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An Exact Method for Constructing Minimal Cost/Minimal SRLG Spanning Trees Over Optical Networks

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Abstract The construction of overlay or broadcast networks, based on spanning trees, over WDM optical networks, with SRLG information has important applications in telecommunications. In this paper we propose a bicriteria optimization model for calculating communication spanning trees over WDM networks the objectives of which are the minimization of the total number of different SRLGs of the tree links (seeking to maximise reliability) and the minimization of the total bandwidth usage cost. An exact algorithm for generating the whole set of non-dominated solutions and methods for selecting a final solution in various decision environments are put forward. An extensive experimental study on the application of the model, including two sets of experiments based on reference transport network topologies, with random link bandwidth occupations and with random SRLG assignments to the links, is also presented, together with a discussion on potential advantages of the model.

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1 Introduction and Motivation

1.1 Introduction and Related Works

The construction of overlay networks, that can be regarded as “logical transport networks” in the sense defined in [32], namely composed of a fraction of the links of the underlying network, is of great importance in various areas of telecommunication network design. Relevant applications of this concept are peer-to-peer networks (see e. g. in [27]) and VPNs (Virtual Private Networks). Different types of approaches can be considered for the design of overlay networks, in different application environments. Examples of proposals of this nature are [32] concerning a general overlay structure or [27], concerning peer-to-peer networks. In any case the obtained network topologies are normally either tree-based or sparse topologies. One common approach, in this context, is shortest path based routing, the practical attractiveness of which results from the features of the OSPF (Open Shortest Path First) routing protocol and its evolutions. It was shown in [30] that assuming certain statistical distributions for the link weights/costs, in certain conditions, the overlay network (formed by the union of all shortest path trees in the underlying network) is a minimum spanning tree, i. e. an optimal tree that includes all network nodes.

A possible type of design method is the construction of overlay networks based on minimal cost spanning trees in the context of a capacity aware QoS routing method. This type of approach is proposed in [12] for application to MPLS networks using a bicriteria model, that uses as metrics to be optimised load balancing cost and average delay bound, a model that also may be used for traffic broadcasting in MPLS.

Furthermore, various applications in communication networks require the calculation of spanning trees, often designated as broadcast trees such as in video broadcasting services or data management.

Note that all previously mentioned models consider a single-layer network representation where the overlay network or broadcast tree is constructed. A more realistic type of approach, when one seeks to introduce resilience aspects explicitly in the model, has to take into account the multilayered nature of telecommunication networks, such that a single failure at a lower level often corresponds to multiple failures at an upper level. For example, a failure risk may represent a fibre cut, a card or a software failure at a node, which may affect more than one link at an upper functional network layer. These concerns are particularly relevant when the overlay/broadcast trees are to be calculated directly over optical networks (physical layer). In this context it is normally necessary to use the concept of Shared Risk Link Group (SRLG), that may be defined as a subset of the functional (or logical) network links which may be affected by a certain failure risk.

These issues have led to the formulation of models for obtaining broadcast trees with reliability requirements in association with problems of design of *reliable collective communication* structures in the sense defined in [36]. The concept of collective communication in optical networks was originally addressed in [2] and many models, solved by heuristic procedures, specially concerning multicast trees where only a subset of network nodes are interconnected, have been proposed. An overview of contributions on multicast trees with reliability requirements can be seen in [36].

Concerning the importance and challenges associated with resilient network design problems of which our model is a very specific case, reference [29] provides an analysis of these issues and an identification of relevant research problems. The author also puts in evidence the importance of a multilayer network representation, in particular crosslayer mapping, in this context. A recent paper [28] proposes a general methodology for evaluating network resilience by using a combination of various techniques, namely topology generation, analytical simulation and experimental emulation. A particular model focused on the construction of overlay multicast networks, using predefined routing trees, is presented in [20]. The formulated problem aims at obtaining maximal throughput with survivability constraints associated with the limitation of throughput losses for single failures in virtual links.

Note that although, from an application point of view, broadcasting can be seen as a special extension of multicasting, it is a distinct problem in terms of combinatorial optimization and as such, its mathematical properties can be explored taking into account the nature of the objectives we seek to optimise, as will be discussed in this paper. Since the focus of our work is the calculation of broadcast/spanning trees in telecommunication networks with reliability/minimal SRLG requirements we will refer to some earlier works more closely related to the specific issue of reliable broadcast trees on optical networks. Several works focussed on the problem designated as node-protected multicast tree pair problem(NP-MTP), involving the calculation of two trees such that at least one of them remains connected in the event of any node or edge failure [17]. Heuristics for solving this type of problem were developed in [22,34,1]. Concerning the special case of broadcast trees for solving the NP-MTP problem, an algorithm for calculating a low cost NP-MTP is given in [35]. The work by Zhu and Jue [36] proposes a maximal reliable collective communication model for optical networks with SRLG and failure probability information. The formulated optimization problem is NP-hard and seeks to obtain a spanning tree which minimises a reliability function. The authors propose 'a greedy' heuristic for solving this problem and provide an experimental study to analyse its performance in networks with random distributions of the SRLGs assigned to the logical network links, including a comparison with the results of an Integer Linear Programming formulation of the problem.

Regarding other telecommunication network optimization models based on spanning trees, reference [14] compares integer programming directed formulations of the capacitated minimal spanning tree problem (see [31]), involving the determination of a rooted spanning tree with minimal cost such that each of the subtrees of the root node contains at most a given number of K nodes; this corresponds to a common formulation of the terminal layout problem in local network design. This work proposes a single flow formulation of the problem with coupling constraints and analyses its possible advantages. The use of generalized capacitated trees for formulating the problem of layout and clustering in local network topology design was later addressed in [15]; the authors proposed a capacitated single-commodity network flow formulation of this problem and presented two heuristics for tackling it, considering LP relaxation. Paper [25] addresses a load balancing optimization problem concerning a routing method based on the calculation of multiple spanning trees (where one spanning tree is assigned to each demand), with applications to Ethernet type networks. A lexicographic optimization formulation of the problem, considering two load balancing objective functions, is proposed, and exact methods and heuristics for its resolution, are described. A model to construct locally overlay routing structures of ad-hoc wireless networks, based on a family of distributed algorithms to build spanning trees on the underlying network, is described in [21].

In general, multiple routing problems in modern telecommunication networks involve the calculation of network sub-graphs, typically paths or trees, satisfying technical constraints (namely QoS constraints) and seeking to optimise relevant network performance and/or cost metrics. Therefore, we think that in many such problems there are potential advantages in developing explicitly multicriteria routing approaches, carefully adapted to the envisaged routing framework and the most relevant network features.

From a methodological point of view the inherent advantage of multicriteria formulations stems from the fact that these enable trade-offs among the distinct objective functions (metrics to be optimised) to be represented and explored in a mathematically consistent manner. The resolution of a multicriteria optimization problem consists of calculating and selecting non-dominated solutions, also known as Pareto optimal solutions. A non-dominated solution is a feasible solution such that there is no other feasible solution which can (in minimization problems) decrease the value of an objective function without increasing the value of at least one of the other objective functions. Note that, if the objective functions are conflicting, usually the so called ideal optimal solution, which minimises simultaneously all objective functions, is unfeasible. A resolution approach is said to be exact if it enables the exact calculation of all non-dominated solutions of the problem. A review on multicriteria routing models for communication networks including multicast routing is in [9]. A survey on multicriteria minimum spanning tree problems, presenting theoretical results and algorithms, is in [24]. A review on multicriteria path and tree problems including a discussing on exact algorithms and applications is presented by Clímaco and Pascoal [10]. A proposal of a generic conceptual framework for the development of consistently multicriteria routing models in IP/QoS networks is described in [33].

A bicriteria minimum spanning tree routing model for MPLS/overlay networks is presented in [12]. The aim of the model is to calculate non-dominated spanning trees seeking simultaneously to optimise load cost and average delay bound, on MPLS networks. An exact solution to the problem, based on an algorithm in [16] which is a specialised version, for spanning trees, of the NISE (Non Inferior Set Estimation) classical approach in [11], is also described.

1.2 Contributions of the Paper

In this paper we propose a bicriteria model for constructing communication spanning trees over optical WDM networks the objectives of which are the minimization of the bandwidth usage of all the tree links and the minimization of the number of different SRLGs assigned to all the links. The first objective seeks the selected links to be the least loaded, ensuring an increased global traffic carrying capability, and it is a type of metric previously used in single criterion and multicriteria routing optimization models for point to point connections over WDM networks such as in [6] (this model uses a lexicographic approach considering as metrics to be optimised the path length and congestion as a secondary criterion for path selection) and [13] (in this explicitly bicriteria model, topological paths are calculated seeking to optimise simultaneously the link bandwidth usage cost and the number of hops). Bicriteria models for routing over WDM networks, as these, have addressed only point to point routing problems (unicast routing) and used a single layer network representation, so that no resilience objectives are included. The second objective of the model proposed in this paper is the minimization of the number of different SRLGs of the tree edges, hence seeking to maximise the reliability of the tree. Note that the minimization of this objective function alone is the so called 'cardinality version' of the reliable collective communication problem formulated in [36], assuming that all SRLGs have the same failure probability. Although we don't address the more general formulation of "the most reliable collective communication problem" involving the calculation of a spanning tree with maximal reliability for a general probability distribution of the SRLG failures, as treated in [36], the cardinality version of this problem is solved exactly as a by-product of our resolution procedure as shown in this paper. We developed an exact resolution approach for the formulated bicriteria spanning tree problem, based on an extension of the algorithm proposed in [7] for the minimal cost/minimal label spanning tree problem. Suggestions on possible applications of minimum label spanning tree problems to communication networks were outlined in [4] and [10]. Note that while the minimal cost spanning tree problem (MCST) can be solved in polynomial time by using, for example, the classical algorithms by Kruskal [19] or by Prim [23], the minimum label spanning tree problem (which seeks to determine a spanning tree with the minimal number of different labels, assuming that each edge of the network is associated with a label) introduced in [5], was proven in this work to be NP-hard. The algorithm used for solving the proposed bicriteria optimization spanning tree problem is based on an extension of the one in [7] and enables dealing with multiple labels (corresponding to SRLGs) per link and the exact calculation of all non-dominated solutions in relatively short times for most practical ranges of networks, in off-line applications. Note that the addressed model and the associated bicriteria optimization problem is not only substantively different but also more complex, in terms of combinatorial optimization, than the one addressed in [12] previously mentioned, since this considers two additive metrics while in our problem the second metric (number of different SRLGs) is not only non-additive but also the assignment of the SRLGs to each link is a multivalued function.

We also present two sets of application experiments with the proposed model. The first set of experiments concerns the application of the model to obtain bicriteria spanning trees on a virtual network for obtaining tree-based VPNs, constructed as overlay networks over realistic transport optical networks described in the report [3]. In these experiments the SRLGs assigned to the edges of the virtual network reflect the structure of the underlying optical fibre links so that a two layer network representation is explicitly defined. In these experiments the link occupancies are randomly generated according to empirical statistical distributions. In the second set of experiments we considered virtual networks constructed over the reference 14-node US NFS network, included in [3] and in many resilient routing studies on optical networks. In these experiments the SRLGs assigned to the links of the logical network are randomly generated, considering different distributions of the SRLGs, defined in terms of the total number of SRLGs and the mean number of SRLGs per link, similarly to the experimental study in [36] on reliable broadcast trees. The experimental results will show that not only the bicriteria approach is justified, since the trade-offs between spanning tree costs and number of SRLGs/tree resilience can be fully analysed and explored, but also this can be done in relatively short times compatible with a wide range of application environments, namely typical national backbone optical networks.

The major contributions of this work may be summarised as follows:

- proposal and mathematical formulation of a bicriteria optimization model for constructing broadcast/spanning trees over optical networks with SRLG information seeking to minimise the bandwidth usage cost and the number of different SRLGs (hence tending to maximise reliability);
- development of an exact algorithmic approach (based on a previous exact algorithm for the minimal cost/minimal label spanning tree problem) for generating the whole set of non-dominated solutions of the model;
- extensive experimental study involving, on the one hand the calculation of VPNs using bicriteria spanning trees constructed over realistic reference optical networks, considering random link occupancies and, on the other hand, the calculation of spanning trees built over a reference transport network, with random SRLGs assignments; the number of non-dominated solutions and CPU times in the two sets of experiments will be presented in order to assess the applicability and potential advantages of the model;
- specification of two alternative methods for selecting a final trade-off solution, considering either an interactive procedure or an automated selection procedure.

Concerning the assumptions underlying the application of the model, we consider, as in previous studies in this area (namely [36]) that: the optical networks are bidirectional, the communication spanning trees are one-to-many directional; only single failures may occur at a given time; wavelength conversion is applied so that feasible broadcast trees are not limited by wavelength continuity constraints.

The contents of the paper are as follows. The next section presents the notation and assumptions of the model and formulates the bi-criteria spanning tree problem. Section 3 describes the exact algorithm developed for calculating the set of non-dominated solutions, after reviewing the main steps of the algorithm in [7] and its mathematical foundations. Section 4 has three parts. The first part describes the experiments regarding the application of the model to the calculation of VPNs built over two reference transport networks: the US optical network (with 14 nodes) and the Germany optical network (with 15 nodes) given in [3], according to certain rules enabling the virtual links and the associated SRLGs, corresponding to optical links to be specified, and for random link occupancies. The second part describes and discusses the results obtained with virtual broadcast networks constructed over the US–NSF network topology, considering various distributions of random SRLG assignments to the links and given the link occupancies. Two alternative methods for selecting a final solution will be given in the third part of this section by considering two application scenarios. Finally the conclusions of this study will be drawn in the last section.

2 Model Description

2.1 Notation and Assumptions

Let us consider an undirected network $(\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ denotes the set of n nodes and $\mathcal{A}$ the set of m edges, defining the logical (or virtual) network topology. Each edge $a \equiv \{i, j\}$ connecting the nodes i and j has an associated bandwidth usage cost depending on the total capacity C_a of the logical edge (or logical link), expressed in the total number of wavelengths (λs) in the associated optical fibre link (s) and on the current number of the available λs (i. e. non occupied λs) in the link a, b_a.

Let R be a set that represents the risks associated with failure situations in the physical (or transport) network that may affect the operational state of the edges, for example a fibre cut or a card failure. Let us denote by A_r the set of edges in $\mathcal{A}$ which can be affected by risk $r \in R$. Thence A_r defines the SRLG associated with r. The set of risks which may affect edge a is denoted by R_a and can be obtained straightforwardly from A_r $(r = 1, 2, \dots, \|R\|)$:

$$R_a = \{r : a \in A_r\} \tag{1}$$

We assume that complete SRLG information is given in the form of $\{A_r\}$. This assumes that either an explicit or an implicit two-layer network representation is given. In the former case the specification of the mapping of physical links (and of the corresponding risks of failure or of failure of the adjacent physical nodes) is given and enables $\{A_r\}$ to be obtained. In the latter case an *a priori* knowledge of the $\{A_r\}$ associated with the edges of the logical network is assumed.

Hereafter, we will designate as *logical network with SRLG information* the structure represented mathematically by $(\mathcal{N}, \mathcal{A}, \mathcal{C}, \mathcal{R})$ where $(\mathcal{N}, \mathcal{A})$ is the logical network topology, $\mathcal{C}$ is the set of edge capacities C_a and $\mathcal{R}$ denotes the set $\{R_a : a \in \mathcal{A}\}$

It is further assumed that only single failures occur at any given time and that wavelength conversion is applied so that the availability of a certain number of wavelengths in any logical link is not limited by wavelength continuity constraints. In these conditions the bandwidth usage of an edge may be simply expressed in terms of b_a, the number of λs available in a.

2.2 Bicriteria Model

Now we will describe the bicriteria optimization model proposed for calculating broadcast trees, defined as spanning trees in the logical network with SRLG information. A spanning tree $\mathit{\Gamma}$ is specified by a loopless subgraph $(\mathcal{N}, \mathcal{A}')$ of $(\mathcal{N}, \mathcal{A})$ with $\mathcal{A}' \subseteq \mathcal{A}$ and the SRLGs associated with the tree edges are defined straightforwardly by the corresponding risk set:

$$R(\mathit{\Gamma}) = \{r \in R : \exists_{a \in \mathcal{A}'} : a \in \mathcal{A}_r\} \tag{2}$$

or

$$R(\mathit{\Gamma}) = \bigcup_{a \in \mathcal{A}'} R_a \tag{3}$$

The first objective function Z_1 is expressed in the bandwidth usage of the tree edges and the associated cost coefficient c_a, for each edge a, is the inverse of the available bandwidth, b_a, i. e. $c_a = 1/b_a$. This type of additive metric has been used in multiple routing models in WDM networks and the minimization of the corresponding objective function seeks a balanced traffic distribution throughout the network, hence favouring the maximisation of the total traffic carried and of the corresponding expected revenue:

$$Z_1 = c(\mathit{\Gamma}) = \sum_{a \in \mathcal{A}'} c_a = \sum_{a \in \mathcal{A}'} \frac{1}{b_a} \tag{4}$$

The second objective function Z_2 is the number of different SRLGs associated with the tree edges. Since there is a one to one correspondence between risks and SRLGs (indeed each risk r specifies the index of the SRLG A_r), the minimization of the number of SRLGs tends to maximise the tree reliability and it is exactly equivalent to the maximisation of the tree reliability (i. e. the probability of none of its edges becoming inoperational due to any risk/failure) if all the risks have equal probability and are statistically independent.

Therefore the second objective function is:

$$Z_2 = \|R(\mathit{\Gamma})\| \tag{5}$$

Let us denote by D, the set of feasible spanning 'light' trees, as the set of spanning trees with at least one free wavelength in all its links:

$$D = \{\mathit{\Gamma} = (\mathcal{N}, \mathcal{A}') : b_a \geq 1 \quad \forall a \in \mathcal{A}'\} \tag{6}$$

The bicriteria spanning tree optimization problem $\mathcal{P}$ in $(\mathcal{N}, \mathcal{A}, \mathcal{C}, \mathcal{R})$ is formulated as:

Problem $\mathcal{P}$

$$\begin{cases} \min\limits_{\Gamma \in D} Z_1 = \sum\limits_{a \in \mathcal{A}'} c_a & (\mathcal{P}_1) \\ \min\limits_{\Gamma \in D} Z_2 = \|R(\Gamma)\| & (\mathcal{P}_2) \end{cases}$$

The existence of a one–to–one correspondence between risks and SRLGs implies that the minimization problem $\mathcal{P}_2$ is equivalent to a minimal label spanning tree problem, by assigning a label to each risk. Therefore the resolution of $\mathcal{P}$ is equivalent to the resolution of a minimal cost/ minimal label spanning tree problem. The exact algorithm by Clímaco et al. [7] enables the resolution of this problem in a network where each edge is associated with one (and only one) label. Therefore, and noting that each link of the light tree may be associated with more than one SRLG, we developed an extension of this algorithm for dealing with multiple labels per edge, as described in the next section.

3 Resolution Algorithm

We need an algorithm to generate the set of non-dominated spanning tree solutions of the formulated bicriteria problem $\mathcal{P}$. The first objective function is a classical additive cost function and the second one consists in the minimization of the number of different risks/SRLGs associated with the spanning trees. Note that each arc can include several risks. As noted above the implemented algorithm is an extension of the algorithm proposed in [7]. Here we present the theoretical justification as well as the description of the algorithm.

The algorithm in [7] considers just one risk/ label associated with each arc. Having in mind to avoid a complicated notation and to facilitate the explanation of the present algorithm we start by outlining the basis of the former algorithm [7] and then show that the extension is straightforward. In fact it is sufficient when considering an edge in the algorithm to consider all the different risks associated with it and then follow a procedure similar to the original one. Let us associate the risks r_{ij}^m and a cost $c_{ij} \equiv c_a$ with each edge $a \equiv \{i, j\}$ of the undirected logical network with SRLG information $(\mathcal{N}, \mathcal{A}, \mathcal{C}, \mathcal{R})$ such that r_{ij}^m is the m^{th} element of R_a (eq. (1)). The cost of a given spanning tree Γ is $c(\Gamma)$ given by (eq. (4)), while $l(\Gamma) = \|R(\Gamma)\|$ represents the number of distinct risks (or labels) of Γ. We look for spanning trees Γ that simultaneously minimise $c(\Gamma)$ and $l(\Gamma)$ in the set of the feasible spanning trees of the network, $\mathcal{D}$. However, when the two objective functions are conflicting there is no solution that minimises both functions simultaneously. Optimality is substituted by the concept of non-dominance. One solution is non-dominated if there is no other feasible solution which improves one objective function without worsening the other. Given two spanning trees Γ and Γ' it is said that Γ dominates Γ', or that Γ' is dominated by Γ (Γ d Γ') if and only if $c(\Gamma) \leq c(\Gamma')$ and $l(\Gamma) \leq l(\Gamma')$ and at least one of the inequalities is strict. Γ' is said to be dominated if and only if there is another spanning tree Γ such that (Γ d Γ') .

Let c^* and l^* denote the minimal cost and the minimal number of risks of any spanning tree, respectively. Let $\hat{c}$ be the minimal cost of a spanning tree with l^* risks, which corresponds to the maximal cost associated with a non-dominated spanning tree. Let $\hat{l}$ be the minimal number of risks of a spanning tree with cost c^*, which corresponds to the maximal number of risks of a non-dominated spanning tree.

Let us recall a Lemma and a Proposition proved in [7], for the corresponding bicriteria problem, with only one label per edge.

Lemma 1 *Let Γ, Γ' be two spanning trees such that $\Gamma' = \Gamma - \{\{x, y\}\} + \{\{x', y'\}\}$ $\{x, y\}$ being a leaving edge and $\{x', y'\}$ an entering edge. Then $l(\Gamma') = l(\Gamma)$ or $l(\Gamma') = l(\Gamma) \pm 1$.*

Proposition 1 *There is at least a non-dominated tree for any l such that $l \in [l^*, \hat{l}]$, except for those $l \in [l^*, \hat{l}]$ for which there exists at least a spanning tree with $l_2 < l_1$ dominating all spanning trees with l_1 labels. In this case the best of those spanning trees (with l_2 labels) is (or are, in the case of alternative optima) non-dominated.*

From this proposition it is possible to propose a new approach to calculate non-dominated spanning trees such that the number of risks $k \in [l^*, \hat{l}]$. Of course, it is enough to calculate the minimal cost spanning tree corresponding to each $k \in [l^*, \hat{l}]$ and check whether some of the obtained solutions are dominated among them. These solutions have to be eliminated.

We need to extend these results to the case where several risks $r_{i,j}^m$ can be associated with the edge $\{i, j\}$. Of course lemma 1 is no more valid. However proposition 1 can be reformulated in the form of the new proposition 2 and the Algorithm 2 presented next is valid.

Proposition 2 *There is at least a non-dominated tree for any l such that $l \in [l^*, \hat{l}]$, except for those $l_1 \in [l^*, \hat{l}]$ for which there is no feasible spanning tree or for those for which there exists at least a spanning tree with $l_2 < l_1$ dominating all spanning trees with l_1 risks. In this case, the best of those spanning trees (with l_2 risks) is (or are, in the case of alternative optima) non-dominated.*

It is not very interesting to check systematically whether several spanning trees are alternative non-dominated solutions with the same cost and number of risks, specially because the computational cost is high and the added information is not very valuable in most of the cases. However it is possible in practical applications to look for some of these solutions in special interesting cases.

The Algorithm

If there is a non-dominated spanning tree with k risks, $k \in [l^*, \hat{l}]$, then it must be a minimal cost spanning tree on some subnetwork of the original network where the set of edges is restricted to have k distinct risks, otherwise it would have the same number of risks and worse cost. In order to find the non-dominated solutions for each number of risks, all the combinations with k out of the L risks in the network are considered. Then, any algorithm for finding the minimal cost spanning tree can be applied on the subnetwork of $(\mathcal{N}, \mathcal{A})$ containing only the edges with those k risks, as described below. Again, this is an NP-hard problem itself, although for a not very large number of distinct network risks, L, this procedure runs with reasonable execution times, as we shall see in the next section.

In the following Algorithm 1, which has to be used by the main algorithm (Algorithm 2) that calculates the non-dominated spanning trees, minimal cost spanning trees have to be calculated. This can be achieved in polynomial time using, for instance, the algorithms by Kruskal [19] or by Prim [23]. In our implementation we used the Kruskal algorithm.

Algorithm 1: Algorithm to compute the minimum cost spanning tree with at most k risks

- BestCost $\leftarrow +\infty$
- **for** *every subset* B *of* $\{1, \ldots, L\}$ *with* k *elements* **do**
  - $\mathcal{A}' \leftarrow$ subset of $\mathcal{A}$ with all the edges with risks in B
  - $\Gamma \leftarrow$ minimal cost spanning tree in $(\mathcal{N}, \mathcal{A}')$
  - **if** $c(\Gamma) <$ BestCost *and* $l(\Gamma) = k$ **then**
    - BestCost $\leftarrow c(\Gamma)$
    - BestT $\leftarrow \Gamma$

The hardness of the MLSTP (Minimum Label Spanning Tree Problem) makes the value of l^* to be unknown in advance. However, since it is easy to obtain $\hat{l}$, we propose the minimal cost spanning tree to be computed for every number of risks combinations, starting from $\hat{l}$. If for a given k no spanning tree, with k or fewer risks, is found, when all k risk subnetworks are examined, this means the optimal value of l has been found, $l^* = k+1$, and the procedure can be halted, as prescribed in Algorithm 2.

The required algorithm (Algorithm 2) may now be formulated. In the algorithm $\mathcal{T}_{ND}$ denotes a set that contains non-dominated spanning trees.

Algorithm 2: Algorithm to compute non-dominated spanning trees with minimal cost/minimal number of risks

- $\Gamma \leftarrow$ minimal cost spanning tree
- $k \leftarrow l(\Gamma)$; $\mathcal{T}_{\mathcal{ND}} \leftarrow \emptyset$; continue $\leftarrow$ True
- **while** $k \geq 1$ *and* continue **do**
  - $\Gamma \leftarrow$ Obtain a minimal cost spanning tree with at most k risks using Alg. 1
  - **If** Γ *is defined and is not dominated* **Then**
    - $\mathcal{T}_{\mathcal{ND}} \leftarrow \mathcal{T}_{\mathcal{ND}} \cup \{\Gamma\}$
  - **Else If** *no spanning tree was found* **Then**
    - continue $\leftarrow$ False
  - $k \leftarrow l(\Gamma) - 1$

It should be noticed that minimal cost spanning trees, in a network with L risks, might not include all these risks. Then, in a case where there exists an optimum of the cost in a network, for instance with six risks, by using just four risks we can avoid the search for trees with five risks. This enables a potential simplification of the search as provided by the algorithm.

4 Experimental Study

In this section we describe the two sets of experiments on the application of the bicriteria model and analyse their results.

4.1 Experiments with Tree-Based Overlay Networks

The first set of experiments involves the application of the model to obtain bicriteria spanning trees on a logical network, constructed as an overlay network over two realistic transport networks described in the Report [3]. In these experiments the logical networks were obtained by considering that the SRLGs assigned to the logical links reflect the underlying structure of optical fibre links of the physical transport network and considering randomly generated link occupations, using three statistical empirical distributions. That is, the assignment of SRLGs to the tree edges is deterministic (defined according to the physical network structure) and the coefficients $c_a = 1/b_a$ of the bandwidth usage cost Z_1 of the model are random. We considered, in

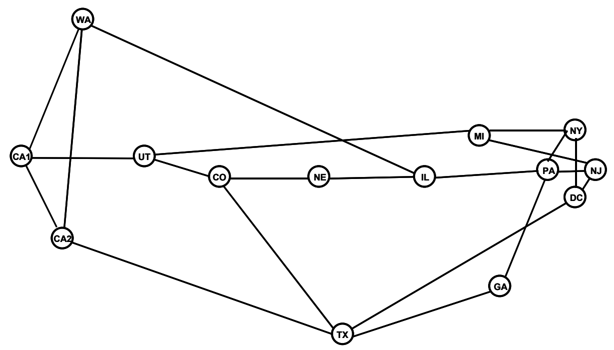

Fig. 1: NSF network [3].

these experiments, only a subset of the transport network nodes so as to simulate the calculation of VPNs encompassing those nodes.

The first experiments were performed taking as transport network the US network based on a former NSF network topology (NSFNET – The National Science Foundation Network, as described in [3]) which has been used in many studies on routing models. This network has 14 nodes and 21 optical links (see Fig. 1) (the average node degree is equal to 3) and in our experiments with the model, the optical link capacities C_a, in terms of wavelengths, were equal to 160 ($C_a = 160\lambda$). The logical network, enabling the simulation of the model application instances, concerning the construction of VPNs based on spanning trees, comprises as node set a subset of the transport network node set N_T, obtained by eliminating three nodes, denoted by UT, NE, GA in Fig. 1, i. e. in the test examples $\mathcal{N} = N_T \setminus \{UT, NE, GA\}$. This might correspond, for example, to a VPN of a corporation with branches in all major cities excepting those three nodes. The logical network edge set and the SRLG assignment were defined according to the following rules: i) each optical link directly interconnecting, in the transport network, two nodes of the logical network, defines an equivalent logical link the SRLG of which is the corresponding optical link; ii) every pair of nodes in $\mathcal{N}$, the physical distance of which is less than or equal to $D_{max} = 2500$ km, is connected by a logical link the SRLGs of which are specified by the sequence of the physical links of the corresponding shortest path (in terms of number of hops) defined in the transport network; iii) an additional logical link was introduced between the nodes CA1 and MI comprising as risks the two physical links that go through the physical node UT (eliminated in the logical network).

In this manner the failure risks/SRLGs reflect, in the defined logical network with SRLG information ($\mathcal{N}$, $\mathcal{A}, \mathcal{C}, \mathcal{R}$), possible failure risks in the transport network

	I_0	I_1	I_2	I_3
Distr. 1	25%	25%	25%	25%
Distr. 2	70%	15%	10%	5%
Distr. 3	18%	18%	18%	46%

Table 1: Empirical distributions.

links. The resulting logical network has $\|\mathcal{N}\| = 11$, $\|\mathcal{A}\| = 26$ and $\|\mathcal{R}\| = 21$ (the average node degree is 4.73).

The available bandwidths, b_a, in the edges of the logical network are randomly generated in four sets of values I_i, $\cup_{i=0}^{3} I_i = \{1, 3, \ldots, 157\}$ where it is assumed that at least 1λ and at most 157λs are available:

$$I_i = \{1 + 2k : k = 20i, \ldots, 20(i+1) - 1\} \quad (i = 0, 1, 2)$$
$$I_3 = \{1 + 2k : k = 60, \ldots, 78\} \tag{7}$$

Three empirical statistical distributions for specifying the percentage of values of b_a in each interval were considered as shown in table 1. The first distribution corresponds to a uniformly loaded network, the second distribution to a heavily loaded network and the third to a lightly loaded network. For each distribution 10 different sets $\{b_a\}$ were randomly generated and the corresponding instances of the bicriteria problem $\mathcal{P}$ were solved.

The results are shown in table 2, indicating for each solution the type of the solution, the value of the bandwidth usage cost Z_1 multiplied by 10^3, the number of SRLGs Z_2 and the total CPU time for obtaining all solutions in each instance of the problem.

A summary of the types of obtained solutions, for the three distributions, is shown in table 3.

The results show that in these experiments the number of non-dominated solutions is typically 2 or 3 excepting for distribution 1, a case in which there are 80% of optimal solutions, that is feasible solutions which minimise simultaneously Z_1 and Z_2. The fact that the number of non-dominated solutions is low results from the manner in which the logical network was constructed, reflecting very closely the topology of the physical network, which tends to originate spanning trees, at logical level, which correspond, in many cases, to spanning trees in the physical network. Noting that the risks/SRLGs of each logical link correspond to one or more physical links and that spanning trees in a graph have a fixed number of edges (equal to $n - 1$ where n is the number of nodes), it can be concluded that there is a very limited variation on the number of risks assigned to the spanning trees of the logical network. This explains why the obtained solutions have a number of SRLGs between 10 and 12, in all cases. In the particular case of distribution 1 (see table 2) the majority of the

Distribution 1				Distribution 2			
Type	Cost	# SRLG	CPU_t(s)	Type	Cost	# SRLG	CPU_t(s)
opt	422.0	10	1.115	n.d.	197.56	11	
				n.d.	382.85	10	1.688
opt	273.95	10	1.059	n.d.	170.53	12	
				n.d.	171.71	11	
				n.d.	197.08	10	2.284
opt	455.16	10	1.062	n.d.	205.94	11	
				n.d.	334.34	10	1.676
opt	288.31	10	1.066	n.d.	169.99	12	
				n.d.	172.04	11	
				n.d.	201.18	10	2.283
n.d.	294.14	11		n.d.	179.85	11	
n.d.	1227.40	10	1.682	n.d.	232.43	10	1.727
opt	244.43	10	1.068	n.d.	158.68	11	
				n.d.	188.17	10	1.698
opt	233.59	10	1.061	n.d.	166.85	11	
				n.d.	200.55	10	1.698
opt	251.64	10	1.056	n.d.	164.70	11	
				n.d.	188.92	10	1.695
n.d.	310.93	11		n.d.	166.03	12	
n.d.	500.73	10	1.704	n.d.	168.65	11	
				n.d.	187.39	10	2.312
opt	226.07	10	1.091	n.d.	187.44	12	
				n.d.	188.73	11	
				n.d.	203.38	10	2.280

Distribution 3			
Type	Cost	# SRLG	CPU_t(s)
n.d.	439.03	11	
n.d.	444.68	10	1.680
n.d.	337.07	12	
n.d.	340.21	11	
n.d.	346.70	10	2.264
n.d.	482.80	11	
n.d.	489.48	10	1.692
n.d.	365.57	12	
n.d.	369.09	11	
n.d.	373.13	10	2.268
n.d.	337.30	12	
n.d.	342.46	11	
n.d.	1275.72	10	2.258
n.d.	265.32	11	
n.d.	271.04	10	1.687
n.d.	337.48	11	
n.d.	342.38	10	1.685
n.d.	287.10	11	
n.d.	291.70	10	1.685
n.d.	338.51	12	
n.d.	343.70	11	
n.d.	533.49	10	2.273
n.d.	268.55	11	
n.d.	276.92	10	1.762

Table 2: Solutions for NSF based logical network (opt: optimal solution; n.d.: non-dominated solution).

Distr.	Opt. Sol.	2 Non-Domin. Sol.	3 Non-Domin. Sol.
1	80%	20%	–
2	–	60%	40%
3	–	60%	40%

Table 3: Percentage of optimal/non-dominated solutions for the NSF based logical network.

ideal optimal solutions are feasible which can be further explained by a very low variability of the total link bandwidth usage cost since here we are considering a uniform distribution of available bandwidths. That is, for this distribution, it is very likely that a minimal SRLG spanning tree (with 10 SRLGs) may correspond to a minimal cost spanning tree.

In this type of application of the model the choice of a final solution by the network designer is greatly facilitated by the limited number of Pareto solutions. If there is no optimal solution then the practical option is simply between a minimal SRLG tree (with l^* associated risks, $l^* = 10$ in these experiments) with higher cost and a tree with lower cost and $l^* + 1$ associated risks.

As for the total CPU time (obtained in a Dual Core AMD Opteron at 2.7 GHz – 4GB RAM) it varies between 1.056s and 2.312s, with an average value of 1.68s. These values are clearly compatible not only with off-line applications but also with automated dynamic applications – assuming that up-dates of the bandwidth accupations in the links are periodically collected – with low up-dating periods.

The second experiment with the model was of similar type as the one described above, but now considering as physical network the reference hypothetical German backbone network described in [3] also used, for example, in [18].

This network, depicted in figure 2, has 17 nodes and 26 optical links and 3.059 average node degree. The logical network, used for constructing VPNs, was obtained by considering a subset of the set N_T of nodes of the transport network, $\mathcal{N} = N_T \setminus \{HB, B, Do, F, S\}$. The logical links were defined using the rules i) and ii) as in the previous experiment but now considering a maximal distance $D_{max} = 280$ km for the logical links, so as to reflect a smaller distance scale. The assignment of SRLGs to the logical links uses the same rules, hence reflecting the failure risks in the underlying transport network links. Three additional logical links were included: between nodes K and L (comprising as risks the two physical links (K,F) and (F,L)); between nodes K and N (comprising as risks the two physical links (K,F) and (F,N)) and between nodes HH and L (comprising as risks the physical links (HH,B) and(B,L). The resulting logical network has $\|\mathcal{N}\| = 12$, $\|\mathcal{A}\| = 20$ and $\|\mathcal{R}\| = 22$ (the average node degree is 3.33).

The experiment also considered random bandwidth occupations in the links, obtained with the same empirical statistical distributions as in table 1, for the same capacity $C_a = 160\lambda$.

The results are shown in table 4, for distributions 1, 2 and 3 and were obtained for 10 instances of the problem $\mathcal{P}$, for each distribution. A summary of the types of solutions obtained for the three distributions, is in table 5.

The number of non-dominated solutions varies between 2 and 3, excepting in the case of distribution 2 where there were two instances (20%) also with feasible optimal solution.

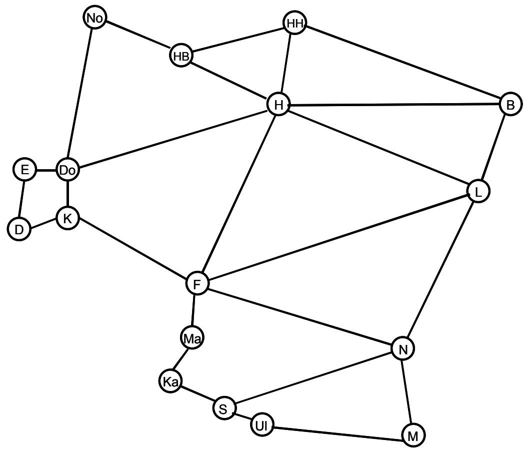

Fig. 2: Germany reference network[3].

Distribution 1				Distribution 2			
Type	Cost	# SRLG	CPU$_t$(s)	Type	Cost	# SRLG	CPU$_t$(s)
n.d.	334.13	15		n.d.	239.03	15	
n.d.	350.58	14	0.706	n.d.	245.32	14	0.704
n.d.	368.42	15		n.d.	244.81	15	
n.d.	378.32	14	0.707	n.d.	245.72	14	0.722
n.d.	367.39	15		n.d.	238.03	15	
n.d.	379.61	14	0.688	n.d.	240.43	14	0.715
n.d.	584.72	15		n.d.	242.72	15	
n.d.	604.13	14	0.762	n.d.	249.12	14	0.690
n.d.	1256.35	15		n.d.	271.49	15	
n.d.	1273.90	14	0.696	n.d.	272.22	14	0.690
n.d.	293.39	15		n.d.	244.15	15	
n.d.	309.07	14	0.696	n.d.	245.81	14	0.710
n.d.	444.43	15		opt	245.89	14	0.578
n.d.	454.13	14	0.665				
n.d.	476.74	15		opt	244.50	14	0.581
n.d.	494.18	14	0.658				
n.d.	1340.86	15		n.d.	234.37	15	
n.d.	1362.93	14	0.646	n.d.	246.01	14	0.694
n.d.	1559.88	15		n.d.	250.98	15	
n.d.	1577.71	14	0.898	n.d.	255.36	14	0.699

Distribution 3			
Type	Cost	# SRLG	CPU$_t$(s)
n.d.	450.18	15	
n.d.	770.27	14	0.692
n.d.	549.46	15	
n.d.	735.23	14	0.685
n.d.	473.78	15	
n.d.	603.28	14	0.711
n.d.	680.83	15	
n.d.	777.75	14	0.721
n.d.	1520.78	16	
n.d.	1596.53	15	
n.d.	1694.40	14	0.746
n.d.	501.40	15	
n.d.	553.78	14	0.682
n.d.	712.08	15	
n.d.	755.60	14	0.732
n.d.	633.85	15	
n.d.	670.09	14	0.675
n.d.	1474.11	15	
n.d.	1508.39	14	0.668
n.d.	1737.19	15	
n.d.	1766.44	14	0.676

Table 4: Solutions for the Germany based logical network.

Distr.	Opt. Sol.	2 Non-Domin. Sol.	3 Non-Domin. Sol.
1	–	100%	–
2	20%	80%	–
3	–	90%	10%

Table 5: Percentage of optimal/non-dominated solutions for the Germany based logical network.

The results for these experiments follow broadly the same patterns as in the experiments based in the US network. The variability in the number of non-dominated solutions is higher than in the former experiments but it is still relatively limited, although there are significantly fewer optimal solutions. The explanation for this type of results is basically the same, resulting, as previously explained, from the way in which the logical network was constructed so that many spanning trees in the logical network correspond to the spanning trees in the physical network, hence limiting the variability in the number of risks associated with logical trees.

As for the CPU times, they vary between 0.578s and 0.89s and are still lower than in the previous experiments. This has to do mainly with the lower connectivity of the German reference transport network (0.191) as compared with the US reference transport network (0.2307), leading to lower number of candidate solutions to the problem. This is highly compatible with dynamic application environments with relatively short up-dating periods.

Concerning the choice of a final solution, the same type of option applies in these experiments as in the former ones. The choice, when the optimal solution is unfeasible, is again between the minimal SRLG spanning tree (with $l^* = 14$ associated risks in these experiments) and a tree with l^*+1 associated risks and higher bandwidth usage cost.

4.2 Experiments with Random SRLGs

In the second set of experiments we considered virtual networks with the topology of the reference 14-node US NFS network, in Fig. 1, and also used in many resilient routing studies on optical networks. In these experiments the SRLGs assigned to the links of the logical network are randomly generated, considering different distributions of the SRLGS, defined in terms of the total number of SRLGs $L = \|R\|$ and the mean number of SRLGs per link, α, similarly to the experimental study in [36] on reliable broadcast trees, with the difference that a random set of the available bandwidths $\{b_a : 2 \leq b_a < c_a, \forall a \in \mathcal{A}\}$ is considered for each of the three distributions specified in table 1, leading to random values of the coefficients of the objective function Z_1 (bandwidth usage cost). The SRLGs are assigned to the links according to uniform distributions with parameters L, α and 10 sets of SRLG assignments were randomly calculated for each specification of (L, α), with $L \in \{15, 20, 25\}$ and $\alpha \in \{4, 6, 8, 10\}$.

The minimal, average and maximal number of non-dominated solutions obtained for each distribution of the available bandwidth and each value L, in terms of α, are depicted in figure 3. The percentages of cases with minimal, average and maximal number of non-dominated solutions for each of the sub-sets of experiments in figure 3, are shown in figure 4.

For fixed $L = 15$ SRLGs the number of non-dominated solutions varies from one (feasible optimal solution) to 4, for $\alpha = 4$, for the three distributions and is minimal for $\alpha = 10$ in all cases. Note that for $\alpha = 10$ the average number of SRLGs per link is $2/3$ of the total number of SRLGs, so that it is likely that a relatively large number of spanning trees are minimal SRLG trees, so that is very likely that at least one of them is also a minimal cost tree. Therefore for $L = 15$ the percentage of optimal solutions tends to increase with α and is maximal for $\alpha = 10$.

For $L = 20$ SRLGs the number of non-dominated solutions has a wider range of variation from 1 (feasible optimal solution) to 5 and the percentage of cases with non-optimal solutions increases with respect to the experiments for $L = 15$. This results from the fact that α/L, the fraction of the SRLGs assigned in average to each link, is lower than for $L = 15$ so that the likelihood of some maximally reliable tree being also a minimal cost tree decreases. In most cases the maximal, average and minimal number of non-dominated solutions tends to decrease as α increases.

For $L = 25$ SRLGs the number of non-dominated solutions has an even wider range of variation, from 1 to 7. This may be explained by the decrease in the ratio α/L as compared with the previous sets of experiments. The total percentage of cases with non-optimal solutions is also larger than in the previous experiments, for similar reasons.

Considering all these experiments, it can be said that the most important factors conditioning the number of non-dominated solutions of the problem instances are the parameters L and α, specifying the SRLG distribution, rather than the distribution of the available bandwidths. Nevertheless, the uniform distribution of available bandwidth will favour, statistically, the occurrence of feasible optimal solutions.

Globally one can conclude that the consideration of a bicriteria formulation of the spanning tree problem is clearly justified since in most cases there is more than one non-dominated solution, so that eventual trade-offs

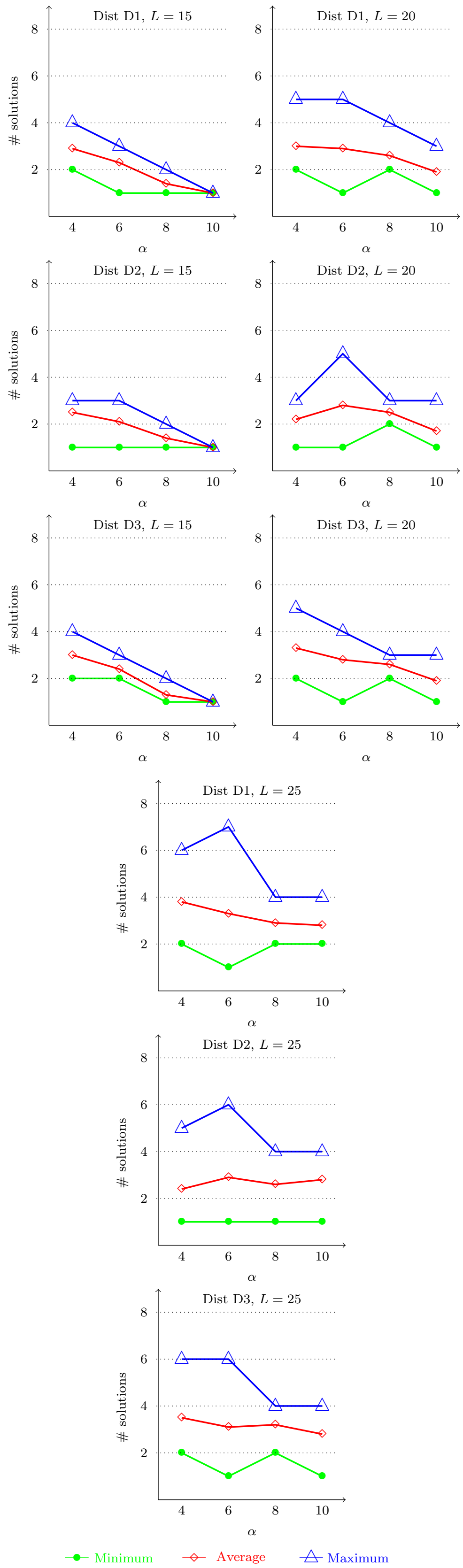

Fig. 3: Number of non-dominated solutions for random SRLG assignment.

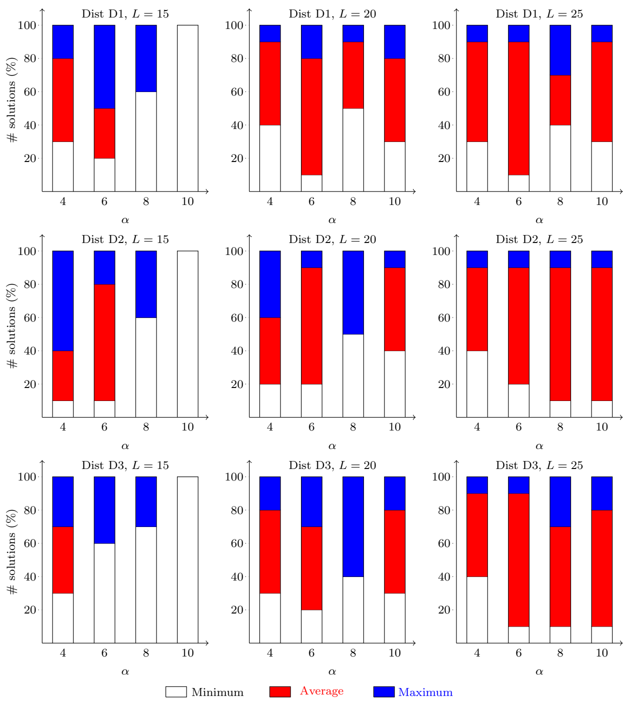

Fig. 4: Percentage of non-dominated solutions for random SRLG assignment.

between the two objective functions can be systematically analysed.

The minimal, average and maximal CPU times obtained for the previously mentioned distributions of SRLGs and different traces of available bandwidths are shown in figure 5. The CPU times for $L = 15$ SRLGs are omitted since the maximal values were always less than 0.02 s (20 ms) and can be disregarded in comparison with the results obtained for $L = 20$ and $L = 25$. The CPU times for $L = 20$ are at most 0.2 s (for $\alpha = 4$) and are less than 20 ms in the other cases, for all distributions of the available bandwidths, and tend to decrease as α increases. For $L = 25$ the CPU times are in the worst case ($\alpha = 4$) equal to 11 s and fall rapidly as α increases.

The observed pattern of variation of the CPU times is tendentially congruent with the variation of the number of calculated non-dominated solutions, which might be expected, taking into account the features of the proposed exact resolution algorithm.

4.3 Solution Selection Methods

In our view, the method to be used for choosing a solution in the set $\mathcal{T}_{ND}$ of non-dominated solutions calculated by the algorithm will have to take into account two essential features of the decision environment,

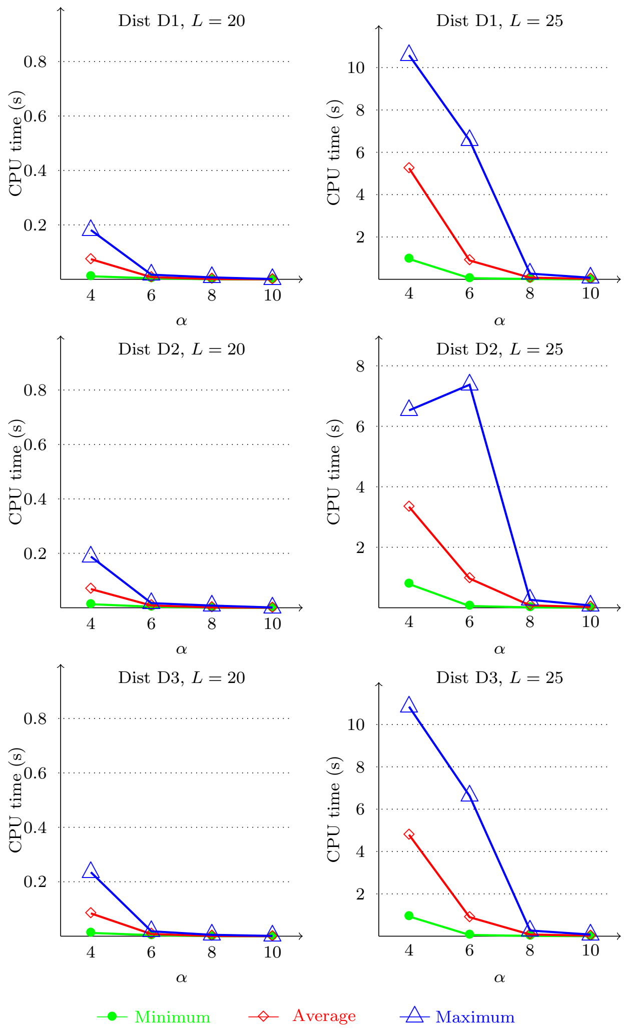

Fig. 5: CPU times for random SRLG assignment.

The first aspect to be taken into account is the network loading status. If the network is lightly loaded or in nominal loading conditions, for which it was designed, then the reliability metric, corresponding to the number of different SRLGs, should be given priority. In these situations the network designer might choose either the minimal SRLG tree Γ^* with l^* SRLGs and bandwidth cost $\hat{c}$ (the minimal cost of the minimal SRLG spanning trees, considering the possibility of multiple optimal solutions to min Z_2) or, as a second alternative, the spanning tree Γ_2 with $l_2 = \|R(\Gamma_2)\|$ immediately greater than l^* if he/she thinks the bandwidth cost reduction $\Delta_c = c(\Gamma^*) - c(\Gamma_2)$ is sufficient to justify the penalty in terms of the increase in the number of associated risks, $\Delta_l = l_2 - l^*$, usually equal to 1 in most practical instances of the problem. Examples of this type of situation were shown in section 4.1. Note that the identification of Γ_2 is immediate since the algorithm

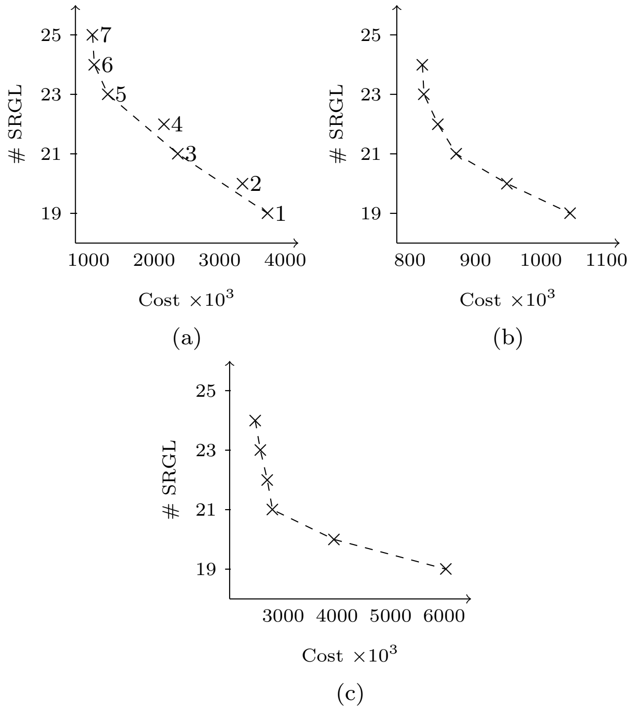

Fig. 6: Non-dominated solutions for selected problems with random SRLG assignment.

enables the ranking of non-dominated solutions by increasing value of l. The choice of the solution, in this case, could be made directly by the network designer by looking at the table with the solution features or, in an automated manner, by setting a threshold value for $\varDelta_c/c(\varGamma^*)$, for considering the alternative solution $\varGamma_2$.

This selection method might correspond to an *interactive decision procedure*, involving a simple interaction between the network designer and the computational algorithm, compatible with an off-line application of the model.

To illustrate this issue of solution selection, examples of solution sets $\mathcal{T}_{ND}$ for three instances of the problem in experiments with random SRLGs (section 4.2), represented in the objective function space, are shown in figure 6. Table 6 shows the objective function values of the solutions for the case of figure 6(a). In this case $\varGamma^*$ corresponds to the point 1 and $\varGamma_2$ to the point 2 in the figure. Note that, in the example in figure 6(a), one can identify supported non-dominated solutions (i. e. solutions located on the boundary of the convex hull of the feasible solution set) and two unsupported non-dominated solutions, corresponding to the points 4, 2 (solutions located in the interior of that hull). This illustrates that the algorithm, being exact, enables the finding of unsupported non-dominated solutions whenever they exist. Furthermore, note that in this decision scenario, one could stop the algorithm execution after obtaining $\varGamma^*$ and $\varGamma_2$, which would substantially reduce the computational execution time. This might be important in applications to networks of great dimension and great connectivity, where execution times may become a relevant factor in the evaluation of the efficiency of the developed approach.

A second decision scenario might occur in heavily loaded networks, situations in which the establishment of spanning tree based overlay networks (on a semi-permanent base) corresponds to a significant occupation of the transmission resources, which represents a limitation of the traffic carrying capability of the network, namely concerning future optical connection requests (point to point light paths or multicast connections). In this type of decision scenario the bandwidth usage may be considered an objective to be taken with a priority similar to the one associated with the number of failure risks. In this case we could consider an interactive decision procedure or a fully automated procedure for final solution selection.

Sol.	Cost$\times 10^3$	# SRLG
1	3731.48	19
2	3350.00	20
3	2362.40	21
4	2150.47	22
5	1295.36	23
6	1088.26	24
7	1060.63	25

$l^* = 19$
$l_{\text{req}} = 22$
$l_{\text{acc}} = \hat{l} = 25$

$c^* \times 10^3 = 1060.6276$
$c_{\text{req}} \times 10^3 = 2396.0515$
$c_{\text{acc}} \times 10^3 = \hat{c} \times 10^3 = 3731.4754$

Table 6: Objective function values and preference thresholds for the example in Figure 6(a).

The *automated selection procedure* could be, in this case, an adaptation of the selection method for non-dominated light paths in WDM networks proposed in [13] which is partially based on the reference point based approach in [8]. The method combines the use of preference thresholds for the objective functions, defining priority regions in the objective functions space, with a Chebyshev distance to reference points.

Taking into account the discrete nature of the function Z_2, the preference thresholds corresponding to required (with index *req*) and acceptable (with index *acc*) levels for the number of SRLGs and the bandwidth usage cost, are given by:

$$\begin{cases} l_{req} = \left\lfloor \frac{l^*+\hat{l}}{2} \right\rfloor \\ l_{acc} = \hat{l} \end{cases} \quad \begin{cases} c_{req} = \frac{c^*+\hat{c}}{2} \\ c_{acc} = \hat{c} \end{cases} \tag{8}$$

where $\lfloor x \rfloor$ denotes the integer part of x and:

$$l^* = \min Z_2 = \|R(\varGamma^*)\| \tag{9}$$

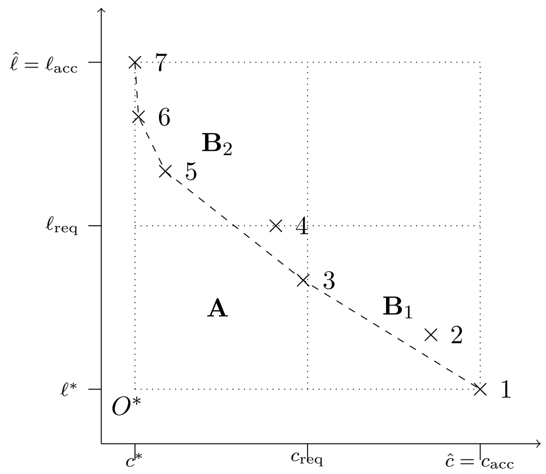

Fig. 7: Automated choice of the solutions in Figure 6(a).

$$c^* = \min Z_1 = c(\Gamma')$$
$$\hat{c} = \min c(\Gamma^*), \ \forall \ \Gamma^* : l(\Gamma^*) = l^*$$
$$\hat{l} = \min l(\Gamma'), \ \forall \ \Gamma' : c(\Gamma') = c^*$$

In this manner we define priority regions in the objective function space, according to figure 7. Region A is the first priority region where the requested values for the two functions are satisfied simultaneously. In the second priority regions $B1$ and $B2$, only one of the requested value is guaranteed while the acceptable value for the other metric is also satisfied. A further preference order between these regions is introduced by giving preference to solutions with fewer SRLGs, that is solutions in $B1$ are given preference over solutions in $B2$.

As for the selection of a solution when there is more than one non-dominated solution in a higher priority region S, we would use a reference point type approach, and consider that the 'form' of the region where solutions are located reflects the user's preferences. This leads to a reference point based procedure as proposed in [8], by considering as reference point the 'left bottom corner' of region S, which coincides with the optimal ideal solution corresponding to point O^*, if $S = A$.

In general, reference type approaches minimise the distance of the solution images, in the objective function space, to a specific point, by recurring to a scalarizing function. In this context we would use a weighted Chebyshev metric proportional to the size of the 'rectangle' $S \in \{A, B1, B2\}$. Therefore, the procedure would choose the solution:

$$\Gamma_f = \arg \min_{\Gamma \in S^c_{\mathcal{N}}} \max_{i=1,2} \{w_i |Z_i(\Gamma) - \overline{Z}_i|\} \tag{10}$$

where $S^c_{\mathcal{N}}$ is the set of non-dominated solutions which correspond to the points in S, $Z_1(\Gamma) \equiv c(\Gamma)$, $Z_2(\Gamma) \equiv l(\Gamma)$ and $(\overline{Z}_1, \overline{Z}_2)$ is the considered reference point, the left bottom corner of S. The weights w_i are calculated in order to obtain a metric with dimensional free values:

$$w_i = \frac{1}{M_i - m_i} \tag{11}$$

where $m_i = \overline{Z}_i$ and (M_1, M_2) is the right top corner of S.

In the illustrative example of figure 7 the reference point for A corresponds to the ideal optimal solution $O^* = (c^*, l^*)$, for region $B1$ is (c_{req}, l^*) and for region $B2$ is (c^*, l_{req}). Applying this method, in this case, (see threshold values in table 6) the selected solution would be solution 3.

In this instance of the problem if now we considered a 'standard' situation of not heavily loaded network, according to the proposed criterion one would select either Γ^* corresponding to solution 1 (minimal SRLG tree with $l^* = 19$ or Γ_2 corresponding to solution 2, with $l_2 = 20$ if the relative cost decrease $\Delta_c / c(\Gamma^*)$, equal to 10.2%, was considered by the network designer to justify the 'penalty' of increasing by one the number of associated risks/SRLGs.

The proposed model could also be used in the context of tree-based routing architectures such as the Viking architecture [26] intended for Ethernet metropolitan areas and cluster networks, with built-in rerouting mechanisms for failure protection. In this case, or in similar application contexts, all the spanning tree solutions would be kept in memory and, in the event of a failure (corresponding to risk r') in a link of the selected solution Γ', the routing mechanism would seek a backup spanning tree. This spanning tree, Γ'', to be chosen in the set $\mathcal{T}_{ND}$ of non-dominated solutions, should not include the faulty element corresponding to r' and have maximal reliability, i. e.:

$$\Gamma'' = \arg \min_{\Gamma \in \mathcal{T}_{ND} : r' \notin R(\Gamma)} \|R(\Gamma)\| \tag{12}$$

5 Conclusions

We have proposed a bicriteria model for obtaining spanning trees constructed over optical WDM networks with SRLG information, seeking the minimization of the number of different SRLGs and the minimization of the total bandwidth usage cost of all the tree links. The first objective seeks the maximisation of the tree reliability while the second objective seeks the selected links to be the least loaded, thence increasing the global traffic carrying capability.

An exact algorithm was developed for solving the bicriteria model, based on an extension of the algorithm proposed in [7] for the minimal cost/minimal label spanning tree problem, by considering multiple labels per link.

Two sets of application experiments with the proposed model, were presented. In the first set of experiments, the model was applied to obtain bicriteria spanning trees on a virtual network, for obtaining tree-based VPNs constructed as overlay networks over two realistic transport reference networks, used in earlier routing studies on WDM networks. In these experiments the SRLG assignment reflects the structure of the underlying optical network and the link bandwidth occupancies were randomly generated, so as to reflect possible resource occupation states. In these experiments the number of non-dominated solutions was relatively low and there were various cases for which ideal optimal solutions were feasible. These features of these experiments result from the manner in which the logical network was constructed, so as to reflect the structure of the underlying optical network, in which spanning trees (that by definition have a fixed number of links) to which the failure risks were assigned, may also be minimal cost solutions, in some cases.

In the second set of experiments we considered virtual networks with the topology of a 14–node reference network, used in many studies on resilient routing models. The SRLGs assigned to the logical links were randomly generated, considering different distributions specified in terms of maximal number of SRLGs and average number of SRLGs per link. In these experiments the bandwidth occupations were fixed traces of the considered empirical link state distributions and the number of non-dominated solutions substantially increased with respect to the first set of experiments, depending on the parameters of the SRLG distributions.

Globally we may conclude that the consideration of a bicriteria formulation of the spanning tree construction problem is potentially advantageous since, in a large number of cases, there is more than one non-dominated solution, so that trade-offs between the two objective functions can be systematically analysed. Furthermore, note that the developed exact algorithm, by providing an exact solution to the minimal SRLG spanning tree problem, always supplies as one of the possible solutions the optimal solution to the cardinality version of the "most reliable collective communication problem" previously formulated in [36] (and tackled by a heuristic), assuming that all SRLGs have the same failure probability.

Concerning computational requirements, the algorithm enabled the calculation of all non-dominated solutions in the experiments with random SRLG assignment, with CPU times varying from a small fraction of a second to less than a dozen seconds, in the worst cases. This shows that, for these types of networks, the model may be readily used not only in off-line applications but also in dynamic applications, assuming that updates of the link wavelength occupations are periodically collected, even with low updating periods.

Finally we have discussed and proposed different methods, either interactive or fully automated, for selecting an efficient solution, taking into account essential features of the network designer decision environment. The proposed selection methods, having in mind the efficiency of the resolution algorithm and the features of the methods, indicate the great flexibility in the application of the model. In fact, even in the method for automated selection of a final solution, the network designer may always adjust the level of priority given to reliability 'versus' bandwidth usage cost, by adjusting the specification of the required and/or acceptable levels of these metrics, by point-wise alterations in the corresponding formulae.

References

1. R. Balasubramanian and S. Ramasubramanian. Minimizing average path cost in colored trees for disjoint multipath routing. In *Proc. IEEE ICCCN 2006.*, pages 185–190, Arlington, VA, 9-11 October 2006.
2. J.-C. Bermond, L. Gargano, S. Perennes, A. A. Rescigno, and U. Vaccaro. Efficient collective communication in optical networks. In Friedhelm Meyer and Burkhard Monien, editors, *Automata, Languages and Programming*, volume 1099 of *Lecture Notes in Computer Science*. Springer Berlin Heidelberg, 1996.
3. A. Betker, C. Gerlach, R. Hülsermann, M. Jäger, M. Barry, S. Bodamer, J. Späth, C. Gauger, and M. Köhn. Reference transport network scenarios. Technical report, MultiTeraNet Project, July 2003.
4. R. Cerulli, A. Fink, M. Gentili, and S. Voß. Extensions of the minimum labelling spanning tree problem. *Journal of Telecommunications and Information Technology*, (4):39–45, 2006.
5. R.-S. Chang and S.-J. Leu. The Minimum Labeling Spanning Trees. *Information Processing Letters*, 63:277–282, 1997.
6. X. Chu and B. Li. Dynamic routing and wavelength assignment in the presence of wavelength conversion for all-optical networks. *Networking, IEEE/ACM Transactions on*, 13(3):704 – 715, june 2005.
7. J. Clímaco, M. E. Captivo, and M. Pascoal. On the bicriterion – minimal cost/minimal label – spanning tree problem. *European Journal of Operational Research*, 204(2):199 – 205, 2010.
8. J. Clímaco, J. Craveirinha, and M. Pascoal. An automated reference point-like approach for multicriteria shortest path problems. *Journal of Systems Science and Systems Engineering*, 15(3):314–329, 2006.
9. J. Clímaco, J. Craveirinha, and M. Pascoal. Multicriteria routing models in telecommunication networks –

overview and a case study. In Y. Shi, D. Olson, and A. Stam, editors, *Advances in multiple criteria decision making and human systems management: knowledge and wisdom*, volume edited in honor of Milan Zeleny, chapter 1, pages 17–46. IOS Press, 2007.
10. J. Clímaco and M. Pascoal. Multicriteria path and tree problems: discussion on exact algorithms and applications. *International Transactions in Operational Research*, 19(1-2):63–98, 2012.
11. J. L. Cohon. *Multiobjective Programming and Planning*. Mathematics in Science and Engineering. Academic Press, 1978.
12. J. Craveirinha, J. Clímaco, L. Martins, C. G. Silva, and N. Ferreira. A bi-criteria minimum spanning tree routing model for MPLS/overlay networks. *Telecommunication Systems*, 52(1):203–215, 2013.
13. T. Gomes, J. Craveirinha, J. Clímaco, and C. Simões. A bicriteria routing model for multi-fibre WDM networks. *Photonic Network Communications*, 18(3):287–299, 2009.
14. Luis Gouveia. A comparison of directed formulations for the capacitated minimal spanning tree problem. *Telecommunication Systems*, 1(1):51–76, 1993.
15. Luis Gouveia and Maria João Lopes. Using generalized capacitated trees for designing the topology of local access networks. *Telecommunication Systems*, 7(4):315–337, 1997.
16. H.W. Hamacher and G. Ruhe. On spanning tree problems with multiple objectives. *Annals of Operations Research*, 52:209–230, 1994.
17. V. Holopainen and R. Kantola. Tackling the delay-cost and time-cost trade-offs in computation of node-protected multicast tree pairs. In ChoongSeon Hong, Toshio Tonouchi, Yan Ma, and Chi-Shih Chao, editors, *Management Enabling the Future Internet for Changing Business and New Computing Services*, volume 5787. Springer Berlin Heidelberg, 2009.
18. R. Huelsermann and M. Jaeger. Evaluation of a shared backup approach for optical transport networks. In *Optical Communication, 2002. ECOC 2002. 28th European Conference on*, volume 1, pages 1–2, 8-12 September 2001.
19. J. B. Kruskal. On the shortest spanning subtree of a graph and the traveling salesman problem. In *Proceedings of the American Mathematical Society*, volume 7, pages 48–50, February 1956.
20. Michał Kucharzak and Krzysztof Walkowiak. Modeling and optimization of maximum flow survivable overlay multicast with predefined routing trees. *Telecommunication Systems*, doi: 10.1007/s11235-013-9823-x(Available online):1–19, 2013.
21. Ricardo Marcelín-Jiménez. Locally-constructed trees for ad-hoc routing. *Telecommunication Systems*, 36(1-3):39–48, 2007.
22. M. Médard, S. G. Finn, R. A. Barry, and R. Gallager. Redundant trees for preplanned recovery in arbitrary vertex-redundant or edge-redundant graphs. *IEEE/ACM Trans. Netw.*, 7(5):641–652, October 1999.
23. R. C. Prim. Shortest connection networks and some generalizations. *Bell System Technical Journal*, 36:1389–1401, 1957.
24. S. Ruzika and H. W. Hamacher. A survey on multiple objective minimum spanning tree problems. In Jürgen Lerner, Dorothea Wagner, and Katharina Anna Zweig, editors, *Algorithmics of Large and Complex Networks*, volume 5515 of *Lecture Notes in Computer Science*, pages 104–116. Springer, 2009.
25. Dorabella Santos, Amaro Sousa, Filipe Alvelos, Mateusz Dzida, and Michał Pióro. Optimization of link load balancing in multiple spanning tree routing networks. *Telecommunication Systems*, 48(1-2):109–124, 2011.
26. S. Sharma, K. Gopalan, S. Nanda, and T. Chiueh. Viking: a multi-spanning-tree ethernet architecture for metropolitan area and cluster networks. In *INFOCOM 2004. Twenty-third AnnualJoint Conference of the IEEE Computer and Communications Societies*, volume 4, pages 2283–2294, 2004.
27. M. Srivatsa, B. Gedik, , and L. Liu. Scaling unstructured peer-to-peer networks with multi-tier capacity-aware overlay topologies. In *In Proc. 10th Int.l Conference on Parallel and Distributed Systems*, 2004.
28. James P.G. Sterbenz, Egemen K. Çetinkaya, Mahmood A. Hameed, Abdul Jabbar, Shi Qian, and Justin P. Rohrer. Evaluation of network resilience, survivability, and disruption tolerance: analysis, topology generation, simulation, and experimentation. *Telecommunication Systems*, 52(2):705–736, 2013.
29. David Tipper. Resilient network design: challenges and future directions. *Telecommunication Systems*, doi:10.1007/s11235-013-9815-x(Available online):1–12, 2013.
30. P. Van Mieghem and S. van Langen. Influence of the link weight structure on the shortest path. *Physical Review E*, 71:056113, 2005.
31. Stefan Voß. Capacitated minimum spanning trees. In Christodoulos A. Floudas and Panos M. Pardalos, editors, *Encyclopedia of Optimization*, pages 225–235. Springer US, 2001.
32. H. Wang and P. Van Mieghem. Constructing the overlay network by tuning link weights. In *International Conference on Communications and Networking in China (ChinaCom2007)*, Shanghai, China, August 22-24 2007.
33. A. P. Wierzbicki and W. Burakowski. A conceptual framework for multiple-criteria routing in QoS IP networks. *International Transactions in Operational Research*, 18(3):377–399, 2011.
34. G. Xue, L. Chen, and K. Thulasiraman. Quality-of-service and quality-of-protection issues in preplanned recovery schemes using redundant trees. *Selected Areas in Communications, IEEE Journal on*, 21(8):1332 – 1345, October 2003.
35. W. Zhang, G. Xue, J. Tang, and K. Thulasiraman. Faster algorithms for construction of recovery trees enhancing QoP and QoS. *Networking, IEEE/ACM Transactions on*, 16(3):642 –655, june 2008.
36. Y. Zhu and J. P. Jue. Networking, IEEE/ACM transactions on. *Reliable Collective Communications With Weighted SRLGs in Optical Networks*, 20(3):851 –863, june 2012.

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