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On the gains of using high frequency data and higher moments in Portfolio Selection

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Abstract

In this paper we conduct an empirical analysis on the performance gains of using high frequency data in portfolio selection. Within a CRRA-utility maximization framework, we suggest the construction of two different portfolios: a low and a high frequency portfolios. For ten different risk aversion levels, we compare the performance of both portfolios in terms of several out-of-sample measures. Using data on fourteen stocks of the CAC 40 stock market index, from January 1999 to December 2003, we conclude that the "fight" is always "won" by the high frequency portfolio for all the considered performance evaluation measures.

Keywords: portfolio selection, utility maximization criteria, higher moments, high frequency data, out-of-sample analysis

JEL Classification: C44; C55; C58; C61; C63; C88; G11

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1. Introduction

A normal distribution of the returns and a quadratic utility function for the investors' preferences are sufficient conditions for the use of the classical mean-variance analysis (Markowitz, 1952). However, these conditions are seldom verified in practice. At least since Mandelbrot (1963), we know that one of the stylized facts of financial time series is that the return's distribution exhibits fat tails. Moreover, it has been shown that investors have preference for positive skewness (see, e.g., the seminal work of Arditti, 1967) and dislike high kurtosis (see, e.g., the empirical work of Maringer and Parpas, 2009). In fact, several empirical studies suggest that there are performance gains when higher moments are taken into account (namely skewness and kurtosis) in the portfolio choice (see, e.g., Athayde and Flôres, 2004, Maringer and Parpas, 2009, and Harvey et al., 2010).

During many years, GARCH type estimators (see Engle, 1982, Bollerslev, 1986, and Nelson, 1991) and stochastic volatility models (see Taylor, 1986) have been used in the financial services industry. However, in the begin of the century, motivated by the increasing availability of high frequency data, the works of Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002) paved the way for the use of realized estimators. Since then, both researchers and quants began to dedicate special attention to the estimation of the realized variance, i.e., began to use intraday data to estimate the variance as the sum of squared returns. The realized variance, in theory, offers a great estimation power, since it is a model-free measure and converges to the quadratic variation. In fact, it was early observed by Merton (1980) that the accuracy of the variance estimation increases with the sample frequency (because the sample path of the variance is continuous). An approach like the one used for realized variance can be designed for the estimation of skewness and kurtosis. Thereby, we can define the realized skewness as the sum of the 3rd power of intraday returns (see Neuberger, 2012) and the realized kurtosis as the sum of the 4rd power of intraday returns (see Amaya et al., 2015).

Concerning the variance estimation, there are already some studies suggesting the existence of benefits in using high frequency data (see, e.g., Flemming, Kirby, and Ostdiek, 2003, and Liu, 2009). On the other hand, Amaya et al. (2015) found a negative effect of skewness and a positive effect of kurtosis on weekly stock returns. However, an important question remains open: are there performance gains, for portfolio selection purposes, in the joint use of the three realized moments (variance, skewness and kurtosis)? This paper contributes empirically, to answering this question. Motivated by the work of Brandt, Santa-Clara, and Valkanov (2009), we consider a CRRA-utility framework to incorporate, not only the first two moments of the returns distribution, but also the skewness and kurtosis into the portfolio selection problem. The methodological design is the following: Firstly, for a given risk aversion level we build two utility-maximizing portfolios - one based on daily data (which we designate by low frequency portfolio) and the other based on intraday data (the high frequency portfolio). Then, we compare the out-of-sample performance of the low and high frequency portfolios for ten different risk aversion levels, using seven measures: the out-of-sample utility, mean, variance, skewness, kurtosis, Sharpe ratio and turnover. The analysis is conducted on a dataset of fourteen stocks from the CAC 40 stock market index for a five-year period (January 1999 to December 2003). These data were provided by the European Financial Institute (EUROFIDAI). The empirical evidence is very clear: the high frequency portfolios outperform the low frequency portfolios for every measure and for any risk aversion coefficient.

The remainder of the paper proceeds as follows. Section 2 formulates the basic investor's problem in portfolio selection and Section 3 develops that problem including higher moments. Section 4 explains the procedures for estimating higher moments using high frequency data. Section 5 presents an empirical application on fourteen stocks of the CAC 40 stock market index. Section 6 concludes the paper.

2. The investor's problem

Suppose that the investor has a certain wealth to invest in a set of N stocks. In this setting, a portfolio is defined by an $N \times 1$ vector, w, of weights representing the proportions of the total wealth invested into the N stocks. Let $E_t(R_{i,t+1})$, $i = 1, \dots, N$, denote the expected return of stock i at time $t+1$. The portfolio is assumed to be linear in $w_1, \dots, w_N$, and thus the portfolio expected return, at time $t+1$, is given by $E_t(R_{p,t+1}) = \sum_{i=1}^{N} w_i E_t(R_{i,t+1})$.

According to the utility maximization criteria and denoting the investor's utility by $U(\cdot)$, the investor's problem can be formulated as

$$
\begin{aligned}
&max_{w \in R^N} \quad E_t\left[U\left(R_{p,t+1}\right)\right] = E_t\left[U\left(\sum_{i=1}^{N} w_i R_{i,t+1}\right)\right] \\
&such\ that \quad \sum_{i=1}^{N} w_i = 1 \\
&\qquad\qquad\quad w_i \geq 0, \ \ i = 1, \dots, N
\end{aligned}
\tag{1}
$$

We have decided not to allow for short-selling, in Problem (1), since in real markets there are some practical and regulatory constraints on short trading positions (especially within the European Union). However, we must point out that allowing for short selling would not change the rationale of the model.

3. The investor's problem with higher moments

Supposing that the investor has CRRA preferences, her/his utility is given by

$$
U\left(R_{p,t+1}\right) = \begin{cases} \dfrac{\left(1 + R_{p,t+1}\right)^{1-\gamma} - 1}{1-\gamma} & if \quad \gamma > 1 \\ log\left(1 + R_{p,t+1}\right) & if \quad \gamma = 1 \end{cases}
\tag{2}
$$

where γ represents the relative risk aversion coefficient (a higher value of γ implies more risk aversion). A CRRA utility allows the incorporation of preferences toward higher moments in a parsimonious manner (Brandt, Santa-Clara, and Valkanov, 2009). Considering the fourth order Taylor expansion of the expected utility, $E_t\left[U\left(R_{p,t+1}\right)\right]$, around the expected return of the portfolio, $E_t\left(R_{p,t+1}\right)$, we have

$$
\begin{aligned}
E_t\left[U\left(R_{p,t+1}\right)\right] \quad &\approx \quad U\left[E_t\left(R_{p,t+1}\right)\right] + \frac{1}{2!}U''\left[E_t\left(R_{p,t+1}\right)\right]V_t\left(R_{p,t+1}\right) \\
&+ \quad \frac{1}{3!}U'''\left[E_t\left(R_{p,t+1}\right)\right]S_t\left(R_{p,t+1}\right) \\
&+ \quad \frac{1}{4!}U''''\left[E_t\left(R_{p,t+1}\right)\right]K_t\left(R_{p,t+1}\right)
\end{aligned} \tag{3}
$$

where $V_t\left(R_{p,t+1}\right) = E_t\left[R_{p,t+1} - E_t\left(R_{p,t+1}\right)\right]^2$, $S_t\left(R_{p,t+1}\right) = E_t\left[R_{p,t+1} - E_t\left(R_{p,t+1}\right)\right]^3$ and $K_t\left(R_{p,t+1}\right) = E_t\left[R_{p,t+1} - E_t\left(R_{p,t+1}\right)\right]^4$ denote the variance, the skewness and the kurtosis of the portfolio, respectively.

For notational simplicity, let us consider that $a = U\left[E_t\left(R_{p,t+1}\right)\right]$, $b = -U''\left[E_t\left(R_{p,t+1}\right)\right]/2$, $c = U'''\left[E_t\left(R_{p,t+1}\right)\right]/6$ and $d = -U''''\left[E_t\left(R_{p,t+1}\right)\right]/24$. Then the investor's problem with higher moments can be formulated as

$$
\begin{aligned}
&max_{w \in R^N} \quad a\left(R_{p,t+1}\right) - b\left(R_{p,t+1}\right)V_t\left(R_{p,t+1}\right) + c\left(R_{p,t+1}\right)S_t\left(R_{p,t+1}\right) - d\left(R_{p,t+1}\right)K_t\left(R_{p,t+1}\right) \\
&such\ that \quad \sum_{i=1}^{N} w_i = 1 \\
&\qquad\qquad\quad w_i \geq 0, \quad i = 1, \dots, N
\end{aligned}
$$

In this paper the solutions of Problem (4), where the moments and co-moments are estimated using daily returns, are called low frequency portfolios and are denoted by $w^{(low)}$.

4. The investor's problem with higher realized moments

Arguably the use of high frequency data reduces the estimation error of the parameters in the portfolio selection problem. Therefore, inspired by the works of Andersen et al. (2001), Neuberger (2012) and Amaya et al.

(2015), we use the realized variance, the realized skewness and the realized kurtosis of the portfolio as inputs in Problem (4).

Supposing that at day $t+1$ there are K intraday sampling periods, the realized variance of stock i (with $i=1,\dots,N$) is defined as

$$RV_{i,t+1}^{K}=\sum_{k=1}^{K}R_{i,t+(k/K)}^{2} \tag{5}$$

where $R_{i,t+(k/K)}$ represents the return of stock i in the intraday period $t+(k/K)$.

Analogously, the realized skewness, at day $t+1$, of stock i can be defined as

$$RS_{i,t+1}^{K}=\sum_{k=1}^{K}R_{i,t+(k/K)}^{3} \tag{6}$$

and the corresponding realized kurtosis is defined as

$$RK_{i,t+1}^{K}=\sum_{k=1}^{K}R_{i,t+(k/K)}^{4} \tag{7}$$

According to the previous definitions, the portfolio realized variance can be computed as

$$RV(w)=w^{T}\Sigma w \tag{8}$$

where Σ (of dimension $N \times N$) is the realized covariance matrix, where each entry, σ_{ij}, is given by

$$\sigma_{ij} = \sum_{i,j=1}^{N} \sum_{k=1}^{K} R_{i,t+(k/K)} R_{j,t+(k/K)} \tag{9}$$

Following Athayde and Flôres (2004), the daily portfolio realized skewness can be computed as

$$RS(w) = w^T \Phi(w \otimes w) \tag{10}$$

where $\otimes$ represents the Kronecker product and Φ (of dimension $N \times N^2$) is the realized coskewness matrix. The realized coskewness matrix can be seen as the composition of N matrixes $A_{ijl,t+1}$ each one with dimension $N \times N$ such that

$$\Phi = \begin{bmatrix} A_{1jl,t+1} & A_{2jl,t+1} & \cdots & A_{Njl,t+1} \end{bmatrix} \tag{11}$$

where each element, $a_{ijl,t+1}$, is given by

$$a_{ijl,t+1} = \sum_{i,j,l=1}^{N} \sum_{k=1}^{K} R_{i,t+(k/K)} R_{j,t+(k/K)} R_{l,t+(k/K)} \tag{12}$$

Similarly, the daily portfolio kurtosis is given by

$$RK(w) = w^T \Psi (w \otimes w \otimes w) \tag{13}$$

where Ψ (of dimension $N \times N^3$) is the realized cokurtosis matrix. The Ψ matrix corresponds to N^2 matrixes $B_{ijlm,t+1}$ of dimension $N \times N$ such that

$$\Psi = [B_{11lm,t+1} \quad B_{12lm,t+1} \quad \cdots \quad B_{1Nlm,t+1} \quad | \quad B_{21lm,t+1} \quad B_{22lm,t+1} \quad \cdots \quad B_{2Nlm,t+1} \quad | \quad \cdots \quad | \quad B_{N1lm,t+1} \quad B_{N2lm,t+1} \quad \cdots \quad B_{NNlm,t+1}] \tag{14}$$

where each element

$$b_{ijlm,t+1} = \sum_{i,j,l,m=1}^{N} \sum_{k=1}^{K} R_{i,t+(k/K)} R_{j,t+(k/K)} R_{l,t+(k/K)} R_{m,t+(k/K)} \tag{15}$$

Bearing in mind that, for estimating the daily mean return using intraday data, only the first and last price observations will matter, the investor's problem with higher realized moments can be posed as

$$\begin{aligned} & max_{w \in R^N} \quad a(w) - b(w)RV(w) + c(w)RS(w) - d(w)RK(w) \\ & such\ that \quad \sum_{i=1}^{N} w_i = 1 \\ & \qquad\qquad\quad w_i \geq 0, \ \ i = 1, \dots, N \end{aligned} \tag{16}$$

A solution of Problem (16) is referred as an high frequency portfolio and is denoted hereafter as $w^{(high)}$.

5. Empirical analysis

5.1. Data description

We compared the performance of the low frequency portfolio (solution of Problem (4)) with that of the high frequency portfolio (solution of Problem (16)) using a dataset from the CAC 40 Index (Euronext Paris). The dataset was provided by the European Financial Institute (EUROFIDAI) and corresponds to intraday price observations of fourteen stocks (see Table 1). These intraday data were gathered during each trading session (09:00 a.m.-17:30 p.m., local time), from January 1999 to December 2003 (1260 trading days). In the raw dataset, the intraday price observations were not synchronized. Such non-synchronization can lead to serious biases in the estimation of the moments and co-moments of stock returns (see Campbell, Lo, and MacKinlay, 1997, pp. 84-98, for further details). To synchronize the data, we used a well-known algorithm, the all refresh-time method (described in Barndorff-Nielsen et al., 2011).

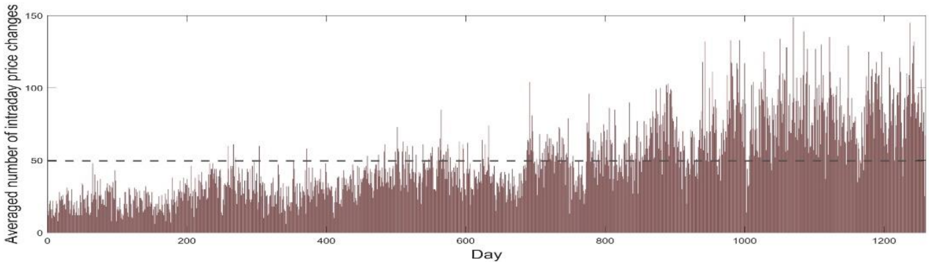
Table 1. The fourteen stocks from the France Stock Market Index (CAC 40)

Stock Designation	
AIR LIQUIDE	LVMH
AXA	MICHELIN
CARREFOUR	PERNOD RICARD
DANONE	SAINT-AVENTIS
ESSILOR INTL	SANOFI-AVENTIS
FRANCE TELECOM	TOTAL
L'OREAL	UNIBAL

This table lists the composition of the dataset used in the empirical analysis. The intraday data on these stocks, from January 1999 to December 2003, were provided by the European Financial Institute (EUROFIDAI).

After the synchronization procedure, there are on average about 50 prices changes per day (see Figure 1), which corresponds to an average trading frequency of 10-minutes. From Figure 1 it is visible an increasing trend in the trading frequency during the period under analysis.

Figure 1. Average number of intraday price changes per day



This figure reports the average number of intraday price changes (on the fourteen stocks) per day. The horizontal axis corresponds to the number of trading days. The dashed horizontal line represents the overall average number of price changes per day.

5.2. Out-of-sample performance

To compare the performance of the low frequency portfolio $\left(w^{(low)}\right)$ with that of the high frequency portfolio $\left(w^{(high)}\right)$, we used a rolling-sample approach for a total of 255 evaluation periods (days) (see, e.g., DeMiguel, Garlappi, and Uppal, 2009). Firstly, for each risk aversion level (with $\gamma = 1, \ldots, 10$), we computed the low frequency portfolio (solution of Problem (4)) and the high frequency portfolio (solution of Problem (16)), for the in-sample window, from the first trading day of January 1999 to the last trading day of December 2002. We held fixed each portfolio and observed their returns over the next trading day (first trading day of January 2003). Then we discarded the first trading day of January 1999 and included the first trading day of January 2003 into the sample. We repeated this process until exhausting the 255 trading days of 2003. With this procedure, we recorded the time series of daily returns for each of the 10 different $w^{(low)}$ portfolios and for the corresponding 10 $w^{(high)}$ portfolios.

From the recorded out-of-sample daily returns for each portfolio ($w^{(low)}$ and $w^{(high)}$) we computed the out-of-sample utility, $\widehat{U}$, given by

$$\hat{U} = \begin{cases} \dfrac{(1+\hat{\mu})^{1-\gamma} - 1}{1-\gamma} & if \quad \gamma > 1 \\ log(1+\hat{\mu}) & if \quad \gamma = 1 \end{cases} \tag{17}$$

where $\hat{\mu}$ represents the out-of-sample mean return. The results are reported in Table 2. We can observe that, for all the ten different risk aversion levels, the high frequency portfolio always outperforms the low frequency portfolio in terms of out-of-sample utility.

Table 2. Utility

Risk Aversion Level	Low Frequency Portfolio $\left(w^{(low)}\right)$	High Frequency Portfolio $\left(w^{(high)}\right)$
$\gamma = 1$	-13.0	-6.92
$\gamma = 2$	-1.39	4.87
$\gamma = 3$	1.17	9.36
$\gamma = 4$	2.38	12.4
$\gamma = 5$	7.64	17.5
$\gamma = 6$	12.5	21.0
$\gamma = 7$	16.0	22.9
$\gamma = 8$	18.4	24.9
$\gamma = 9$	20.6	25.8
$\gamma = 10$	22.2	26.8

This table reports the out-of-sample utility $\hat{U}$ of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample utility values are multiplied by a factor of 10^5.

The investor wants to achieve the portfolio with the highest mean and skewness and the lowest variance and kurtosis, therefore the superiority of the high frequency portfolios may be the result of its dominance in any of these dimensions. Strikingly, regardless of the risk aversion coefficient, the high frequency portfolio is able to outperform the low frequency portfolio in terms of out-of-sample mean (see Table 3), out-of-sample variance (see Table 4), out-of-sample skewness (see Table 5) and out-of-sample kurtosis (see Table 6).

Table 3. Mean

Risk Aversion Level	Low Frequency Portfolio $\left(w^{(low)}\right)$	High Frequency Portfolio $\left(w^{(high)}\right)$
$\gamma = 1$	-13.0	-6.92
$\gamma = 2$	-1.39	4.87
$\gamma = 3$	1.17	9.36
$\gamma = 4$	2.38	12.4
$\gamma = 5$	7.64	17.5
$\gamma = 6$	12.5	21.0
$\gamma = 7$	16.0	22.9
$\gamma = 8$	18.4	24.9
$\gamma = 9$	20.6	25.8
$\gamma = 10$	22.2	26.8

This table reports the out-of-sample mean of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample mean values are multiplied by a factor of 10^5.

Table 4. Variance

Risk Aversion Level	Low Frequency Portfolio $\left(w^{(low)}\right)$	High Frequency Portfolio $\left(w^{(high)}\right)$
$\gamma = 1$	22.2	19.4
$\gamma = 2$	19.2	17.8
$\gamma = 3$	18.2	17.2
$\gamma = 4$	17.4	16.7
$\gamma = 5$	16.6	15.7
$\gamma = 6$	16.2	15.0
$\gamma = 7$	15.9	14.5
$\gamma = 8$	15.7	14.0
$\gamma = 9$	15.5	13.6
$\gamma = 10$	15.4	13.3

This table reports the out-of-sample variance of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample variance values are multiplied by a factor of 10^5.

Table 5. Skewness

Risk Aversion Level	Low Frequency Portfolio $\left(w^{(low)}\right)$	High Frequency Portfolio $\left(w^{(high)}\right)$
$\gamma = 1$	-50.3	-18.5
$\gamma = 2$	-24.1	-8.89
$\gamma = 3$	-18.0	-6.49
$\gamma = 4$	-13.5	-5.63
$\gamma = 5$	-11.0	-5.05
$\gamma = 6$	-9.71	-4.75
$\gamma = 7$	-8.92	-4.45
$\gamma = 8$	-8.32	-4.02
$\gamma = 9$	-7.73	-3.72
$\gamma = 10$	-7.33	-3.51

This table reports the out-of-sample utility skewness of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample skewness values are multiplied by a factor of 10^7.

Table 6. Kurtosis

Risk Aversion Level	Low Frequency Portfolio $\left(w^{(low)}\right)$	High Frequency Portfolio $\left(w^{(high)}\right)$
$\gamma = 1$	77.7	30.0
$\gamma = 2$	34.1	15.9
$\gamma = 3$	24.5	12.3
$\gamma = 4$	17.9	10.5
$\gamma = 5$	14.1	8.93
$\gamma = 6$	12.3	8.03
$\gamma = 7$	11.3	7.40
$\gamma = 8$	10.5	6.85
$\gamma = 9$	9.95	6.45
$\gamma = 10$	9.56	6.16

This table reports the out-of-sample kurtosis of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample kurtosis values are multiplied by a factor of 10^8.

These results present quite strong evidence in the sense that for any possible out-of-sample performance measure, involving any of the four moments (mean, variance, skewness and kurtosis), the high frequency portfolio will always exhibit a better performance than the low frequency portfolio. For instance, we may consider the out-of-sample Sharpe ratio,

$$\hat{S} = \frac{\hat{\mu}}{\hat{\sigma}} \tag{18}$$

where $\hat{\sigma}$ represents the out-of-sample standard deviation. Note that when the numerator (the out-of-sample mean) of $\hat{S}$ is negative, the ratio should be refined to achieve a correct rank of the portfolios. The most widely used methodology to refine the Sharpe ratio is presented by Israelsen (2005):

$$\hat{S}_{ref} = \frac{\hat{\mu}}{\hat{\sigma}^{\hat{\mu}/abs(\hat{\mu})}} \tag{19}$$

where $abs(\cdot)$ is the absolute value function. The $\hat{S}_{ref}$ is equal to $\hat{S}$ when the out-of-sample mean is non-negative. Table 7 presents the results for the refined Sharpe ratio. The results show that the low frequency portfolio always underperforms the high frequency portfolio, for any of the considered risk aversion levels.

Table 7. Refined Sharpe ratio

Risk Aversion Level	Low Frequency Portfolio $(w^{(low)})$	High Frequency Portfolio $(w^{(high)})$
$\gamma = 1$	-0.002	-0.001
$\gamma = 2$	-0.000	3.650
$\gamma = 3$	0.866	7.145
$\gamma = 4$	1.799	9.621
$\gamma = 5$	5.925	13.97
$\gamma = 6$	9.826	17.12
$\gamma = 7$	12.71	19.03
$\gamma = 8$	14.68	21.03
$\gamma = 9$	16.55	22.17
$\gamma = 10$	17.94	23.27

This table reports the out-of-sample refined Sharpe ratios ($\hat{S}_{ref}$) of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the out-of-sample refined Sharpe ratios values are multiplied by a factor of 10^3.

Finally, we also compare the portfolios' turnover. The turnover may be seen as a metric for the trading costs implied by the investment strategies, and is here defined as the average, over all time periods, of the absolute changes in weights across the N available stocks:

$$turnover = \frac{1}{\#periods} \sum_{t=1}^{\#periods} \sum_{i=1}^{N} \left(|w_{i,t+1} - w_{i,t}^{h}| \right) \tag{20}$$

where $w_{i,t}^{h} = w_{i,t-1} \frac{1+R_{i,t}}{1+R_{p,t}}$. The results are reported in Table 8.

Table 8. Turnover

Risk Aversion Level	Low Frequency Portfolio $(w^{(low)})$	High Frequency Portfolio $(w^{(high)})$
$\gamma = 1$	95.2	79.7
$\gamma = 2$	64.0	57.5
$\gamma = 3$	59.3	50.8
$\gamma = 4$	55.0	44.6
$\gamma = 5$	48.3	38.2
$\gamma = 6$	42.7	33.5
$\gamma = 7$	37.9	31.9
$\gamma = 8$	34.8	30.0
$\gamma = 9$	31.9	27.7
$\gamma = 10$	29.1	25.4

This table reports the turnover of each low frequency portfolio ($w^{(low)}$) and high frequency portfolio ($w^{(high)}$) for ten different risk aversion levels. All the turnover values are multiplied by a factor of 10^3.

The same pattern, presented in the previous out-of-sample performance evaluation measures, was found here, i.e., for the ten different relative risk aversion levels the high frequency portfolios outperform the low frequency portfolios, meaning that the high frequency portfolios provide a saving in proportional trading costs, implying that the superiority of these portfolios increase after considering trading costs.

We also highlight that, for all the performance evaluation measures, a surprising pattern was found: the out-of-sample performances, both for the low and high frequency portfolios are increasing functions of the risk aversion level (γ). A possible explanation for this puzzling pattern can lies on the fact that with the increase of the risk aversion level, the constructed portfolios become closer to the minimum variance portfolio, which tends to exhibit a superior out-of-sample performance (see, e.g., Jagannathan and Ma 2003 and DeMiguel, Garlappi, and Uppal 2009).

6. Conclusions

Nowadays the use of big datasets seems to offer a competitive advantage in many fields. Particularly in Finance, the increasing availability of high frequency data encourages the emergence of new investment strategies built on all that information.

In this paper we have analysed the practical benefits of using intraday information in portfolio selection. We have considered a general framework where the investor wants to maximize her/his CRRA utility. The expected utility was modelled using not only the two first moments of the returns distribution but also higher moments, namely the skewness and the kurtosis. Within this framework, for a given risk aversion level, we have constructed two portfolios: a low frequency portfolio, solution of the portfolio choice problem where the inputs are obtained from daily data, and a high frequency portfolio, solution of the portfolio choice problem where the inputs are obtained from intraday data.

The empirical results, based on fourteen stocks from the CAC 40 Index, showed a superior daily out-of-sample performance of the high frequency portfolio over the low frequency portfolio. For ten different risk aversion levels, each high frequency portfolio outperformed the corresponding low frequency portfolio in terms of several out-of-sample measures (utility, mean, variance, skewness, kurtosis, Sharpe ratio and turnover). This empirical evidence suggests the existence of practical real gains in using high frequency data for portfolio selection. This is in accordance with one elementary principle in statistics: *ceteris paribus*, more data is desirable to less.

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