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Landau Damping and One-Body Dissipation in Nuclei

CARLOS FIOLHAIS

Departamento de Física, Universidade de Coimbra, P-3000 Coimbra, Portugal

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In the framework of the theory of Fermi liquids, the wall formula describes a type of Landau damping of the collective motion. It is found that in this model the ratio between width and energy of the nuclear giant resonances is of the order of 1, essentially independent of the multipolarity. This result, which is in contradiction with the experimental findings, is due to the fact that the boundary is externally imposed. With the exception of the compression modes, the Landau damping is not the appropriate process for describing the damping of the low-multipole giant resonances, being however very effective for deformations corresponding to high multipolarities (a modified wall formula predicts overdamping to occur at $\lambda = 7$).

1. INTRODUCTION

Following the experimental identification of new collective excitations in atomic nuclei (giant resonances), their damping mechanism has received much attention in the last years (see the reviews [1, 2] and references therein). As the concept of elementary excitations has a widespread usefulness in other fields of physics (e.g., helium, plasma, and solid state physics) we think that we may learn something about nuclear dissipation from establishing analogies with the different many-body systems where quantal collective motion has been observed. It is a purpose of the present paper to develop such a correspondence.

Only recently attempts have been undertaken to formulate a consistent microscopic description of the width of nuclear excitations, with a result which in some cases falls short, from the quantitative viewpoint. As the theoretical understanding of damping is still embrionary, it is usual to resort to phenomenological methods [3, 4, 5].

The phenomenological approaches to nuclear friction may be divided in two main types:

1. One-body dissipation/long-mean free path (wall formula)
2. Two-body dissipation/short-mean free path (viscosity).

The wall formula [6, 7, 8] has been introduced due to the recognition that a theory based on a short-mean free path, like hydrodynamics, is not justified for the nuclei. Surprising enough, the comparison with experiment of formulae 1 and 2 is in favor of the latter [5].

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From the microscopic treatment it has become apparent that the main damping mechanism for small amplitude collective motion is neither of type 1 nor 2, but is a two-body dissipation adequate to systems with a long-mean free path. Similar behavior is found in helium-3 [9] and in plasmas [10, 11].

In the theory of Fermi liquids [12] the damping of elementary collective oscillations, like helium phonons, electronic plasmons, and lattice phonons, may occur due to the following processes:

1. Landau damping [13].
2. Collision damping [14].

The first corresponds to the direct decay of the coherent *ph* excitation into a separated $1p-1h$, while the second corresponds to the decay into $2p-2h$ configurations or processes of higher order (see Fig. 1). We will use the terminology of Fermi liquids to refer to nuclear damping, as an alternative to the common designations of one- and two-body dissipation.

In this work we are interested is ascertaining the eventual role of Landau damping in nuclear collective motion. We show that the wall formula describes processes of the Landau damping type, in accordance with recent work by Yannouleas [15, 16]. Furthermore, we argue that Landau damping cannot be the main mechanism in the damping of the observed electric giant resonances with multipolarity $\lambda = 1, 2$, and 3.

A simple analysis of the phase space available for decay shows that small Landau damping at low temperatures is characterized by a proportionality between the width and the energy of the excitation, while the width corresponding to small collision damping depends quadratically on the energy, for the same range of temperatures. The wall formula when applied to the giant resonances gives indeed a linear dependence between width and energy, but the rate of damping in this model is too high. An analogy with the Landau damping present in the jellium model points out that the unrealistic assumption in the wall formula lies in the rigid character of the wall, which is strange to the particles. The incorporation of self-consistency in the wall formula leads to a reduction of damping for the low multipoles and an increase for the multipoles $\lambda \geqslant 6$.

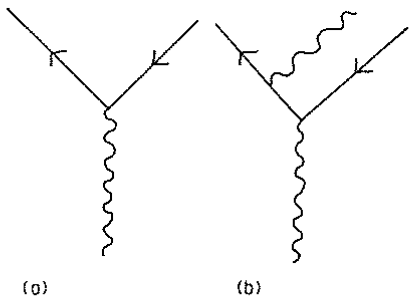

FIG. 1. Feynman diagrams corresponding to a) Landau damping and b) collision damping (with one of the final *ph*-states correlated).

The Landau damping is not able to account for the width of the giant resonances with $\lambda = 1, 2$, and 3, due to the simple fact that these self-sustained low multipole oscillations have an energy which corresponds to a gap of the spectrum of single-particle excitations (the dipole is an exception, but its negative parity avoids the direct decay into a $1p-1h$ pair). On the other side, the role of Landau damping is rather effective for higher multipolar modes, destroying them at the very beginning.

In Section 2 we present a short review of the damping processes found in helium-3 and plasmas. The phase space analysis is developed in Section 3 and used to show that the wall formula represents a process of Landau damping. The discussion of Section 4 is intended to explain the non-applicability of one-body friction to the small amplitude nuclear motion with low multipolarities. The conclusions are collected in Section 5. In Appendix A, we present the calculation of the integral which measures the phase space available for one-body decay.

2. Damping Mechanisms in Fermi Liquids

The Landau theory of Fermi liquids [14] is based on the concept of a self-consistent mean field for infinite systems, in which quasi-particles are allowed to move according to the Pauli principle. The Landau kinetic equation for the distribution function of quasi-particles is the Vlasov equation supplemented by a collision term of the Uehling–Uhlenbeck type.

In normal liquid helium-3 at low enough temperatures collisions scarcely take place, and any disturbance can only be propagated through the mean field. This phenomenon is called zero-sound to contrast with the normal or first sound, which propagates due to frequent particle collisions. The physics of zero-sound is just the same as the one which has been successfully proposed to account for the vibrational motion of nuclei [17].

Mathematically the zero-sound modes are found as solutions of the linearized Vlasov equation, which is valid for small wavenumbers ($k \ll k_F$). The existence of solutions depends on the effective interaction between quasi-particles. In the crudest approximation, which consists in taking only the first Landau parameter (F_0 in the isoscalar spin independent channel), the velocity of zero-sound, in units of the Fermi velocity, $s = v_{\text{phase}}/v_F = \omega/kv_f$, is the solution of

$$1 + \frac{s}{2}\log\left|\frac{s-1}{s+1}\right| + i\frac{\pi}{2}s\theta(1-|s|) = -\frac{1}{F_0}, \tag{2.1}$$

with $\theta(x)$ the step function. If $F_0 > 0$ (repulsive force) $s > 1$ is real. As the phase velocity of the wave is larger than the Fermi velocity, no direct energy transfer from the wave to the particles is possible. If $-1 \leqslant F_0 \leqslant 0$ (weak attractive interaction) the solution is complex, meaning that the modes are Landau damped. If $F_0 < -1$ (strong attractive interaction) the solutions are purely imaginary, meaning instability of the ground state.

TABLE I

Landau Parameters for Helium-3 and Nuclear Matter

	Scalar		Spin		Isospin		Spin–isospin	
	F_0	F_1	G_0	G_1	F_0'	F_1'	G_0'	G_1'
Helium-3[a]	10.07	6.04	−0.67	−0.67				
Nuclear Matter[b]	−0.37	−0.69	0.45	0.76	0.56	0.45	1.29	0.07

[a] Experimental results at $P = 0$ atm, according to J. C. Wheatley, *Rev. Mod. Phys.* **47** (1975), 415.
[b] Brueckner calculation based on a realistic bare interaction, according to D. Sjöberg, *Ann. Phys.* **78** (1973), 39 and S. Bäckman *et al.*, *Nucl. Phys. A* **321** (1979), 10.

The experimental values of the first Landau parameters for helium-3 are displayed in Table I. The force is strongly repulsive in the scalar channel and weak attractive in the spin channel. The zero-sound at $T = 0$ should therefore be free from Landau damping, which should take place for the spin modes. In Fig. 2 the zero-sound dispersion relation is schematically represented, illustrating the fact that the inexistence of Landau damping for small k is due to the position of the collective branche above the band of single particle excitations.

If, however, we extrapolate the results of the Landau theory to the range of higher k, we find that there is a critical value of k, for which the collective mode plunges into the continuum of single particle excitations and therefore strong Lan-

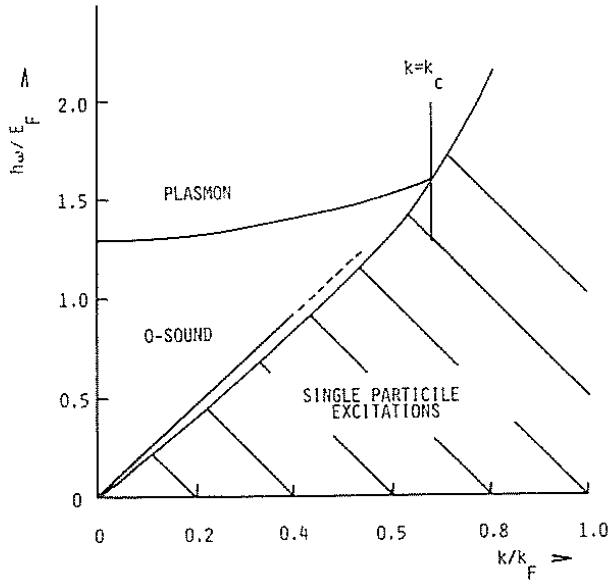

FIG. 2. Schematic representation of the dispersion relation $\omega = \omega(k)$ of the zero-sound for neutral and for a charged Fermi liquid.

dau damping begins to occur. This sound attenuation has been observed through neutron scattering on helium-3 [18].

For values of k smaller than the critical, the damping is only due to collisions between quasi-particles. The collisional width of the Landau theory of Fermi liquids at $T=0$ is proportional to the square of the sound frequence [14].

Helium-3 may serve as a guide for the study of nuclear matter, for which the Landau parameters are not exactly known. Table I shows a set of Landau parameters obtained with the aid of Brueckner calculations. As the interaction is weak attractive in the isoscalar spin-independent channel, the Landau damping should forbid the existence of the corresponding collective mode. In principle spin, isospin and spin-isospin modes might exist. It is an open question to know whether these modes are allowed in a range of k, wide enough to consider them as well-defined modes.

Recently, Yukawa and Kurazawa [19] have proposed an approach to the sound in nuclear matter in terms of the Wigner transform of the RPA equation, with the result that weak Landau damping sets up even for small values of k and not very repulsive interactions. The degree of damping they obtain is very sensitive to the force, indicating that more progress in nuclear effective forces is needed before one can make definitive statements about the propagation of sound in nuclear matter.

For a charged Fermi liquid like the electron gas, the zero-sound mode has a finite mass due to the long range Coulomb interaction. The oscillation (plasmon) is undamped for a large of k, if processes of higher order than the RPA are excluded (see Fig. 2). The RPA provides the following dispersion relation for a plasmon with small k at $T=0$

$$\omega = \omega_p \left[1 + \frac{3}{10} \left(\frac{k}{k_c} \right)^2 \right], \tag{2.2}$$

with $\omega_p = (4\pi\rho e^2/m)^{1/2}$ the classical plasmon frequence and $k_c = \omega_p / v_F$ the critical wavenumber for the onset of Landau damping. The sudden broadening of the plasmon at $k = k_c$ has been observed by electron loss spectroscopy [11]. A typical experience is 50 KeV electron scattering on *Al*, with the result that the plasmon with energy $\hbar\omega_p = 15.8$ eV becomes very wide at $k_c = 0.74\, k_F$. Note that this cutoff is much smaller than the geometrical constraint $k_{\max} = 1.7\, k_F$, which arises from equating the half-wavelength with the mean distance between the electrons.

The only possible type of damping before the cutoff k_c is the collision damping. The collision damping width has been calculated by DuBois with the following result in first order and for small k [20]

$$\Gamma^{\mathrm{CD}}(k) = 9.30 k^2 \omega_p^2 (1 + 0.79\omega_p). \tag{2.3}$$

The strong supression of the collision width (2.3) for $k \simeq 0$ should be remarked.

For finite temperatures, Landau damping does exist both for helium-3 and the electron gas at all wavenumbers. It is due to the quasi-particles of the thermal distribution whose velocity is nearly identical to the phase velocity of the wave. These quasi-particles may "steal" energy from the wave, without spoiling it completely.

In helium-3 the thermal Landau damping is obscured by the much stronger collision damping and is usually ignored.

For the electron gas at $T \neq 0$ and small k the plasmon frequence and the Landau damping width are given respectively by [21]

$$\omega(k, T) = \omega_p \left[1 + \frac{3}{2} \left(\frac{k}{k_D} \right)^2 \right] \tag{2.4}$$

$$\Gamma^{LD}(k, T) = \hbar\omega_p \left(\frac{\pi}{2} \right)^{1/2} \left(\frac{k_D}{k} \right)^3 \exp\left[-\frac{1}{2} \left(\frac{k_D}{k} \right)^2 \right], \tag{2.5}$$

where $k_D = (4\pi\rho e^2/k_B T)^{1/2}$ is the reciprocal of the Debye shielding length. The ratio between width and energy, which depends only on k and T, is exponentially small, but nevertheless the effect has been confirmed by experiment. An approximate upper limit is obtained putting $k = k_D/2$, with the result $\Gamma^{LD}/\hbar\omega = 0.987$ (for $k > k_D/2$ the Eq. (2.5) no longer holds).

3. The Wall Formula as Landau Damping

The width of the giant resonances in finite nuclei due to the coupling of the RPA modes to more complicated configurations has been shown to correspond to the collision damping at low temperatures described by the theory of infinite matter [1, 22]. The microscopic calculations seem to reproduce the experimental width, with the exception of the breathing mode.

It is purpose of this section to examine the differences between the less known Landau damping and the collision damping and to establish a relationship with the current phenomenological models. It is already apparent beforehand, that the wall formula should correspond to some limit of Landau damping. In fact, in both cases there is a direct energy transfer from the wave to the particles.

The difference between Landau and collision damping with respect to the energy and temperature dependence may be explained in terms of phase space available for decay.

The width of a given collective state $|c\rangle$, with the excitation energy $E = \hbar\omega$ may be evaluated using Fermi's golden rule

$$\begin{aligned} \Gamma &= \frac{2\pi}{\hbar} \sum_f |\langle f | V | c \rangle|^2 \, \delta(\hbar\omega - E_f) \\ &\simeq \frac{2\pi}{\hbar} \overline{|\langle f | V | c \rangle|^2} \frac{dn_f}{dE}, \end{aligned} \tag{3.1}$$

where $|f\rangle$ are the final states with energy E_f, V is the interaction responsible for the decay, the bar denotes a mean value of the transition probability, and dn_f/dE is the density of final states with the prescribed energy $\hbar\omega$.

The structure of final states is distinct for Landau and collision damping. In the first case the final states are uncorrelated $1p-1h$ excitations, while in the second are $2p-2h$ states (Bortignon and Broglia [23] have emphasized the role of the diagram of Fig. 1b, where one $1p-1h$ is correlated).

Let us consider first the Landau damping in infinite matter at an arbitrary temperature. The damping is assumed to be small enough in order to legitimate the perturbative formula (3.1).

The density of $1p-1h$ states having the given energy is proportional to the integral

$$I_{1p-1h} = \iint d\varepsilon_1\, d\varepsilon_2 n(\varepsilon_1)[1-n(\varepsilon_2)]\,\delta(\hbar\omega-\varepsilon_2+\varepsilon_1), \tag{3.2}$$

where $n(\varepsilon)$ is the Fermi distribution function

$$n(\varepsilon) = 1/\{\exp[(\varepsilon-\mu)/k_B T]+1\} \tag{3.3}$$

with μ the chemical potential. The integral (3.2) can be evaluated exactly (see Appendix A). The final result reads as

$$I_{1p-1h} = \frac{\hbar\omega}{1-\exp(-\hbar\omega/k_B T)}. \tag{3.4}$$

The Landau damping width for homogeneous matter is therefore

$$\Gamma^{\mathrm{LD}} \alpha\, \overline{W(\mathbf{k}_1,\mathbf{k}_2)}\, \frac{\hbar\omega}{1-\exp(-\hbar\omega/K_B T)}, \tag{3.5}$$

where $\overline{W(\mathbf{k}_1,\mathbf{k}_2)}$ is a mean one-body interaction obtained from the scattering amplitude in the medium. For small temperatures, the width (3.5) is proportional to the collective energy, being independent of the temperature

$$\Gamma^{\mathrm{LD}} \alpha \hbar\omega, \qquad \hbar\omega \gg k_B T, \tag{3.6}$$

while for large temperatures increases linearly with the temperature, being independent of the frequency

$$\Gamma^{\mathrm{LD}} \alpha k_B T, \qquad k_B T \gg \hbar\omega. \tag{3.7}$$

The frequency and temperature dependence (3.6) and (3.7) contrasts to that of the collision damping width. The density of $2p-2h$ states with energy $\hbar\omega$ may be measured by the integral

$$\begin{aligned} I_{2p-2h} = {} & \iiiint [n(\varepsilon_1)\, n(\varepsilon_2)\, n(-\varepsilon_3)\, n(-\varepsilon_4) - n(\varepsilon_3)\, n(\varepsilon_4)\, n(-\varepsilon_1)\, n(-\varepsilon_2)] \\ & \times \delta(\hbar\omega-\varepsilon_3-\varepsilon_4+\varepsilon_1+\varepsilon_2)\, d\varepsilon_1\, d\varepsilon_2\, d\varepsilon_3\, d\varepsilon_4 \\ = {} & \frac{2}{3}\pi^2\hbar\omega(k_B T)^2\left[1+\left(\frac{\hbar\omega}{2\pi k_B T}\right)^2\right]. \end{aligned} \tag{3.8}$$

The mean two-body interaction contributes with a factor $(\hbar\omega)^{-1}$, leading to the result [14]

$$\Gamma^{\mathrm{CD}} \alpha (\hbar\omega)^2, \qquad \hbar\omega \gg k_B T, \tag{3.9}$$

$$\Gamma^{\mathrm{CD}} \alpha (k_B T)^2, \qquad k_B T \gg \hbar\omega. \tag{3.10}$$

For $k_B T \gg \hbar\omega$ the Landau damping does not increase so fast with the temperature as the collision damping, due to the reduced phase space available for decay. Which of the two processes prevails is a question which depends on the interaction of the particular system under study.

For finite nuclei and $\hbar\omega \gg k_B T$ the dependence of Γ^{LD} and Γ^{CD} on $\hbar\omega$ may not be so simple as in (3.6) and (3.9), but it is expected that the dominant behavior is still given by the infinite matter result.

We wish now to consider the relationship between the width given by the phenomenological models of nuclear friction and the energy of the giant resonances. For the isoscalar electric modes with $\lambda \geqslant 2$, Bertsch [24] has recognized that the high frequency response of nuclei to external probes should be accounted for by a stiffness coefficient characteristic of an elastic body, rather than the stiffness provided by the standard liquid drop model (similar behavior has been pointed out for helium-3 [25, 26]).

The restoring force constant

$$C_\lambda = 2\frac{\lambda - 1}{\lambda} mA\langle v^2 \rangle \tag{3.11}$$

and the irrotational hydrodynamical mass

$$M_\lambda = \frac{3}{\lambda(2\lambda + 1)} mAR_0^2 \tag{3.12}$$

lead to the collective energy

$$E_\lambda = \hbar\left(\frac{C_\lambda}{M_\lambda}\right)^{1/2} = \hbar\left[\frac{2}{3}(2\lambda + 1)(\lambda - 1)\right]^{1/2} \frac{\bar{v}}{R_0}$$

$$= \hbar\left[\frac{2}{5}(2\lambda + 1)(\lambda - 1)\right]^{1/2} \frac{v_F}{R_0}, \tag{3.13}$$

where Am is the nuclear mass, R_0 is the nuclear radius, and where we have used the relation between the mean square velocity $\bar{v} = (\langle v^2 \rangle)^{1/2}$ and the Fermi velocity: $\bar{v} = (\frac{3}{5})^{1/2} v_F$. The expression (3.13) gives a very good overall account of the dependence of the resonance energies on A, without any adjustable parameters. We obtain for the quadrupole and the octupole

$$E_2 = \sqrt{2}\,\hbar\frac{v_F}{R_0} = 1.58\,\hbar\omega_0 \tag{3.14}$$

$$E_3 = \sqrt{\frac{28}{5}}\,\hbar\frac{v_F}{R_0} = 2.64\,\hbar\omega_0, \tag{3.15}$$

where $\hbar\omega_0 = 41A^{-1/3}$ MeV $= 0.895\hbar v_F/R_0$ is the usual shell energy spacing. For ^{208}Pb the agreement of (3.14) and (3.15) with the experimental data is nearly perfect ($E_2^{\exp} = 10.87 \pm 0.10$ MeV and $E_3^{\exp} = 18.16 \pm 1.35$ MeV, these values being averages over the results of different laboratories [27]).

The wall formula relies on the assumption of a rigid wall, which is the container of a rarefied gas (Knudsen gas). The wall is able to transmit energy to the particles once a disturbance is imposed from outside. The rarefaction is in agreement with the requirement of a long-mean free path which is known to be adequate in nuclei. The wall formula should therefore be applied to the nuclear elastic vibrations rather than to the hydrodynamical modes which require by their very definition a short-mean free path.

The wall formula for the friction coefficient referring to multipolar distortions is

$$\gamma_\lambda^{\mathrm{WF}} = \frac{4\pi}{2\lambda+1}\rho\bar{v}R_0^4, \tag{3.16}$$

where ρ is the nuclear density. In this context the mean velocity is usually taken to be $0.75v_F$ but we shall employ the slightly different mean square root $\bar{v} = 0.77v_F$. Dividing (3.16) by the collective mass (3.13) we obtain the width

$$\Gamma_\lambda^{\mathrm{WF}} = \hbar\frac{\gamma_\lambda^{\mathrm{WF}}}{M_\lambda} = \hbar\lambda\frac{\bar{v}}{R_0}. \tag{3.17}$$

The following quotient between $\Gamma_\lambda^{\mathrm{WF}}$ and E_λ is readily achieved

$$\frac{\Gamma_\lambda^{\mathrm{WF}}}{E_\lambda} = \left[\frac{2}{3}(2\lambda+1)(\lambda-1)\right]^{1/2}\cdot\lambda. \tag{3.18}$$

This quantity does not depend on the particular nucleus considered. Table II shows that the dependence on the multipolarity of the oscillation is rather weak.

The linear relation between width and energy characteristic of Landau damping is therefore fulfilled in the wall model. The dependence of $\Gamma_\lambda^{\mathrm{WF}}/E_\lambda$ on λ resembles the dependence on k of the rate of the Landau damping of plasmons (see Eqs. (2.4) and (2.5)).

Table II indicates that the width is of the order of magnitude of the excitation energy, like the maximal Landau damping width of the plasmon (see (2.5)). The smallest value for $\Gamma_\lambda^{\mathrm{WF}}/E_\lambda$ occurs in the limit of very large multipoles. The wall formula has been claimed to be valid only in this limit, due to the randomization hypothesis required in its derivation [6]. We remark that only very big nuclei can support such modes, since the wavelength of a collective oscillation must be larger than the mean distance between the nucleons. In the limit $\lambda \to \infty$ (with $R_0 \to \infty$, $\lambda/R_0 =$ constant) we obtain from (3.18)

$$\lim_{\lambda\to\infty}\frac{\Gamma_\lambda^{\mathrm{WF}}}{E_\lambda} = \frac{\sqrt{3}}{2} = 0.87. \tag{3.19}$$

TABLE II

Ratio between Width and Energy of the Giant Resonances, using Wall, Modified Wall, and Viscosity Formulas

Γ \ λ	2	3	4	5	6	7
Γ^{WF}/E	1.09	0.98	0.94	0.92	0.91	0.90
Γ^{MWF}/E	0.07	0.24	0.51	0.88	1.37	1.95
Γ^{VF}/E	0.17	0.28	0.39	0.50	0.61	0.72

It is well known that the transition from damped periodic (underdamping) to aperiodic motion (overdamping) of an oscillator takes place when the imaginary part of the frequency $\omega = \omega_r - i\omega_i = (E - i\Gamma/2)/\hbar$ is equal to the real part, i.e., when $\xi = \omega_i/\omega_r = 1$. Within the wall model this parameter assumes values which go from $\xi_2^{WF} = \Gamma_2^{WF}/2E_2 = 0.55$, for the quadrupole mode, to

$$\lim_{\lambda \to \infty} \xi_\lambda^{WF} = \frac{\sqrt{3}}{4} = 0.43. \tag{3.20}$$

As one has always $\xi_\lambda^{WF} < 1$, the giant resonances are underdamped with the wall friction. This statement contrasts to earlier verifications, that the liquid drop oscillations are overdamped in the wall model [3, 6].

It is however clear that the condition $\xi_\lambda \ll 1$ (or $\Gamma_\lambda \ll E_\lambda$) which is expected for well-defined modes, is not satisfied by the wall friction. For all multipolarities, the time to reduce the amplitude of the oscillation to half its initial value is comparable with the period. In the framework of the wall model, the mode should therefore disappear after very few periods. It is known from the experimental data that the dissipation of the low multipoles is not so strong as indicated by Eq. (3.18). A typical quadrupole vibration, for instance, lasts at least 4 periods before damped out (the giant 2^+ of ^{208}Pb has an experimental ratio between width and energy $\Gamma_2^{exp}/E_2^{exp} = 2.73/10.87 = 0.25$ [27]).

We must resort to the conclusion that there are assumptions in the derivation of the wall formula, which cannot be justified in coherent collective motion. It is not difficult to recognize that a wall external to the particles is not realistic in the present context. Yannouleas [16] and Griffin and Dworzecka [28] have recently discussed this issue.

The validity of the wall formula may be better understood if analogies with other many-body systems are developed.

For helium-3, besides the small Landau damping taking place in the bulk, there is strong Landau damping at the boundary [29], originated by the transfer of energy from the solid container to the particles in the interior. The particles are in this case strange to the source of energy as in the nuclear wall formula.

There is bulk Landau damping in the jellium model of a plasma. The model consists of an ion-electron gas. The ionic plasmons are Landau damped through energy transfer to the electrons, because there are plenty of electrons available with velocity equal to the phase velocity of the wave. The only hindrance, which keeps the process weak, lies in the mismatch between the electron and the ion masses. The ratio between the imaginary and the real parts of the collective frequency is [12]

$$\frac{\omega_i}{\omega_r} = \frac{\pi}{12}\left(\frac{3mZ}{M}\right)^{1/2}, \tag{3.21}$$

where Z is the number of electrons per ion, m is the electronic mass, and M is the ionic mass. An upper bound for (3.30) is $\pi\sqrt{3}/12 = 0.45$, corresponding to $mZ = M$. This value is remarkably close to the wall formula result (3.20). Attention should however be paid to the fact that for $mZ = M$ the Born–Oppenheimer-like approximation, which allows the separation of the electronic from the ionic problem, breaks down.

The last examples show, as a common feature of the wall model, the circumstance that the collective modes are not made up of the particles which receive the energy, but are external to them.

A modification of the wall formula has been proposed by Sierk, Koonin, and Nix [30] with the scope of implementing self-consistency, at least in an approximate way. In the modified wall formula the dissipation rate is calculated, replacing the normal surface velocity by the relative normal velocity between the surface and the matter inside. The result reads as

$$\gamma_\lambda^{\mathrm{MWF}} = \frac{4\pi}{2\lambda+1}(\lambda-1)^2\,\rho\bar{v}a^2R_0^2, \tag{3.22}$$

with a a parameter with the dimensions of a length, which must be of the order of magnitude of the range of nuclear forces. The associated width is

$$\Gamma_\lambda^{\mathrm{MWF}} = (\lambda-1)^2\,\lambda\hbar\bar{v}\frac{a^2}{R_0^3} = (\lambda-1)^2\left(\frac{a}{R_0}\right)^2\Gamma_\lambda^{\mathrm{WF}}. \tag{3.23}$$

In comparison with the wall width (3.17), there is a correction in the order of magnitude ($a \ll R_0$) and a modification on the multipolar dependence (this dependence is much stronger in (3.23)). If we take $(a/R_0)^2 = 0.06$, corresponding to $a = 1.73$ for ^{208}Pb [30], we obtain for (3.23) the range of values shown in Table II. There is a very weak damping of the first multipoles and a strong damping of the $\lambda \geqslant 6$ modes. The transition to overdamping occurs approximately at $\lambda = 7$ ($\xi_7^{\mathrm{MWF}} = 0.98$). The next mode is already overdamped ($\xi_8^{\mathrm{MWF}} = 1.35$). The situation is very similar to that found in helium-3 and in the electron gas, where the collective mode disappears due to Landau damping at some critical value of k.

The maximal value for the multipolarity of a surface oscillation in ^{208}Pb is $\lambda = 11$, as it may easily be verified equating half wavelength $2\pi R_0/2\lambda$ with the mean distance between nucleons $d = 2$ *fm*. The mode $\lambda = 11$ is strongly overdamped ($\xi_{11}^{\mathrm{MWF}} = 2.61$).

This picture of one-body damping is much more reasonable than the one provided by the simple wall formula. The modified wall formula may be supported by several arguments. On one hand, a weak one-body damping of symmetrical shapes (even zero for the quadrupole) has been found in a classical linear response calculation by Koonin and Randrup [31]. On the other hand, Griffin and Dworzecka [28], based on the work of Yannouleas [16], have proposed a quantal wall formula giving a damping which is less than 10% of the standard wall result. The modified wall formula of Sierk *et al.* satisfies this constraint and seems therefore to simulate in a simple way the real quantal damping.

Let us now discuss the experimental information on the width of giant resonances. The width of the dipole and of the quadrupole is fitted by the following formula [32]

$$\Gamma_\lambda^{\mathrm{exp}} = 0.029 E_\lambda^{1.9}, \qquad \lambda = 1, 2. \tag{3.24}$$

This empirical relationship indicates that the prevailing mechanism of damping is collisional, although strong shell effects obscure the smooth trend. If one tries to describe the experimental data by a straight line, the slope should be approximately $\Gamma_2/E_2 = 0.25$. Nix *et al.* [33] claim that the renormalization of the wall formula by a factor 0.27 may describe both the quadrupole and the octupole widths. This approach has the drawback of failing to predict the rapid decay of high multipoles and its consequent non-observability. On the contrary, a formula based on the modified wall formula shows up this feature. Recent results on the detection of the octupole strength suggest that the high energy octupole is much more spread than it was supposed to [27].

As the modified wall formula only accounts for 28% of the experimental quadrupole damping, the rest should be attributed to two-body processes. Let us then consider the dissipation given by the viscosity formula. The viscosity friction constant is

$$\gamma_\lambda^{\mathrm{VF}} = 8\pi \frac{\lambda - 1}{\lambda} R_0^3 \mu, \tag{3.25}$$

where μ is the viscosity coefficient, which is related to the mean free path L by

$$\mu = \frac{1}{3} \rho \bar{v} L. \tag{3.26}$$

The whole approach depends on the condition $L \ll R_0$. This requirement is not met in practice in nuclear physics, but we may consider (3.25) and (3.26) merely as a

phenomenological prescription for the two-body dissipation. Inserting (3.26) in (3.25) we obtain

$$\gamma_\lambda^{VF} = 2\frac{\lambda-1}{\lambda} mA\bar{v}L. \tag{3.27}$$

The viscosity width is obtained dividing (3.27) by (3.12)

$$\Gamma_\lambda^{VF} = \frac{2}{3}\hbar(\lambda-1)(2\lambda+1)\bar{v}\frac{L}{R_0^2}. \tag{3.28}$$

This width increases with λ much faster than the wall width (3.17), in agreement with the physical picture we have of the damping process.

It is straightforward from (3.28) and (3.13) to arrive at the following quadratic dependence of the viscosity width on the energy:

$$\frac{\Gamma_\lambda^{VF}}{E_\lambda^2} = \frac{L}{\hbar\bar{v}}. \tag{3.29}$$

This value is not only independent of the nuclear radius but also of the multipole degree.

The relation (3.29) is similar to (3.9), which characterizes collision damping at low temperatures, although the physics involved is different. At low temperatures Fermi liquids display a long-mean free path, which is incompatible with the idea of frequent particle collisions.

The fact that the viscosity width shows the same dependence on the frequency as the collisional width of zero sound may explain some qualitative agreement with experiment of the viscosity width [4, 5]. Actually, the viscosity coefficient is adjusted from some data (in general on the kinetic energy of fission fragments). According to (3.26) this fitting corresponds to choosing an "effective mean free path" L. The viscosity $\mu = 0.03 \pm 0.01 TP$, which is adequate for fission, leads to $L = 0.65 \pm 0.22 fm$, in disagreement with the experimental knowledge on the mean free path ($L \gtrsim R_0$, with $R_0 = 7.0 fm$ for ^{208}Pb). Nevertheless, we remark that the dissipation predicted by (3.29) with $L = 0.65 fm$ is essentially the difference between the experimental and the one-body (modified wall) results for the quadrupole in ^{208}Pb

$$\frac{\Gamma_2^{VF}}{E_2} = \sqrt{\frac{10}{3}}\frac{L}{R_0} = 0.17. \tag{3.30}$$

In this way we obtain a comprehensive view of the damping of giant resonances as a sum of one- and two-body contributions. In Table II the ratio $\Gamma_\lambda^{VF}/E_\lambda$ is shown for $2 \leqslant \lambda \leqslant 7$, keeping L constant.

We note that (3.28) gives underdamping of all the admissible multipoles. The coefficient

$$\xi_\lambda^{\mathrm{VF}} = \left[\frac{1}{6}(2\lambda+1)(\lambda-1)\right]^{1/2}\frac{L}{R_0} \tag{3.31}$$

is 0.09, 0.31, and 0.58 for $\lambda = 2$, 6, and 11, respectively. Wong [34] has proposed to take as lower and upper bounds for the nuclear viscosity the values which give overdamping of the $\lambda = \lambda_{\max}$ and the $\lambda = 2$ modes of the liquid drop model, respectively: $0.0568 \leqslant \mu \leqslant 0.0687$ *TP*. The coefficient $\mu = 0.03$ does not satisfy the first inequality since the two-body dissipation we have in mind is adequate to describe the damping of giant resonances and not of the low-lying modes. Although the viscosity we are using is not sufficient to assure overdamping of the maximal mode $\lambda_{\max} = 11$, the combined effect of one- and two-body damping provides overdamping at already $\lambda = 6$ for ^{208}Pb (see Table II).

4. Irrelevance of Landau Damping for the Giant Resonances $\lambda = 2$ and 3

In this section we will show why Landau damping cannot be the adequate process for the description of the damping of the lowest surface multipolar modes. One-body damping is only pertinent for high distorted shapes and for the compression modes.

We may look into the problem from a classical or from a quantal perspective:

(i) Classical picture. Landau damping can only occur if the velocity of the wave is slightly smaller than the velocity of the particle. The phase velocity of the wave corresponding to the giant surface modes is, in units of v_F, $v_{\mathrm{phase}}/v_F = \omega/kv_F = [\frac{2}{5}(2\lambda+1)(\lambda-1)]^{1/2}/\lambda$. For $\lambda = 2$, the result is 0.71. There is in this case a serious discrepancy between the wave velocity and the velocity of the most external particle and a large energy exchange between wave and particle cannot take place. The phase velocity is however increasing with λ. For $\lambda = 7$, $v_{\mathrm{phase}}/v_F = 0.85$, indicating that Landau damping is possible. Note that the corresponding "critical" wavenumber for ^{208}Pb is $k = 0.78k_F$, a value which is very close to the critical wavelength in the given example of Landau damping in a metal.

(ii) Quantal picture. If we take as shell basis an harmonic oscillator, the quadrupole motion is build up from $0\hbar\omega_0$ and $2\hbar\omega_0$ single particle excitations. The effect of the residual forces is to pull down the isoscalar mode and to pull up the isovector mode. The isoscalar mode $\lambda = 2$ has been predicted using self-consistency arguments to be at $E_2 = \sqrt{2}\hbar\omega_0$, while on the same basis the high frequency octupole should be at $E_3 = \sqrt{7}\hbar\omega_0$ ([17], compare with (3.14) and (3.15)). Both the giant quadrupole and octupole are therefore situated in the middle of gaps of the single-particle excitation spectrum (see Fig. 3). There is no possibility of direct energy transfer from the collective to single-particle degrees of freedom.

The "dispersion relation" $E = E(\lambda)$ represented in Fig. 3 contrasts with those of Fig. 2 for infinite matter systems, due to the quantization of both collective and single-particle spectra. But there does exist a resemblance in the fact that for long wavelengths the collective modes do not superimpose with single-particle ones.

Although the real situation is much more involved than in the schematic model, mainly due to the spin-orbit splitting which mixes different shells, the microscopic RPA calculations show a clear-cut concentration of strength for the quadrupole and octupole giant modes (the octupole has a low-lying collective component, which carries a fair ammount of strength, but this mode is outside the scope of the macroscopic treatment of Sect. 3). The shell structure together with the residual forces assure the existence of self-sustained modes.

It is apparent from the RPA results that the strength of the multipole operators becomes gradually very fragmented with increasing λ [32]. After $\lambda = 6$ it is impossible to recognize any peaks. This fact is a clear manifestation of Landau damping. For $\lambda = 6$, the centroid energy predicted by the macroscopic formula (3.13) is $E_6 = 5.70\hbar\omega_0$. This value is inside the $6\hbar\omega_0$ excitation "band", which should be rather large due to the increasing number of configurations which are possible at high excitation energies. The formula (3.13) is supported by RPA sum-rules for $\lambda \leqslant 6$, but it should be noted that it does not hold anymore for $\lambda > 6$ [35]. This effect should be interpreted as a manifestation of Landau damping, which affects not only the width but the centroid energy of the resonance, lowering it. The numbers $\Gamma_\lambda / E_\lambda$ displayed in Table II and calculated on the basis of (3.13) are only conservative estimates of the real damping for $\lambda = 7$.

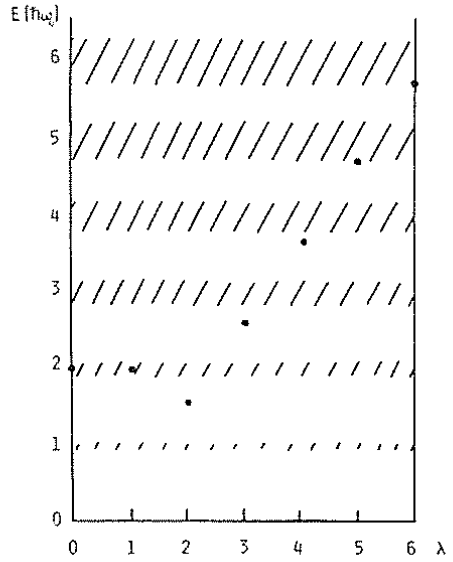

FIG. 3. Schematic representation of the "dispersion relation" $E = E(\lambda)$ for the first nuclear giant resonances. The broadening of the single-particle excitation "bands" is intended to simulate the increasing number of configurations at high energy.

For the isovector dipole mode, the strong isovector interaction displaces the energy from the unperturbed value to approximately $2\hbar\omega_0$ in heavy nuclei. There is however no possibility for an extended energy exchange through Landau damping due to the different parity of collective and single particle excitations (the spin-orbit force causes some intrusions of levels with negative parity into this energy range).

The monopole is a special case since its *ph* components lie at $2\hbar\omega_0$ and the observed energy is roughly at the same position (the experimental 0^+ for ^{208}Pb is at 13.72 ± 0.21 MeV, having a width of 2.67 ± 0.37 MeV). We can therefore expect that Landau damping plays a role for the monopole. Corroborating this possibility, we may add some arguments:

(a) Microscopic calculations give only a collisional width of 1 MeV [1], which is too low when compared with the experimental result.

(b) It has been reported by some authors [36] that the monopole has as appreciable width ($\simeq 2$ MeV) already in the RPA.

(c) The data systematics [32] on the monopole width does not follow the empirical rule (3.24).

We think that a proper combination of one- and two-body damping processes may explain the monopole width. As suggested in [1], the decay branches of the monopole should be investigated.

5. Conclusions

As in infinite Fermi liquids there are two main types of damping of nuclear collective motion: Landau or collisioless (one-body dissipation) and collision damping (two-body dissipation).

We have concentrated in this work on the Landau damping. A criterium for well-defined collective motion is that the Landau damping width is small when compared with the collective energy: $\Gamma^{\mathrm{LD}} \ll E$. If this condition is not met, it is not worth to extend the calculations to the further level of complexity.

We have shown that in infinite systems the Landau and the collision widths have different dependencies on energy and temperature. The two most common phenomenological approaches to nuclear friction display the energy dependence required by such general considerations.

The wall formula does not satisfy the condition of the smallness of the width when compared to the energy. This shortcoming is due to the assumption of a wall as an external source of energy. The modified wall formula, which incorporates self-consistency, is able to describe the Landau damping of high multipoles. The occurrence of overdamping after $\lambda = 7$ indicates that there is a critical multipolarity for which the collective behaviour is essentially replaced by incoherent single-particle response.

Although this work confirms other studies about the inadequacy of the wall formula in some circumstances [16], it should be emphasized that the formula was

not originally intended to describe small disturbances but the large amplitudes found in fission and heavy ion collisions. In these cases the nuclear shapes include very high multipolar components.

Finally we have shown the reason for processes of Landau type to be irrelevant when studying the decay of the $\lambda = 2$ and 3 giant modes. The self-consistency provided by the residual interactions conjugated with the existence of shells guarantee that the collective energies do not coincide with the energy of any single-particle excitation of the same parity. The Landau damping in nuclear as well as in other many-body systems manifests itself in strongly damping modes with short wavelengths.

The broadening of giant resonances in deformed nuclei, which may be accounted by a RPA description, is due to the dense bunches of levels characteristic of deformed shell models. This Landau damping in the deformed basis, should be equivalent to the $\omega \to 0$ limit of the collision damping in the spherical basis, since the deformed shape may be interpreted as a very slow, adiabatic vibration.

A recent interest on the question of collective motion at finite temperatures has emerged from the experimental detection of the giant dipole in compound nuclei [37, 38]. In this situation Landau damping should appear or become more accentuated, but its relative role in comparison with the collision damping is unclear. The one-body width increases with the temperature certainly due to the increase of deformation of the system. We would expect a weak direct T-dependence of both Landau and collision damping for the range of temperatures available ($k_B T < 2$ MeV, $\hbar\omega \simeq 15$ MeV, and therefore $k_B T \ll \hbar\omega$). Further data is needed to determine the main mechanism of damping of collective energy in thermal excited nuclei.

Appendix A

In this Appendix, we calculate the integral (3.2):

$$I_{1p-1h} = \iint d\varepsilon_1 \, d\varepsilon_2 n(\varepsilon_1)(1 - n(\varepsilon_2)) \, \delta(\hbar\omega - \varepsilon_2 + \varepsilon_1).$$

We change variables from ε and ω to respectively $x = (\varepsilon - \mu)/k_B T$ and $\xi = \omega/k_B T$ obtaining

$$I_{1p-1h} = k_B T \iint n(x_1)(1 - n(x_2)) \, \delta(\xi - x_2 + x_1) \, dx_1 \, dx_2$$

$$= k_B T \iint \frac{1}{e^{x_1} + 1} \frac{1}{e^{-x_2} + 1} \delta(\xi - x_2 + x_1) \, dx_1 \, dx_2$$

$$= k_B T \int_{-\infty}^{+\infty} \frac{1}{e^{x_1}+1} \frac{1}{e^{-x_1-\xi}+1} dx_1$$

$$= k_B T \int_0^{+\infty} \frac{1}{u+1} \frac{1}{ua+1} du$$

$$= k_B T \int_0^{+\infty} \frac{du}{au^2+(a+1)u+1},$$

where we have introduced $u = e^{-x_1}$ and $a = e^{-\xi}$.

The result is the expression (3.4)

$$I_{1p-1h} = -k_B T \frac{\log a}{1-a} = \frac{\hbar\omega}{1-\exp(-\hbar\omega/k_B T)}.$$

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