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Leonardo's Vitruvian Man Drawing: A New Interpretation Looking at Leonardo's Geometric Constructions

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Abstract Generally speaking, today's scientific community considers that the famous figure drawn by Leonardo da Vinci at the end of the fifteenth century was made using the Golden ratio. More specifically, the relationship established between the circle's diameter and the side of the square is a consequence of the geometric relationship probably discovered by Euclid, but made famous by Luca Pacioli in his *De Divina proportione*. Aware of the close working relationship between Leonardo and Pacioli, namely in the writing of this last book, the theory that establishes a close relationship between these two figures, making use of this remarkable mathematical relationship, has gained credibility. In fact, the use of the *Divina proporzione*, despite being a very stimulating construction on an intellectual level, presents too great a margin of error, especially for such a competent geometrician as Leonardo da Vinci was. For that reason, the relationship between these two figures (square and circle) is grounded on a much simpler geometric relationship than the one found at the base of the definition, for instance, of Le Corbusier's *Modulor*.

Keywords Leonardo da Vinci · Vitruvian man · Vesica piscis · Golden number · Proportion

Introduction

> *No human investigation may claim to be a true science if it has not passed through mathematical demonstrations, and if you say that the sciences that begin and end in the mind exhibit truth, this cannot be allowed, but must be*

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> *denied for many reasons, above all because such mental discourses do not involve experience, without which nothing can be achieved with certainty.*
> (Leonardo 1989, p. 14)

The Vitruvian man is, undoubtedly, Leonardo da Vinci's most famous and widely reproduced folio (Fig. 1). This representation, which objectively reflects the human body's proportional basis, is historically associated with the Roman architect Marcus Vitruvius Pollio, who explained the principle in his book *De Architectura* at the beginning of the first century AD. If this drawing objectively represents the Leonardian reflection on the canon that apparently regulates the different parts of the human body, in reality this figuration has been recurrently used to illustrate the Renaissance idea of man as a symbolic microcosm, thus praising his role as the centre of the universe.

It is certain that this fascination with the human body is transversal throughout history, and that it probably had its beginning when, in the caves of Pech Merle (about 10,000 BC) in France, man indelibly left the mark of his footstep and

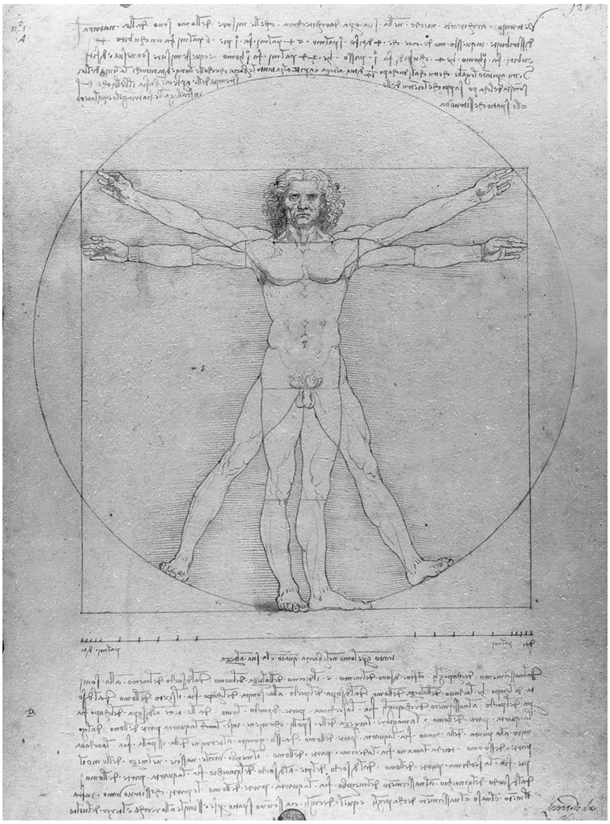


Fig. 1 Leonardo da Vinci, study of human proportions in the manner of Vitruvius (c. 1490), Gallerie dell'Accademia, Gabinetto dei Disegno e Stampe, n. 228, Venice

fingerprint. Unwittingly or in a conscious manner, he left a record and gaugeable metric base against which to determine, exactly, some of the dimensions of men from that pre-historic age (Fig. 2). There was at the time no idealized or stable system regarding the dimensioning of things in their relation with humans, but this concern can be found a few centuries later in the unfolding of the Egyptian civilization, with the adoption of metrological systems (Badawy 1965, pp. 36–38) as well as graphic and spatial modulations in the definition of art in general and architecture in particular.[1] During the Greek period we can already find a stable use of measurement units taken from human body parts, thus shedding light on Protagoras' precept that man is the measure of all things. In parallel, and in the field with more of an artistic calling, Polykleitos develops a written canon that allegedly demonstrates the harmonious principle between the parts as well as between them and the whole of the human body,[2] and he would eventually materialize this rule in the celebrated statue named Doryphoros (Fig. 3).[3] For this reason, one is not surprised by the confident and very concrete manner in which Vitruvius defines, through his *homo bene figuratus*, a rigorous proportional base for the human figure. In particular, he defines the metrical foundations from the finger, palm, foot, and cubit and characterizes the relationship between these different parts.[4] Zealously, at the base of his Vitruvian man drawing and certainly bearing the Roman architect's text in mind, Leonardo graphically materializes a scale that allows the foot and palm measurements to be identified, on both sides of the representation, and they somehow establish a graduated system against which to measure the real man and his presented figuration. This probably becomes one of the first presentations where

[1] Among the extensive bibliography on Egyptian art about modular diagrams and Egyptian dimensions cf., for instance, Ruiz de la Rosa (1987, pp. 60–74) or Rossi (2004, pp. 2–56); a classic source on the relationship between architecture and human anatomy is always Schaller de Lubicz (1949).

[2] Given that the book is missing, the descriptions that come closer to what Polykleitos' *canon* would be, with eminently anthropomorphic characteristics of comparison between parts of the human body, are the ones referred to by Galen in Placitis Hippocratis et Platonis. Galen's most important quote is mentioned by Torrini (2009, p. 36).

[3] For a more encompassing approach on the relation between the human body and the metric systems cf. Murtinho (2006, pp. 180–191). In the specific case of the proportional base at the origin of the Doryphoros statue cf., for instance, the analysis conducted by Pierre Gros on the copy existing in Naples which corresponds to the most widely celebrated reproduction using Polykleitos' *canon*. This analysis intends to confirm the principle that the various parts of the human body, starting with the smallest, are obtained, according to Polykleitos, from the relation of dynamic squares whose common element is the fact that the side of one corresponds to the diagonal value of another, and so on. Architects often used this proportional relation, historically very common, in different historic periods. One example of its application can be found in Villard de Honnecourt's notorious medieval album, the cloister design method, where the relation between covered and uncovered parts is regulated by the proportion founded on the square root of two principle, which technically corresponds to the *ad quadratum* construction. On Polykleitos cf. Gros (2006, pp. 81–83); on Villard and the *ad quadratum* construction cf. the first article cited in this note.

[4] "And further, as the foot is one sixth of man's height, the height of the body as expressed in number of feet being limited to six, they held that this was the perfect number, and observed that the cubit consisted of six palms or twenty-four fingers. (…) They found their authority for this in the foot. For if we take two palms from the cubit, there remains the foot of four palms; but the palm contains four fingers." (Vitruvius 1914, p. 74). For a better understanding of the measures according to Vitruvius, we recommend reading Cunha (2003, pp. 22–24).

Fig. 2 Prehistoric painting of human hand, Pech Merle cave, France

a scale can be determined from the drawing between measure on paper and real-life size.

In fact, the original Vitruvian description of human proportions appears at the beginning of Book III in relation to symmetry regarding the composition (design) of temples (Vitruvius 1914, p. 72). Even though in detail Leonardo and Vitruvius' execution differ in some aspects, both converge on the fact that the navel is the centre of a circle that inscribes the whole body when the man has his arms and legs outstretched.[5] In Leonardo's folio about the human body proportions he writes: "If you open your legs so much as to decrease your height 1/14 and spread and raise your arms till your middle fingers touch the level of the top of your head you must know that the centre of the outspread limbs will be in the navel and the space between the legs will be an equilateral triangle. The length of a man's outspread arms is equal to his height" (Leonardo 1970 (1), p. 182) This construction that establishes the navel, in a given human position, as a convergence point represents a commonplace in the classical period, as referenced, in a different context and about a century earlier, by Varro in his *De Origine Linguae Latinae libri III*.[6] Similarities between the Vitruvian and Leonardian figure can be found, also, in the fact that the

[5] For instance in the treatise of Vitruvius he wrote: "Then again, in the human body the central point is naturally the navel. For if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centred at his navel, the fingers and toes of his two hands and feet will touch the circumference of a circle described therefrom. And just as the human body yields a circular outline, so too a square figure may be found from it. For if we measure the distance from the soles of the feet to the top of the head, and then apply that measure to the outstretched arms, the breadth will be found to the same as the height, as in the case of plane surfaces which are perfectly square" (Vitruvius 1914, pp. 72–73). In Leonardo's folio about the human body proportions he writes: "If you open your legs so much as to decrease your height 1/14 and spread and raise your arms till your middle fingers touch the level of the top of your head you must know that the centre of the outspread limbs will be in the navel and the space between the legs will be an equilateral triangle. The length of a man's outspread arms is equal to his height." (Leonardo 1970 (1), p. 182)

[6] "The *umbilicus*, they say, was so called from our *umbilicus* navel, because this is the middle place of the lands, as the navel in us" (Varro 1938, p. 285).

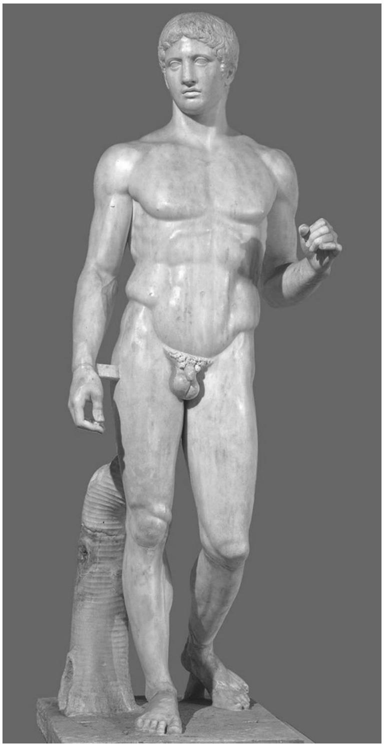


Fig. 3 Doryphoros, Roman marble copy after a Greek original, Museo Archeologico Nazionale di Napoli, Naples

face, from chin to top of the forehead, as well as the length of the hand, measured from the wrist, correspond to a tenth of the total height. Regarding Leonardo's subdivision of the square into ten equal parts, it is given particular significance since this module is openly formalized by a ruler at the bottom of his drawing. As for the distance from the top of the shoulders to the top of the forehead, it corresponds to one-eighth of the human height. For Vitruvius, in his turn, the foot corresponds to one-sixth of the total height, the chest to a quarter of the same general measurement and the top of the sex organ to half that value. In the Leonardian figure, only the first of these measurements differs from the Vitruvian reference, in that the foot corresponds to one-seventh of the human height. For both authors, the extremities of the man's body, when placed in the crucified Christ position, define a square whose side corresponds to one-eighth of the human head.

However, nowhere in the Vitruvian treaty is there a clarification of the proportional system that establishes the relational factor between the square and the circle. This situation has led to immense geometric and symbolic speculation in terms of the search for and definition of the rules that will have guided Leonardo to the drawing of his Vitruvian man.

In the development of the Vitruvian text, a few numbers are considered capable of conveying exquisiteness. First with that status is the number ten, called the perfect number by the ancients and *mónades* by the Greeks, corresponding to the number of fingers or toes, and which is at the base of the entire archaic metric system. In fact, the number ten can be obtained from units, namely being the sequence sum of one, two, three and four as represented by the tetractys. The first is a simple unit and the other three amounts to compound or aggregate units. Another number that presents some distinction, particularly for mathematicians, is, according to Vitruvius, the number six, which results from the sum of the first three numerals, i.e., one, two and three. This perfection attributed to the numbers six and ten has led some authors to conclude that, in the Roman architect's definition of the proportional system, he had considered that in the sectioning of the square where the human figure is inscribed, when one divided into sixteen parts the part corresponding to ten modules, that would coincide with the navel and thus give us the radius for the circle where the human figure is inscribed when presented with feet and hands outstretched in the form of an X.[7]

Vitruvian Man Constructions

The original measures commonly defined for Leonardo da Vinci's Vitruvian man drawing are one hundred and eighty-one millimetres and a half (181.5 mm) for the length of the square and one hundred and ten millimetres (110 mm) for the circumference's radius (Fig. 4). However, some imprecisions in terms of the representation of these figures are noticeable: the square's vertical sides are slightly skewed, by about 1 mm, and the circumference is built from successive arcs which consequently correspond to slightly different centres.[8] In this context, had Leonardo's understanding of man's proportions been in keeping with those attributed to Vitruvius, we would have for the same square, a circumference diameter of about two hundred and twenty-six point eight millimetres (226.8 mm), which represents a considerable deviation (Fig. 5). As a curiosity, if this were the proportional relation and the circumference touched the midpoint of the square's lower side, it would be practically tangent to the drawing's top vertices.

As a matter of fact, in the absence of sound evidence and having no way of using Vitruvius' text to accurately establish a credible relation between the square and the circle in Leonardo's drawing, it seems reasonable that efforts made to understand that connection start from other drawings by Leonardo himself, or with the attempt to ascertain the veracity and rigour of interpretations made by other authors. One of these analyses is the construction developed by the philosopher Otto Helbing, in which he argues that the relationship between the afore-mentioned figures is rendered harmonious through the use of a heptagon (Helbing 2005). In this situation, by inscribing a heptagon in the circumference and placing its inferior side horizontally, the figure's two (non-consecutive) vertices located immediately below the top vertex (A and B) define a horizontal line that, according to Helbing,

[7] Cf., for instance, the correspondence induced by Valli (2008, p. 179).

[8] The folio with Leonardo's drawing is 345 × 246 mm, which corresponds to a root 2 proportion, and the dimensions extracted from the original are 110 mm for the circumference's radius and 181.5 mm for the side of the square (cf. Valli 2011, pp. 71–73).

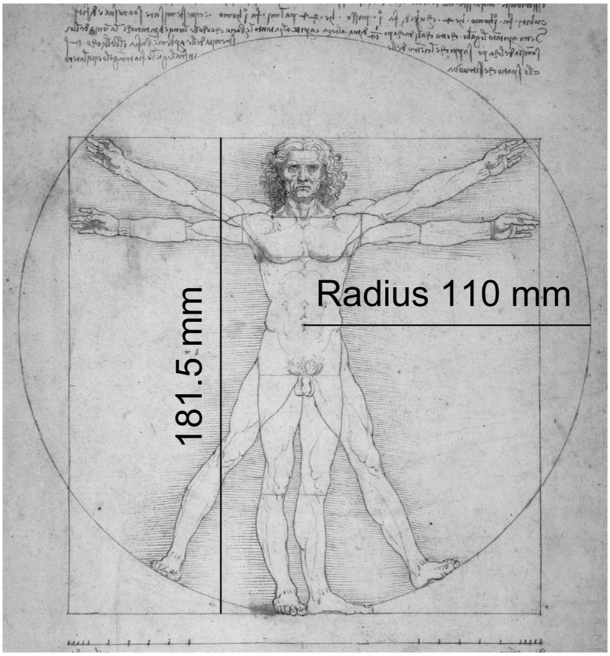
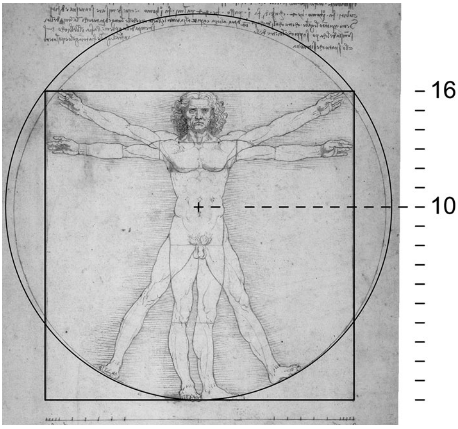


Fig. 4 Leonardo's drawing dimensions of *square* side (181.5 mm) and circumference radius (110 mm)

Fig. 5 Relation between Leonardo's *original circle* and *circle* with 10/16 radius of the side of the *square*



coincides with the top side of the reference square (Fig. 6). According to Helbing's construction, admitting it was made from the circumference with a diameter of two hundred and twenty millimetres (220 mm), the noticeable points that would yield the dimensions for the side of the square would only allow its length to be one hundred and seventy-eight millimetres and a half (178.5 mm). If one considers the square to be stable, in order to fulfil the condition that two of the heptagon's vertices

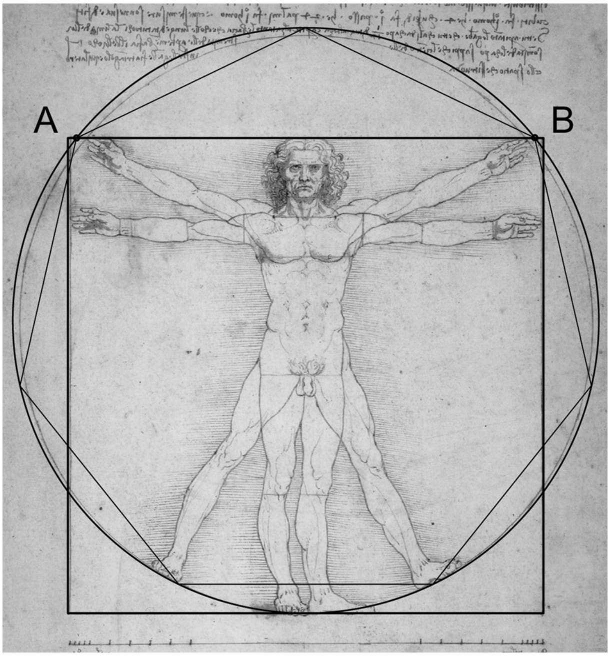


Fig. 6 Relation between Leonardo's drawing and Helbing's construction

are over the square's upper side, the circumference's diameter should be two hundred and twenty-three point six millimetres (223.6 mm).

One of the most enticing constructions for scholars of Leonardo's Vitruvian man drawing has undoubtedly been the one which presupposes that the relation between the square and the circle is established by the use of the commonly named Golden ratio, and which corresponds to proposition thirty in Book VI of Euclid's *Elements*.[9] However, in the event of the Golden ratio having had any influence over Leonardo that would certainly be owed to Friar Luca Pacioli. Indeed, Pacioli would come to name this peculiar proportion *De Divina Proportione* and it would be the pretext behind the formulation of an in-depth text on the topic that, having reportedly been completed in Milan in December 1498, would only have its first edition printed in Florence in 1509. There remains no doubt concerning the influence and friendship between Leonardo and Pacioli, nor regarding the fact that they both will have discussed the book's contents, since Leonardo himself drew about sixty illustrations of regular geometric solids that supplement this work. It seems that the meeting between these two remarkable Renaissance personalities took place under the auspices of Ludovico Maria Sforza, *dito il Moro* in Milan in the year 1496 (Torrini 2009, pp. 23–24).[10]

[9] "To cut a given finite straight line in extreme and mean ratio" (Euclid 1956, p. 267).

[10] See Pacioli (1978, pp. 116–117). On the reciprocal influence between Leonardo and Pacioli, as well as their peculiar relation in Milan, cf. González (1991, pp. 10–18).

The particularity described above is of the utmost importance, for the Leonardian drawing of the *homo ad quadratum* and *homo ad circulum* was reportedly made around 1490.[11] This means that the most reliable research places the production of the Vitruvian man in a period before Leonardo's contact with Pacioli's book and even before they had had more direct and regular contact.[12]

The Golden ratio has the peculiarity of establishing an important geometric principle, for if we define a rectangle by this proportion, and from that figure remove a square whose side length is equal to the figure's smaller side, we will still obtain a new Golden rectangle. Now, it is precisely this particularity that turns this proportion into a success factor in terms of geometric drawings.

Hence the fact that, historically, when dealing with an interesting geometric relationship, it is an element that one recurrently tries to find in countless different situations, always working as an element of valorisation and intellectuality. Moreover, Agrippa also executed, in his *Occult Philosophy*, cabbalistic drawings that place the human figure within square and circles, and one of the representations in particular was made in such a way that a regular pentagon whose vertices correspond to the five human extremities is superimposed on the figure (Fig. 7) (Agrippa 2003, pp. 345–352).[13] And it is known that this figure produces a configuration of five diagonals such that each divides in Golden section.

This interest in the divine proportion becomes even more evident when precisely one of the fundamental architecture theorists in the twentieth century methodologically proposes the use of a proportional system grounded on man and having the Fibonacci series for a numeric base, but which is illustrated using the Golden ratio's geometric construction. In its turn, the Corbusian *Modulor* system (Le Corbusier 1983) was built under the influence of Adolph Zeising, who in 1854 had published the book *Neue Lehre von den Proportionen des menschlichen Körpers* and who would somehow influence for posterity the supremacy of the Golden number.[14] An example of this influence is the fact that the most widely used book in the architectural community, the *Neufert Architects' Data*, has in its beginning an illustration of the human dimensions, grounded on this German mathematician's study and which obviously establishes a principle that the main measures of the human body are marked out by the divine proportion (Fig. 8). In parallel, many of the analyses conducted on the human canon described by Albrecht Dürer incline to

[11] The drawing's first dating is owed to Pedretti (1978, p. 159), who places it in the year 1490. Rocco Sinisgalli too, for instance, considers the same date to be plausible: "The Vitruvian man of Leonardo dates 1490 circa and measures 344 × 245 mmm; the author was inspired by Vitruvius in the relationship between proportions, as he himself reports." (Sinisgalli 2010, p. 5).

[12] In the *Divina Proporzione*, Pacioli said explicitly: "Comme apien in le dispozioni de tutti corpi regulari e dependenti di sopra in questo vedete, quali sonno stati fatti dal degnissimo pittore, prospettivo, architetto, musico e de tutte virtù dotato Lionardo da Vinci florentino nella cità de Milano, quando a li stipendii dello eccellentissimo duca di quello, Ludovico Maria Sforza Anglo, ci trovavamo nelli anni de mostra salute 1496, final'99; donde poi da siemi per diversi sucessi in quelle parti ci partemmo e a Firenze, pur insiemi, traemmo domicilio et cetera." (Pacioli 1978, p. 117).

[13] For a good analysis of the symbolic role of this image and about the consonance of human body parts with the microcosm see Heninger (2004, pp. 144–158).

[14] On this author cf. Herz-Fischler (2004, pp. 45–82).

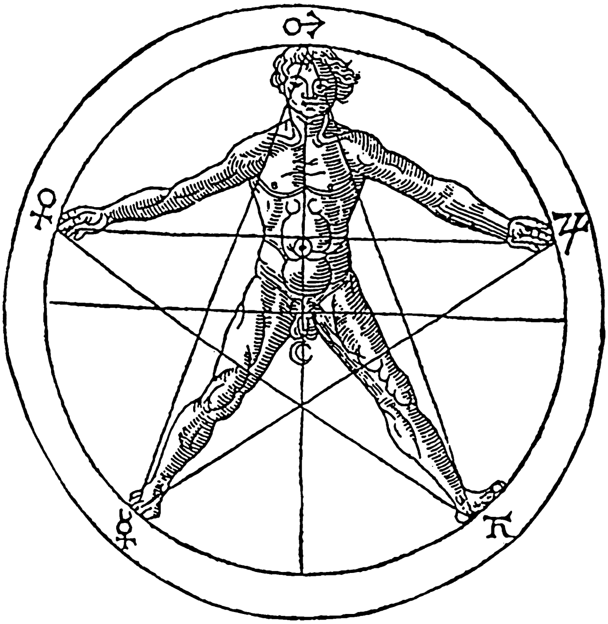


Fig. 7 Man inscribed in a *circle* according to Agrippa, which is drawn from the centre of the groin, with extremities into *five equal parts*, constituted a perfect *pentagon* (1533)

situations where some of the proportions are established according to the Golden number (Lawlor 2002, p. 86).

Indeed, the use of the geometrical procedure as a conceptual instrument for form is an effect that has been alluring both from the architects' and artists' point of view, and from that of individuals who mean to analyse and study works of art. It is in this context and from this perspective that we understand both the work developed by Le Corbusier and the analysis conducted by Charles Bouleau to cite only two examples, one in each direction.[15] Whereas the first one refers to the definition of a methodology and process aimed at the production of works of art, the second one corresponds to a more speculative process. In fact, if there is no evidence, any proposal concerning the process that led to the definition of the final form will always result in a speculative act. So, in any subsequent process of analysis, be it about method or process, one must be aware of its inherent limitations and the relativity of conclusions reached. This analysis can and should however be substantiated on other evidence that may aid in defining a given conclusion.

The generalized recognition of the Golden number's importance due to its geometrical particularities and the assumption that Leonardo knew and mastered this harmonious proportion, has practically turned the possibility that he made use

[15] Regarding Charles Bouleau, he proposes the specific analysis of numerous works of art using the superposition of complex geometric layouts which we believe in some cases to be more forceful than what was truly at its genesis and in the mind of the artist when he created it (on this matter, cf. Bouleau 1963).

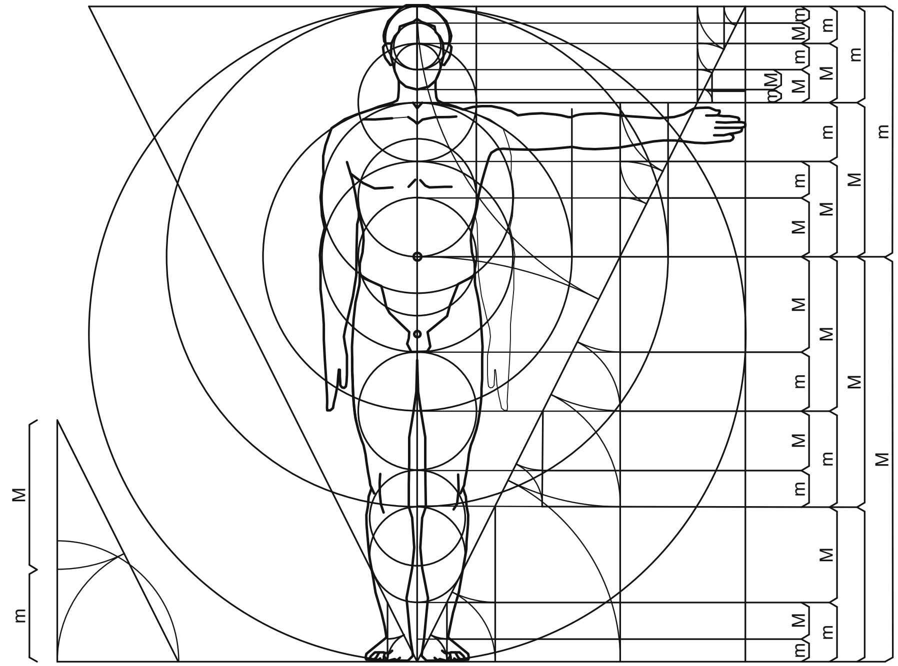


Fig. 8 Neufert's human proportion according to Zeising with geometrical division of length employing the Golden section

of this construction to define the relation between his *homo ad quadratum* and *homo ad circulum* into a credible fact.[16] Even despite the fact that, in the geometrical application of this relationship, the construction presents some degree of error, perhaps too great a one, this drawing has gradually been assumed as something believable and correct. And the fact that Leonardo would not have proposed a design that presented such gross errors as the one seen here, given his training as a rigorous geometer, has systematically been ignored.

In fact, if one starts building the circumference (centre C) from the square in Leonardo's Vitruvian man, we will have a diameter of two hundred and twenty-four point five millimetres (224.5 mm). This means the construction would present an error of over 2 % in relation to the original circumference. And when this mistake is represented on paper, one can see it is too great a discrepancy, which leads us to conclude that, alluring as this hypothesis is, it could not have been the construction used by Leonardo to define the relationship between these two figures, the square and the circumference (Fig. 9).

More plausible given the approximation between the two figures is the interpretation proposed by Lionel March (1998, p. 108). Aware of the inconsistencies that the construction based on the Golden number presents, March believes

[16] One of the authors who has more widely spread this idea is Rocco Sinisgalli (cf. for instance Sinisgalli 2010).

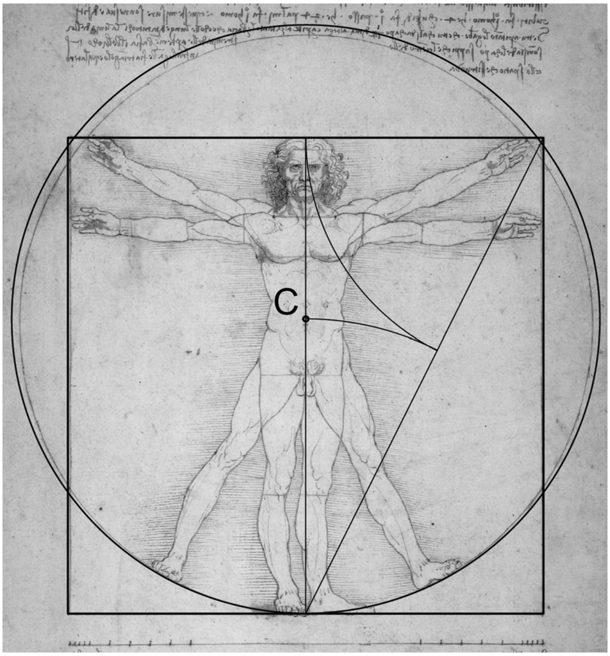


Fig. 9 Leonardo's Vitruvian man and divine proportion

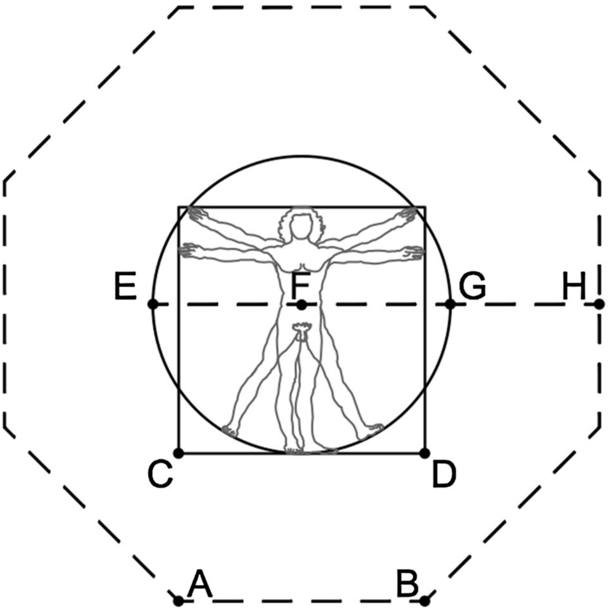


Fig. 10 Alternative construction of Leonardo's man according to Lionel March

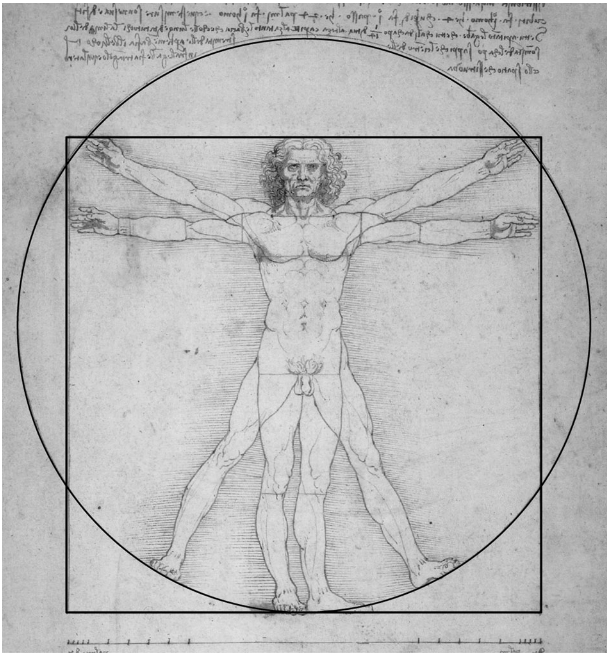


Fig. 11 Leonardo's Vitruvian man and Lionel March construction

that if one considers a regular octagon, its side length (AB) equal to the side of the square (CD), the distance between the centre of the figure and one of the sides' midpoint (FH) corresponds to the diameter of the circumference in Leonardo's drawing (EG). Starting from the drawing's side length, this construction would give us a circumference with a diameter of two hundred and nineteen point one millimetres (219.1 mm). In face of all the constructions already described, this is the one which presents the smallest discrepancy (Figs. 10, 11). However, this figure and the way it is formalised comes nowhere close to a possible methodology adopted by Leonardo,[17] even if, as Mark Reynolds has already shown, he has used the figure recurrently (Reynolds 2008, pp. 51–76). In our understanding, the degree of proximity between March's proposal and Leonardo's drawing is a happy coincidence.

That being said, the Vitruvian man's representation should be analysed according to two aspects. First, one that will allow us to discern the evidence in the drawing itself and see in what way we can objectively arrive at the existing outline. Second, one which must obviously start from the plenitude of different geometric constructions Leonardo carried out in his various folios and the way these or their analysis can provide insight into the decoding of the Vitruvian man.

[17] There is no great sense to Leonardo's use of a much higher figure (in this case the octagon), to obtain a dimension whose determination is not straightforward or direct. Normally the geometric shapes that Leonardo uses have direct visibility, or are apparent, in his drawings.

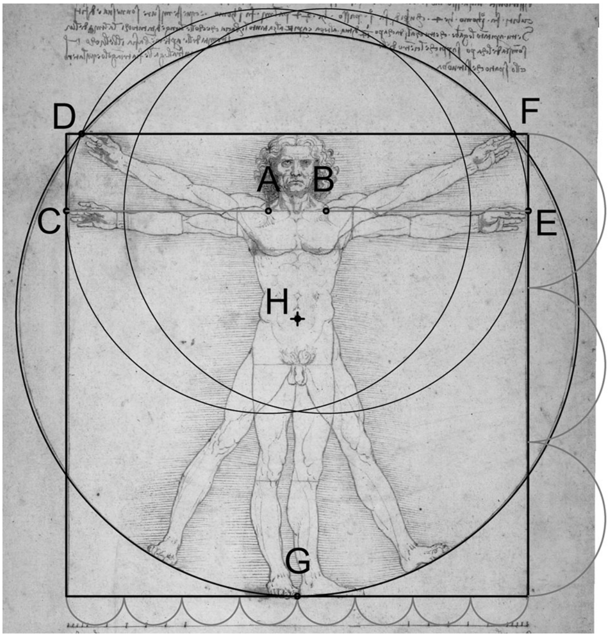


Fig. 12 *Square* and *circle* construction using the arms dimension and Leonardo's man drawing

If we objectively analyse Leonardo's drawing, we can see that, graphically, the contact points between the two figures—the square and the circumference—may be elements to take into consideration. Combining this evidence with the two positions of the human figure, one can easily draw a few conclusions. The most important one is, namely, that the transformation of the square into circumference takes place through the shifting of the position of the arms from horizontal to oblique (position CA to DA and position BE to BF) (Schröer 2007, pp. 104–112) (Fig. 12). And in the adjustment of those positions an individual centre is defined for each rotation (point A and point B). The centre of each rotation is on the fifth horizontal line that subdivides the square into six equal parts (counted from below) and it roughly coincides with the human figure's arm line. The exact point for each of these centres (point A and point B) is placed near the shoulders and the distance between them is three palms (AB), with half this distance for each side of the human figure's vertical axis. Having rotated the distance between the previously defined centre and bringing its radius up to the closest vertical side of the square, in the intersection with the same figure's horizontal side we can find the point of concurrence that belongs to the Vitruvian man's circumference. After that, if we consider the symmetrical points obtained on the upper side of the square (D and F) and the midpoint of the figure's lower side (point G), we have found three points belonging to the reference circumference which enables this form's final construction and thus geometrically to obtain the circumference's centre (H). By executing this construction over the Leonardian square of one hundred and eighty-one point five millimetres

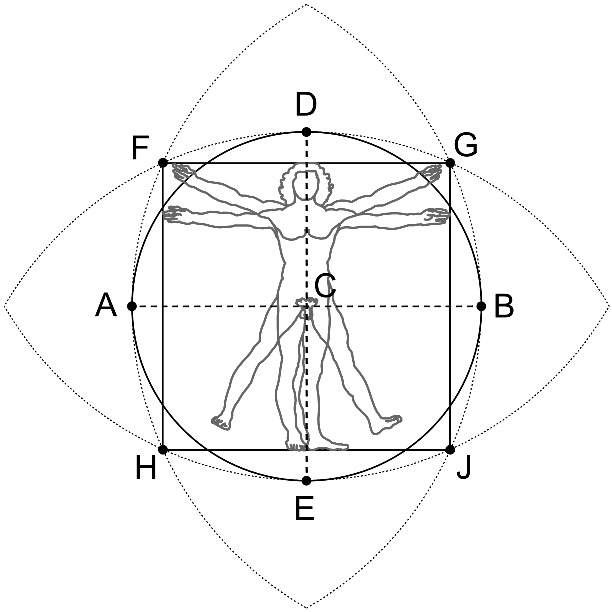


Fig. 13 Double vesica piscis construction. With *square* and *circle*

(181.5 mm), we will obtain a circumference diameter of two hundred and twenty-one point one millimetres (221.1 mm). This hypothesis, geometrically more plausible in terms of the assessment made directly over the drawing, raises the issue that there is an objective discrepancy between both rotation centres for the drawing of the arms and the actual point where the shoulders fit, between the humerus and the shoulder blade. And this is no negligible aspect, for Leonardo was an expert on human anatomy and would not have disregarded it.

Therefore, this centre probably corresponds to a technical point which aids and favours the movement of the human figure's arms, but it allows us to conclude that this relationship between square and circumference is related to another geometrical construction. From that perspective, and following a serious analysis of the geometric outlines carried out by Leonardo, a much generalized tendency stands out to explore drawings with one or several axes of symmetry and the particularity of having one centralized aspect. In parallel, and intending for the construction between the two Leonardian figures not to be accidental but possessing some intrinsic value, it becomes reasonable to look at the philosophical meaning that the *homo ad quadratum* and *homo ad circulum* have per se. In this regard, it is worth remembering that in the context of his age and of Leonardo himself, given the way he acted and lived, man is undoubtedly the centre of the universe.

Thus assuming that at the time Leonardo develops his Vitruvian man he does not yet know or properly master the Golden ratio's properties, the constructions that could have had a distinctive status, conceptually, would in our opinion be only two. The *ad quadratum* construction, which is clearly outside this sphere, and vesica

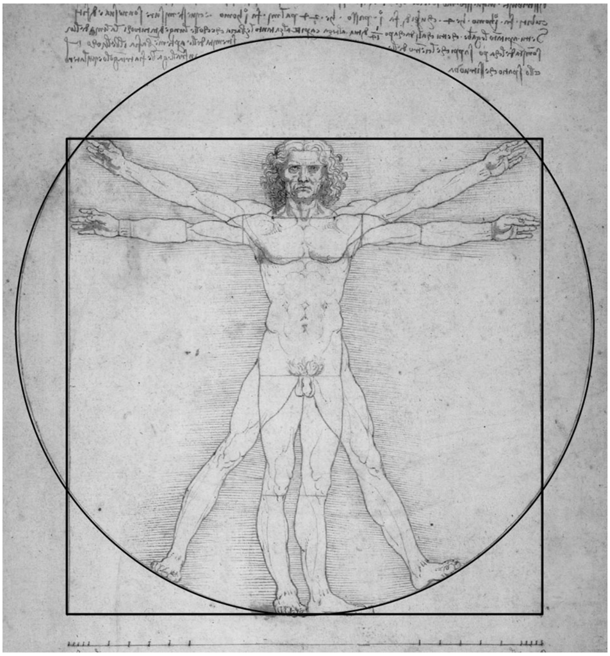


Fig. 14 *Square* and *circle* drawn using a double vesica piscis construction superimposed on Leonardo's drawing

piscis. Now this construction has a very strong symbolic value, but in its formalisation it allows us to obtain a figure that establishes the predominance of an axis over the other. There is however another figure, the double vesica piscis, which, by enabling the drawing of two vesicas oriented in orthogonal axes, becomes a more stable and geometrically more balanced configuration.[18]

Therefore, if we take the circumference and not the square as a starting point and represent two orthogonal axes going through the centre of the figure, we obtain four noticeable points in the circumference (A, B, D and E) that would conceptually enable the representation of a skewed square (Fig. 13). If centred in each of these points (vertices of the imaginary square) we represent a circumference with a radius equal to the diameter (e.g. AB or DE) of the given circumference, and so tangent to it, we will obtain a set of four circumferences that allow us to, in pairs and on opposing situations, draw two vesica piscis, one with a vertical and the other with a horizontal axis. Within this construction, each auxiliary circumference pair will originate an intersection point. These four intersection points will make it possible to draw a square (F, G, H and J) that sections the original circumference (centre C) in eight different points. This square's side length is one hundred and eighty-one point one millimetres (181.1 mm). If the construction is made from the Leonardian

[18] On the double vesica piscis, cf. Rachel Fletcher's excellent article about squaring the circle (Fletcher 2007).

Table 1 Comparison of measurements and discrepancies

Author	Square side (mm)	Circumference diameter (mm)	Discrepancy (mm)
Leonardo	181.5	220	
Vitruvius	181.5	226.8	+6.8
Helbing	181.5	223.6	+3.6
Sinisgalli	181.5	224.5	+4.5
March	181.5	219.1	−0.9
Schröer	181.5	221.1	+1.1
Murtinho	181.5	220.7	+0.7

square, we will obtain a circumference diameter of two hundred and twenty point seven millimetres (220.7 mm) (Fig. 14).

Conclusion

In the following comparative table of measures (Table 1) can be observed the different levels of discrepancy between Leonardo's drawing and the constructions that have been described. From the analysis shown in the table it can be concluded that our construction is the one with the smallest deviation from Leonardo's drawing (about 0.7 mm).

From the above, if there is a construction prior to the Leonardian drawing, admitting that the current folio had a preliminary sketch and that the drawing of the relation between square and circumference was the object of a separate intellectual consideration rather than merely mechanical, the probabilities set forth in this demonstration tend to show that Leonardo da Vinci made use of the double *vesica piscis*, be it for the proportional similarity between the two figures, or the symbolic value of this representation, whose analysis should be developed in an article to be presented subsequently.

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