners Date

Collision Excitation of Atoms (Franck-Hertz Experiment)

Goal

- Build inelastic and elastic energy models of collisions between electrons and gas molecules.
- Use the Franck-Hertz experiment to investigate atom and electron collisions in Neon.
- Build a model that describes the excitation of neon atoms and the energy lost by electrons during collisions.
- Measure the excitation energies for neon and mercury.

Introduction

The experiment you will do today was first performed, using slightly different equipment, by James Franck and Gustav Hertz in 1914. In 1925 Franck and Hertz received the Nobel Prize in physics for their work.

A. Collisions Between Electrons and Atoms

In the tutorial on the photoelectric effect we examined photons interacting with atoms in a metal. In this tutorial we will look at electrons colliding with gas atoms. We will bombard a gas with electrons whose kinetic energy can be varied. The collisions between the electrons and gas atoms can be either elastic or inelastic.

A-1. Predict the effects elastic and inelastic collisions would have on:

- electrons

- gas atoms

Soon after Bohr presented his atomic model in 1913, Franck and Hertz devised an experiment to demonstrate that:

- atoms can be excited by bombardment with electrons,
- the energy is transferred from the electrons to the atoms in discrete amounts, and
- the amounts of energy transferred are consistent with spectroscopic results.

The apparatus Franck and Hertz used is shown schematically in Figure 1. In our experiment we will use neon as the gas instead of mercury but the process is the same.

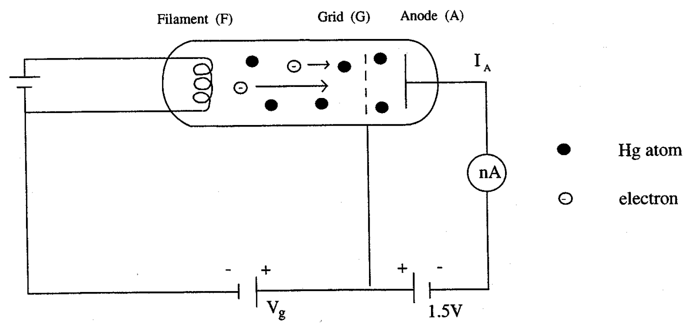


Figure 1. Simplified circuit for the Franck-Hertz experiment

The original Franck-Hertz tube contained droplets of mercury and was baked in an oven so that the mercury vaporized. The hot filament (F) emits electrons thermionically, that is the electrons "boil" off. The grid (G) is at a positive potential with respect to the filament so the electrons are accelerated towards G, and their kinetic energy increases. The potential of G, V_g, can be varied.

A-2. Predict what would happen as V_g is increased.

A-3. Examine Figure 1. What is the potential of the anode with respect to the grid?

FH-2

A-4. Why do you think the experiment is set up in this way? (We don't expect a good answer at this stage; we will revisit this question after you have collected some data using the apparatus.)

B. The Experiment

Equipment: An electron gun apparatus contained in a glass tube with neon, a specialized power supply, and an oscilloscope.

Hints:

In this experiment it is possible to boil off so many electrons that the current becomes too great for the electronics to handle. You will see this happening if the line on the oscilloscope becomes flat at the top. If this happens, just turn down the heater.

The "Gain" setting on the upper left of the power supply amplifies the current. You may need to adjust it to keep it from overwhelming the electronics.

In this experiment you can control the energy of the accelerated electrons by controlling the accelerating voltage (dial marked U_a). You can also measure the current through the tube. As you increase the accelerating voltage, you will see an increase in the current because the electrons are increasing their kinetic energy. Thus, more of them are passing through the wires each second. However, if the electrons lose energy the current will decrease.

The oscilloscope is measuring and plotting the accelerating voltage on the horizontal axis and the electron current on the vertical axis. The power supply has a switch in the middle of the panel with two options illustrated:

 and 

The first allows you to manually adjust the voltage and the second automatically ramps through the voltage. Select the manual option.

FH-3

Gradually increase the accelerating voltage and observe changes in the neon tube. As you increase the voltage you should see the neon begin to glow. However, the neon does not glow all the way through the tube. The glow begins at a certain distance from the cathode.

B-1. Think about how the accelerating electrons gain energy and explain why the tube does not glow uniformly.

As you increase the accelerating voltage further, you will see the glowing region increase. Look carefully and you should be able to see a dark band in the middle of the glowing region.

B-2. Explain this observation in terms of the energies of the electrons and the neon gas.

Depending on the values of the various currents and voltages, you may be able to see a second dark band. Try it although it may be difficult to achieve.

B-3. Dial through the accelerating voltage and describe what happens to the current as you increase the voltage.

B-4. How are the changes in current related to the onset or changes in the light emitted by the neon gas?

FH-4

For the next part of the experiment set the power supply to the “ramp” option. On the oscilloscope, voltage is the horizontal axis while current is displayed vertically.

B-5. Sketch below what you see on the screen. Be sure to include values for the current and voltage.

Adjust the accelerating voltage, oven temperature, and gain so you can see the maximum number of peaks.

B-6. Record the peak number for several different voltages in the table below.

Peak Number	Accelerating Voltage

Table 1. Data Table of Peak Number and Accelerating Voltages

FH-5

C. Analysis – Putting It All Together

C-1. Using your observations of the glowing neon, the sketch you made and the table of peak voltages, describe what is happening when the electrons collide with the neon gas. You should refer to the predictions you made in questions A-1 and A-2 and resolve any differences.

C-2. What does the minima in electron current mean? Describe how these minima are related to the quantized energy levels in the neon atom.

C-3. The dips occur regularly in a plot of current versus voltage. Using your sketch and the table of values, determine the excitation energy of the neon gas.

FH-6

C-4. The dips in the oscilloscope plot of current versus voltage are not perfectly sharp. Why is this the case?

C-5. Why is the first peak at about 30V instead of 19V?

C-6. Using the energy diagram provided in Appendix A, determine the most likely transitions resulting from the collision excitation of neon.

Below is a table showing the peak number and the accelerating voltage for a typical Franck-Hertz tube using mercury vapor instead of the neon you used here.

Peak number	Accelerating voltage (V)
1	6.7
2	11.5
3	16.5
4	21.25
5	26.25

Table 2. Table of Typical Results Using a Mercury Vapor Tube

C-7. Calculate the excitation energy of mercury and the corresponding wavelength of light emitted.

C-8. Determine the work function of the filament in this experiment.

C-9. Now that you have collected some data, explain why the anode is set at a lower potential compared to the cathode.

FH-8

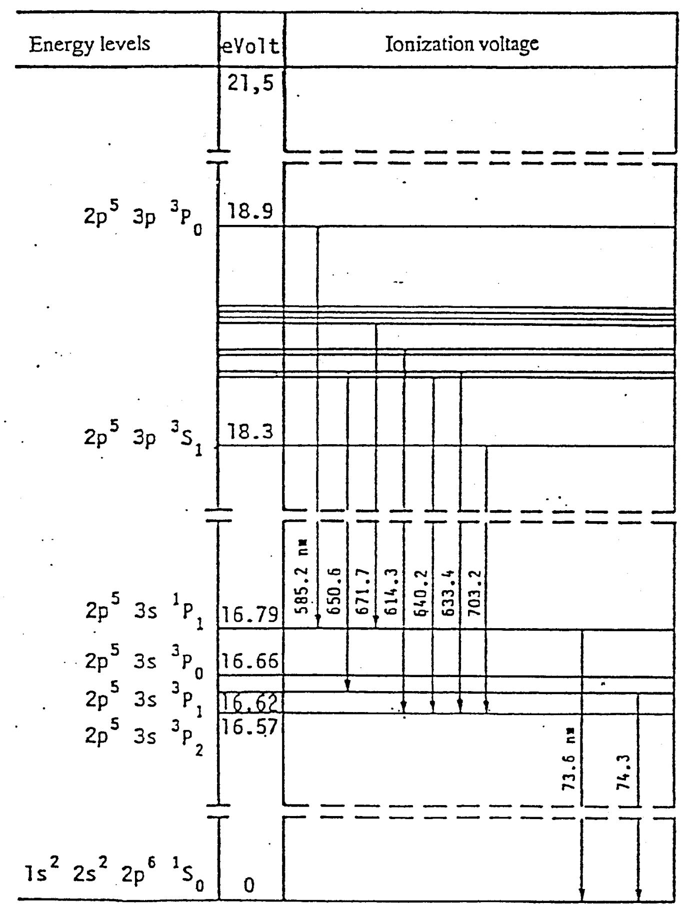


Figure 2. Partial energy diagram of neon

FH-9

Experiência de Rutherford

Manual da "Leybold"

Atomic and nuclear physics The structure of matter: Rutherford scattering	6.1.3-1/3

Rutherford scattering: Measuring the angle dependence N (Θ) in Rutherford scattering

Alpha particles of uniform energy are scattered onto a gold foil. The dependence of the scattering rate N and the scattering angle Θ is measured and is compared with Rutherford scattering formula.

If alpha particles meet on a gold foil, they are deflected from their path ("scattering"). The majority of alpha particles are scattered by a "scattering angle Θ" of less than 1°. A few particles have a substantially larger Θ, in the extreme case up to 180° (back scattering). These initially qualitative observations can only be explained by assuming that the gold atoms have a very small nucleus, containing practically the whole atomic mass, and is positively charged.

On the basis of this idea, Rutherford calculated the angular distribution N (Θ) of the scattering rate. This is the number N of α particles, which is scattered in a time period of a determined interval dΘ by an average angle Θ. The result of the calculation is the "Rutherford's scattering formula". Except for proportionality factors, which are kept constant in our experiment, it supplies us with the relationship for the angular dependence of the scattering rate:

$$N(\theta) \sim \sin^{-4}\left(\frac{\theta}{2}\right) \qquad (1)$$

This proportionality is verified in our experiment.

Because of the very low range of alpha particles in the air, this experiment must be carried out in a vacuum.

Fig. 1 shows the geometrical arrangement of the components of the scattering chamber, Fig. 2 shows the geometry of the experiment.

The alpha particles emitted from the Am-241 preparation fall through an aperture of 1 mm width onto the gold foil and leave this gold foil with various scattering angles. The scattered alpha particles are identified with a detector. By swinging the detector in steps of 5°, for example, the scattering rate can be determined for all scattering angles from 5° to 60°. With the setup we are going to use, the detector is not swung, but rather the preparation, slit and gold foil, which are attached on a common swivel arm.

The α detector is firmly attached to the side wall of the chamber.

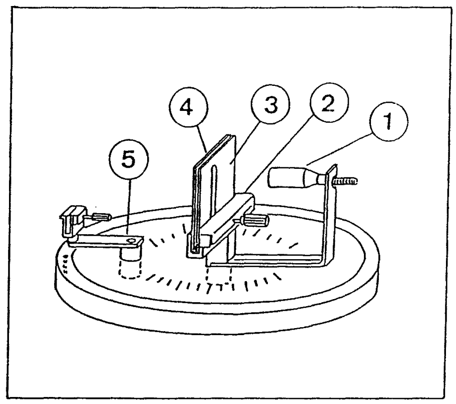


Fig. 1: The scattering chamber
(1) Preparation (4) Gold foil
(2) Holder (5) Arm to swivel end
(3) Slit

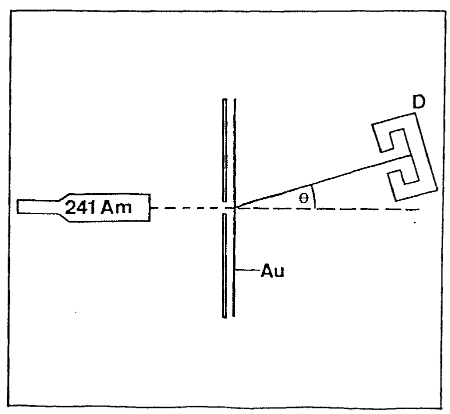
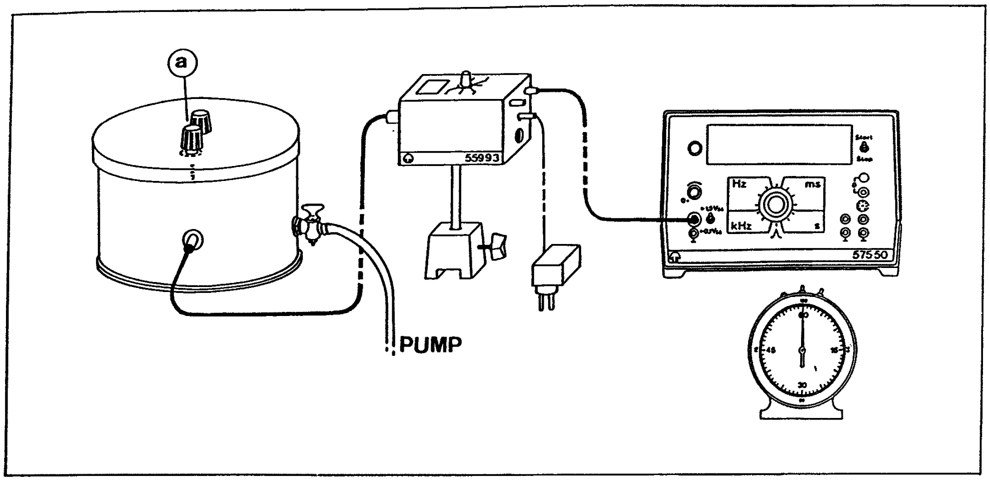


Fig. 2: Scattering geometry with preparation, collimator slit, gold foil and detector (D).

1



. ıg. 3: Experiment setup for Rutherford scattering, electrical connection

Apparatus:

1	Rutherford scattering chambers	559 56
1	Pump S 1.5, 220 V,50 Hz	101 01
1	Vacuum rubber hose 2 m	307 68
1	Digital counter	575 59
1	Discriminator preamplifier...........	559 93
1	Power supply unit, plug-in, 9.2 V	530 88
2	HF cables	501 02
1	Americum, AM-241 preparation, 333 kBq (9 μCi)	559 82
1	Saddle base	300 11
1	Stop clock	313 05

:ting up:

⚠ Never touch the gold foil!

Air the scattering chamber very carefully (see operating instructions 559 56), otherwise you may destroy the sensitive gold foil.

Set up and connect the instruments as shown in Fig. 3. Turn the potentiometer on the discriminator preamplifier all the way to the left. Air the scattering chamber and take off the lid. Set the digital counter to "⁄\⁄\".

Preparing the scattering chamber (Fig. 1):

Tnsert the preparation into the 4mm socket of the swiveling holder. Place the 1 mm slit and the plastic sheet containing the gold foil on top of one another and insert them both into the holder so that the slit points towards the preparation. Swing the holder for swinging in (not needed during this measurement) to the top of the cover. Make sure the aperture slit of the detector (on the inner wall of the chamber) is perpendicular (with the mark at the top). Close the chamber and evacuate it.

Carrying out the experiment:

Note:
Protect the detector from light while measuring (especially from fluorescent light). If necessary, cover the chamber with a black cloth or similar.

Each time Θ = 5°, 10°, ... count at least 20 particles ($n(\Theta) \geq 20$). Note the measuring time Δt needed and calculate the counting rate $N(\Theta) = n(\Theta)/\Delta t$. When Θ is 30°, replace the 1 mm slit by a 5 mm slit and repeat the measurement for 30°, 40°, 50° and 60°.

Measurement example:

For the following series of measurements, 100 to 200 α-particles were counted per angle setting in order to keep the statistical error small.

Table 1

θ	n(θ)	Δt/min	N(θ)/min	
10°	170	5	34	1 mm slit
15°	136	15	9,07	1 mm slit
20°	140	33	4,24	1 mm slit
25°	103	40	2,58	1 mm slit
30°	101	105	0,962	1 mm slit
30°	124	16	7,75	5 mm slit
35°	170	40	4,24	5 mm slit
40°	190	80	2,38	5 mm slit
45°	133	93	1,43	5 mm slit
50°	96	100	0,96	5 mm slit
55°	84	120	0,70	5 mm slit
60°	78	200	0,39	5 mm slit

Θ : Scattering angle
$n(\Theta)$: Counting rate in the time Δt for the scattering angle Θ

$N(\theta) = \frac{n(\theta)}{\Delta t}$

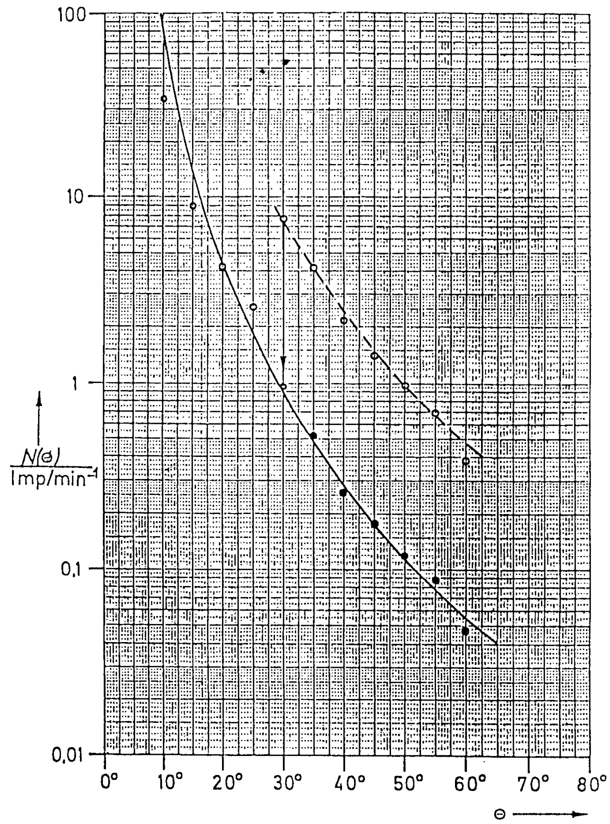


Fig. 4: N as a function of Θ
Open circles: Measured values
Closed circles: Measured values when $\theta > 30°$ converted to 1 mm slit
Conversion factor: The ratio of the counting rate at 30° with a 5 mm slit and a 1 mm slit
Solid line: Theoretical curve

Evaluation and results:

When $\Theta = 30°$, an eight times higher counting rate was measured with the 5 mm slit than with the 1 mm slit. The scattering rates for $\Theta > 30°$ must then be divided by 8, in order to convert them to "1 mm slit conditions".

The corrected measured values, and those for $\Theta < 30°$, are shown in Fig. 4 in floating point notation. In order to compare the measured values with the theoretically predicted $\sin^{-4}(\frac{\Theta}{2})$ - dependence of the scattering rate, the curve

$$f(\theta) = \sin^{-4}(\frac{\theta}{2})$$

is plotted on a second sheet of logarithmic paper with the same division of the axes.

The measured points and the theoretical curves must be made to coincide by displacement parallel to the ordinate. This parallel displacement corresponds to multiplication of the overall function with a corresponding proportionality constant (observe the logarithmic division of the axes). As can be concluded from Fig. 4, the measured points can easily be made to coincide with the theoretical curves.

Note:
If we remove the preparation from the scattering chamber and darken the chamber, the digital counter does not register anything, even after several hours.

False counts can occur when electrical devices (e.g. the pump) are turned on or off during a measurement and interfering impulses from the power source influence the digital counter.

Measurements of angles over 30° turn out especially well when the holder with the foil and the 5 mm slit is turned by 30° beyond the 90° alignment (Fig. 5). This prevents the α-particles from losing too much energy after the scattering process by having too long a path in the gold foil (Fig. 6).

If the 4 mm pin of the preparation is bent (the preparation turns in a circle when it is pushed in and then it turns around its longitudinal axis), a constant error of the angle can appear. In this case, it is necessary to displace the theoretical curve also parallel to the abscissa of the measuring diagram (Fig. 4), so that the curves $f(\Theta) = \sin^{-4}(\theta/2)$ and the measured points coincide.

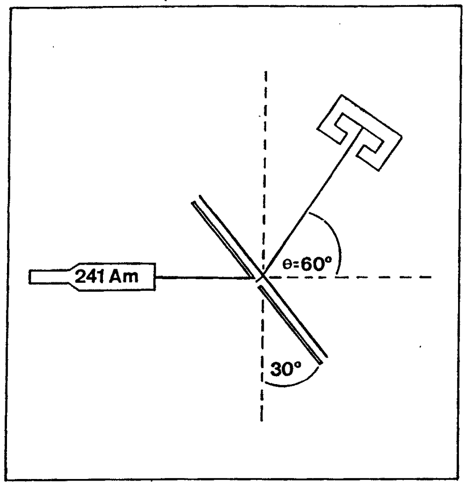


Fig. 5: Position of the preparation, slit, foil and detector during the measurement for large scattered angles. In the shown example: Θ = 60°.

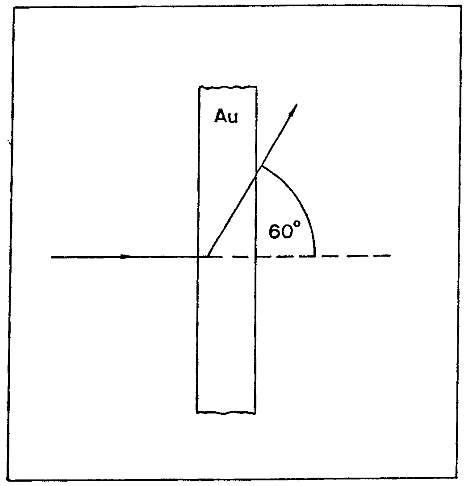


Fig. 6: The path of the α-particles in the gold foil with a scattering around 60°without the movement of slit and foil shown in Fig. 5