Graph Theory in Higher Order Topological Analysis of Urban Scenes

J.-P. de Almeida [a, b, 1], J.G. Morley [a], I.J. Dowman [a]

[a] Dept of Geomatic Engineering, University College London, Gower Street, LONDON WC1E 6BT, UK
(almeida, jmorley, idowman)@ge.ucl.ac.uk
Tel. +44 (0)20 76792740 Fax +44 (0)20 73800453
http://www.ge.ucl.ac.uk
[b] Geomatic Engineering Section, Dept of Mathematics, Faculty of Science and Technology, University of Coimbra, Largo D. Dinis, Apartado 3008, 3001-454 COIMBRA, Portugal
Tel. +351 239 791150 Fax +351 239 832568
http://www.mat.uc.pt

Abstract

Interpretation and analysis of spatial phenomena is a highly time consuming and laborious task in several fields of the Geomatics world. That is why the automation of these tasks is especially needed in areas such as GISc. Carrying out those tasks in the context of an urban scene is particularly challenging given the complex spatial pattern of its elements. The aim of retrieving structured information from an initial unstructured data set translated into more meaningful homogeneous regions can be achieved by identifying meaningful structures within the initial collection of objects, and by understanding their topological relationships and spatial arrangement. This task is being accomplished by applying graph theory and by performing urban scene topology analysis. For this purpose a graph-based system is being developed, and LiDAR data are currently being used as an example scenario. A particular emphasis is being given to the visualisation aspects of graph analysis, as visual inspections can often reveal patterns not discernable by current automated analysis techniques. This paper focuses primarily on the role of graph theory in the design of such a tool for the analysis of urban scene topology.

Keywords: GIS, Topology, Graph theory, Analysis, Visualisation, Understanding

---

1. Introduction

Interpretation and analysis of spatial phenomena is a highly time consuming and laborious task in several fields of the Geomatics world. This is particularly evident given the more accurate, but also the increasingly large, spatial data sets that are being acquired with the new technologies continuously being developed, *e.g.* LiDAR data, (Anders *et al.*, 1999). In addition, these tasks become extremely complex when the

---

[1] Corresponding author.

starting point is an unstructured data set. That is why the automation of these tasks is especially needed in areas such as Geographical Information Science (GISc).

According to some authors (including Eyton, 1993, and Barr & Barnsley, 1996, both cited in Barnsley & Barr, 1998), "the classification process of spatial information to produce land-cover maps for urban areas can be considered fairly straightforward if we compare it with the process of deriving information from those maps on urban land-use, which is normally much more problematic". Carrying out this sort of analysis in the context of an urban scene is particularly challenging given the great number of component elements (*e.g.* buildings, roads and intra-urban open spaces) and their generally complex spatial pattern.

The aim of retrieving structured information translated into more meaningful homogeneous regions, for instance from an initial unstructured data set, can be achieved by identifying meaningful structures within the initial random collection of objects and by understanding their spatial arrangement (Anders *et al.*, 1999). It is believed that the task of understanding topological relationships between objects can be accomplished by applying graph theory and carrying out graph analysis.

A graph-based system for urban scene analysis is still being developed, and this paper describes primarily the role of graph theory in the design of such a tool. The paper is structured as follows. After giving a brief overview of the context of this research in sections 1.1 and 1.2, the theoretical background of our concepts is presented in section 2: the individual steps for the preparation of the unstructured data are identified in section 2.1; details of the construction of the network of connectivity are given in section 2.2, *e.g.* retrieval of polygon adjacency information and how adjacency graphs are represented in the computer; finally, the bases for the analytical analysis method are described in section 2.3. To conclude the paper, an outlook on the next steps of our work is given (especially aspects of the visualisation of the urban scene topology and its analysis are presented).

1.1 Topology

Topology is a particularly important research area in the field of GISc, for it is a central defining feature of a geographical information system (GIS) (Reed, 1999; Theobald, 2001). But, as far as topological relationships between spatial objects are concerned, generally speaking contemporary desktop GIS packages do not support further information beyond the first level of adjacency (Theobald, 2001).

Therefore, one of the first motivations of the research work described in this paper was to focus on scene analysis by building up a technique for the better understanding of topological relationships beyond the first level of adjacency, between GIS vector-based objects.

1.2 Graph theory

Another initial interest of this research was to investigate further the possible use of graph theory for the purposes mentioned in the previous section. Concepts from the mathematical areas of topology and graph theory are valuable for revealing the spatial structure of geographical entities and their spatial arrangement. In fact, these mathematical frameworks have been used so far in different applications of a wide range of fields for that purpose (Barr & Barnsley, 1997; Barnsley & Barr, 1998; Kim & Muller 1999; Bunn *et al.*, 2000; Roberts *et al.*, 2000; Bauer & Steinnocher, 2001; O’Sullivan & Turner, 2001; Nardinocchi *et al.*, 2003; Steel *et al.*, 2003; Barr & Barnsley, 2004).

Graph theory is said to be fairly powerful and elegant based only on a few basic simple principles (Temperley, 1981). Laurini and Thompson (1992) have maintained that this particular tool is "extremely valuable and efficient in storing and describing the spatial structure of geographical entities and their spatial arrangement". Theobald (2001) added that "concepts of graph theory allow us to extend the standard notion of adjacency".

For topological analysis purposes, some geographical entities can be represented by vertices in a graph, and the connections between them by edges in a graph. The combination of vertices and edges forms a graph (Temperley, 1981; Gibbons 1989; Wilson, 1996; Gross & Yellen, 1999). In such a topological graph-based representation of a geographic dataset, information referring, for instance to line shape, compass orientation or line length, is normally thrown away concentrating on the structural components: junctions and connections (Laurini and Thompson, 1992).

2. The graph-based analysis tool

In most applications developed so far, the starting point is to some extent a meaningful data set in terms of the scene. We seek to explore and investigate whether it is possible to start at a level further back, before meaningful data sets are obtained, and hence in this case no prior knowledge of the spatial entities is being assumed.

2.1 Preparation of the polygon data base

To start with, LiDAR data are being used as an example scenario to test the graph-based technique. It is an unstructured data set with no patterns pre-defined and meaningless in terms of urban scene. The data set currently being used has 3m point spacing and contains both ground points and object points reflected from trees, buildings and other small objects above ground level. The data set refers to an area (1470x1530m^2) in Kew, southwest London, including the National Archives building and its neighbourhood, comprising a total of 169819 laser points. To give an idea of the spatial distribution of the range data in vertical terms, the cloud of points is colour coded in Figure 1 according to the points' height.

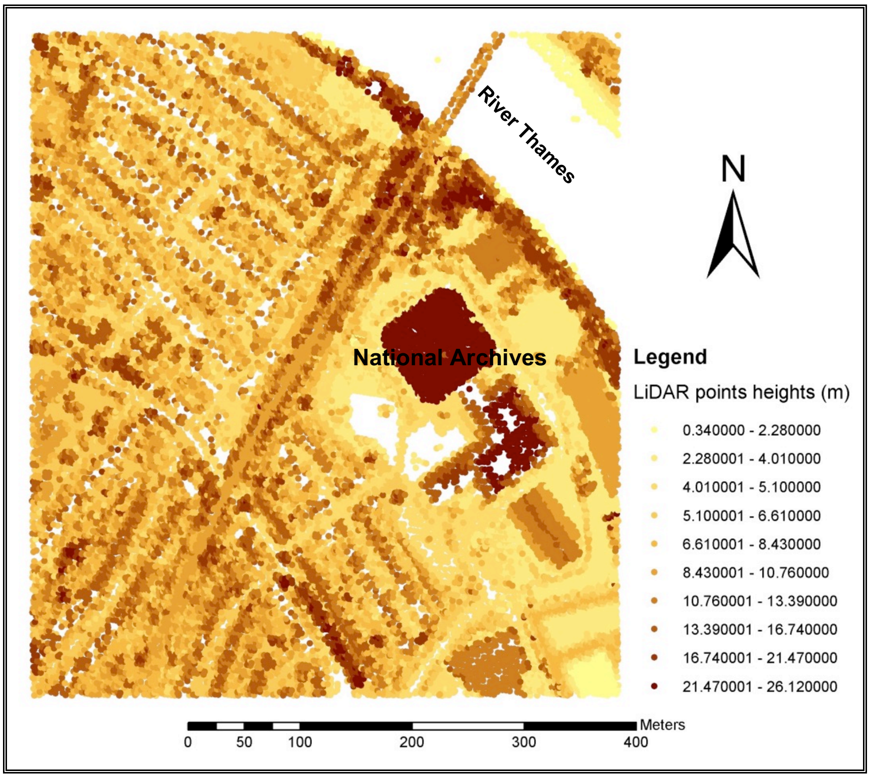


Figure 1. LiDAR data set being used - Kew, southwest London.
Data is colour coded in elevation range.
(After de Almeida *et al.*, 2004a, 2004b)

In order to start structuring information and make it more explicit, some topological information was brought in by establishing a triangulated irregular network (TIN) through the given data set (*vd*. Figure 2), (Toussaint, 1980a; Urquhart, 1982; Edelsbrunner *et al.*, 1983; Kirkpatrick and Radke, 1985). In fact, the generation of the TIN was based upon the Delaunay triangulation which, given the fact that it is a maximal planar description of the given point set internal structure (Kirkpatrick and Radke, 1985), expresses proximities and neighbourhoods between the LiDAR points.

This was accomplished with the 3D Analyst extension of ArcMap (ArcGIS 8.3 environment), using an ArcInfo coverage containing the range point set described above as the input.

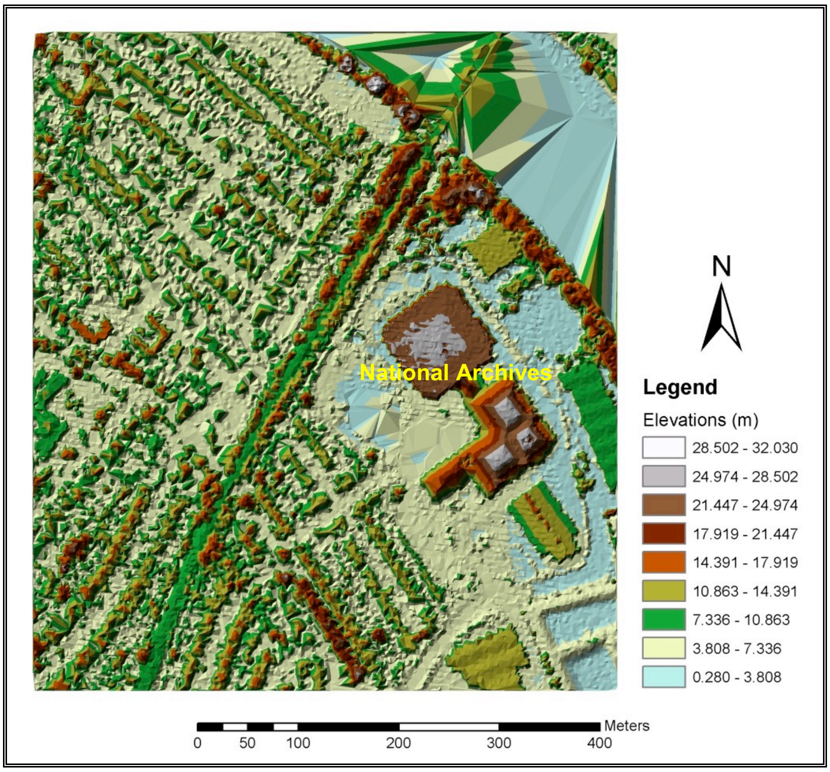


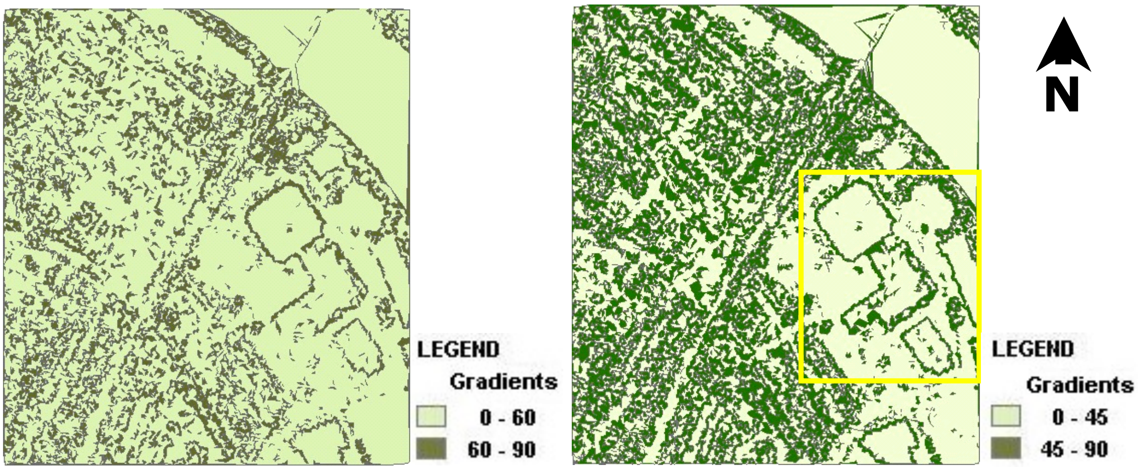
Figure 2. TIN generated from the LiDAR point set.
(After de Almeida *et al.*, 2004a, 2004b)

In terms of GIS analysis, a TIN translates original unstructured point data into what can be defined as a set of "first order connections" in vector domain, *i.e.* spatial relationships between nearest neighbours. By using a graph-based approach the initial aim is to build up networks of connectivity through these data sets, and hence to perform higher-order connectivity analysis. In other words, we seek to investigate and understand the spatial relationships between objects within the context of the whole scene rather than within the context of their own neighbourhood.

After the generation of the TIN a classification was applied to its facets based on their attributes. As the point spacing of this data set is about 3m on the plane, and supposing that the average height of an urban feature is about 5m, a TIN facet against an urban feature and the local terrain has roughly a 60° gradient.

The 60° gradient value was considered for a binary thresholding of the facet gradients; more classes could have been considered, but the problem is clearly simplified by considering only two gradient classes. In considering two classes of gradients, *i.e.* “flat” and “steep” TIN facets, it is expected that the most important urban features (*e.g.* man-made structures and vegetation) are enclosed by the steep facets.

Several polygonal regions were then generated by aggregating TIN facets in accordance to the binary classification mentioned above, *i.e.* facets of the same class meeting on edges were merged; facets of the same class meeting at a node were preserved. As it can be seen in Figure 3a), building features are not well defined and this is more evident in the eastern area containing the National Archives building and surrounding buildings. This fact was probably caused by the variation in facet gradient given the non uniform distribution of the LiDAR points. Moreover, after having a look at the TIN facets slope statistics graph, it was realized that the great majority of the facets have a gradient less than 30°. Therefore, a second experiment was carried out using a lower gradient threshold; however the trouble in using such a low value is that more noise is brought in. Given this fact, the usage of a 45° gradient threshold was considered and an equal interval binary classification was performed. The result obtained with this new classification is shown in Figure 3b).



a) using the 60° gradient threshold

b) using the 45° gradient threshold

Figure 3. Binary classification of TIN facets.
(Box in yellow shows area extracted for analysis in Figure 4)

Given the large size of the initial data set and the complexity of the map of polygonal regions displayed in Figure 3b), a case study area was chosen (*vd*. yellow box in Figure 3b)) comprising the National Archives building and its neighbourhood, where urban features, like buildings and some trees, are clearly standing on their own and hence easier to analyze.

2.2 Construction of the network of connectivity

The next step was the establishment of a network of connectivity throughout this map of polygonal regions by using graph theory: each merged polygon is represented by one vertex in the graph, and graph edges link graph vertices corresponding to adjacent polygons. Figure 4 shows the corresponding graph of adjacencies for the case study area.

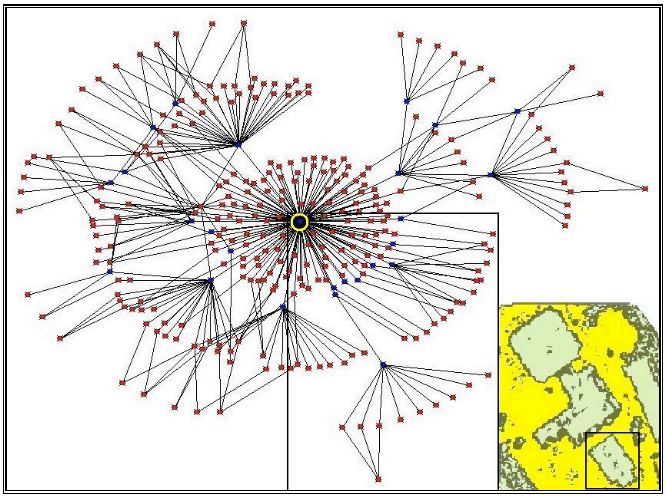
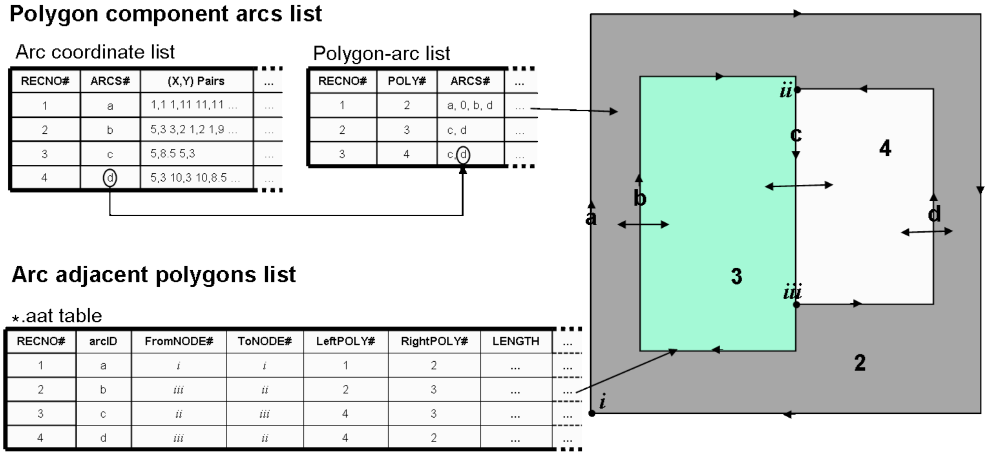


Figure 4. Graph of adjacencies for the case study area shown.
Boxes refer to the areas enlarged in Figure 6.
(Graph generated using Ucinet 6 for Windows; Borgatti *et al.*, 2002)

2.2.1 Polygon adjacencies retrieval

It should be pointed out at this stage that, for the purpose of this work, the spatial relationship of *adjacency* between polygons means that two polygons are adjacent if and only if they share at least one arc. As far as the spatial relationship of *containment* is concerned, our understanding is broader than the usual notion of containment between polygons. It is clear that a polygon is contained by another polygon if the former is completely surrounded and enclosed by the latter. However, in our case it is also possible to have a polygon surrounded by a ring of two or more polygons of the same class, which were not merged into one single entity because they happen to meet only at nodes. In this case, the former polygon is said to be *contained* by this ring of polygons.

A routine was implemented in *Arc Macro Language* (AML) to access polygon and arc attribute tables of the polygon coverage, and hence to retrieve polygon adjacencies. Given the GIS environment that is being used, this task implied a combination of information spread over two lists (*vd.* Figure 5): Polygon Component Arcs list (information referring to area definition) and the Arc Adjacent Polygons list (information referring to *connectivity* of arcs and *contiguity* of polygons), (ESRI, 1995, 2004).



Polygon component arcs list

Arc coordinate list

RECNO#	ARCS#	(X,Y) Pairs	...
1	a	1,1 1,11 11,11 ...	...
2	b	5,3 3,2 1,2 1,9 ...	...
3	c	5,8.5 5,3	...
4	d	5,3 10,3 10,8.5 ...	...

Polygon-arc list

RECNO#	POLY#	ARCS#	...
1	2	a, 0, b, d	...
2	3	c, d	...
3	4	c, d	...

Arc adjacent polygons list

⋆.aat table

RECNO#	arcID	FromNODE#	ToNODE#	LeftPOLY#	RightPOLY#	LENGTH	...
1	a	*i*	*i*	1	2	...	...
2	b	*iii*	*ii*	2	3	...	...
3	c	*ii*	*iii*	4	3	...	...
4	d	*iii*	*ii*	4	2	...	...

Figure 5. Combination of information to retrieve polygon adjacencies.

2.2.2 Graph representation

There are two common methods to represent graphs in a computer (Sedgewick, 1988, 1998, 2002): the *adjacency matrix* representation; and the *adjacency lists* representation. Both graph representations are arrays of simpler data structures, one for each vertex describing the edges incident on that particular vertex. The adjacency matrix is the simpler data structure and is implemented as an indexed two-dimensional array; the implementation of adjacency lists is based upon an array of linked lists. Because the graphs of adjacencies obtained in our case are fairly sparse, the adjacency lists representation appears to be the most appropriate data structure to use (Kruse *et al*. 1991, Schalkoff 1992, all cited in Barr and Barnsley, 1997; Sedgewick, 1988, 1998, 2002). It allows us to process all the edges of a graph in time proportional to $V+E$ (*i.e.* the total number of vertices plus the number of edges).

The processing was implemented in an application developed in C foreseeing the advantages and potentialities of pointer structures in C for graph analysis (Kelley *et al.*, 1990).

2.3 Bases for the analytical analysis method

2.3.1Preleminaries

In Figure 4 there is a representation of the graph of adjacencies for a particular area within the initial data set. Different levels of adjacency are indicated. Polygon *3*, highlighted on the bottom right, whose corresponding vertex is located right in the centre of the graph (*vd.* yellow circle), is the most connected flat polygon and the one with the longest perimeter. It corresponds indeed to the ground polygon and therefore constitutes the *useful external border* (Nardinocchi *et al.*, 2003), with which the graph drawing was started, and from where sequences of adjacencies/containments make most sense in terms of the urban scene.

It is possible to retrieve further geographical information by analysing different paths within the generated graph of adjacencies**.** Starting from the useful external border, a simple visual observation of the represented sequences of levels of adjacency between vertices along some graph paths, may tell us that, for instance, a vertex in the end of a path, representing the highest level of adjacency in that particular graph path, is a candidate to be either a hole in the ground or something on top of an urban feature (de Almeida *et al.*, 2004a, 2004b).

To give an example, let us go through the graph path highlighted in Figure 6 (a detail of Figure 4). Starting from polygon *3*, at the first level of adjacency the steep polygon *198* is found which is contained by previous polygon *3*. That, in turn, contains flat polygon *200* at the second level of adjacency. Polygon *200* contains several others and, in particular, contains steep polygons *250*, *256*, *260* which all together form a ring *containing* flat polygon *257*, belonging to the fourth and last level of adjacency. In terms of urban scene, the meaning of this sequence of spatial relations of adjacency and containment is the existence of a building (pictured on the bottom right of Figure 6), whose boundary is almost shaped by the *rectangular* dark green polygon displayed (de Almeida *et al*., 2004a, 2004b).

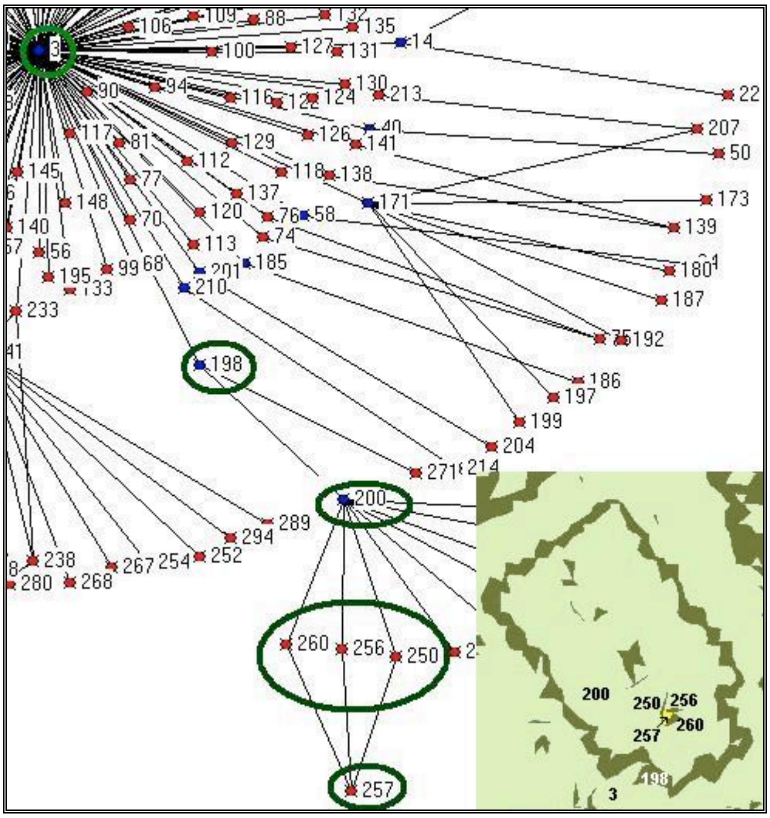


Figure 6. Detail of Figure 4: an example of the geographical information that may be inferred from a graph path in the context of the urban scene.

In the example given above polygons *250*, *256* and *260* are separate entities though they belong to the same class. This fact is mainly due to the GIS package being used; as explained in section 2.2.1, two polygons are considered adjacent if both share at least one arc (*vd.* Figure 7). What happens in this case is that polygon *250* meets polygon *256* at a node, *256* meets *260* at another node, and *260* in turn closes the ring meeting *250* at a third node. These polygons were not merged into one polygon because of the reasons described in section 2.1. Given this fact, there are no edges in the graph of adjacencies linking their corresponding vertices. Shown on top of the polygons in Figure 7 is the graph structure linking the vertices representing the polygons.

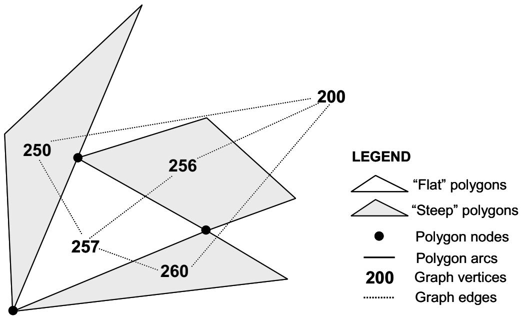


Figure 7. Detail of Figure 6: a special case of the spatial relation containment, between a ring of polygons and a single polygon.

This is a particular example of a situation which can be detected across the whole map of polygons. In reality, this fact constitutes an issue in the process of the graph analysis, for this particular case of containment has to be derived as it is not explicit in the graph. As far as this particular example is concerned, polygons *250*, *256* and *260* shall be considered component parts of a single entity (the ring of polygons) to be represented in the graph of adjacencies by only one vertex, as illustrated by a green ellipse in Figure 6. This might be achieved in ArcGIS by considering the three-polygon ring as a composite feature, such as a *region* in ESRI's coverage data model, comprising three polygonal components.

2.3.2 Depth-first search vs. breadth-first search

Given the complexity of the urban scene, typically with a high density of small size features, the generated graph of adjacencies for the map of polygonal regions is also complex. In order to be interpreted, the graph has to be explored and its properties determined by systematically examining each of its vertices and edges. Carrying out this task is cumbersome and equivalent to exploring a maze. Should one be interested in determining some simple graph properties, like computing the degrees of all vertices, this can be easily accomplished by examining each edge. But many other more complex properties of a graph are related to its paths. Those can be learnt by moving through the graph, from vertex to vertex along its edges, and by understanding its properties as we go. Indeed, this abstract model is used by most of the graph-processing algorithms (Sedgewick, 2002).

Therefore, it is believed that the retrieval of further geographical information, which was described above, constitutes that sort of analysis that is possible to carry out if based upon the graph traversal. In the literature two different algorithms are available to accomplish this task: the *depth-first search* (DFS) and the *breadth-first search* (BFS), (Sedgewick, 1988, 1998, 2002). Although both algorithms visit systematically all the graph nodes, the manner they operate is different. Briefly, depth-first search "moves from node to node, backing up to the previous node to try the next possibility whenever it has tried every possibility at a given node", it can be compared to "a single searcher probing unknown territory as deeply as possible" (Sedgewick, 1998). In contrast, breadth-first search "exhausts all the possibilities at one node before moving to the next", it amounts to "an army of searchers fanning out to cover the territory" (Sedgewick, 1998). From the implementation point of view, DFS can be either recursive or can use an explicit pushdown stack (in our application the recursive function implementation was used), whereas BFS uses FIFO (*first in, first out*) queues for its implementation.

As far as DFS is concerned, "the resulting spanning tree depicts the sequence of the traverse function calls"; whereas "BFS spanning tree provides a compact description of the dynamic properties of this level order search, corresponding one branch to each connected component" (both citations from Sedgwick, 2002). In both cases each tree vertex corresponds to each graph vertex and a tree edge corresponds to a traversed graph edge, thus non-traversed edges are not considered.

For illustration purposes, Figure 8 represents a simulated map of simple polygons, not classified in any manner, with the respective graph of adjacencies drawn on top of them. Let us choose polygon *2* as the root: the resulting spanning trees, both DFS and BFS, are pictured in Figure 9.

For the sake of flexibility, both graph-search algorithms, DFS and BFS, were implemented in our application in C, giving the user the choice of which algorithm to run. However, given the way the respective algorithms are conceived, it seems that the spanning tree resulting from the BFS traversal (broad and shallow, *vd.* Figure 9) is more meaningful in terms of the urban scene: if we look at its several short branches, these indeed appear to correspond to different urban features. In contrast, the DFS spanning tree (slim and deep, *vd.* Figure 9) does not seem to be as easily related to urban features as the previous one, for the interpretation of its long deep path does not appear to be straightforward.

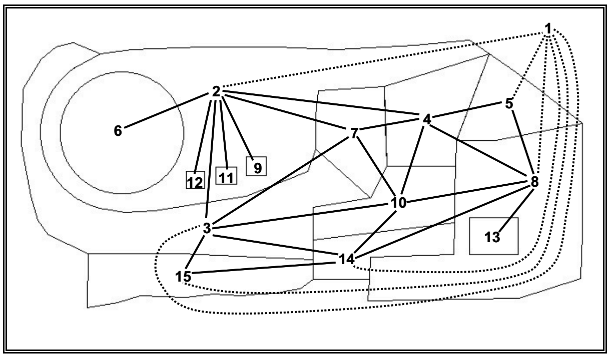


Figure 8. Example of a map of a simulated scene and respective graph of adjacencies.

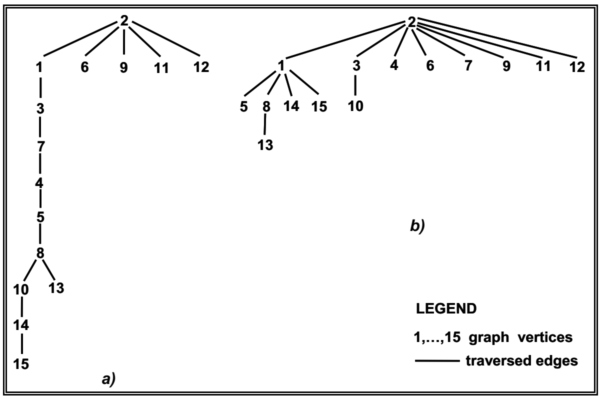


Figure 9. Example of two different traversal trees for the graph in Figure 8, both having the same root: *a)* depth-first search; *b)* breadth-first search.

Both DFS and BFS were implemented in such a way that, again for the sake of flexibility, it is possible to traverse the graph starting from any of its vertices. Nevertheless, we might be ultimately interested in considering this analysis starting from the useful external border (ground polygon), from where the sequences of adjacency (which in some cases represents containment too) make most sense in terms of the scene.

Although it is possible at the moment to visualise either spanning tree as an interim result of analysis, the main aim being sought is to extend the analysis algorithm and eventually being able to visualise its results. The ongoing developments for this purpose are being based in particular on the BFS and comprise the implementation of further analytical rules; they take into account namely, the valence (or degree) of each vertex in the initial graph, and the level of adjacency of each vertex in the tree. For instance, while traversing the whole graph, the application counts the levels of adjacency and analyzes the depth of the tree; hence, it is possible to know how many levels of adjacency/containment a graph vertex is away from the root in the tree generated by a specific search.

3. Conclusions and further work

3.1 Conclusions

The use of graph theory is becoming an increasingly important tool for the analysis and understanding of complex urban scenes. Starting from initially unstructured geospatial data sets of urban areas (thus, no prior knowledge of the spatial entities is assumed), this paper shows how a graph-theoretic approach can be applied in these circumstances towards the analysis of urban scene spatial topology.

The theoretical and practical methods developed so far to analyze entities within a LiDAR data set of urban environment have been presented. In particular, the merits of depth-first and breadth-first search algorithms in analysing the structure of the urban spatial topology were discussed. Given the different ways both algorithms operate in traversing a graph, it was noted how BFS results are more meaningful in terms of the urban scene: the BFS tree branches are connected components of the original graph, and represent the shortest path between the root and their leaf (Sedgewick, 2002); it seems that they can be related to potential urban features.

Thus, the implementation of the graph analysis procedure is being based upon BFS. It traverses the graph looking for sequential relationships of containment amongst the sequences of adjacency: *containment-first search* (CFS). In fact, where containment occurs there is a high likelihood of an urban feature being present.

However, for an effective interpretation of the urban scene topology, developing CFS simply based on BFS is not sufficient. The CFS procedure has to be extended in order to be able to detect the spatial relation of containment in a broader sense (*vd.* 2.2.1). Indeed, there are some particular cases of containment which are not explicit in the graph (*vd.* example in Figure 7). In order to address this issue, we propose that the spatial relation of *touching* between steep polygons should be taken into consideration, so as these particular cases can be derived by defining polygon-ring containments. Further investigation of this aspect will be object of future work.

3.2 Further work

Currently, the system is being extended to the visualisation of graphs, and more importantly of the resulting traversal trees. A visual representation of the urban topological analysis is also under consideration. In fact, the human brain is sensitive to the visual representation of real scenes, and visual analysis can often reveal patterns not discernable by current automated analysis techniques.

Thus, it reveals relevant the incorporation of capabilities for visual representation of both, the urban scene topology and its analysis, in the application under development. In particular, a graph traversal tree is a simpler representation of the initial graph that is well worthy of careful study (Sedgwick, 2002). The interesting aspect in visualising a traversal tree is that it contains almost the same information as the initial graph, but it is displayed in a slightly different way, making the graph structure somewhat more explicit. Given the dimension and complexity of the original graphs of adjacencies, we believe that the observation of the traversal trees is useful in detecting the existence of urban structures.

Also, the possibility of linking up the graph analysis application with the GIS environment is being investigated. In fact, it is believed that the utility of the visual representation of the topological data structures described in section 2.3.2 should be enhanced in terms of scene analysis if the visualisation tool is coupled with the original map. The ultimate goal is the implementation of functionalities to display dynamically the initial map of polygonal regions according to the results of the urban topology analysis.

For this purpose, we are currently working on the development of an interactive tool. This is being implemented in ArcMap (ArcGIS 8.3) using its embedded programming language, Visual Basic for Applications (VBA). Developing the application in this original environment has the advantage of being able to perform graph analysis based upon some of the polygon attributes, which can be withdrawn from the respective Polygon Attribute Table (PAT). In turn, this table can also be updated to integrate numerical results of the analyses carried out.

At the moment, the user can carry out any particular visual inspection, say in the original map of polygons, and simultaneously being able to obtain the respective traversal tree starting from the chosen root (*vd*. Figure 10). Other functionalities, like accessing directly a polygon's attributes when its respective vertex is selected in the spanning tree, or when selecting a vertex in the tree the corresponding polygon being highlighted on the map, are also being implemented.

Furthermore, the interactive capabilities should be useful in dealing with complex scenes, like the existence of a discontinuous ground polygon. In this situation, the corresponding graph of adjacencies will consist of different sub-graphs connected to each other by single linking edges. In such an interactive tool, the capability of analyzing which sub-graph corresponds to which area on the map, and simultaneously obtaining the respective traversal tree, appears to be interesting.

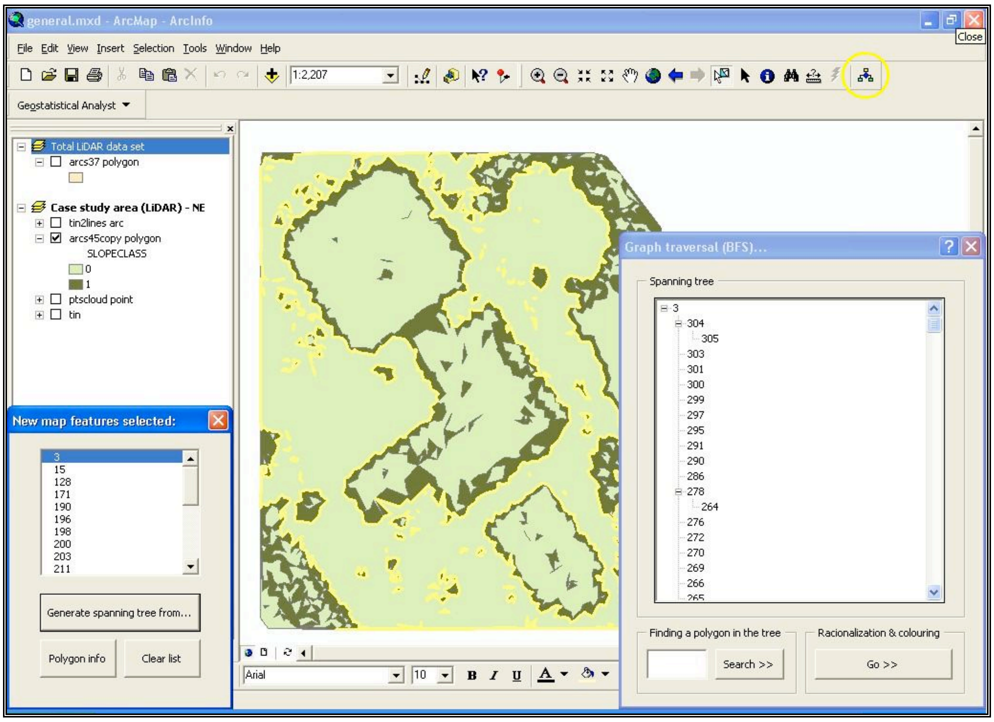


Figure 10. An interactive tool for the visualisation of topological data structures generated from the analyses of the adjacency graph. (Root – polygon *3*, selected in yellow)

Further work will also entail the implementation of other possible rules to enable the analysis process explained in section 2.3, eventually leading to the aggregation of graph vertices into identified meaningful structures. These, in turn, should be clustered into homogenous regions (Forberg & Raheja, 2002). After the delineation of cluster shapes, an analysis process will have to be accomplished, either by pattern recognition or interpretation procedures (Toussaint, 1980b). The aim of the ultimate cluster shapes analysis is the retrieval of higher-level information, *e.g.* sets of buildings, vegetation areas, and say land-use parcels.

We note that this application is at the same time to investigate rules for urban scene analysis and graphic representation of results. We expect the resulting system to be useful to support land-use mapping, image understanding or, in more general terms, to support clustering analysis and cartographic generalisation processes.

References

de ALMEIDA J.-P., MORLEY J. G., DOWMAN I. J. (2004a). A graph-based scene analysis technique. *Proceedings of GISRUK 2004*, 365-369 (Norwich, UK).

de ALMEIDA J.-P., MORLEY J. G., DOWMAN I. J. (2004b). A graph-based approach for higher order GIS topological analysis. *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, Vol. XXXV, Part B4, Commission IV*, 1024-1028 (Istanbul, Turkey: XX Conference of ISPRS).

ANDERS, K.-H.; SESTER, M.; FRITSCH, D. (1999). Analysis of settlement structures by graph-based clustering. *Semantische Modellierung, SMATI 99*, 41-49 (Munich, Germany).

BARNSLEY, M.J.; BARR, S.L. (1998). Distinguishing urban land-use categories in fine spatial resolution land-cover data using a graph-based, structural pattern recognition system. *Comput., Environ. And Urban Systems*, Vol. 21, No. 3/4: 209-225 (Elsevier Science Ltd).

BARR, S.L., BARNSLEY, M.J. (1996). Inferring urban land-use from satellite sensor images using kernel-based spatial reclassification. *Photogrammetric Engineering and Remote Sensing*, No. 62: 949-958.

BARR, S.L., BARNSLEY, M.J. (1997). A region-based, graph-theoretic data model for the inference of second-order thematic information from remotely-sensed images. *International Journal of Geographical Information Science*, Vol. 11, No. 6: 555-576 (Taylor & Francis Ltd).

BARR, S.L., BARNSLEY, M.J. (2004). Characterising the structural form of an urban system using built-form connectivity model concepts. In *Spatial Modelling of the Terrestrial Environment* (Kelly, R.E.; Drake N.A.; Barr, S.L., eds.): 201-226 (Chichester, UK: Wiley).

BAUER, T.; STEINNOCHER K. (2001). Per-parcel land use classification in urban areas applying a rule-based technique. *GeoBIT/GIS 6*: 24-27.

BORGATTI, S.P.; EVERETT, M.G.; FREEMAN, L.C. (2002). *Ucinet 6 for Windows: Software for Social Network Analysis*. (Harvard, MA: Analytic Technologies).

BUNN, A.G.; URBAN, D.L.; KEITT, T.H. (2000). Landscape connectivity: A conservation application of graph theory. *Journal of Environmental Management*, no.59, 265-275 (Academic Press, USA).

EDELSBRUNNER, H.; KIRKPATRICK, D.G.; SEIDEL, R. (1983). On the shape of a set of points in the plane. *In IEEE Transactions on Information Theory*, Vol.29, no.4: 551-559.

ESRI (1995). *Understanding GIS, The ARC/INFO Method*, Lesson2. ESRI, Inc., (Cambridge, England - UK).

ESRI (2004). ArcGIS Topology [online]. Available from http://www.esriuk.com/products/support/ArcGIS%20UK%20EXT/Topology.asp [Accessed 10 June 2005].

EYTON, J.R. (1993). Urban land-use classification and modelling using cover-type frequencies. *Applied Geography*, Vol. 13: 111-121.

FORBERG, A.; RAHEJA, J.L. (2002). Generalization of 3D settlement structures on scale-space and structures recognition. Work Report, Institut für Photogrammetrie und Fenerkundung (Universität der Bundeswehr München), Lehrstuhl für Kartographie (Munich, Germany: Technische Universität München).

GIBBONS, A. (1989). *Algorithmic Graph Theory*. (Cambridge, UK: Cambridge University Press).

GROSS, J.; YELLEN, J. (1999). *Graph Theory and Its Applications* (Boca Ratón-Florida, USA: CRC Press).

KELLEY, A.; POHL, I. (1990). *A Book on C - Programming in C*. 2nd ed (Redwood City – CA, USA: Benjamin Cummings).

KIM, T.; MULLER, J.-P. (1999). Development of a graph-based approach for building detection. *Image and Vision Computing*, 17: 3-14 (Elsevier Science B.V.)

KIRKPATRICK, D.G.; RADKE, J.D. (1985). A framework for computational morphology. *Computational Geometry* (TOUSSAINT, G.T., ed.), 217-248 (Amsterdam, The Netherlands: North- Holland).

KRUSE, R.L.; LEUNG, B.P.; TONDO, C.L. (1991). *Data Structures and Program Design in C* (Prince Hall, New York - USA).

LAURINI, R.; THOMPSON, D. (1992). *Fundamentals of Spatial Information, 5* (London, UK: Academic Press).

NARDINOCCHI, C., GIANFRANCO, F., ZINGARETTI, P. (2003). Classification and filtering of laser data. ISPRS Workshop on *3D reconstruction from airborne laser scanner and InSAR data* (Dresden, Germany).

O'SULLIVAN, D.; TURNER, A. (2001). Visibility graphs and landscape visibility analysis. *International Journal of Geographical Information Science*, Vol.15, No.3: 221-237 (Taylor & Francis Ltd).

REED, C. (1999). GIS Users Shouldn't Forget About Topology [online]. In *GeoWorld Readers' Forum*. Available from http://www.geoplace.com/gw/1999/0499/499form.asp [Accessed 29 January 2004].

ROBERTS, S.A.; HALL, G.B.; CALAMAI, P.H. (2000). Analysing forest fragmentation using spatial autocorrelation, graphs and GIS. *International Journal of Geographical Information Science*, Vol.14, No.2: 185-204 (Taylor & Francis Ltd).

SCHALKOFF, R. J. (1992). *Pattern Recognition: Statistical, Structural and Neural Approaches*. (New York, USA: John Wiley).

SEDGEWICK, R. (1988). *Algorithms*. 2nd ed (Reading-MA, USA: Addison-Wesley).

SEDGEWICK, R. (1998). *Algorithms in C, Parts 1-4*. 3rd ed. Chap. 3, 4, 5 (Reading-MA, USA: Addison-Wesley).

SEDGEWICK, R. (2002). *Algorithms in C, Part 5*. 3rd ed. Chap. 17, 18 (Reading-MA, USA: Addison-Wesley).

STEEL, A.M.; BARNSLEY, M.J.; BARR, S.L. (2003). Inferring urban land use through analysis of the spatial composition of buildings identified in LiDAR and multispectral image data. *Remotely-Sensed Cities* (MESEV, T.V., ed.), (London, UK: Taylor & Francis Ltd).

TEMPERLEY, H. (1981). *Graph Theory and Applications* (Chichester, UK: Ellis Horwood Ltd).

THEOBALD, D.M. (2001). Topology revisited: representing spatial relations. *International Journal of Geographical Information Science*, Vol. 15, No. 8: 689-705 (Taylor & Francis Ltd).

TOUSSAINT, G.T., 1980a. The relative neighbourhood graph of a finite planar set. *Pattern Recognition*, Vol.12, No.4: 261-268. (Pergamon Press Ltd).

TOUSSAINT, G.T., 1980b. Pattern recognition and geometrical complexity. Proceedings of *V International Conference on Pattern Recognition*: 1324-1347. (Miami, USA).

URQUHART, R., 1982. Graph theoretical clustering based on limited neighbourhood sets. *Pattern Recognition*, Vol.15, No.3: 173-187. (Pergamon Press Ltd).

WILSON, R, 1996. *Introduction to Graph Theory* (4th ed.), (Harlow, UK: Longman).

Acknowledgements

Faculdade de Ciências e Tecnologia da Universidade de Coimbra (Faculty of Science and Technology of the University of Coimbra, Portugal) and *Fundação para a Ciência e Tecnologia* (Science and Technology Foundation, Portuguese Ministry of Science, Technology and Graduate Education - scholarship SFRH/BD/9909/2002) which support financially this research.

An acknowledgement is also due to Dr. João Coutinho-Rodrigues (Department of Civil Engineering, Faculty of Science and Technology, University of Coimbra, Portugal), J-P de Almeida's PhD co-supervisor at the University of Coimbra.