On-off scheduling schemes for power-constrained electric vehicle charging

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Abstract

In this paper, we study the problem of establishing a dynamic charging schedule of electric vehicles (EVs) at a charging station, assuming that limited power implies that only a limited number of EVs can charge simultaneously. The only control we assume to be available to the charging station is the ability to (at any given time) turn on or off the power supply to any EV, with this tool we want to develop a charging schedule that will satisfy the energy demands of the EVs in their intended deadlines. We propose two distinct approaches to this problem: a discretized time version, based on a greedy-like algorithm, and a continuous time version, based on linear programming. We compare these two approaches and numerically study the improvement they yield in the efficiency of the charging procedure, when applied to simulated data based on real parking data. Finally, we illustrate the flexibility of the models by sketching several possible extensions.

1 Introduction

The automotive industry has, in recent years, strongly invested in the production and marketing of electric vehicles (EVs) and hybrid electric vehicles. The energetic efficiency and positive environmental impact have been some of the key selling points [12]. Since these vehicles are charged directly of the electric grid, a strong adoption of such vehicles will introduce stress in the production and distribution of electricity. It is, therefore, not surprising that the impact of EVs in the electric grid, and how to minimize it, has been an important early focus of research as exemplified by [17], [9], and [5]. This perspective is a high-level one, directed to systemic topics like energy consumption peaks, distribution grid limitations and other grid-wide phenomena. Other important commonly studied topic is the integration of renewable energies and its coordination with EV charging [11], [26].

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As far as planning and managing the charging itself, many approaches have been proposed, varying greatly in objectives and methods. Examples of that are Clement *et al.* [6] that minimizes energy loss using stochastic programming while Zhang *et al.* [26] minimizes waiting times at a charging station with recurse to Markov Chains and Alonso *et al.* [1] using a genetic algorithm searches for the daily optimal scheduling of EV demand to flatten the loading profile.

To introduce the work in this paper, it is relevant to mention some previous uses of optimization in the EV charging problem. In [19], quadratic optimization is used to minimize costs with varying electricity price while, with a more technical point of view, [21] focuses on the behavior and limitations of batteries, and schedules charging based on those constrains. Other authors explore different approaches like quadratic constraint programs [23] or a mixed-integer linear programming models [7]. While somewhat close in spirit with the problem we treat, the planning is done taking the vehicles as a coordinated fleet and not random individual arrivals to the system as treated in this paper.

We focus in a smaller scale energy management problem. We want to tackle the decision process at the level of a charging station (or a set of charging stations sharing the same power source). From that point of view, we have a restricted source of power that allows only a certain fixed number of EVs to charge simultaneously, and as new EV arrive to the system to charge, we need to manage their charging schedule. We are assuming that EVs have a fixed deadline to charge known *a priori*. They also have each a fixed amount of energy required, and the only control that we are assuming the charging station to have is the capability of switching on and off the power supply to any vehicle at any given time.

What we want is to develop a simple decision tool that, upon the arrival of a EV, will verify the demands on the system imposed by the EVs charging at that moment and develop an on-off charging schedule that will accommodate them and those of the new EV, or show it to be impossible. We also want the method to have good dynamical properties. In other words, it has to perform well when confronted with the multiple vehicle arrivals that will happen throughout a typical day.

On-off charging strategies are part of the general smart charging paradigm currently being proposed. The integration of EV charging with smart grids is expected to allow in the near-future much more control over the load profile it imposes on the electrical network, a very important step if EVs are to become a significant portion of the circulating cars. There is extensive literature on on-off strategies for charging. A significant part of it tends to focus on large-scale load profile shaping i.e., on setting a priori charging strategies system wide as opposed to the local problem of reacting dynamically to demand at a charging station, as addressed by the method proposed in this paper. Examples of such studies include probabilistically interrupting cars charging during expected peak-times [10], [2]. Deterministic local strategies are considered in papers like [14] and [25], but they assume a known demand throughout the day, which is not true in our case study. In [13] an interesting dynamic method is proposed where knowledge of the load profile is assumed but each EV is scheduled as it arrives, but once

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more the focus is on large scale systems. Finally [8] casts on-off strategies in the more ample context of modulation strategies.

The paper is organized as follows. In section 2, we propose two methods to solve this problem: one based on its discretization and using a dispatch rule, the other a continuous approach that uses linear programming at its core. In section 3, we evaluate these methods. To do that we develop a simulator for two scenarios of EV behavior and compare the efficiency of several approaches, showing that there is a significant improvement versus a standard unbroken charging schedule strategy. The continuous approach is further explored in section 4, where we illustrate its flexibility by developing variations to deal with some possible extensions of the problem. In the same section we also study the number of charging cycles induced by both strategies, and basic strategies to reduce it. In the last section, we present some concluding remarks.

2   Scheduling schemes

The basic building block to our schemes is a static scheduling problem: given a number N of EVs such that vehicle i needs to charge for an amount of time C_i and be done in time T_i, we have to find a plan where all that is satisfied without charging more than P vehicles simultaneously at any given time, or show that it does not exist. The idea is that we will follow this plan until the arrival of a new EV, at which time we will interrupt the process, run the problem with the new data and start running the new charging plan. If at any given time the resulting problem is unfeasible, we reject the new vehicle and this EV will not be included with the new demands that will arrive to the system in the future. We assume no a priori knowledge of when will EVs arrive to the system, and our goal is to define charging schedules that, in the long term, will result in the rejection of as few EVs as possible.

We will introduce, in this section, two options for dealing with this basic problem.

2.1   Discrete approach

In this model, we will discretize the time, and therefore consider C_i and T_i to be integers. With this setting, the basic problem can be reinterpreted as a *job-shop scheduling problem* with precedences, if we consider each "unit of power" a machine and charging a vehicle for a unit of time a task. Vehicle i will then contribute with C_i tasks that have to be performed in a certain order of precedence until time T_i. This suggests the use of dispatch rules ([16] and [4]).

Recall that a dispatch rule is simply a rule to rank the priorities of all pending tasks, so that when a machine becomes available, the highest priority pending task will be processed. A common such rule is the *slack first* dispatch rule, that order tasks by their time slacks, defined as the difference between the time they have to be processed and the amount of time they will take to process. These rules are used in several contexts as they have a good dynamic behaviour

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in terms of producing the least violations to the deadline constraints. We will show that in the context of the basic problem, the slack first rule will actually always return a feasible schedule if such schedule exists, making it a perfect baseline to which compare other methods.

To formalize this method, we need to introduce some notation. We want to find binary decision variables $Z_{ij} \in [0,1]^{T_i}$ that will codify the charging schedule by

$$Z_{ij} = \begin{cases} 1 & \text{if vehicle } i \text{ charges at instant } j, \\ 0 & \text{otherwise.} \end{cases}$$

This is a valid schedule if

$$\sum_{j=1}^{T_i} Z_{ij} = C_i, \ i = 1, ..., N \tag{1}$$

and

$$\sum_{i=1}^{N} Z_{ij} \leq P, \ j = 1, ..., \max\{T_i : i = 1, ...N\}. \tag{2}$$

Note that the restrictions (1) guarantee that EVs charge for the exact time needed before their deadline, while restrictions (2) guarantee that the power used never surpasses the available power. In this context, the slack of a vehicle is simply given by

$$S_i = T_i - C_i, \ i = 1, ..., N.$$

The slack first method consists of charging, at any given instant, the vehicles with the least slack. Then, decrease by 1 the C_i of the charged vehicles and decrease by 1 the T_i of all vehicles, and repeat the procedure. This process ends when all C_i are zero (every vehicle was charged successfully) or if, at any given time, the slack of a vehicle becomes negative (not enough time left to charge it). It turns out that if this method fails to find a solution then none exists.

Theorem 2.1. *If there is a feasible scheduling plan satisfying (1) and (2) then the slack first method is successful.*

Proof. It is enough to prove that any feasible solution can be transformed into a slack first feasible solution. Let Z be a feasible solution and let j be the first instant where we are not charging the P least slack vehicles. Let i_1 and i_2 be respectively the EV with the largest slack that is charging ($Z_{i_1 j} = 1$) and the EV with the least slack that is not charging ($Z_{i_2 j} = 0$). Let $\mathcal{S}_i$ be the set of instants from j to T_i on which EV i is not charging. We know by hypothesis that $|\mathcal{S}_{i_1}| \geq |S_{i_2}|$ hence there exists $l \in \mathcal{S}_{i_1} \setminus S_{i_2}$ and we have two possibilities:

1. There exists $l > j$ for which $Z_{i_1 l} = 0$ and $Z_{i_2 l} = 1$. In this case we can simple change all values $Z_{i_1 j}, Z_{i_1 l}, Z_{i_2 j}$ and $Z_{i_2 l}$ and that will not affect any of the constrains.

2. There exists $l > T_{i_2}$ for which $Z_{i_1 l} = 0$. If there are least than P EVs being charged in l then we can just delay the charging of vehicle i_1 to instant l and use the freed slot at time j to charge EV i_2, so we just need to worry about what happens if P vehicles are charging at instant l. Let $j < t < l$ be an instant where $Z_{i_1 t} = Z_{i_2 t} = 1$ (it must exist, otherwise we would have been in case 1). There must be a vehicle i_3 that is charging in l but not in t, so we can cyclicly change between the EVs schedules, making i_2 charge at j, i_1 and i_3 at t and i_1 at l. These changes in time slots again do not change constrain satisfaction.

We just saw that we can always fix a slack reversal without losing feasibility, so doing it repeatedly we will end up with a slack first compatible schedule, proving the result. $\square$

Note that the slack first scheduling procedure is actually very fast (it runs in $O\left(\sum_{i=1}^{K} T_i\right)$) if the discretization units are not too small.

2.1.1 Example

To illustrate the procedure, suppose we have 6 EVs with available time and charge demands of $T = (17, 18, 22, 22, 24, 25)$ and $C = (13, 8, 19, 8, 4, 16)$, respectively. Additionally, assume that the available power only allows us to charge 3 EVs simultaneously ($P = 3$). The slack first method will return the scheduling indicated in Figure 1.

Figure 1: Charging schedule for the slack first method.

2.2 Continuous approach

While the discrete approach is elegant and provides a very good baseline for testing scheduling methods, it has several drawbacks. First, it would either require the chosen units of time discretization to be small, which would make the scheduling process cumbersome, or it would force to round the data, introducing error. Second, and most important, it is not a very flexible scheme. We have very little leeway in the scheduling plan attained, so if we want to extend the procedure to confront slightly more sophisticated aspects of the problem, we need something else. This motivates the introduction of a linear programming based approach to the basic problem.

The main idea will be to divide time in few intervals, with each time interval being the time when a certain fixed subset of vehicles are available to charge. We will then try to determine the percentage of time in each fixed interval on which a given EV is charging. To formalize this idea assume that $T_1 \leq \cdots \leq T_N$, i.e. the EVs are in ascending order of available time to charge. Our decision variables will be $z_{ij}, 1 \leq j \leq i \leq N$, and represent the percentage of time in interval $[T_{j-1}, T_j]$ (where T_0 is defined as 0) in which vehicle i is charging.

We then obtain the LP feasibility problem of finding z such that:

$$\sum_{i=j}^{N} z_{ij} \leq P, \quad j = 1, ..., N; \tag{3}$$

$$\sum_{j=1}^{i} z_{ij}(T_j - T_{j-1}) = C_i, \quad i = 1, ..., N; \tag{4}$$

$$z_{ij} \in [0,1], \quad i = 1, ..., N, \quad j = 1, ..., i. \tag{5}$$

Here, constrains (3) guarantee that the power limits are respected, while constrains (4) force every vehicle to be fully charged. This approach grants us a great deal of flexibility, since we can introduce many different types of objective function to pick among feasible solutions the more desirable ones, or add all kinds of different constrains to deal with different types of scenarios like energy costs variations or layered client types. This shows a potential advantage of this type of method against pure queuing methods (used, for example, in [18]). Some of these ideas will be explored later in this paper.

This LP method is, however, still incomplete: knowledge of the percentages z_{ij} does not immediately translate to a scheduling. So we need to introduce a conversion method that will tell us in which moments should we switch on or off each of the EVs.

2.2.1 From LP solution to charging schedule

We will create our charging scheduling sequentially for each interval $[T_{j-1}, T_j]$, $j = 1, ..., N$. Assume for simplicity that we are working in the interval $[0, 1]$. One way to visualize the conversion is to think of the available energy at that interval as a bar with length P, subdivided in P sections of length 1. We then mark in it blocks of length z_{ij}, one for each vehicle that charges during that interval. Let the block corresponding to some vehicle i be the interval $[a, b] \subseteq [0, P]$. If a and b have the same integer part (the interval is contained in a single subdivision of the bar) we will charge vehicle i in the time interval $[\mathrm{fr}(a), \mathrm{fr}(b)]$ (where $\mathrm{fr}(x)$ stands for the fractional part of a x). If a and b have different integer parts (the interval is divided between two subdivisions of the bar), we will charge the vehicle i in the time intervals $[0, \mathrm{fr}(b)]$ and $[\mathrm{fr}(a), 1]$.

The geometric interpretation of this conversion makes it clear that if the z_{ij} are feasible for the LP, then there will never be more than P vehicles charging at any given time. The method is also fast and easy to implement. If there are

no more than P vehicles in the system, we just charge them all from the start, and not run the conversion.

2.2.2 Example

Let us get back to the situation in Example 2.1.1, where we had six EVs charging. Running the LP feasibility problem for that data we get the (non-unique) solution presented in Figure 2.

EV1	76, 47%				
EV2	41, 18%	100, 00%			
EV3	88, 24%	0, 00%	100, 00%		
EV4	23, 53%	0, 00%	100, 00%		
EV5	0, 00%	0, 00%	50, 00%	100, 00%	
EV6	70, 59%	0, 00%	50, 00%	100, 00%	0, 00%

$$0 \qquad 17 \qquad 18 \qquad 22 \qquad 24 \qquad 25$$

Figure 2: Percentages of activity at each interval.

We can see, for example, that only one vehicle (EV1) charges at the second interval in this particular solution already pointing to some inefficiencies in the method as is. Let us now apply the conversion method described in 2.2.1. We will start by visualizing its application in the first time interval ($[0, 17]$). Figure 3 shows the bar of length 3, the maximum power we are considering, subdivided as previously described, and the resulting schedule for each EV. Initially, it is showed on a 0 to 1 scale, and after that, already converted to the $[T_0, T_1]$ time segment. Here it is visible that only 3 EVs charge simultaneously.
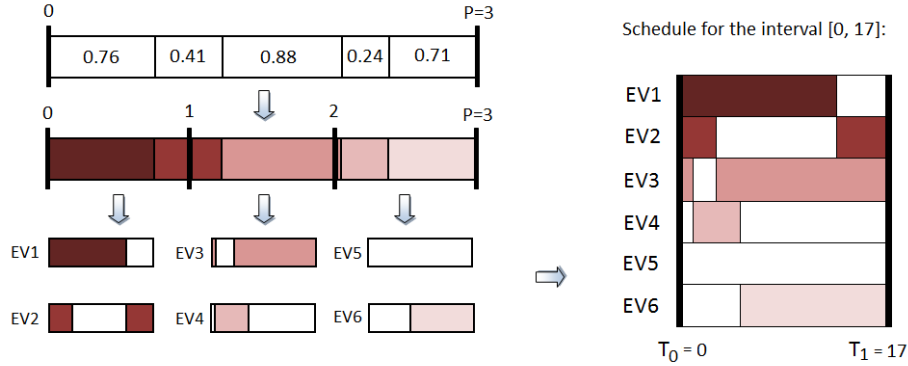

Figure 3: Conversion of the first interval.

Extending this to the other four intervals and gathering the respective results, we obtain the final schedule pictured in Figure 4.
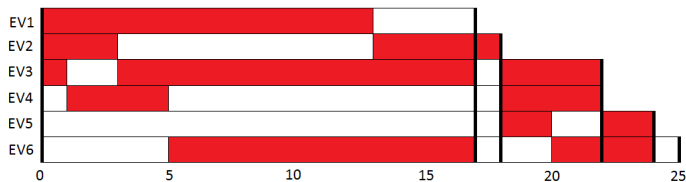
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Figure 4: LP approach scheduling.

2.2.3 Sequential optimization

As seen above, the use of an LP feasibility problem has some drawbacks. For one, the feasible point found is highly dependent on the algorithm used to solve the LP but, more importantly, it can lead to time waste as we saw in the example, where at some points there were EVs waiting while free power was available. We want to avoid that by pushing all charging to be as early as possible, so that if new vehicles arrive, we will have as much free power going forward as possible. The more direct way to do this is to do it sequentially, interval by interval.

For the j-th interval, we would solve the LP maximizing the objective function $f_j(z) = \sum_{i=1}^{N} z_{ij}$, attaining some maximum f_{j-1}^*. We would then add the restriction

$$\sum_{i=1}^{N} z_{ij-1} \geq f_{j-1}^* - \varepsilon_{\text{tolerance}} \tag{6}$$

to the LP, and proceed to optimize the objective function $f_{j+1}(z) = \sum_{i=1}^{N} z_{ij+1}$ and so on. The $\varepsilon_{\text{tolerance}}$ introduced is simply an artifact to deal with numerical instability. Applying this method to the previous example we get the scheduling in Figure 5.
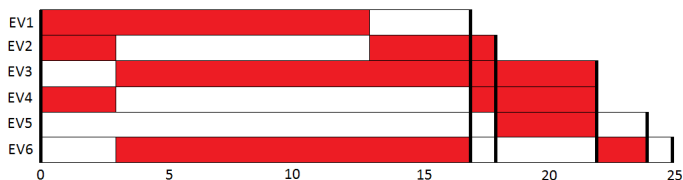

Figure 5: Sequential LP approach scheduling.

We can see that in the interval $[0, 22]$ there are always three EVs charging at each given time, and it results in better overall performance (EV5 is fully charged at $t = 22$ instead of $t = 24$ as was happening in the original feasibility approach). This method has some disadvantages: we have to solve several LPs instead of just one, and we have some error build up form interval to interval that demands great care in implementation. To work around these issues we will

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now present a compromise solution, that preserves some of the good qualities of this approach but avoids its pitfalls.

2.2.4 Global optimization

To address this need to push charging to as early as possible but still solve a single LP, we will introduce an objective function that gives less weight to energy charged in later intervals than to that charged in the early ones. A simple possible function is given by

$$f(z) = \sum_{i=1}^{K} \sum_{j=1}^{i} \frac{1}{2^i} z_{ij} (T_j - T_{j-1}).$$

Computational experiments show that the results attained are in general very close to the ones given by the sequential approach (as will be visible in the Table 1 of the section 3.3). Applying it to the running example, for example, gives rise to the precise same solution as was obtained in Figure 5 using the sequential approach. This method is faster, since only one LP is ran, and proves to be much more stable than the previous one. In the numerical experiments both alternatives will be applied to the generated data but, since no advantages result from the sequential approach, regarding the study of some extensions of the models in section 4 the global approach will be adopted preferentially to the sequential method.

3 Numerical experiments

We now have several ways to tackle the basic problem. However, we are interested in solving not a single instance of that problem but the sequence of problems that will be generated as new EVs arrive to the system and attempt to charge. Recall that every time that happens, a new scheduling problem is generated. A feasible solution, if found, will be implemented, while if the problem is unfeasible the new vehicle will be turned down. We want to make sure that we will be able to satisfy as much of the energy demand as possible, and that will be our main criterion for comparison.

Before we engage into more detail, some aspects about the context of the problem should be clarified or, in some cases, reviewed. Regarding the type o vehicles considered, we will assume that they can be pure electric or hybrids but are slow charging. The moments at which they arrive are unpredictable and, therefore, each charging plan only takes into account the requirements of the EVs already on the system. These intend to mimic the expected usage in a urban public charging station, or a set of such stations connected to a common transformer station, the relevant cases for the project mobiOS from which this study emerged.

3.1 Uninterrupted charging schedule

In order to have a baseline to compare to the performance of our methods, we have to introduce a simple, easy to implement scheduling technique making no use of the on-off switches. This will simply consist of, upon arrival of an EV to the system, see when will some power be available to charge the vehicle, and if after that moment there will be enough time until its deadline to attain full charge. If so, proceed with that plan, if not, reject the EV. This is a very natural charging scheme and, without using interruptions, the best possible fitting our framework.

3.2 Data generation

Due to the scarcity of usable real world data on the variation of charging demand during a day, we designed a simulator that will randomly generate vehicles, and give each an arrival time, a charging need and a deadline. We will use it to generate a fixed number n of vehicles per day, for as many days as we want (typically we will generate a demand profile for a period of 365 days, to simulate a year of data). In order to be able to compare between the discrete and continuous approaches, we will use the minute as the discretization unit of time, and generate all data with an integer number of minutes.

We propose two different simulator models.

3.2.1 Model A

The first model proposed is a simple control model, where we just pick for each car an arrival time from a uniform distribution in the day, a deadline (T) picked uniformly random between 2 and 8 hours and the time needed to charge (C) picked uniformly from 1 to T hours.

3.2.2 Model B

For results with more real world significance, we use parking data as a proxy to the typical variation of charging demand during a day in a business area. To do this we used a geo-coded mobility survey that was conducted in Coimbra, Portugal, between October of 2008 and March of 2009 [22]. This survey tracked the daily commute of 10000 drivers, and allows us to extract parking patterns throughout the city. We concentrated our attentions in a particular commercial and business focused area of the city with high traffic affluence.

As can be seen in Figure 6, there are clear peaks of parking at around 11am and 4pm, with a noticeable decrease between 1 and 2pm, and almost no parked vehicles at night. To emulate this behavior we generate several types of EV: some with arrival time given by a Gaussian centered at 9am and others with a Gaussian centered at 2pm. For the ones arriving in the morning, we generated their deadlines also by a Gaussian model, but allowed two behaviors: a shorter stay (modeling the EVs leaving before 1pm) or a longer stay (modeling the EVs staying until the end of the working hours). For the ones arriving in
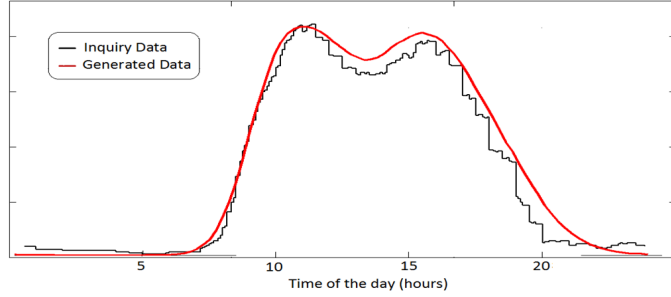
Figure 6: True number of cars in a commercial/business area of Coimbra and, in red, the simulated data.

the afternoon, we modeled their deadlines similarly, simulating leaving at the end of the working hours. We ignored all generated deadlines smaller than 2 hours, since in a short stay we do not expect drivers to charge their vehicles. Charging demand was generated analogously to the previous model. Fixing the parameters we obtain the behavior seen in Figure 6, that strongly mimics the real world data we have in hand.

3.3 Results

All tests were done using MATLAB 7.9.0 R2009b. In Figures 7 and 8, we present the results obtained with different scheduling methods for the percentage of energy delivered in a year as we vary the number of cars arriving each day. In Figure 7, we are considering Model A, with a power limit $P = 15$, while in Figure 8 we consider Model B, with a power limit $P = 25$.
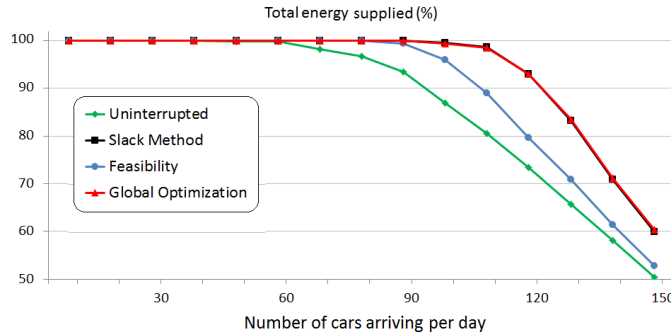

Figure 7: Percentage of delivered energy - Model A with $P = 15$.

The methods we tried were the uninterrupted method introduced as a baseline, the discrete slack-first method and the three versions of the continuous approach: one using only the feasibility problem, other with the global objec-
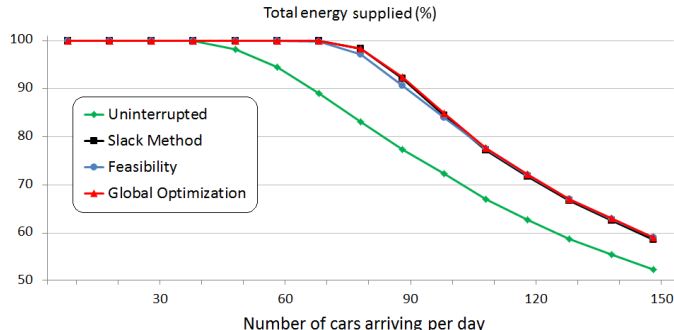
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Figure 8: Percentage of delivered energy - Model B with $P = 25$.

tive function introduced and, finally, the sequential perspective (which was not included in the Figures 7 and 8 but is in Table 1). We can see in the results that the proposed approaches consistently improve upon the base uninterrupted method. Moreover, the discrete and the global optimization methods are almost indistinguishable in term of results, while the feasibility approach lags behind in the Model A simulation. In general we can see that, depending on the parameter settings, we attain gains of up to 20 percentage points. Particularly worthy of notice is that the application of the best performing methods as an alternative to the uninterrupted one allows, in Model B, an increase from 40 to 70 daily clients, without compromising a perfect satisfaction of all demand, a 75% increase in the capacity of the system.

We have chosen percentage of energy supplied as the evaluation criteria, as a proxy for economic return. However, the picture would remain almost precisely the same if we had instead considered percentage of cars accepted into the system, as can be seen in Table 1. Also seen in the table are the results for the sequential method, omitted from the graphs. One can see there that the sequential approach has no benefits over the global approach in terms of practical performance therefore, since the computational cost and numerical sensitivity of the sequential approach are much higher, we will always omit it from further analysis.

For more insight on this improvement, we focused on Model B and varied both the number of daily arrivals and the power constrains. We then simulated a year for each combination and applied the uninterrupted and the slack first discrete method to the data recording the efficiency of each. We omit the global continuous approach since its results follow closely to the discrete results. The obtained numbers for the discrete approach are represented chromatically in Figure 9.

Also visible in that figure is a black line highlighting the boundary of full satisfaction of demand in the discrete approach, and a green line representing the same for the uninterrupted approach. We can see a large range of parameters inside the two lines, corresponding to settings where changing from uninterrupted

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	Uninterrupted	Global Opt.	Slack	Sequential Opt.
n	% cars / % en.	% cars / % en.	% cars / % en.	% cars / % en.
30	100/100	100/100	100/100	100/100
40	99.9/99.9	100/100	100/100	100/100
50	98.6/98.2	100/100	100/100	100/100
60	95.7/94.5	100/100	100/100	100/100
70	91.3/89.0	99.9/99.9	99.9/99.9	99.9/99.9
80	86.3/83.1	98.2/98.4	98.2/98.4	98.1/98.3
90	80.9/77.3	91.3/92.4	91.0/92.2	90.7/92.0
100	76.2/72.3	83.1/84.8	82.7/84.5	82.6/84.3
110	71.4/67.0	76.3/77.7	75.8/77.3	76.0/77.2
120	67.3/62.8	71.3/72.2	70.9/71.8	70.9/71.8
130	63.1/58.9	66.5/67.1	66.3/66.8	66.3/66.8
140	59.9/55.6	63.0/63.0	62.7/62.7	62.7/62.7
150	56.6/52.4	59.3/59.1	59.0/58.7	59.0/58.7

Table 1: Comparison of results for Model B $P = 25$ in terms of percentage of cars accepted and energy supplied for the uninterrupted, global optimization, slack heuristic and sequential methods.
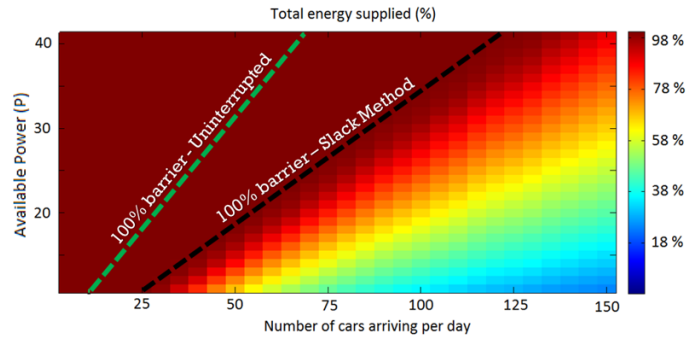

Figure 9: Slack-first results as P and n vary - Model B.

to one of the proposed alternatives will lead to full satisfaction of demand.

4 Extensions

In this section we describe two examples of how to use the flexibility of the LP formulation to increment the model and adapt it to more complicated situations. These are simple illustrations, but highlight the capability of this approach to support more sophisticated refinements.

4.1 Reduction of the number of charging cycles

It is well known that battery wear is dependent of the number of charging cycles ([3] and [15]). Breaking a charging cycle in intervals, may therefore decrease battery life. There is, to our knowledge, no conclusive study done with regards to the impact of smart charging to battery live, but some preliminary work in [24] seems to suggest that it might not be too high. It could still be useful, however, to study the typical number of charging cycles that our algorithm imposes, and to try to reduce it when it is high.

4.1.1 Discrete approach

In the discrete approach, one could attempt to formulate an integer programming with the number of charging cycles as an objective, but the resulting problem would be too computationally expensive to be solved in real time in average sized systems. An alternative is to employ a post-processing heuristic.

The idea will be to apply the scheduling strategy developed in section 2.2.1 for the continuous approach to readjust the discrete schedule attained by the slack first algorithm: Let TO be the sorted vector of the available charging times, the schedule obtained via slack first is divided in several subintervals, from 0 to $TO(1)$ and $TO(i-1)$ to $TO(i)$ with $i = 2, ..., N$. For each subinterval, we know the number of time units each EV will charge, so we reorganize the schedule by starting the first EV and, when its finished, we immediately start the second one. If there is not enough time to charge the whole amount, the missing charge will be added at the start the same time interval. The process is then repeated for the rest of the vehicles. This way, we ensure that the number of charging cycles for the i-th EV is not superior to $2 \times i$.

4.1.2 Continuous approach

Regarding the reduction of charging cycles, no extension was developed using the continuous approach. Despite that, it's important to mention that the optimization method (and even the solver) have a big influence on this metric. As expected the simplex method gives better results than the interior point method, since the latter reaches an optimal solution inside the feasible region, leading to a more even distribution of energy throughout time that forces more interruptions. Below we quantify this impact.

4.1.3 Results

The data used was the same as the one previously generated using Model B and the tests were, once again, conducted in MATLAB. The options analysed in this section are global optimization with both simplex and interior point method (I.P.), the slack first heuristic and the mentioned post-processing rule.

As expected, the average number of charging cycles grows with the increase of demand. However, when the system begins to reject EVs due to overload (n
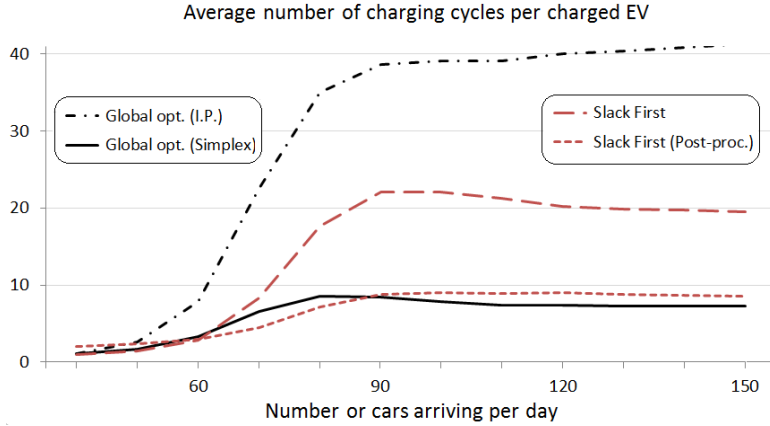
Figure 10: Average number of charging cycles according to each approach for $P = 25$.

bigger than 80), the numbers tend to stabilize. Through Figure 10, it is also visible the improvement provided by the post-processing heuristic and the positive impact that the simplex method has when applied to the global optimization alternative. Note that the other two criteria, energy supplied and successfully charged EVs, showed no significant differences for the several alternatives studied.

4.1.4 Discrete approach interval size

Another possible way of reducing the number of charging cycles in the discrete approach, is to increase the size of the smallest indivisible unit of time considered. In the previous tests, we always took it to be one minute, meaning that only once every minute would the system be able to change the charging status of any EV. By increasing that number, we are slightly reducing the amount of usable charging time for any EV, but we are forcing larger minimal charging cycles. The results of this trade-off are visible in Table 2. For theses computations we used the simulated Model B data for a year with both 80 and 100 cars per day and power $P = 25$.

4.2 Tiered client system - energy reserve

Assuming one wants to have a class of priority premium clients, one could achieve this is by reserving some quantity p of the available power P to be used only by the premium clients. If we assume the non-premium clients are the first M of the N EVs wanting to charge in the original LP formulation (see section

	Interval Size (minutes)	1	5	10	15	20	25	30
n=80	% energy supplied	98.4	98.1	97.6	96.9	96.2	95.5	94.6
	Av. number of cycles	17.7	4.9	3.2	2.6	2.3	2.1	1.9

	Interval Size (minutes)	1	5	10	15	20	25	30
n=100	% energy supplied	84.5	83.6	82.6	81.5	80.6	79.7	78.8
	Av. number of cycles	22.0	5.5	3.3	2.6	2.2	2.0	1.9

Table 2: Impact of the interval size on the performance of the slack method for $n = 80$ and $n = 100$ in Model B.

2.2) we just have to modify the problem by adding the constrain

$$\sum_{i=j}^{N} z_{ij} \leq P - p, \quad j = 1, ..., M. \tag{7}$$

This adds no difficulty to the original LP problem. We simulated this approach using Model B, assuming 20% of the cars to be premium, $n = 80$ and $P = 20$. We are interested in the efficiency obtained in the two classes of clients as we vary p. The result can be seen in Figure 11.
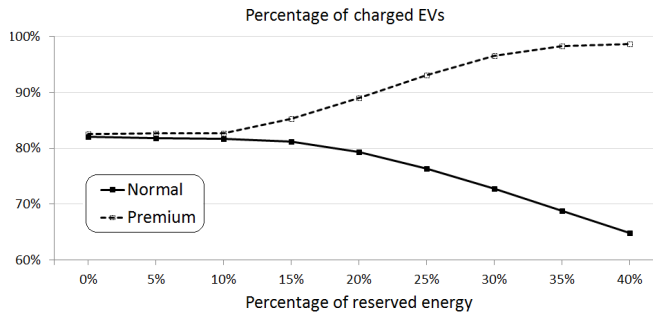

Figure 11: Percentage of charged EVs for $n = 80$ (with 20% premium), $P = 20$ and different values of p (as a percentage of P).

4.3 Delay penalties

We could also allow the violation of the deadlines, with the trade-off of paying some penalty compensation. This can easily be implemented by adapting our model into a mixed integer linear program. The basic idea is that for each vehicle i we can add a potential delay d_i. When considering our time subintervals we would consider, not only the values T_i but also the values $T_i + d_i$. Therefore instead of N variables z_{ij} for each vehicle i, we would get $2N$ such variables at

most. For a vehicle i, let I_i be the set of j so that z_{ij} represents an interval before T_i, and let J_i be the set of j so that z_{ij} represents an interval between T_i and $T_i + d_i$. We will introduce a binary decision variable w_i that is set to 1 if we need to use the delay. Considering $\overline{T}$ (the vector with size $\overline{N}$ of all T_i and $T_i + d_i$ sorted entries), we can then modify the original LP constrains to

$$\sum_{i=1}^{N} z_{ij} \leq P, \quad j = 1, ..., \overline{N}; \tag{8}$$

$$\sum_{j=1}^{\overline{N}} z_{ij}(\overline{T}_j - \overline{T}_{j-1}) = C_i, \quad i = 1, ..., N; \tag{9}$$

$$w_i \geq z_{ij}, \quad j \in J_i, \quad i = 1, ..., N; \tag{10}$$

$$z_{ij} \in [0,1], \quad i = 1, ..., N, \quad j = 1, ..., \overline{N}; \tag{11}$$

$$w_i \in \{0,1\}, \quad i = 1, ..., N. \tag{12}$$

If we associate a cost q_i to a violation of the deadline of car i we would then have to solve the MILP of minimizing $\sum_{i=1}^{N} q_i w_i$. Note that a mixed integer linear program is notoriously harder than a pure LP, but since these problems are solved only once per EV arrival, a solution time in the order of seconds is good enough. Since the problems we are considering are not too large, this is perfectly attainable with an off-the-shelf MILP solver.

4.4 Minimizing energy costs

As mentioned earlier, a typical question in scheduling EV charges (generally in a larger setting like fleet management) is that of minimizing the cost of the energy supplied (whether its focused only on the electricity costs [20], [7] or not [23]). If we can predict in advance how will the prices vary during the day, one can discretize the price function and feed it directly to the LP formulation. Every time we have to solve our base scheduling problem we would then minimize the price of energy supplied, while keeping the original constrains.

5 Conclusion

The main contribution of this paper is the proposal of on-off schemes for dealing with the problem of scheduling EV charging in the presence of restricted power capacity. The case we are interested to address is that of a charging station with limited power that has to satisfy an uncertain daily demand, as is the case of a public charging station. As EVs gain acceptance, their charging will cause stress in power supplies, so this is a problem that needs to be addressed. This is also true for corporations converting their existing fleets to electric vehicles, that have their own facilities on which to charge them. Optimizing their charging schedules can lower the power needs of such a facility.

In this paper, we propose two models, one discrete and one continuous, that take advantage of the capability of interrupting the charging of an EV and we carry on numerical tests with simulated data based on real parking data that suggest a significant improvement in efficiency. These methods are simple, fast and easy to implement. The continuous method in particular is very well suited for customization, a fact that we illustrate by developing extensions of it to deal with several additional complexity layers in the model. The discrete model, being a dispatch heuristic, lacks an objective function that can be adjusted, so it is less versatile, but is simpler to implement and very fast, so maybe better for a more straightforward implementation.

The direct applicability of this work has some limitations, since we used several simplified assumptions: the linearity of charging with respect to time is not verified in practice, so charging demand is in reality more nuanced than the "charging time needed" simplified model of demand used here. Furthermore, the battery wear that could potentially be caused by more frequent energy supply interruptions needs further study, and we only tried to address this problem by controlling and reducing the number of charging cycles induced by the methods. We feel that even with these simplifications, the models proposed and the accompanying analysis makes a good case for the big impact of these types of schedulings on instances of EV charge planning with limited available power and suggests that even in a more realistic framework they can potentially be very useful.

6   acknowledgements

References

[1] Alonso, M., Amaris, H., Germain, J.G., Galan, J.M.: Optimal charging scheduling of electric vehicles in smart grids by heuristic algorithms. Energies **7**(4), 2449–2475 (2014)

[2] Baek, S.J., Kim, D., Oh, S.J., Jun, J.A.: A queuing model with random interruptions for electric vehicle charging systems. In: Consumer Electronics (ICCE), 2011 IEEE International Conference on, pp. 679–680 (2011)

[3] Bashash, S., Moura, S.J., Forman, J.C., Fathy, H.K.: Plug-in hybrid electric vehicle charge pattern optimization for energy cost and battery longevity. Journal of Power Sources **196**(1), 541 – 549 (2011)

[4] Blackstone, J.H., Phillips, D.T., Hogg, G.L.: A state-of-the-art survey of dispatching rules for manufacturing job shop operations. The International Journal of Production Research **20**(1), 27–45 (1982)

[5] Camus, C., Silva, C., Farias, T., Esteves, J.: Impact of plug-in hybrid electric vehicles in the portuguese electric utility system. In: Power Engineering, Energy and Electrical Drives, 2009. POWERENG'09. International Conference on, pp. 285–290. IEEE (2009)

[6] Clement, K., Haesen, E., Driesen, J.: Coordinated charging of multiple plug-in hybrid electric vehicles in residential distribution grids. In: Power Systems Conference and Exposition, 2009. PSCE'09. IEEE/PES, pp. 1–7. IEEE (2009)

[7] Franco, J.F., Rider, M.J., Romero, R.: An milp model for the plug-in electric vehicle charging coordination problem in electrical distribution systems. In: PES General Meeting— Conference & Exposition, 2014 IEEE, pp. 1–5. IEEE (2014)

[8] González Vayá, M., Baringo, L., Krause, T., Andersson, G., Almeida, P.M.R., Geth, F., Rapoport, S.: In: 23rd International Conference and Exhibition on Electricity Distribution (CIRED)

[9] Hadley, S.W., Tsvetkova, A.A.: Potential impacts of plug-in hybrid electric vehicles on regional power generation. The Electricity Journal **22**(10), 56–68 (2009)

[10] Harris, C., Dusparic, I., Galvn-Lpez, E., Marinescu, A., Cahill, V., Clarke, S.: Set point control for charging of electric vehicles on the distribution network. In: Innovative Smart Grid Technologies Conference (ISGT), 2014 IEEE PES, pp. 1–5 (2014)

[11] Li, C.T., Ahn, C., Peng, H., Sun, J.: Synergistic control of plug-in vehicle charging and wind power scheduling. Power Systems, IEEE Transactions on **28**(2), 1113–1121 (2013)

[12] Li, Y., Kaewpuang, R., Wang, P., Niyato, D., Han, Z.: An energy efficient solution: Integrating plug-in hybrid electric vehicle in smart grid with renewable energy. In: Computer Communications Workshops (INFOCOM WKSHPS), 2012 IEEE Conference on, pp. 73–78. IEEE (2012)

[13] Mets, K., Verschueren, T., Turck, F.D., Develder, C.: Evaluation of multiple design options for smart charging algorithms. In: Communications Workshops (ICC), 2011 IEEE International Conference on, pp. 1–5 (2011)

[14] Nguyen, V.L., Tran-Quoc, T., Bacha, S., Nguyen, B.: Charging strategies to minimize the peak load for an electric vehicle fleet. In: Industrial Electronics Society, IECON 2014 - 40th Annual Conference of the IEEE, pp. 3522–3528 (2014)

[15] Peterson, S., Apt, J., Whitacre, J.: Lithium-ion battery cell degradation resulting from realistic vehicle and vehicle-to-grid-utilization. Jornal of Power Sources **195**(8), 2385–2392 (2010)

[16] Pinedo, M., Chao, X.: Operations scheduling with applications in manufacturing and services. In: Irwin/McGraw-Hill, Boston (1999)

[17] Putrus, G., Suwanapingkarl, P., Johnston, D., Bentley, E., Narayana, M.: Impact of electric vehicles on power distribution networks. In: Vehicle Power and Propulsion Conference, 2009. VPPC'09. IEEE, pp. 827–831. IEEE (2009)

[18] Qin, H., Zhang, W.: Charging scheduling with minimal waiting in a network of electric vehicles and charging stations. In: Proceedings of the Eighth ACM international workshop on Vehicular inter-networking, pp. 51–60. ACM (2011)

[19] Shrestha, G., Ang, S.: A study of electric vehicle battery charging demand in the context of singapore. In: Power Engineering Conference, 2007. IPEC 2007. International, pp. 64–69. IEEE (2007)

[20] Soares, F., Almeida, P.R., Lopes, J.P.: Quasi-real-time management of electric vehicles charging. Electric Power Systems Research **108**, 293–303 (2014)

[21] Sundström, O., Binding, C.: Optimization methods to plan the charging of electric vehicle fleets. In: Proceedings of the International Conference on Control, Communication and Power Engineering, pp. 28–29 (2010)

[22] System, M.R.M.: Mobility survey in the mondego region (2009). Database metadata

[23] Trippe, A.E., Arunachala, R., Massier, T., Jossen, A., Hamacher, T.: Charging optimization of battery electric vehicles including cycle battery aging. In: Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2014 IEEE PES, pp. 1–6. IEEE (2014)

[24] Vroey, L.D., Jahn, R., Omar, N., Mierlo, J.V.: Impact of smart charging on the ev battery ageing - discussion from a 3 years real life experience. In: 28th International Electric Vehicle Symposium and Exhibition (EVS28) (2015)

[25] Wu, T., Wu, G., Bao, Z., Yang, Q., Yan, W.: Demand dispatch of smart charging for plug-in electric vehicles. In: Control Engineering and Communication Technology (ICCECT), 2012 International Conference on, pp. 803–806 (2012)

[26] Zhang, T., Chen, W., Han, Z., Cao, Z.: Charging scheduling of electric vehicles with local renewable energy under uncertain electric vehicle arrival and grid power price. Vehicular Technology, IEEE Transactions on **63**(6), 2600–2612 (2014)