Luís Manuel Lobato Macedo

AN INQUIRY INTO THE VALIDITY OF TECHNICAL ANALYSIS IN FINANCIAL MARKETS WITH THE USE OF EVOLUTIONARY TECHNIQUES

Doctoral Thesis of the Doctoral Programme in Management-Decision Aiding Science, supervised by Professor Doctor Pedro Manuel Cortesão Godinho and Professor Doctor Maria João Teixeira Gomes Alves and submitted to the Faculty of Economics of the University of Coimbra

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FEUC FACULDADE DE ECONOMIA
UNIVERSIDADE DE COIMBRA

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Abstract

Technical Analysis (TA) has been subject of debate from some decades over. In the late 1920's, the debacle of Financial Stock Markets in the US and to some extent all over Europe (in particular London) raised questions concerning the ability of Fundamental Analysis to explain price movements. In the 1970's and 1980's, with the development of new infrastructures and new informatics platforms, the markets became accessible to more retail investors looking for new methods of market analysis to support their trading practices. TA, for its simplicity and ease to use, since it is mostly based in price fluctuation and price-based indicators, was chosen by several investors.

After the beginning of the internet revolution in the mid 1990's to the late 2010's, the expansion of trading, either by direct trading accounts in a Brokerage firm or indirectly through a fund in an Investment house, has been exponential and rose it to levels never seen before, increasing subsequently the use of TA. But the main problem of TA persists. Although many academic studies have emerged, there is still no significant support to the notion that TA is an effective tool to improve trading results.

With this work we intend to draw some conclusions about the relevance of TA. For that purpose, TA-based systems were developed with the use of optimization evolutionary techniques, namely Genetic Algorithms and Multiobjective Evolutionary Algorithms. These methodologies were applied to the Forex Market and Worldwide Stock Markets to infer the value of TA indicators in reaching meaningful results.

The outcome obtained in the three most relevant Forex crosses (EUR/USD, GBP/USD and USD/JPY) showed that the studied TA indicators presented limited value as a predicting tool for trading after including realistic trading costs. On the other hand, with respect to the Stock markets, a method of portfolio optimization was developed and results after trading costs vary from almost non-profitable markets (the most efficient, like the US) to interestingly profitable markets (such as Belgium, Portugal and Greece) when considering Bollinger Bands trading rules. The obtained results are also consistent and support to some extent the Adaptive Market Hypothesis theorized by Professor Andrew Lo.

Keywords: Finance; Technical Analysis; Trading; Portfolio Optimization; Genetic Algorithms (GAs); Multiobjective Evolutionary Algorithms (MOEAs).

Resumo

A Análise Técnica (AT) tem sido alvo de debate desde há já algumas décadas. No final dos anos 1920, a derrocada dos Mercados Financeiros de Acções nos Estados Unidos e um pouco por toda a Europa (em particular em Londres) levantou questões sobre a capacidade da Análise Fundamental para explicar os movimentos de preços. Nos anos 1970 e 1980, com o desenvolvimento de novas infraestruturas e novas plataformas informáticas, os mercados tornaram-se acessíveis a mais investidores individuais, à procura de novos métodos de análise do mercado para fundamentar as suas práticas de *trading*. A AT, pela sua simplicidade e facilidade de utilização, uma vez que se baseia essencialmente nas flutuações de preços e em indicadores baseados no preço, foi seleccionada por vários investidores.

Após o início da revolução da internet desde meados dos anos 1990 até finais dos anos 2010, a expansão do *trading*, quer por transacção directa em contas junto de Correctoras quer indirectamente através de fundos de investimento geridos por Sociedades especializadas, tem sido exponencial e aumentou-o para níveis nunca antes vistos, aumentando subsequentemente a utilização da AT. Mas o problema principal da AT persiste. Apesar de muitos estudos académicos terem emergido, ainda não existe suporte significativo para a ideia da AT ser uma ferramenta efectiva para a melhoria dos resultados de *trading*.

Com este trabalho pretendem-se obter algumas conclusões sobre a relevância da AT. Para esse propósito, foram desenvolvidas metodologias baseadas em AT em combinação com o uso de técnicas de optimização evolucionárias, mais propriamente Algoritmos Genéticos e Algoritmos Evolucionários Multiobjectivo. Estas metodologias foram aplicadas no mercado Forex e nos Mercados de Acções Mundiais para averiguar a utilidade de indicadores de AT na obtenção de resultados significativos.

Os resultados obtidos em três dos principais pares de divisas do Forex (EUR/USD, GBP/USD e USD/JPY) mostraram-nos que os indicadores de AT estudados apresentam um valor limitado como ferramenta previsional para o *trading*, após inclusão de custos de transacção realistas. Por outro lado, no que respeita aos mercados de Acções, foi desenvolvido um método de optimização de portfólios e os resultados, após inclusão de custos de transacção, variam desde mercados próximos de não rentáveis (os mais eficientes, como dos EUA) até mercados com rentabilidades interessantes (como Bélgica, Portugal e Grécia) quando consideradas as regras de *trading* de *Bollinger Bands*. Os resultados obtidos são também consistentes e sustentam até certo ponto a Teoria dos Mercados Adaptativos do Professor Andrew Lo.

Palavras-chave: Finanças; Análise Técnica; *Trading*; Optimização de Portfólios; Algoritmos Genéticos; Algoritmos Evolucionários Multiobjectivo.

Acronyms

%R – Williams' %R

AMEX – American Stock Exchange

AMH – Adaptive Market Hypothesis

ANN – Artificial Neural Networks

AR – Auto Regressive Framework

ATR – Average True Range

B&H – Buy and Hold Trading Strategy / Scenario

BB – Bollinger Bands

BF – Behavioural Finance; Behaviourist School of Finance

CAPM – Capital Asset Pricing Model

CR – Calmar Ratio

CVaR – Conditional Value-at-Risk

DMX – Directional Movement Index

EA – Evolutionary Algorithm

EF – Evaluation Function

EMA – Exponential Moving Average

EMH – Efficient Market Hypothesis

EP – Evolutionary Programming

ES – Expected Shortfall

EST – Evolution Strategy

FA – Fundamental Analysis

Forex – Foreign Exchange Market

FR – Financial Ratio

GA – Genetic Algorithm

GP – Genetic Programming

IR – Information Ratio

IS – In Sample

LPM – Lower Partial Moments

MACD – Moving Average Convergence/Divergence

MAD – Mean Absolute Deviation

M-LPM – Mean – Lower Partial Moment Framework

M-MAD – Mean Return - Mean Absolute Deviation Framework

MOEA – Multiobjective Evolutionary Algorithm

MOGA – Multiple Objective Genetic Algorithm

MOP – Multi-Objective Problem(s)

M-SV – Mean-Semivariance Framework

M-V – Mean-Variance Framework

M-VaR – Mean-Value at Risk Framework

NASDAQ – National Association of Securities Dealers Automated Quotations (Stock Exchange)

NPGA – Niched Pareto Genetic Algorithm

NSGA-II – Non-dominated Sorting Genetic Algorithm II

NYBOT – New York Board of Trade, recently acquired by ICE – InterContinental Exchange

NYSE – New York Stock Exchange

OOS – Out of Sample

OR – Operations Research

PAES – Pareto Archived Evolutionary Algorithm

PESA – Pareto Envelope-based Selection Algorithm

RSI – Relative Strength Index

SMA – Simple Moving Average

SO – Stochastic Oscillator

SPEA-2 – Strength Pareto Evolutionary Algorithm 2

SR – Sharpe Ratio

SV – Semivariance

TA – Technical Analysis

TR – True Range

VaR – Value-at-Risk

VEGA – Vector Evaluated Genetic Algorithm

WMA – Weighted Moving Average

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1 Introduction

The present work is intended to be an inquiry on the validity of technical analysis (TA) in the attainment of above normal profits. Most often, the banking industry and especially the trading industry have sometimes claimed that there are advantages in using TA-based indicators and rules to improve financial management and overall financial results of asset portfolios. There is in this area an institutionalized mind-set prone to take for granted the postulated benefits of TA, most times without a critical view on the factual existence of such benefits. This work delves into the thematic of TA and tries to elicit evidence either for or against it.

TA indicators rely on a set of parameters that are used in combination with TA trading rules, inducing the execution of trading actions (*buy*, *sell*, *stay* or *get out* of the market) which TA practitioners have been claiming to produce above normal financial outcomes. An adequate way to optimize the mentioned parameters is through the use of Evolutionary Algorithms (EAs) and Multiobjective Evolutionary Algorithms (MOEAs), because of their flexibility, limited execution time and ability to rapidly search large solution spaces.

The use of EAs and MOEAs is justified by several reasons: 1) the kind of optimization problems (either in a single or multiobjective case) is usually too complex to use an exact solving method; 2) the flexibility and adaptability of these evolutionary techniques; 3) the nature of EAs allows us to achieve great savings in computational resources and time, avoiding a thorough and exhaustive search along the feasible search space of solutions. Adding to these motives, EAs are some of the most used techniques to solve financial *return-risk* optimization problems.

The thesis is devoted to the study of the most relevant, TA indicators and oscillators, integrated into distinctive trading models, and assess their predictive power. The specific objectives of this work are:

1. To compare the performance of diverse markets, to know how efficient markets are;

2. To find out if TA is effectively useful for trading, both for portfolio management and direct account trading activities;
3. To determine which of the studied TA indicators shows more potential to be used as a predicting tool;
4. To assess if evolutionary techniques help in any way to improve financial performance (via parameters fine-tuning), with reference to traditional parameters used in TA;
5. To compare performances of up-to-date MOEAs in portfolio optimization problems, which involve risk-return trade-offs and where a Pareto front can be obtained;
6. To verify if existing costs affect significantly the attained results in all studied markets.

After this brief introduction, a literature review follows in Chapter 2, presenting the major theoretical approaches to market analysis and asset valuation. In this chapter, we will draw an insight into the philosophical perspectives (mostly, Efficient Market Hypothesis, Behavioural Finance and the Adaptive Market Hypothesis), as well as the general methodologies of market analysis – Fundamental Analysis (FA) and Technical Analysis (TA). In this particular subject we will consider the most relevant indicators for each methodology. A more detailed analysis will be devoted to TA indicators, since they are core to this work and it is important to establish the reasoning underlying their conception and the rules applied to each indicator. In this chapter, we will also devote our attention to financial *risk-return* analysis, with eight major frameworks being selected: Mean-Variance (M-V), Mean-Semivariance (M-SV), Mean – Lower Partial Moments (M-LPM), Mean – Value-at-Risk (M-VaR), Mean – Expected Shortfall (M-ES or M-CVaR), Mean – Mean Absolute Deviation (M-MAD), Sharpe Ratio (SR) and Calmar Ratio (CR).

Chapter 3 concerns with evolutionary techniques applied to finance. A survey on the most well-known single objective and multiobjective evolutionary optimization algorithms is conducted. In the single objective case, we will explore the categories of Genetic Algorithms (GA), Genetic Programming (GP), Evolutionary Strategies (EST) and Evolutionary Programming (EP). Regarding multiobjective optimization, particular attention will be devoted to Multi-Objective Evolutionary Algorithms (MOEAs). We will start with the presentation of the common concepts to all these population-based evolutionary algorithms, continue with a more detailed view of the four mentioned evolutionary techniques and conclude the chapter with a characterization of the two MOEAs used in part of the empirical work, the *Nondominated Sorting Genetic Algorithm II* (NSGA-II), (Deb et al., 2002), and the *Strength Pareto Evolutionary Algorithm 2* (SPEA-2), (Zitzler et al., 2001). Some other important MOEAs will also be mentioned in a final section.

Chapter 4 is devoted to the presentation of the first part of our empirical work, which is the outcome of a Forex market study, namely in the three most important crosses, the EUR/USD, GBP/USD and USD/JPY. The trading model, including all the considered assumptions, the mechanics of trading and the explanation of the used GA will be presented in Section 4.1. The model tries to replicate the trading environment in the Forex market, with a direct trading approach through a limited amount escrow account, with the possibility of short selling and a 1:100 leverage level, and considering typical TA indicators and trading strategies commonly used by the industry. The purpose is to obtain and compare optimized solutions in three different TA categories (*momentum*, *trend* and *breakout*). An original GA was developed for this purpose. In the subsequent section a presentation of the empirical results and a discussion will follow.

Chapter 5 presents the stock portfolio optimization process within a Mean-Semivariance approach, where we will assess the performance of four different kinds of markets that try to depict four different stages of development, from underdeveloped *market Tier 1* (Argentina, Brazil and South Africa), *market Tier 2* – peripheral developed countries (Greece, Portugal and Belgium), to *market Tier 3* – fully developed countries (UK, Australia and The Netherlands), and finally, the *US market*. In this chapter we will use the NSGA-II and the SPEA-2 to make a comparative analysis of portfolio optimization and discuss the results.

Chapter 6 presents the most relevant conclusions of this work and suggests the most sensible future directions taking into consideration all gathered information and the present context in this field of knowledge.

2 Theoretical Approaches to Market Analysis and Asset Valuation

In this chapter, a review of the most relevant approaches to financial markets analysis and perspectives of valuation will be drawn. The market behaviour theories try to capture *how the market works*. In the following sections, we will address the major perspectives on market functioning, starting by the classic *Efficient Market Hypothesis* (EMH), moving to the other extreme framework of market interpretation, the *Behaviourist School of Finance* (BF), and ending in a conciliatory tone, with Andrew Lo's *Adaptive Market Hypothesis* (AMH).

The methodologies of market analysis centre their scope on *what kind of data* should be used and *how to use it* in order to predict future market price moves: while *Fundamental Analysis* (FA) makes use of the economic theoretical fundaments represented in a varied array of information to predict future prices, *Technical Analysis* (TA) assumes all information is translated in a single variable, *price*, and therefore future prices can be predicted through the study of past market data. These methodologies are supported by the Behaviourist School of Finance and the Adaptive Market Hypothesis frameworks, but not by the Efficient Market Hypothesis perspective (at least in its purest form – the *strong* form). In subsection 2.2.1 (FA), attention will be focused on several financial ratios, commonly used all over the financial industry to predict market direction and future prices. Regarding TA, we will study the major indicators used in the financial markets and make a review of the most well-known TA theories available today.

Financial analysis possesses two major vectors of study, translating the greatest concerns of investors and traders: one is financial *return*; the other is *risk*. All financial analysts centre their activity on these two important objectives and their interconnection, for the simple assessment of one without taking into consideration the other may induce biased results. Also a return-risk analysis will be presented in this chapter, both in a portfolio modelling and single asset contexts.

2.1 Market Behaviour Theories

2.1.1 The Efficient Market Hypothesis

It is not certain when the first signs of the *Efficient Market Hypothesis* (EMH) arose in Academia. We cannot state a specific date or work as the beginning landmark, since this theory emerged within a cauldron of enabling ideas. One of the most important of them was the concept of *randomness* in price movement, which was first noticed in a survey by the acknowledged statisticians Maurice Kendall and Bradford Hill– (Kendall and Hill, 1953). In this research paper evidence of the random nature of stock price movements was presented, something no less than anathema at the time, since prices were considered, under the theoretical and intuitive economic view, to move in cycles and counter-cycles and fundamentally driven (according to economic laws). This notion of market price randomness was later theorized by (Samuelson, 1965).

Another landmark article which contributed to this theory was (Muth, 1961), a work that applied the tenets of rational expectations into economic analysis translated into market behaviour. The impact of the subjects approached in this article were very relevant to the development of the EMH, in particular the role of *information* as a key factor to define market efficiency: all available market information is used and embedded in the economic agents' formation of expectations.

In this context, it was no surprise how the succession of ideas projected stock market analysis to a new theoretical perspective, information-based, pro-efficiency, where stock prices ought to move in a random-like fashion, i.e., the EMH. As we may see, this is no longer a scattered set of free ideas, but instead a set of postulates in a stable theory, something that offered consistency and logic to the explanation of market behaviour. Eugene Fama took care of the full embodiment of this emerging theory when he published (Fama, 1965a) and (Fama, 1965b), presenting the *Random Walk Model* and *independence* of stock price movements, later revised by (Fama et al., 1969) where he discusses how market efficiency holds under the stock splitting and dividend payment scenario, and (Fama, 1970) where he acknowledges the existence of different market efficiency forms: *weak*, *semi-strong* and *strong*, adding more credibility to the EMH.

Nevertheless, the emergence of the EMH was not peaceful. Different academics contested the core ideas of the theory. Among them, (Lucas, 1978), one of the most significant, examined the stochastic behaviour of prices and arrived at the conclusion that *randomness* in price formation is not an ubiquitous trait in all markets and does not reflect market efficiency *per se*. Therefore, equilibrium prices are more a function of rational expectations interactions than of luck. (Grossman and Stiglitz, 1980), on the other hand, centre the analysis on the *arbitrage* feature in markets' price formation, presenting a demonstration on the impossibility of informationally efficient markets, and enhancing the importance of the costs of acquiring information, which distort efficiency to some extent.

Another step followed in this discussion on the feasibility of the EMH: Malkiel pointed how flaws and insufficiencies in the very nature of the rationality-based economic models limits – in particular applied to portfolio analysis – and deteriorates the cause-effect rationale employed by academics. Both in (Malkiel, 1996) and (Malkiel, 2003), the author exposes doubts on the significance of the fundamentally driven economic models relevant role in explaining market behaviour. For Malkiel the fundamentals of economic theory are not always good explaining variables of price movement in Stock markets. Malkiel concludes for the validation of the *Random Walk Theory* (RWT) and the EMH, considering previous economic models as over-appreciated. Regarding this claim by Malkiel, (Lo and MacKinlay, 1999) distinguish these two fields (EMH and RWT) for, according to their research, they only coincide under very special circumstances (for example, in the case of risk-neutrality).

(Timmermann and Granger, 2004) explore the meaning of market efficiency and its implications in trading activities. Their classic general idea (purest notion) of market efficiency states markets are indeed efficient if, and only if, it is not possible to take profits from information on that market. The consequences of this postulate are considerable: price prediction or any other kind of forecasting effort for superior profit seeking activities is not compatible with the notion of market efficiency (any strategy in that line of thought will be, according to the authors, short lived and suppressed). In this paper, the authors point out a curious paradox: in a market with no profit opportunities there is no incentive whatsoever to intervene and promote its efficiency – the *arbitrageur*'s role; so the very existence of arbitrageurs contradicts the core premise of the EMH. Therefore, at its most basic notion, the EMH negates the possibility of market arbitrage opportunities. But without arbitrageurs, any market will seldom be considered efficient.

A brief history on the emergence and evolution of the EMH is excellently portrayed by (Lo, 2007), drawing a remarkable review of the complete journey of the EMH since its inception, and including reactions and criticisms.

2.1.2 Behavioural Finance

Behavioural Finance (BF) is «the study of how psychology affects finance» - (Shefrin, 2002). In the behaviourists' perspective, a market is an aggregation of individual investors and traders beliefs that interact with each other in the process of price formation. They represent the confrontation between *bulls* and *bears* and incorporate their beliefs, anxieties, opinions, concerns. Markets embed all the individual investors' emotions – *greed* and *fear*, translated into prices. Therefore, price formation is subject to all delusions inherent to human perception, the biases that characterize every human being: *representativeness* (similarity to stereotypes), *availability* (ease of how we remember things), *adjustment* and *anchoring* (errors of perception based on relative positioning in reference to a predetermined assumption or starting point) – (Tversky and Kahneman, 1974); the *regression to the mean*, the *gamblers' fallacy* (the ostensive prediction of a market turn), *overconfidence* and *expert judgement*, *aversion to ambiguity* – (Shefrin, 2002). The very risk awareness may change attitude towards capital gains and losses. People react differently before the menace of incurring losses, according to the level and kind of risk involved in each situation, or in other words, according to their *individual preferences* – (Kahneman and Tversky, 1979), (Tversky and Kahneman, 1981).

For behaviourists, trading the Markets is not a simple mechanical task. It is not linear, it is much more complex, and it requires different approaches than an inflexible econometric translation of market behaviour. (Bondt and Thaler, 1985), for instance, discuss the role of *overreaction* to unexpected dramatic events in trading activity. With collected stock data from the NYSE, the authors obtained evidence consistent with the overreaction hypothesis: previously winner portfolios tend to present lower returns, in about 5 percentage points (p.p.), in relation to the market average; on the other hand, prior losers show some resilience, outperforming the market by 19.6 p.p. and prior winners by 25 p.p., on a 3 year time span. The overreaction hypothesis states that, as investors focus on stocks with more extreme price performances, the subsequent price reversals will be also pronounced. The collected and treated data confirms the hypothesis. Furthermore, the authors notice how the reversal is also biased (the price reversal is not of the same magnitude whether we are talking about previous portfolio winners or losers).

This perspective represents a radical and dramatic change in relation to the EMH. In this view, efficiency is a mirage and may only be achieved momentarily by chance. Fundamentalists of the EMH were not indifferent to these last publications and Fama, on the turn of the century, reacts with an article – (Fama, 1998) – pointing out the existence of *under-reactions* just as often as *over-reactions* over the short-term, something consistent with the concept of randomness and the EMH. Over the long run, although these anomalies might not be as significant, evidence suggests differences in results are due to methodologies or statistical approaches, and consequently may be attributed to chance.

2.1.3 The Adaptive Market Hypothesis

The *Adaptive Market Hypothesis* (AMH) has been developed by Professor Andrew Lo, from the MIT Sloan School of Management, as a conciliatory view on market behaviour. In Lo's notion (Lo, 2004), both the perspective of the EMH and those of the Behaviourists have merits and do not necessarily need to be in conflict. In his attempt to discern financial markets' movements, Lo explored the validity of previous behaviourist studies adding more depth and preventing bias by introducing *objective measurement* into their analyses.

In the context of a historical review of both the EMH and BF, (Lo, 2004) presents his arguments in favour of each of the perspectives and the rationale that allows a consensual interpretation of financial markets. For instance, it is acknowledgeable that some particular markets present a greater level of efficiency than others, and some may in fact present themselves as efficient markets in the *strong* or *semi-strong* forms depicted by Fama. Different circumstances affect the development of a particular market. Liquidity is among the most important. Another aspect may be the kind of investor operating in that market. Traits such as socioeconomic background, literacy, access to sophisticated trading tools, intelligence, play an important role in the characterization of a market. It is also undeniable how the human nature always present in the markets (trading decisions are human-based or programmed by humans) can introduce biases and inefficiencies to the trading systems. Studies – (Lo and Repin, 2002) and (Lo et al., 2005) – suggest there is a statistically significant difference in physiological characteristics of relevant market events (high price volatility) compared to normal volatility periods along a trading session. Physiological responses also change with the trader's level of experience. As a result, these studies reinforce the notion that market action is not all about rationality, mechanical (re)action, efficiency, but instead, may be understood as a process involving cognitive emotions.

Later, in (Lo, 2004), (Lo, 2005) and (Lo et al., 2005), Andrew Lo explains his perspective on market functioning, depicting the implicit rationale of market movement and its various possible stages. Lo named his theory the Adaptive Market Hypothesis (AMH). Highly influenced by important advances in "evolutionary psychology", this approach applies evolutionary principles (competition, reproduction, natural selection) to the study of market action. In his view, markets are in constant change, or in better words, are always *evolving*, according to the confrontation of market forces ("species" of investors) in an everlasting iteration of profitability and loss cycles. This theory also invokes the general idea of survival (i.e., attaining a good fitting solution, neglecting the optimum) to admit some human decision-making processes are indeed biased, establishing a compromise between rationality and emotion, efficiency and bias. In some sense, as Lo himself considered, the AMH may be seen as the «*new* EMH» with an added evolutionary flavour.

Empirical evidence supports this new theory of the AMH. (Neely et al., 2009) tested the validity of these claims for the FOREX market, by studying the existence of abnormal excess returns to technical trading rules for the time span of 1970-1980, and found out that the obtained results are consistent with the AMH but not the EMH.

2.2 Methodologies of Market Analysis

2.2.1 Fundamental Analysis

Fundamental Analysis (FA) is based on economic theory to predict market price motion. It assumes there are established economic relations between explanatory variables (independent variables, also considered the *foundation* or the *fundamentals* of economic behaviour) and the dependent or explained variables (i.e., the security prices or returns). FA admits the possibility of forecasting market prices and, therefore, considers that the dependent variable (price) is not fully random. To FA, a fair assessment of a price can be accomplished by resorting to the relevant variables that measure economic performance. There are several ways to put FA into practice.

2.2.1.1 Discounted Cash Flows

Discounted Cash Flows of a given asset is a method of valuation where future expected inflows and outflows are discounted to a present value according to pre-established assumptions. Expected values of future cash flows are estimated and the interest rate applied in the model may be determined by common sense within reasonable bounds, which is prone to subjectiveness that may lead to inaccuracies, or by more objective methods, such as the *Capital Asset Pricing Model* (CAPM), granting more sustenance to the obtained figures.

So regarding exchange traded assets valuation, there may arise divergences between two different investors, according to their expectations and assumed premises. In the case of a corporation, its shareholder net present value may be determined by discounting the future dividends' estimates (Dividend Theory). The shareholders' net present value divided by the number of shares gives us an estimation of the stock price.

In spite of its shortcomings, this methodology remains as an important way to compute corporate valuation and, therefore, it is no surprise to see all major bibliography in financial analysis making a detailed explanation of it. (Damodaran, 2002) or (Brealey et al., 2011) provide good examples.

2.2.1.2 Financial Ratios

Financial Ratios (FR) may be used isolated or as variables in a more complex model. These ratios are almost exclusively applied to corporation financial health assessment. There are numerous kinds of ratios, depending on the specific subject in study. For instance, (Brealey et al., 2001), (Brigham and Daves, 2007) and (Brealey et al., 2011) classify FR into *leverage* (or debt), *liquidity*, *efficiency* (operational), *profitability* and *market value* ratios. Alternatively, (Brigham and Gapenski, 1996) and (Ehrhardt and Brigham, 2011) identify *liquidity*, *asset management* (operational), *debt management*, *profitability*, and *market value* ratios.

The use of ratios in the analysis of the financial strength of corporations dates back to the early beginnings of the stock markets. This form of value assessment became the target of rigorous scrutiny long after the great depression of 1929/30, since the great crash in stocks exposed the fragility in share-evaluation methods – everyone was asking if stocks had not been overrated. It was in this context that one of the best books in financial analysis emerged (Graham, 1934) under a fundamental perspective (*Security Analysis*, which became a reference for corporate stock, bond and warrant analysis). It is important to say that FRs for themselves might not suffice for the explanation of the financial situation of a corporation under analysis; it is recommended a thorough comparative analysis of the ratios within the context of the industry in which the corporation competes, in order to get a more accurate perspective of the studied figures. Following the nomenclature portrayed in the aforementioned books, we will discuss the most preeminent ratios in each category.

2.2.1.2.1 Leverage Ratios

Leverage Ratios represent the relevance of debt in a corporation. They try to grasp how leveraged is the company in terms of external capital. These ratios are of paramount importance, particularly when in recession times, perilous periods in which all debt-related costs may bring a corporation into serious financial problems if it is highly leveraged. So the importance of these indicators resides not only in their absolute and comparative figures within the industry, but also in the economic scenario the corporation / industry is facing at the moment.

The first leverage ratio presented intends to translate the corporation's long term debt structure. It is designated as *Long-Term Debt Ratio* (LTDR) and may be stated as:

$$LTDR = \frac{Long\ Term\ Debt}{Long\ Term\ Debt\ +\ Equity}$$

The LTDR represents the proportion of structural debt within the context of all raised capital (internal and external) at the corporation's disposal. This figure may be adjusted as follows into another similar ratio, the long term *Debt to Equity Ratio*:

$$D/E = \frac{Long\ Term\ Debt}{Equity} = \frac{Total\ Liabilities}{Total\ Assets\ -\ Total\ Liabilities}$$

$$= \frac{LTDR}{1 - LTDR}$$

which is another way of presenting similar information, but now, instead of giving the proportion of long term debt in the context of overall capital, this new indicator shows the relation between long term debt and the corporation's equity, i.e., the comparison between raised capital from outside the corporation and internal capital.

Another leverage measure is *Times Interest Earned Ratio* (TIE), which represents how the corporation earnings can support interest costs, the cost of assuming debt. It shows in perspective how many times earnings can cover interest costs, and therefore it may also be designated as *Interest Cover Ratio*. On the other hand, it may also be interpreted as an equivalence measure: the earnings measured in total interest turns, and is important to know how balanced a firm is financially. The formula may be stated as:

$$TIE = \frac{EBIT}{Interest\ Payments}$$

where EBIT = Earnings before interest and taxes = Sales revenues − Operating costs.

However, earnings do not mean net cash flows, and so if we want to assess financial health in a broader sense, we must acknowledge the existence of other terms that do not represent a cash outflow of the corporation, namely Depreciation and Amortization. With this new perspective in mind we can devise a new leverage ratio, the *Cash Coverage Ratio* (CCR), which may be formulated as:

$$CCR = \frac{EBIT\ +\ Depreciation\,\&\,Amortization}{Interest\ Payments} = \frac{EBITDA}{Interest\ Payments}$$

In this way we may assess the effective proportion of cash available to serve the amount of debt-related financial liabilities (interests) in a more realistic way.

It is important to notice that all these leverage ratios use book-value data and not market-value. This situation may generate biases and inaccuracies in the determination of a corporation's financial status, since not always the value presented in the accounting system represents a true assessment of the respective real value. This is particularly important when valuating assets.

2.2.1.2.2 Liquidity Ratios

Liquidity Ratios focus on the firm's ability to release cash for incoming financial commitments. They measure how easily a corporation can release financial liquidity to pay immediate emerging liabilities. The *Net Working Capital to Total Assets Ratio* (NWC/TA) refers to the proportion of assets that are allocated to the net working capital, which equals the difference between *current assets* and *current liabilities*[1]. The formula of the ratio may be stated as:

$$\frac{NWC}{TA} = \frac{Net\ Working\ Capital}{Total\ Assets} = \frac{Current\ Assets - Current\ Liabilities}{Total\ Assets}$$

[1] Current assets and current liabilities are those available and due within a time span of a year, respectively.

This measure presents the net short term financial position of the firm and gives a good idea of its ability of endure an unexpected situation of financial stress. Another way to present this information would be in the form of the *Current Ratio*, which shows the quotient between current assets and current liabilities:

$$Current = \frac{Current\ Assets}{Current\ Liabilities}$$

The ratio shows the balance between assets and liabilities over the short term.

Another important liquidity ratio is the *Quick* (or *Acid-Test*) *Ratio*, that translates into measure the relation amongst Cash and Equivalents to Cash (including Securities and short term Receivables) on one hand and Current Liabilities on the other:

$$Acid\ Test = \frac{Cash\ +\ Marketable\ Securities\ +\ Receivables}{Current\ Liabilities}$$

$$= \frac{Current\ Assets - Inventories}{Current\ Liabilities}$$

If the analyst wants to be more rigorous in his appreciation of the firm's short term solvability, he will study the *Cash Ratio*, where only most liquid type of assets (Cash and Securities) are considered in the numerator. In this way, the analyst may assess the ability of the corporation to meet the financial immediate responsibilities almost overnight, in case of urgency. The formula may be depicted as:

$$Cash\ Ratio = \frac{Cash\ +\ Marketable\ Securities}{Current\ Liabilities}$$

2.2.1.2.3 Efficiency Ratios

Efficiency Ratios or *Asset Management Ratios* measure how efficiently employed assets are being managed. One of the most well-known is the *Inventory Turnover Ratio* (ITOR), which measures the velocity at which Inventories turn within the company. For instance, if an ITOR assumes value 5, this means, during the period at question, stocks were replaced (or turned over) 5 times on average. For this assessment, it looks more sensible to use the Cost of Goods Sold in the numerator, but other variants are also common, for instance, the use of Sales instead. There are also variants for the numerator of the ITOR. For example we may use the Inventory figures at the beginning of the period, at the end of the period, a daily average, and so on. The formula may be stated as:

$$ITOR = \frac{Cost\ of\ Goods\ Sold}{Average\ Inventory}$$

If we want to extend the analysis to all assets and have a good grasp of the number of times the value of all assets is replaced within a certain period of time, we may use the Asset Turnover Ratio (ATOR):

$$ATOR = \frac{Sales}{Average\ Total\ Assets}$$

This ratio also may be interpreted as the amount of euros a firm generates in sales with one euro of assets. There are also variations to this ratio. We may consider Fixed Assets Turnover instead of Total Assets, or use different forms of compute the Average Total Assets. Nevertheless, the reasoning remains and the interpretation is similar.

The *Receivables Turnover Ratio* (RTOR) stands for the number of times sales of a determined period represent in terms of normal or average Receivables stock within the same period. It is calculated as the quotient between Sales and Average Receivables:

$$RTOR = \frac{Sales}{Average\ Receivables}$$

It depends both on the Sales amount and the kind of credit policy maintained by the corporation. If the corporation gives credit for many days, this will reflect negatively in the RTOR. However, these situations are not always under control of the company, and, for instance, a firm may be forced to extend credit for more days in order to capture clients from the competition. It is therefore more reasonable to view this measure comparatively within the industry in which the corporation operates.

Another efficiency related ratio is the *Average Collection Period* (ACP), or *Days Sales Outstanding* (DSO), which measures the average number of days the corporation takes to collect its receivables and is calculated by the following formula:

$$ACP = DSO = \frac{Average\ Receivables}{Average\ Daily\ Sales}$$

2.2.1.2.4 Profitability Ratios

Profitability Ratios are meant to assess the earning power of a firm. *Net Profit Margin* (NPM) is one of the most important and of common use by fundamental analysts. It measures the proportion of Net Income comparatively to overall Sales:

$$NPM = \frac{Net\ Income}{Sales}$$

The purpose of the corporation's financial manager is to improve this ratio by increasing its positive value. However, this must be done in a sustainable way over the long run. For instance, if the firm is basing its profit margin on high prices, this policy may hurt sales in the near future, and, although the NPM might be high, the corporation could be better off with lower prices and higher absolute Net Income (even though with a lower NPM). Therefore, for the fundamental analyst, the ratio must not be interpreted in isolation and must be contextualised within a set of the firm's financial and operating information.

Another important ratio and also a reference for corporate financial analysis is *Return on Assets* (ROA). It gives us a notion of the proportion of the Net Income relatively to Total Assets (average for the period in analysis). It shows how Assets are being financially productive in terms of Net Income:

$$ROA = \frac{Net\ Income}{Average\ Total\ Assets}$$

The comparison might also be established with relation to Common Equity instead of Assets, thus generating another important ratio, the *Return on Equity* (ROE), which is the quotient between Net Income and Equity. This shows how the capital invested in the corporation is being financially productive, what is the financial gain the Capital owners are attaining by placing their capital in the corporation. The formula is:

$$ROE = \frac{Net\ Income}{Average\ Equity}$$

But corporate earnings might be of little interest by Capital owners if they cannot translate that into individual financial gains, i.e., if the earnings are not distributed. Particularly for institutional investors, the *Payout Ratio* is of great importance, a ratio that translates the proportion of earnings given to capital owners at the end of the year in the form of dividends.

$$Payout = \frac{Dividends}{Earnings}$$

2.2.1.2.5 Market-Value Ratios

Market-Value Ratios use information related to the corporation's stock price (a variable dependent on the market and therefore on the opinion of outside investors and analysts on the true value of the firm, independently from what the accounting systems might show) and the respective number of shares available for trade.

One of the most typical ratios used is the *Price/Earnings Ratio* (P/E), which compares the price of a single share with the earnings per share that corporation obtained in the recent past. Assuming annual earnings, at any given moment and considering the current market stock price, the P/E shows the number of years necessary for the corporation to generate earnings that cover the value of its shares. This measure gives in an instant idea of the financial effort versus financial gain, and the analyst may ponder with more accuracy if indeed the share is worth of buying. The formula is:

$$P/E = \frac{Price\ per\ Share}{Earnings\ per\ Share}$$

One other important market-value ratio establishes the comparison of a share price with the average Cash-Flow per share, the *Price/Cash-Flow Ratio* (P/CF). It presents the relation between the cash needed to buy one share and the cash the same share releases with its operating activity, and is represented as:

$$P/CF = \frac{Price\ per\ Share}{Cash\text{-}Flow\ per\ Share}$$

The *Market/Book-Value Ratio* (M/BV), by its token, compares the Market Price per share, i.e., what the external investor/analyst considers the fair value per share, with the Book-Value of a single share, that is, the value per share in accounting terms. This represents a confrontation of perceptions: the external perceived value versus the corporate's internal declared value per share. The formula is stated below:

$$M/BV = \frac{Market\ Price\ per\ Share}{Book\ Value\ per\ Share}$$

2.2.2 Technical Analysis

The crisis of 1929/30 had an impact on the way analysts used FA: a large number of investors, losing confidence in fundamental methodologies, which had performed poorly, was eager to try other ways of asset valuation or market price direction forecasting – a great deal of traders turned into Technical Analysis, the study of price movement.

Technical Analysis (TA) typically deals with *price*. In the TA spectrum, all effects are mostly aggregated into this single variable, which is considered to embed all the information required to understand market action. Naturally, one might argue TA also includes the use of *open interest* and/or *volume*, and that may be true but only to some extent, for the indispensable variable remains *price* and the others possess a complementary nature. Hence, TA attempts to predict market price moves fundamentally on the basis of price history. TA assumes three generic tenets – (Murphy, 1999):

1) market action discounts everything,
2) prices move in trends, and

3) History repeats itself through time.

It is interesting to notice the paradox of TA versus the EMH about the assumption that price discounts every bit of information: while the first considers it to be a condition to use price as the single studied variable and part of the reason why price can be predicted (it encloses in itself all the market information needed to perform an estimation of its future value, or at least the direction of the anticipated movement), the second takes this premise as a condition to invoke the existence of market efficiency as well as the random character of price and therefore a reason why prices cannot be predicted. The same assumption leads to two contradictory conclusions.

(Menkhoff and Taylor, 2007), in a literature review article, analyse the predictive capabilities of TA and the major possible reasons for its wide utilization in the forex market, verifying four distinct explaining hypotheses or arguments: 1) TA as reflecting irrational behaviour (traders may not reflect in their activity a full rational reasoning, generating market inefficiencies, allowing for that matter the effectiveness of TA, at least for the short term); 2) TA as exploiting the impact of central bank interventions (distortions/inefficiencies introduced in the markets by central banks); 3) TA as a method of information processing (in the sense that economic policies take time to be completely reflected in forex currency quotes); and 4) TA as providing information about non-fundamental exchange rate determinants (the influence of other reasons beyond economic nature, such as psychological, self-fulfilling beliefs or other). They conclude that it is unlikely that the reasons mentioned in 1 or 2 may have an important role in the use of TA, emphasising reasons 3 and 4.

In TA there are two major branches of analysis – *Chart Analysis* (sometimes called "Subjective TA") and *Indicator Analysis*, or, according to (Aronson, 2007), "Evidence-based TA". In the next few subsections, we will explore in detail these branches of TA, in particular the indicators used in this kind of analysis.

2.2.2.1 Chart and Pattern Analysis

Chart analysis centres in the study of market price patterns depicted in graphs and the prediction of price movement upon their detection. This implies price patterns are recurrent in time and that it is possible to recognize those patterns with relative ease before the whole pattern unfolds, and subsequently exploit any possibility of price prediction and profit taking.

There are different kinds of charts, such as line, candlestick and bar charts. Point-and-figure charts are somewhat different in nature from the others, and their conceptual nuances prevent the generalized pattern formation recognition. This branch of TA is often criticized for the inherent subjectiveness of its procedures.

2.2.2.1.1 General Pattern Formations

The branch of "Chart Analysis" in TA has already a long list of theoretical pattern formations. Among the most common and popular are double tops, double bottoms, head-and-shoulders, wedges, triangles, flags, gaps, pennants, rounding tops, rounding bottoms. (Savin et al., 2006) explore the predictive power of the head-and-shoulders price formation, finding strong evidence that such specific pattern may successfully be used in short sales (as a complement of information, not a stand-alone strategy).

(Bulkowski, 2005) recognizes over sixty different chart formations, presenting a series of pattern-related statistics: general statistics (number of formations, average rise/decline, number of rises/declines over a reference percentage, 45%, benchmark performance - comparative to S&P500 performance); failure rates; breakout and post-breakout statistics; size statistics; volume statistics; a list of trading tips for better performance; and others. For the production of these statistics, Bulkowski used data from 500 stocks of the S&P (5 years each, from 1991 to 1996) and of various stocks from NYSE, AMEX or NASDAQ (time span from 1996 to 2002).

Although acknowledged as genuine by trading industry practitioners, chart analysis has been clearly the most refuted field of knowledge in financial markets analysis, wavering between uneasy acceptance and pure contempt by academics (the latter being a symptom of the academic bias against the so called *charting folklore*, consequence of the poor objectiveness of such analyses).

2.2.2.1.2 Dow Theory

The Dow Theory (DT) is a methodology of market interpretation introduced by Charles H. Dow, a prominent journalist in the New York financial business district of the 19th century. He founded the *Dow, Jones & Co.* and *The Wall Street Journal*. He also developed indexes to assess economic progress (today's well known *DJIA-Dow Jones Industrial Average* and the sectorial *Dow-Jones Rail Average*).

Dow studied the price movement of stocks, bonds and commodities, and found out similarities in their behaviour. With time and empirical observation, three simple principles (or assumptions) that today constitute the core assumptions of the DT, were identified by Dow and later developed by (Hamilton, 1922):

1) Manipulation – price manipulation is possible on a daily basis, but the major trend cannot ever be manipulated;
2) The Averages discount everything – all relevant information is incorporated in the indexes;
3) The theory is not infallible.

This last one leaves room to failure admission, and in spite of the saying «The Exception makes the Rule», it undermines the possibility of considering DT as a real scientifically sustained theory. In addition to these three principles some other hypotheses were added to the theory by William Peter Hamilton, and later resumed and systematized by (Rhea, 1932):

- There are 3 kinds of movements in the Averages – primary movements, secondary reactions, and daily fluctuations;
- To determine a trend, both Averages (Industrial and Rail) must confirm it;
- Volume behaviour must confirm changes of trend;
- An individual stock may not accompany Averages' trend due to specific conditions.

Nowadays, the DT still gathers a significant amount of followers in the trading industry, remaining as one of the major TA techniques available.

2.2.2.1.3 The Wave Principle

The Elliott Wave Theory (EWT), or “The Wave Principle” as called by its author, Ralph Nelson Elliott, is a methodology considered an extension both in content and in depth of the Dow Theory. Although with common aspects with The Wave Principle, the DT differs in nature from the former: while the EWT is based on mass psychology and its inherent behavioural reactions, DT is economic-based oriented. When some critics declare the Wave Principle as being nothing more than a refined Dow Theory, they miss completely the point here – the two theories have different approaches at their core. The similarities between them are formal aspects of pattern recognition and interpretation of technical indicators of wave progression. In fact, despite being at the dawn of technical analysis, DT remains in its essence very fundamentally driven, as if every market price movement could or should be explained by economic reasons. This is expressed in the assumption that the stock market discounts all relevant news. For the EWT, news are not so new, therefore they are irrelevant as predictors, and usually reveal the social and psychological mood of the market arena.

Elliott observed first in (Elliott, 1938), later in (Elliott, 1946) that human actions follow predetermined patterns confined to an established law, since humans are part of Nature, and Nature (or the Universe) works under deterministic laws. Once the character of the natural law is discovered, it can be understood, may be mastered (therefore, predicted) and used towards the trader’s benefit. According to his findings, movements in stock prices depict a 5 wave impulse (motive waves numbered 1-2-3-4-5) followed by a 3 wave correction (corrective waves lettered A-B-C). This forms a complete 8 wave cycle of price movement. Elliott noted a consistent behaviour in these waves, in three particular circumstances: 1) Wave 2 never regresses wave 1, or put in other terms, prices of wave 2 never fall below the beginning of wave 1; 2) Wave 3 is never the shortest wave; in fact it usually is the biggest in length and price expansion; 3) Wave 4 never falls back into the price territory of wave 1. These three conditions are the core assumptions of the wave theory. If any of these rules is broken in a chart wave count, then there must be something wrong with the count under the Elliott Wave perspective.

After being forgotten for a period of two decades, (Frost and Prechter, 2005) recovered the theory and resumed the research in this field. Prechter conquered a renowned position in the trading industry, following the successful application of the theory in the mid 80's. (Poser, 2003) also tries to show how we may apply the theory profitably. In the last few years, Prechter along with Parker, have been working on what they call "Socionomics", exploring the existing strong correlation between economic environment (expansion/recession) and psychological mood (enthusiasm/depression). The common social sciences, such as Sociology and Economics, take for granted a causal relation in which the environment affects social mood; Socionomics, however, propose the inverse: psychological mood changes the way people behave and ultimately induces expansion and contraction moves in the economy – (Prechter, 2003), (Parker and Prechter, 2005), (Prechter and Parker, 2007).

2.2.2.2 Indicators

Indicators comprise the quantitative area of TA. They represent an attempt to introduce the scientific method into this branch of analysis, in the sense that they try to capture relations, tendencies and predictive power in price data. Several authors have devoted their lives to the expansion of knowledge in this field – some invested in trend indicators' analysis and their use for building trading systems, for instance in (Kaufman, 2013), (Katz and McCormick, 2000) and (Schwager, 1996). Other authors dedicated more time to and centred their attention in the study of oscillators – e.g., (Pring, 1993) or (Brown, 1999). There are even authors who made relevant contributions to TA in all categories of indicators and oscillators as they disclosed to the trading industry landmark indicators, such as the RSI, the ADX, Parabolic SAR, ATR – (Wilder, 1978); the MACD – (Appel, 1999). This kind of TA, due to its (more) objective nature, may well be considered evidence-based statistical science – (Aronson, 2007).

This kind of TA uses data (mainly *price data*) to compute values for different indicators, each with a different interpretation, according to the way they are conceived. There are three main categories of TA indicators:

- *Trend Indicators* – try to capture the market tendency;

- *Momentum Indicators*, or *Oscillators* – to identify the market pace, the rhythm at which price flows;
- *Volatility* or *Breakout Indicators* – to detect sudden market volatility variations.

Trend Indicators are based on Moving Averages (MA) that smoothen or flatten price movement. In a certain way, they avoid erratic behaviour, which could, with false signalling, trigger unprofitable decisions when engaging in trading activities (whipsaws; continuous sideway moves). Usually two or three MAs of different time lags are used in a crossover system. The system identifies turning points in price trend motion, the moment of intersection between the short term and the medium/long term MAs. A long position signal is settled when the short MA climbs above the medium/long term MA. A short position signal occurs when the opposite happens: when the short term, more sensitive, MA falls below the medium/long term MA.

Momentum concerns the velocity and intensity of price change and has the implicit idea that rapid and significant changes in one direction are not sustainable over a long period of time; the stronger the move, the more the price will be closer to an extreme market situation (overbought or oversold), and the more likely price will be prone for a reversal. Momentum should not be mistaken with sentiment. *Sentiment* is all about perception of market intervenients (in a direct or indirect fashion – even if not active in trading, still with power to influence the market, such as opinion makers, news stations, survey editors and others) and usually transmits how bullish or bearish they are. Momentum is *price* on the move (or price variation). The difference might seem subtle or irrelevant, but is not – it is almost like *thinking* and *acting* upon it; market is *acting*.

Breakout Indicators use some price-trend or price-band indicator as a reference. When the price shows a determinant movement either up or down passing through the reference line (either price trend or applicable extreme of the band), a signal to buy or to sell is triggered, depending on the pre-established rules of trading.

In a study from the beginning of the century, (Lo et al., 2000) evaluated the efficacy or effectiveness of TA, based on a new approach – new smoothing techniques. They state that TA provides, for certain patterns and with reference to a vast majority of stocks (especially from the NASDAQ), incremental information for decision making purposes. They also claim that TA may be improved by the use of automated algorithms.

(Brown and Jennings, 1989) demonstrated theoretically how, in a model with two periods of time, the usage of past price information presents value to the investor: «Investors use the historical price in determining time 2 demands because the current price does not reveal all publicly available information provided by price histories, that is, investors use technical analysis to their benefit.»

(LeBaron, 2002) acknowledged the existence of sustenance for predictive value of simple technical trading rules, specifically simple moving averages, even though exaggerated by the trading industry. After studying the application of these rules to the forex markets in the 90's, he also found out performances vary throughout the decade, deteriorating along the time span. LeBaron speculates on the origins of these discrepancies, as far as technical trading rules' outcomes are concerned, advancing with some hypotheses: changing markets, the dissemination of the trading rules, forex governmental intervention, the advancement of the internet, reducing trading costs, or, most likely and more importantly, the data snooping bias (data mining or data dredging problem). One main conclusion is that simple technical trading rules present statistical evidence of being useful for profitable price movements' prediction, particularly in longer term time horizons. However, there is strong evidence that forex markets move in an evolving process (we may see a change of regime in successive time-series), and consequently these trading techniques, according to the author, should be used carefully.

(Schulmeister, 2008) assessed the profitability of technical trading models (applied to forex markets – Deutsch Mark, Euro after year 2000, against the US Dollar) for the time span varying from 1973 to 2004. In this work two kinds of models were used:

- a simple unweighted moving average crossover system, in which long signals occur when the fast moving average (1 to 15 trading days' moving window) crosses from below the slow moving average (5 to 40 days), and short position signalling happens in the opposite case (Fast MA crosses the Slow MA from above). The combination of all the different MA windows allows 474 model variations.
- A simple momentum system, represented by the arithmetical difference between currency price at the present moment t and some moment $t - i$ in the past ($M_i =$

$P_t - P_{t-i}$); i varies between 3 and 40 trading days (permitting 38 model variations). In this straightforward trading system, a long position is taken when M_i turns from negative into positive; the reverse (short position) when the opposite ensues.

Schulmeister considered the mentioned variations with and without one trading day lag of execution following the trading signal, summing up to 1024 variants of these two trading model types. The author also considered trading costs of 0.02% per trade, implying a 4 basis points bid-ask spread cost to complete a round turn trade. All of the model variations showed overall positive returns with small p-values for the in-sample period of 1973 to 1999, the majority of which statistically significant with a 95% confidence level. For the out of sample period of 2000 to 2004, over 91% of the models remain profitable. These positive returns are exclusively due to the systematic exploitation of price trends in the considered forex market, since profitable positions last substantially longer periods of time than unprofitable trades. The presented models seem to be quite robust: most of them remain profitable (at similar levels of effectiveness and statistical confidence) for most subsets of sample data. However, it is also clear that the profitability under these models' technical rules declines from late 1980's.

A year later, the same author - (Schulmeister, 2009) – presented a study where trend following and contrarian indicators were tested with S&P 500 spot and index futures trading data. These included moving average / raw momentum indicators transformed into relative measures, and the relative strength index, combined in six models with different sets of trading rules. In his work, Schulmeister uses two different kinds of indicators, that basically are the differential of *price* (either raw or smoothed, as in a moving average) in two distinct moments of time, and he calls them *oscillators* (for fluctuating around zero, through the abscissa axis). There is no limit for these indicators' fluctuations, and their magnitude depends on the absolute variation of price itself. This is not the concept of *momentum indicators* or *oscillators* adopted in this thesis – here, oscillators refer to the measure of the pace at which prices move, translated into the relative strength of *bulls* / *bears*, within a limited ordinate axis scale. It is assumed the more rapidly prices move towards one specific direction and hit extreme levels (overbought/oversold), the more prone they are for a reversal in directional movement.

Two of the models assumed constant active participation in the market, i.e., once a position is taken the trader is always in the market, holding or reversing his position; the other four admit the possibility of being out of the market within two consecutive trades. With these premises, a total of 2580 trading systems (model variations) were conceived, which were tested in daily as well as in 30-minute time lag data. Sub-periods of 3 to 4 years were also taken into consideration for partial performance assessment.

Schulmeister came to these major conclusions:

- As far as daily data is concerned, technical trading models have been presenting a gradually smaller profitability, declining over time; when used with 30-minute frequency data, the models showed more consistency, with higher profitability - an average return of 7.2% per year throughout the complete sample period of 1983 to 2007;
- A residual 2.6% of all 2580 trading systems would have produced negative outcomes over the entire sample;
- The top performance 25 trading systems for the period of 1986 to 2007 attained an average gross return of 14.5%, way above the overall gross rate of return over that same time span (7.5%);
- The shift of profitability patterns provided by the gathered statistical evidence fits and sustains the Adaptive Market Hypothesis by Lo.

(Olson, 2004) reaches similar conclusions about the performance evolution of technical trading systems. Olson tested the profitability of simple moving averages for the forex market (18 different currency crosses against the US dollar, from 1971 to 1995) and found out the 1970's statistically significant positive risk-adjusted returns had vanished in the 1990's, supporting the idea of declining profits over time. The author suggests, for the forex, the progression from circumstantial (temporarily) inefficient markets to a mature efficient market status.

From this brief literature review we may see different assessments on the usefulness of TA indicators as a tool for trading. While in some circumstances it seems to produce relatively good results, indicator-based TA lacks the consistency over time that might allow it to be considered an important trading aiding tool. Our empirical analysis will try to unfold and scrutinize the real contribution of TA indicators to the achievement of above normal profits in the trading industry. In the following subsection we will present the used TA indicators and the associated trading rules, as well as their parameters subject to optimization.

2.2.2.2.1 Simple Moving Averages

There are various types of MA's, depending on the weight given to the observations used for the MA computation. A Simple Moving Average (SMA) uses identical weights for all observations. The value of the SMA reporting to moment t with lag of n_{SMA} observations will be:

$$SMA_t^{n_{SMA}} = \frac{\sum_{i=1}^{n_{SMA}} P_{t-i+1}}{n_{SMA}}$$

The shorter the n_{SMA} value, the quicker the moving average will adjust itself to new price data. Usually SMAs with different numbers of lags are used: an SMA with a smaller number of lags is termed *faster*, and an SMA with a larger number of lags is termed *slower*. The crossing of the resulting curves (fast vs. slow) is seen as a signal of a new trend and is commonly taken as a good entry point in a market for short term trading. In this work, we consider two different simple moving averages crossover indicators: a two SMA crossover (with a maximum moving window of 30 observations) indicator and a three SMA crossover (with moving window length up to 90 observations).

The implicit trading rules vary in these two strategies: while in the first case a simple crossover of the fast MA generates a position signal and, therefore, the trader is always in the market long or short, in the second situation, the fast MA needs to cross both the intermediate and the long term MAs in order to make the trader/system assume a long or short position (provided that it does not happen, we hold out of the market).

For these two different types of trend strategies, the parameters to be object of optimization are:

n_{SMA1}; n_{SMA2} = moving window length for the first and second SMA's,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$ (2 SMA Crossover)

and

n_{TSMA1}; n_{TSMA2}; n_{TSMA3}; = moving window length for the first, second and third SMA's,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 90\}$ (Triple SMA Crossover).

2.2.2.2.2 Exponential Moving Averages

A possible variation of the simple moving average technique might be the weighted moving average. A Weighted Moving Average (WMA) will assume different weights, and may be stated as follows:

$$WMA_t^m = \frac{\sum_{i=1}^{m} w_i \cdot P_{t-i+1}}{\sum_{i=1}^{m} w_i}$$

where w_i stands for the weight of observation i in the WMA and m represents the moving window size.

A special case of the WMA is the Exponential Moving Average (EMA), where the following relation of weights (w_t; w_{t-1}) holds:

$$w_{t-1} = \alpha \cdot w_t \Leftrightarrow \alpha = \frac{w_{t-1}}{w_t} \quad , 0 < \alpha < 1 \qquad (1)$$

and α is the relation of weights w_{t-1} and w_t.

The tendency of traders and analysts in the industry is to give more weight to recent data, bringing more emphasis to new price formations at every moment of time. Usually, weights in this sort of modelling may have a linear rate of decrease or even an exponential rate of decrease. In this context, we may define a n^{th} moment moving average when this smoothing process is applied n times to price data (raw and subsequently transformed data).

There are other ways to compute an exponential moving average. For instance, Matlab calculates a specific EMA where the relation between the weights of the moving window depends on the moving window length itself. In this case, the constant k will be used to calculate the EMA is determined by the following expression:

$$k = \frac{2}{(1 + n_{EMA})}\ , 0 < k < 1$$

where n_{EMA} is the EMA's moving window size,

$$EMA_t^{n_{EMA}} = \left[Close_t - EMA_{t-1}^{n_{EMA}}\right] \cdot k + EMA_{t-1}^{n_{EMA}}$$

$Close_t$ is the closing price at moment t,

and the first possible computable EMA will be equal to the correspondent SMA of the same window size. In this type of indicator (applied to a 2-EMA-Crossover), we have two different kinds of parameters for optimization:

α_{EMA1}; α_{EMA2} = *weight ratios* for the first and second EMAs;

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

n_{EMA1}; n_{EMA2} = *moving window length* for the first and second EMA's,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

The strategy will work just as in any two simple moving averages crossover: start a long position when the fast MA surpasses the slow MA; hold an inverse position when the opposite happens.

2.2.2.2.3 Average True Range (ATR)

The *True Range* (TR), developed by (Wilder, 1978), represents the widest price variation within a complete single moment, i.e., the maximum price variation since the close of the previous day, which is the same as:

$$TR_t = Max \begin{cases} High_t - Low_t \\ High_t - Close_{t-1} \\ Close_{t-1} - Low_t \end{cases} \quad (2)$$

where $High_t$, Low_t and $Close_t$ represent the highest, the lowest and the closing prices negotiated at period t.

The Average True Range is an exponential moving average of n TR observations:

$$ATR_{t,n_{ATR}} = \sum_{i=t}^{n_{ATR}} \frac{w_{t-i+1}}{\sum_{j=1}^{n_{ATR}} w_{t-j+1}} . [max(High_i;\ Close_{i-1}) - min(Low_i;\ Close_{i-1})]$$

where again the weights between two consecutive moments relate with each other according to equation (1) – this means that the averages computed within the moving window are EMAs. The ATR can be used in a breakout trading system, where a long position signal is generated in t when price in moment t moves above the close price EMA in moment t accrued by the ATR value or a multiple thereof. By the same token, a short position signal is ignited when price of moment t decreases below the close price EMA in moment t minus (a multiple of) the ATR value. In the conception of this trading system, the following parameters are subject to consideration:

α_{CPema}; α_{ATR}= weight ratios for the computation of the EMA and ATR values, respectively.

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

n_{CPema}; n_{ATR} = moving window length for the determination of the EMA and ATR values.

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

2.2.2.2.4 Directional Movement Crossover (±DI Crossover)

The *Directional Movement Index* (DMX or ±DI) is an indicator embedded in a trading system developed by an American engineer with a passion for the markets, J. Welles Wilder, Jr., in (Wilder, 1978). To compute this index, the following steps should be met:

1. Calculate the *True Range* (TR) for each day according to

$$TR_t = [max(High_t;\ Close_{t-1}) - min(Low_t;\ Close_{t-1})]$$

which is equivalent to (2);

2. Calculate +DM and –DM, as:

$$+DM_t = High_t - High_{t-1}$$

$$-DM_t = Low_{t-1} - Low_t$$

3. Determine +DI and –DI as exponential averages of +DM and –DM (smoothed indicators) divided by a TR exponential average of the same length:

$$\pm DI = \frac{EMA_t^{n_{DMI}}(\pm DM)}{EMA_t^{n_{DMI}}(TR)}$$

With the ±DI data, a long position signal is triggered when +DI>-DI; a short position signal is generated when the reverse happens, i.e., the moment +DI<-DI. For this kind of indicator, we will proceed with the optimization of the following parameters:

α_{DIatr}; α_{EMA+DM}; α_{EMA-DM} = weight ratios for the TR exponential average, +DM and -DM, respectively;

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

n_{DMI} = moving window length for all three EMA's,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

2.2.2.2.5 *Moving Average Convergence/Divergence (MACD)*

The Moving Average Convergence/Divergence (MACD) is an indicator created and initially published by (Appel, 1999). This indicator is a combination of price exponential moving averages that, through a confrontation with a MACD EMA (the *signalling function*), generates buying and selling signals. In his original configuration, Appel used time periods of (n_{FEMA}; n_{SEMA}; n_{Signal}) = (12; 26; 9), corresponding to the *fast price*, *slow price* and *MACD signal line EMAs* time lengths, respectively.

$$FastEMA_t = EMA_t^{n_{FEMA}} \quad , n_{FEMA} = 12$$

$$SlowEMA_t = EMA_t^{n_{SEMA}} \quad , n_{SEMA} = 26$$

$$MACD_t = FastEMA_t - SlowEMA_t$$

$$Signal_t = EMA_t^{n_{Signal}}(MACD_t) \quad , n_{Signal} = 9$$

The common technical strategy associated to this indicator states we should buy (long position) when the MACD values rises above the Signal line and sell (short position) when the opposite situation occurs. The parameters to tweak in this indicator are:

$\alpha_{FEMA}; \alpha_{SEMA}; \alpha_{Signal}$ = weight ratios for the fast, slow and signal EMAs;

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

n_{FEMA}; n_{SEMA}; n_{Signal} = moving window length for the fast, slow and signal EMAs,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

A note should be made about this indicator: it is not unanimous whether the MACD should be classified as a momentum oscillator or a trend following indicator. Some in the trading industry would assign this to the momentum category; we, conversely, consider a trend following indicator. The subjacent reasoning lies on this: a momentum oscillator deals with the confrontation between buying and selling market forces, and that is better translated in their ability, both in speed and intensity, to invade the opponent's ground (how often and how far each of the powers – *bulls* and *bears* – can overcome the other market faction). This ability is measured in new price extremes (highs and lows). The MACD neglects highs and lows, and focuses only in closing price, through a series of constructed (exponential) moving averages, a clear characteristic of trend indicators.

2.2.2.2.6 Relative Strength Index (RSI)

The Relative Strength Index is a momentum oscillator conceived to measure market conditions (overbought/oversold), i.e., the relative pressure of buyers / sellers in a market. (Wilder, 1978) depicts the structure and the rationale of the index:

$$RSI = 1 - \frac{1}{1 + RS}$$

where RS is the quotient between the upward movements' exponential average for an *n*-day period and the downward movements' exponential average for that same period. At the present time, the commonly used version adopts simple moving averages, version popularized by Cutler:

$$RS = \left[\frac{\sum_{i=1}^{n_{RSI}} U_{t-i+1}}{n_{RSI}}\right] \Big/ \left[\frac{\sum_{i=1}^{n_{RSI}} D_{t-i+1}}{n_{RSI}}\right]$$

being

$U_t =$ price variation of day *t* for when closing higher than the previous day

and

$D_t =$ price variation of day *t* when closing lower than the previous one.

This RS is, as we may see, equal to

$$RS = \frac{\sum_{i=1}^{n_{RSI}} U_{t-i+1}}{\sum_{i=1}^{n_{RSI}} D_{t-i+1}}$$

and again, we may rewrite the RSI as

$$RSI = 1 - 1 \Big/ \left[1 + \frac{\sum_{i=1}^{n_{RSI}} U_{t-i+1}}{\sum_{i=1}^{n_{RSI}} D_{t-i+1}}\right]$$

Typically, this oscillator is built with reference to the past fourteen days ($n_{RSI} = 14$) and Wilder himself used this time frame (the last 14 daily closing prices), but the indicator may be also customized. On one extreme, if there is a rising streak in market prices and the sum of D_t assumes zero value, RS will tend to infinity and RSI to 1 or 100%. This means the market is completely overbought (exaggeratedly bought) and is prone for a reversal. On the other hand, with a continuous falling market, the sum of U_t and RS will tend to zero, generating also a null RSI, i.e., RSI = 0%. In this situation the market is extremely oversold (exaggeratedly sold) and is likely to go up.

Empirical evidence in several major markets over the years since the proposal of this index (RSI has become tremendously popular in the trading industry since its debut) suggests the existence of a general 30%-70% band where the indicator moves most of the time. These figures have been appointed and commonly accepted as oversold and overbought marks. This also depends on the number of days used for calculation. The lower this number, the more sensitive RSI will be to price fluctuations, igniting false overbought / oversold signals. The parameters to be fine-tuned in this indicator are:

ub_{RSI}; lb_{RSI} = upper and lower bounds, respectively, for the RSI signal activation,

$\{ub_i; lb_i \in \mathbb{R}: 0 \leq ub_i \leq 1 \wedge 0 \leq lb_i \leq 1 \wedge lb_i < ub_i\}$

n_{RSI} = moving window length for the two moving averages of U_t and D_t,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

For the purpose of this work, we have used the typical rule and an alternative rule of signal generation. The typical rule implies that a short position will be taken if the RSI hits the *ub* in an ascending movement – the short position will stand until the RSI signal breaks the *lb* barrier on a falling movement; the converse (long position) is triggered when the RSI moves through the *lb* in a descending movement and only changes to a long position when the RSI signal hits the *ub* on the rise, and so forth. This rationale will be applied in Chapter 4, dealing with the Forex market. In Chapter 5 we will use a different reasoning for trading Stock Markets in a portfolio optimization perspective: we assume a long position if the RSI is above the specific *lb* of 30% and hold that position while it remains on the rise; whenever this condition is not met, we stay out of the market.

2.2.2.2.7 Williams %R

The percentage-R oscillator, introduced by Larry Williams, measures market saturation conditions. The proposed formula for the computation of this indicator is as follows:

$$\%R_t = \frac{Close_t - HH_{t:(t-n_{\%R}+1)}}{HH_{t:(t-n_{\%R}+1)} - LL_{t:(t-n_{\%R}+1)}}$$

where

$HH_{t:(t-n_{\%R}+1)}$ refers to the highest high within the previous $n_{\%R}$ days to moment *t*;

$LL_{t:(t-n_{\%R}+1)}$ refers to the lowest low within the previous $n_{\%R}$ days to moment *t*

%R varies from -100% (extreme oversold market conditions) to 0% (extreme overbought conditions). Although with an uncommon scale, its rationale of interpretation is similar to any other oscillator: if the indicator approaches the lower limit (-100%) the market is likely to reverse from a declining price trend; the opposite holds also – if the indicator moves towards the upper limit (0%), the market is prone to react from an advancing trend and move down.

In his studies, Williams considered $n_{\%R}$=10, and -80% and -20% as thresholds for oversold and overbought market conditions, respectively. As trading strategy, Williams adopted a sort of pullback effect: to buy only when -100% was reached and the oscillator subsequently moves to -95% and confirms at -85%; to sell if 0% level is attained and levels -5% and -15% (confirmation) follow. As a market saturation indicator, it may also be used with a simpler strategy – to buy when the index hits the oversold threshold (*%R*=-80%) or the reverse when overbought conditions are met (*%R*=-20%). For this indicator three parameters were identified:

$ub_{\%R}$; $lb_{\%R}$ = upper and lower bounds (thresholds), respectively, for the Williams %R signal generator,

$\{ub_i; lb_i \in \mathbb{R}: 0 \leq ub_i \leq 1 \wedge 0 \leq lb_i \leq 1 \wedge lb_i < ub_i\}$, and

$n_{\%R}$= moving window length for the determination of the $HH_{t:(t-n+1)}$ and $LL_{t:(t-n+1)}$,

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

The rules of trading applicable to this indicator are to adopt a long position when hitting the lower bound (oversold) in a fall; assume a short position in the market when the upper bound (overbought) is hit in a rising movement; hold the position otherwise.

2.2.2.2.8 Stochastic Oscillator

The stochastic oscillator is, in fact, a set of momentum indicators developed and proposed by (Lane, 1984). The concept of this oscillator-based trading system sustains its rationale on the applied smoothing process, by using a 3-day exponential moving average (in two different phases) on a raw indicator of relative saturation or price exhaustion, the *fast stochastic %K*:

$$Fast\%K_t = \frac{Close_t - LL_{t:(t-n_{F\%K}+1)}}{HH_{t:(t-n_{F\%K}+1)} - LL_{t:(t-n_{F\%K}+1)}}$$

From this fast stochastic, we may derive one first signalling line, the *Fast %D* or *Slow %K*:

$$Fast\%D_t = Slow\%K_t = \left[EMA_t^{n_{F\%D}}(Fast\%K_t)\right]$$

The joint use of these two indicators in a crossover system is called "Fast Stochastics". But, because of its acute sensitivity, this system is prone to too much false signalling. Lane saw this caveat / shortcoming and smoothed further the system indicators:

$$Slow\%D_t = \left[EMA_t^{n_{S\%D}}(Slow\%K_t)\right]$$

where typically $n_{F\%K}, n_{F\%D}, n_{S\%D}$ assume value 3,

and $n_{F\%D}$ and $n_{S\%D}$ stand for the faster and slower EMAs moving window lengths, respectively.

The Trading Industry suggests this last smoothing process (and the correspondent use of slow stochastics) avoids the sensitivity trap and respective false signals. Long position signals are triggered when %K moves above %D and short position signals when the opposite happens (either in fast or in slow stochastics crossover).

There is a strict connection between *fast stochastic %K* and *Williams %R*: %K is equivalent to %R with 100 percentage points added to the scale. Considering the following equality:

$$\left(Close_t - LL_{t:(t-n+1)}\right) - \left(Close_t - HH_{t:(t-n+1)}\right) = HH_{t:(t-n+1)} - LL_{t:(t-n+1)}$$

and rearranging, we obtain

$$\frac{\left(Close_t - LL_{t:(t-n+1)}\right)}{\left(HH_{t:(t-n+1)} - LL_{t:(t-n+1)}\right)} - \frac{\left(Close_t - HH_{t:(t-n+1)}\right)}{\left(HH_{t:(t-n+1)} - LL_{t:(t-n+1)}\right)} = 1$$

which is the same as

$$\%K_t - \%R_t = 1$$

or

$$\%K_t = \%R_t + 100\%$$

The Stochastic oscillator has the following parameters subject to optimization:

$\alpha_{F\%D};\ \alpha_{S\%D}$= weight ratios for the computation of *Fast%D* and *Slow%D* EMAs;

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

$n_{F\%K};\ n_{F\%D};\ n_{S\%D}$= moving window length for the determination of *Fast%K*, *Fast%D* and *Slow%D* EMAs, $\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

Also in this indicator we will use the trading rules as previously stated in this subsection, but now with reference to the smoothed indicators (*Fast%D* and *Slow%D*), i.e., assume a long position while *Fast%D* > *Slow%D* and a short position if *Fast%D* < *Slow%D* (out of the market in the remote possibility of *Fast%D* = *Slow%D*).

2.2.2.2.9 Bollinger Bands (BB)

Another tool for market technical analysis are the *Bollinger Bands*. Bollinger Bands are formed by a simple moving average of n-day lag with a confidence interval of k price standard deviations for that same period of n days. Usually, the common adopted parameters are $n_{SMABB} = 20$ days and $k_{BB} = 2$. Once more, parameters n_{SMABB} and k_{BB} may be tuned using evolutionary algorithms.

$$BB_{t,n,k} = SMA_t^{n_{SMABB}} \pm k_{BB} \cdot \sigma$$

This kind of indicator, as widely interpreted by the Trading Industry, is often used in a breakout trading system, where price is compared, at any given moment, with the upper and lower bands; if it crosses the former in upward movement, a long position is signalled, the opposite (short position) occurs if price plunges below the latter.

In order to generate Bollinger Bands, we will have to determine these parameters:

n_{SMABB} = moving window length for the determination of the Bollinger Bands Simple Moving Average and standard deviation

$\{n_{SMABB} \in \mathbb{N}: 2 \leq n_i \leq 30\}$

k_{BB} = number of standard deviations used in the computation of the Bollinger Bands

$\{k_{BB} \in \mathbb{R}: 0 < k_i \leq 5\}$

2.2.2.2.10 EMA ± $k\sigma$ Bands

Instead of using a Simple MA, we may generate a more elaborated indicator structure by adopting an Exponential MA of the closing price.

$$CPEMA_{t,n,k} = EMA_t^n \pm k\sigma$$

This is the main difference regarding the trading system depicted in the previous subsection. The other difference relatively to the *BB* lays in the possible use of diverse k's for the determination of long and short market positions. The signal generation procedures and trading rules remain the same, i.e., a long position signal is activated when the current price moves above Close Price EMA plus $k\sigma$ and the converse whenever price stays below Close Price EMA minus $k\sigma$. For the computation of the indicator it is necessary to determine the following parameters:

$\alpha_{CPemaSD}$= weight ratio for the computation of the Close Price EMA

$\{\alpha_{CPemaSD} \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

$n_{CPemaSD}$= moving window length for the determination of the Close Price EMA

$\{n_i \in \mathbb{N}: 2 \leq n_i \leq 30\}$

$k^{long}_{CPemaSD}$; $k^{short}_{CPemaSD}$= number of standard deviations used for the computation of Close Price EMA ± kσ

$\{k_i \in \mathbb{R}: 0 < k_i \leq 5\}$

2.2.2.2.11 Double EMA Breakout

In a Double EMA Breakout system, current price is compared with two distinct Close Price EMAs, where the trading rules are to take a long position as long as price stays above the two moving averages and a short position while the price remains below the same two averages; whenever the mentioned conditions are not met, the trading system signals to stay out of the market. In order to calculate those two EMAs, it is essential to assume two different figures for each moving average – the weight ratio between two consecutive average observations and the moving window length:

α_{EMA1}; α_{EMA2}= weight ratios for the computation of the two EMAs

$\{\alpha_i \in \mathbb{R}: 0 < \alpha_i \leq 1\}$

n_{EMA1}; n_{EMA2}= moving window length for the determination ofthe two EMAs

$\{n_i \in \mathbb{N}: 1 \leq n_i \leq 30\}$

2.3 Financial Risk-Return Analysis

2.3.1 Portfolio Modelling

2.3.1.1 Mean - Variance

The *Mean-Variance* (MV) analysis in financial context is perhaps the most well-known and most common approach to portfolio building. That is mainly due to the Harry Markowitz's work (Markowitz, 1952) in the projection of this subject in finance. Before Markowitz, portfolio analysis was inexistent both in academic terms. So much so that humorously, Milton Friedman argued he could not give Markowitz the doctoral degree in economics since portfolio theory was not economics. Nonetheless, (Markowitz, 1952) expanded *Variance* as a standard risk measure for financial portfolio optimization, and he still got his PhD diploma.

In this approach, the goal is to maximize the expected portfolio return and minimize its variance. For a given *asset*, the expected future return and variance are usually estimated resorting to the past returns. For the expect return, we define:

$$E(R) = \frac{1}{T}\sum_{t=1}^{T} R_t$$

A measure of variance σ^2 of a determined *asset* is defined as:

$$\sigma^2 = \frac{1}{T}\sum_{t=1}^{T}[R_t - E(R)]^2$$

where

R_t is the asset's rate of return in moment t;

$E(R)$ is the asset's average return for the period from $t = 1$ to $t = T$;

T is the total number of trading days, the length of time.

A measure of the expected return of a *portfolio P* with n assets can be calculated as:

$$E(R_P) = \frac{1}{T}\sum_{t=1}^{T} R_{Pt} = \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{n} w_i R_{it}$$

and a measure of its variance σ_P^2 is:

$$\sigma_P^2 = \frac{1}{T}\sum_{t=1}^{T}[R_{Pt} - E(R_P)]^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_i\, \sigma_j \rho_{ij}$$

where

R_{Pt} is the portfolio rate of return in moment t;

R_{it} is the rate of return of asset i in moment t;

$E(R_P)$ is the expected portfolio rate of return with reference to period from $t = 1$ to $t = T$;

w_i is the weight of asset i in the portfolio;

σ_{ij} is the covariance between asset i and asset j for the period from 1 to T;

σ_i is the standard deviation of asset i for the period from 1 to T;

$\rho_{ij}\left[= {}^{\sigma_{ij}}/_{\sigma_i \cdot \sigma_j}\right]$ is the (Pearson) correlation coefficient between assets i and j, for the period from 1 to T.

Although Markowitz admitted that variance is not a perfect measure of risk, as it portraits the volatility of both unfavourable and advantageous return variations, variance assumed a preeminent strong role in financial risk assessment worldwide, mostly because it requires a calculation effort that is much smaller than a downside risk measure (this was particularly relevant in a time where personal computers and laptops were essentially non-existent in business activities). Thus it became the reference of risk estimation from the mid 1960's to the 1990's. With the advancement of computer technology, and the availability of more computational capabilities, the main reason to use variance as reference measure for risk assessment faded away, and we started to see pioneering works dealing with more realistic and useful risk measures. Nevertheless, even today it is virtually impossible to dismiss Markowitz's variance as an important measure of risk. In fact, it is so established in today's risk analysis that (Kritzman, 2011), in an invited editorial for *The Journal of Portfolio Management* entitled "The Graceful Aging of Mean-Variance Optimization", advocates the robustness of this method for risk-return analysis, dismissing several criticisms (some eventually unfair) and anticipating a still long life for the model.

(Brito et al., 2015) call attention to the sometimes misinterpretation of the sufficient conditions in order to apply the MV model. It is consider sufficient the normality of the return distributions or the existence of quadratic utility functions, as Markowitz has already clarified (Markowitz, 2014). However, it is not uncommon to see these conditions considered as necessary to a valid application of the MV model of Markowitz.

This model has also been criticized by not taking into consideration the use of *leverage*, so common in the trading industry. (Jacobs and Levy, 2013), for instance, developed the Mean-Variance-Leverage model (MVL) and compared with the original Markowitz model, in particular when observing market and trading conditions that may affect the trading activity itself, such as the risk and associated costs with margin calls, losses above capital invested, and bankruptcy. They conclude that the MVL model provides many practical insights and a simpler procedure of portfolio stock selection, allowing investors some volatility and leverage tolerance (mild, not excessive), and therefore giving a more realistic flavour to the model.

Another criticism to empirical MV-based academic works relies on the fact that the model seems to fall short in terms of performance when trading costs are taken into account. Several empirical studies point out how costs may influence the risk/reward panorama for a set of trading strategies and therefore its Pareto frontier, or Pareto front[2]. (Fu et al., 2015) is such an example that tries to demonstrate how relevant the role of trading costs might be.

[2] The Pareto frontier, or Pareto front, represents the set of nondominated solutions in a bi-objective optimization problem. In a two-objective optimization problem, nondominated solutions are those which cannot be improved regarding one objective's performance without degrading the other objective's performance.

The attained results show transaction costs significantly affect the optimal investment decisions.

The MV framework is also pointed out as not being the most efficient way to produce optimal out-of-sample portfolios. (Karagiannidis and Vozlyublennaia, 2016) present such evidence for the mutual fund financial industry. Empirical results show that out-of-sample MV optimized portfolios perform worse than other portfolios defined by the industry, despite being optimal in-sample. These results suggest MV optimization is more adequate when means/variances/covariances are known than when they are unknown and must be estimated using historical data. In the case of the mutual fund industry, therefore, financial managers may not be using the basic MV framework as a reference. And proven management talent may outperform simple mathematical optimization when applied in a context of uncertainty.

(García et al., 2014) propose a multiobjective evolutionary algorithm-based methodology to solve the MV portfolio optimization problem by adding to the classic risk and return goals a third regarding robustness of the optimized solution. This new robustness measure, denoted by Z_p, equals the sum of the Mahalanobis distances[3] between the pair of combinations $\left[E\left(R_p\right);\ \sigma_p\right]$ and G feasible alternative portfolios ($\{[E(R_i);\ \sigma_i]\ , i = 1, 2, \dots, G\}$), all divided by the number G. This approach provides a three-dimensional efficient frontier giving relevant information about the reliability of the pair [risk; return] for every solution. Experimental work showed how the use of the third objective improved solution reliability, stability, consistency and pattern regularity (high stability frontiers out-perform low stability frontiers). Solutions are considered more stable as they show resiliency and little change to significant parameters variation.

Some authors claim that the assumed positive relation between expected return and risk (i.e., the higher the risk, the higher the expected return) does not always hold, and it depends on the buy-sell imbalance. (Yang and Jia, 2016) conclude that when this imbalance is negative (when selling orders outstand buying orders) there is a significant negative

[3] The Mahalanobis distance between two vectors $\bar{x}$ and $\bar{y}$ is equal to $d_M(\bar{x};\bar{y}) = \sqrt{(\bar{x}-\bar{y})^T \cdot \Sigma^{-1} \cdot (\bar{x}-\bar{y})}$, where Σ represents the variance–covariance matrix.

relation in the MV relation; when the imbalance is positive, the MV relation is not significant. The results are robust regarding different variance models and market capitalization. The analysis makes sense – in a strict stock market, where profits can be made only with increasing prices, it is natural that a strong selling pressure will induce market losses and, because of the intensity of the pressure, high market volatility. Nevertheless, this remains as a caveat of the MV optimization framework.

Considering all the aforementioned limitations of the MV approach, other alternatives emerged both in academia and in the financial industry. For example, (Gülpinar and Rustem, 2007) adopt a *min-max* formulation for a set of multi-period risk-return scenarios and compare its performance with the normal Markowitz MV portfolio optimization. The results show advantages for the *min-max* approach, for it performs better and grants more conservative solutions in terms of risk in the optimization process itself, avoiding excessive losses.

Another classic alternative approach is the naïve 1/N (equal weight) portfolio, where N stands for the number of assets included in the portfolio. Usually, the MV portfolio optimization is benchmarked against a simple trading strategy such as this one, to see how the effort of optimization is rewarded with larger returns and/or smaller risk. Several authors have devoted attention to this comparison, with somewhat mixed results. While some, such as in (Georgiev, 2014) admit a superior performance of the MV optimization approach, more recent papers have been questioning this superiority. (Kourtis, 2015), (Yew Low et al., 2016) and (Yan and Zhang, 2017) reached similar conclusions: although in-sample results enforce the previous superiority idea of the classic MV approach, when considering *transaction costs*, the naïve strategy regularly outperforms out-of-sample MV optimization outcomes.

2.3.1.2 Mean - Semivariance

Markowitz was aware of the conceptual limitations of the variance measure and proposed an alternative measure of risk, the *Semivariance* (Markowitz, 1991), which considers only adverse deviations. In this approach, the goal is to maximize the expected return (portfolio) and to minimize risk, translated into the semivariance measure. Portfolio semivariance is mathematically defined as:

$$S_{PC} = E(\min(0, R_P - C)^2) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - C)]^2$$

where

$E(\cdot)$ is the expected value;

R_P is the portfolio return;

R_{Pt} is the portfolio return at moment t;

T is the time length;

C is a benchmark.

S_{PC} stands for Semivariance of portfolio P relative to benchmark C,

A common value for C is $E(R_P)$, equally proposed by (Markowitz, 1991), so that the benchmark reference is the overall portfolio average return, and the values used for the semivariance calculation are those below this portfolio expected value. This constitutes a problem in terms of computation, since the endogenous nature of the portfolio semicovariance matrix prevents the use of a simple optimization method: the semivariance depends on the portfolio expected value and by its token the expected value is conditioned by the asset *weights* of the portfolio which we want to determine; but these weights also affect the semivariance value.

(Estrada, 2008) proposed and alternative method, based not on the endogenous semicovariance matrix but in an exogenous approximation to this matrix. This new exogenous semicovariance matrix is symmetric and allows the same computational procedure applied in the classic MV optimization problem, granting a simple form to solve the problem. Estrada showed that the correlations of the solutions provided by his proxy matrix and those of the real semicovariance matrix are high. However, this still remains an approximation and not the exact solution. This very notion is attested by (Cumova and Nawrocki, 2011), which revealed how in some circumstances the Estrada's methodology may not work. The authors provided also an alternative method for computing the semivariance avoiding the disadvantageously laborious iterative procedure initially proposed by Markowitz.

Another feasible alternative is to use an iterative procedure, where at any given iteration it is possible to compute the expected value of the portfolio, therefore allowing to calculate the portfolio semivariance for that particular iteration with its specific weights. (Zhang et al., 2012) used a Monte Carlo simulation for that purpose, overcoming the difficulties previously mentioned and yielding in the process an efficient optimization method.

The use of semivariance has been claimed as an alternative to the variance, so it is not a surprise to see comparative analyses of both. (Pla-Santamaria and Bravo, 2013) applied both frameworks to the optimization of portfolios composed by Dow Jones stocks (daily prices data set from 2005 to 2009). For the same expected returns, the M-SV model presented lower risk figures than the M-V, showing how the latter model may be overestimating risk. Other articles used semivariance alongside with other measures of risk, for a more complete risk assessment. (Najafi and Mushakhian, 2015), for instance, used two different measures of risk, the semivariance and the Conditional Value-at-Risk (CVaR) or Expected Shortfall (ES), to test the effectiveness of a specific hybrid GA-PSO metaheuristic.

2.3.1.3 Mean - Lower Partial Moments

The *Mean – Lower Partial Moments* (M-LPM) model represents a generalisation of the M-SV approach, and is commonly used with the purpose of improving and expanding the risk-associated information about a portfolio. Therefore, it is not unusual to see Lower Partial Moments (LPMs) of different orders used in the same optimization problem, in order to better characterize the portfolio behaviour in terms of *variability* with reference to a benchmark (frequently the portfolio expected value). These different $LPM(n; B)$ (lower partial moments of the n^{th} order with respect to a *benchmark* B) may include:

- Semi-Variance$[= LPM(2; B)]$, as defined and considered in subsection 2.3.1.2;
- Semi-Skewness $[= LPM(3; B)]$, which includes those contributions below the benchmark B to the skewness measure; skewness represents a deviation from B along the abscissa axis; this deviation is negative for values smaller than B, and positive for larger values; the semi-skewness relative to B represents only the

negative deviations from the value B and therefore gives an indication of the behaviour of the *probability density function* (*pdf*) of returns to the left of the benchmark, i.e., how the curve of the *pdf* extends to the left side of B;

- Semi-Kurtosis$[= LPM(4; B)]$, that represents the contributions of returns to a more flat *pdf* of returns relative to the normal distribution, which presents a kurtosis of 3; therefore, our semi-kurtosis considers the influence of returns below B towards the kurtosis value;

and so forth.

Usually the adopted benchmark is the $E(R_P)$. Mathematically, and considering the premise that $B = E(R_P)$, we have:

$$SV_P = E(\min(0, R_P - E(R_P))^2) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - E(R_P))]^2$$

$$SS_P = E(\min(0, R_P - E(R_P))^3) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - E(R_P))]^3$$

$$SK_P = E(\min(0, R_P - E(R_P))^4) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - E(R_P))]^4$$

or generically

$$LPM(n; B)_P = E(\min(0, R_P - B))^n) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - B))]^n$$

where

$E(R_P)$ is the portfolio expected value;

R_P is the portfolio return;

R_{Pt} is the portfolio return at moment t;

T is the time length;

SV_P is the portfolio semivariance;

SS_P is the portfolio semi-skewness;

SK_P is the portfolio semi-kurtosis;

$LPM(n;B)_P$ is the n^{th} order portfolio lower partial moment;
B is the benchmark.

The M-LPM framework may be used as an alternative or a complement to the traditional M-V in the risk-return analysis. It points out to different perspectives of risk measurement and improves the available information about returns' behaviour. (Brogan and Stidham, 2008), for instance, uses the LPM as an alternative to variance for portfolio risk assessment. (Kuzmina, 2011), on the other hand, develops a risk analysis for the Baltic Insurance market, providing a different theoretical approach to asset allocation by recurring to the M-LMP framework, and deriving the advantages of using downside risk as a more reliable measure for the Insurance industry, particularly after the 2008 financial crisis. (Gao et al., 2014) presents a parallel risk analysis using two different frameworks for portfolio optimization: the M-LPM and the *Mean – Conditional Value-at-Risk*, which are considered by the authors as the most promising downside risk measures. In the next two subsections, we will explore this new downside probabilistic approach to risk assessment.

2.3.1.4 Mean - Value-at-Risk

Value-at-Risk (VaR) measures the maximum amount of expected financial loss at any given period of time considering a predetermined *risk level* (probability) of α or *confidence level* equal to $(1-\alpha)$, providing that all market conditions remain unchanged. The typical levels of risk used are $\alpha = 0.05 = 5\%$ or $\alpha = 0.01 = 1\%$, but other figures may also be adopted. This risk measure assumed great importance after its recommended use in the Banking & Insurance industries by the Basel committee in 1995.

The VaR formula applied to a portfolio (Kellner and Rösch, 2016), (Wang et al., 2013), (Kawata and Kijima, 2007), (Lwin et al., 2017), might be stated as:

$$VaR_{1-\alpha}(R_P) = \inf\{R_P \in \mathbb{R}: F(R_P) \geq \alpha\} = R_P^*$$

where

$(1-\alpha)$ is the confidence level; α is the risk level; both α and $(1-\alpha)$ between 0 and 1;

R_P stands for the variable portfolio return;

$F(R_P)$ is the cumulative probability distribution function of portfolio returns;

$\inf\{\theta\}$ represents the infimum of the set θ;

R_P^* is the amount correspondent to the VaR with the specific level of confidence $(1-\alpha)$ for the variable R_P.

As we may deduct from the formula, the VaR represents the loss that one might expect at a given risk (probability) of α (or confidence level of $1-\alpha$), if we were to trade a portfolio in the financial market at question. This risk measure was considered to be reliable until the last major financial crisis broke out in 2008, and that fact may be due to several VaR inherent characteristics, which were more intensely exposed during those troubled times, particularly when the most affected economic sectors were the Banking and the Insurance industries.

A first implicit characteristic of VaR relates to *how* it is usually calculated. The usual method for calculating the VaR measure assumes normal distribution of the variable R_P, and its mean and volatility are estimated recurring to historical data. This poses a caveat, since we cannot guarantee the normality of returns' distribution and likewise we also cannot assure the past behaviour (mean; variance/volatility) of R_P remains unaltered through time. The problem of returns' varying behaviour is addressed by (Gargallo et al., 2010). It is proposed a new methodology for the VaR computation in the challenging context of heteroscedasticity. The authors attained better results with their new methodology compared to the classic procedures. (Gencer and Demiralay, 2016) estimate VaR with more relaxed assumptions: in this case, the authors used the *Student-t* distribution (symmetric and skewed) as a reference for the returns' behaviour applied to eight emerging stock markets. The skewed version with heavy tails of the Student-t distribution allowed to obtain the VaR estimations with better predictive power. But the good results may be due to the more conservative return distribution implied by the used *Student-t* distribution (which is flatter than the normal distribution). The fat tails lead to some overestimation of the VaR in comparison with the usual estimation methods. This VaR overestimation might be the reason of better results when the VaR is applied to out-of-sample data to verify its predicting power.

A second characteristic is connected to *what* is computed with this measure of risk. The VaR calculates the value of the return representing the threshold of the α^{th} percentile, which means values to the left of R_P^* have a cumulative probability of α or less, and those to the right have a cumulative probability of at least $(1-\alpha)$. Therefore R_P^* does not represent the worst possible feasible value according to past data, it represents the worst for that specific level of confidence. Now considering this second characteristic in conjunction with the first one mentioned above and we may have a serious problem of risk underestimation if the VaR is interpreted as the worst probable return with a risk level of α, not showing the amplitude of worse possible returns with probability of occurrence inferior to α%.

A third characteristic is the following: the VaR measure is non-subadditive, i.e., the general condition $f(x+y) \leq f(x) + f(y)$ does not always hold, and therefore VaR is not a coherent measure of risk. This may generate additional problems for the portfolio optimization process in a mean-VaR framework, i.e., the VaR of a portfolio may be greater than the sum of their individual VaRs.

Some of these caveats were already known before the financial crisis, as (Kawata and Kijima, 2007) reported. According to these authors, many empirical researches suggested the VaR understated the effective 1% quantile, in particular when the methodology is applied to returns with fat tailed probability density functions. Kawata and Kijima propose a new simple regime switching model to estimate VaR that corrects the underestimation problem. (Bellini and Figà-talamanca, 2007) study the computation of VaR under different of distribution assumptions (in particular the tail behaviour). The authors propose a different methodology – the Markov Conditional Value at Risk (MCVaR) – to better deal with diverse return distributions. Their new approach seems to be more resilient and robust to variations in distribution of asset returns. They also note that classic methods such as the Extreme Valuation Theory (EVT), a specific branch of Statistics dealing with extreme deviations, seem to fall short in accuracy (due to underestimation) when it comes to VaR estimation. The analysis is applied to eight distinct stock market indexes (DJIA, SP500, MIB30, CAC40, FTSE, DAX, SMI and NASDAQ) and the suggested method consistently outperforms the EVT approach.

Other issues may emerge in a portfolio optimization problem in a M-VaR context. (Lwin et al., 2017) acknowledges the challenge of this model, since it may lead to a nonconvex non-deterministic polynomial hard problem, which is very costly in terms of computational effort. A common practice to overcome this difficulty is the use of metaheuristics. In this article, a new multiobjective evolutionary algorithm (MODE-GL) was proposed, which eventually ended performing even better in this type of optimization problem than the NSGA-II (Deb et al., 2002) and the SPEA-2 (Zitzler et al., 2001) algorithms, both in solution quality (the shape of the Pareto front) and in execution time.

(Bernard et al., 2017), on the other hand, suggest the application of a "Rearrangement Algorithm" (RA) to prevent inaccuracies in the calculation of the VaR measure due to errors and omissions with the information used in the Banking industry and applied to credit portfolios. The article is very harsh in the criticism to the unconscious way sometimes the VaR is used without understanding the limitations and caveats implicit to its use. The authors mention the need to estimate a worst-case and a best-case VaR in order to determine a safety band/margin or gap. They also assess this gap is typically very high, and the use of their RA approximates the two VaR figures. VaR assessments of credit portfolios performed at high confidence levels remain subject to a great dose of uncertainty and are not robust.

Considering all the limitations associated with the VaR measure, some researchers have proposed its joint use with other risk alternative measures. One in particular has achieved notoriety within this context: the Conditional Value-at-Risk (CVaR) or Expected Shortfall (ES). (Dedu and Şerban, 2015) applied both measures to portfolio optimization in a two stage procedure, first using the VaR for a preliminary optimization and CVaR applied to the solutions obtained from the first stage. The next subsection will present the CVaR methodology.

2.3.1.5 Mean - Expected Shortfall

With the debacle of the VaR in 2008, the Basel Committee felt the need to recommend new risk measures that might overcome the shortfalls of the previous ones. One that meets those criteria, namely regarding the problems of fat tails in the probability density distribution and subadditivity, is the Expected Shortfall, suggested, for instance by (Rockafellar and Uryasev, 2000), which seems to achieve good results in investment risk analysis. The *Expected Shortfall* (or *Conditional Value-at-Risk* or even *Average Value-at-Risk*) at an α risk level (or $(1-\alpha)$ confidence level) represents the expected return of the α% worst cases in the probability density function of the *Return*. The mathematical formula may be stated as:

$$ES_{1-\alpha}(R_P) = CVaR_{1-\alpha}(R_P) = \int_{-\infty}^{R_P^*} f(R_P)\, dR_P = \frac{1}{\alpha}\int_0^{\alpha} VaR_{\varepsilon}(R_P)\, d\varepsilon$$
$$= ES_{1-\alpha}^*$$

where

$f(R_P)$ is the probability density function of portfolio returns;

$ES_{1-\alpha}^*$ is the specific expected shortfall for the $(1-\alpha)$ confidence level; both α and $(1-\alpha)$ are between 0 and 1.

(Rockafellar and Uryasev, 2002) develop further the advantages of ES over VaR: the ES is able to quantify dangers that the VaR measure could not detect (for instance, the VaR determines the worst expected return at an $(1-\alpha)$ confidence level, but it does not anticipate the worst possible return, which will be worse if the left tail is more extended); and it is a coherent measure of risk, namely because the subadditivity condition holds under the ES measure. The use of this measure allows a more accurate estimation of the risk, unlike the use of VaR which usually has the tendency to underestimate the real levels of risk. The authors also propose a method to calculate the ES and perform an empirical study to support their claims.

The advantages of ES over VaR were already known since the turn of the century, but the recommendations of a prestigious and authoritative committee such as the Basel Committee launched VaR as a standard risk measure and prevented the emergence of *official* mainstream alternatives. Other measures were used, such as variance or semivariance, Sharpe Ratio analysis, CALMAR Ratio and others, but VaR remained as the most relevant risk measure, particularly for reporting purposes in the Banking and Insurance industries. Nevertheless, (Tasche, 2002) despite admitting VaR as «(…) the most popular risk measure (…)», presented the advantages of ES over the former. In his view, Tasche considered the VaR failed to promote or reward diversification and was not subadditive, a characteristic already mentioned.

(Yamai and Yoshiba, 2005) reveals another perspective about the VaR. It is pointed that this measure focuses its attention exclusively in a predetermined risk level (the α percentile of the distribution). The VaR does not really assess the risk beyond that specific figure. This characteristic may cause serious problems of risk underestimation when dealing with distributions with extended left tails, a problem the authors designated by "*tail risk*". In their view, ES solves this problem quite effectively, even though it requires a larger sample size than VaR to provide the same accuracy of risk assessment. Several other articles explored the advantages of ES over VaR, and how the implicit pressure of the Basel Committee recommendations to follow the latter as a standardized measure for risk assessment in financial management promoted VaR as the reference measure. (Elliott and Miao, 2009) while acknowledging the central role of VaR in the financial area (despite all the criticisms), shows how the ES, by being coherent and convex, presents itself as a better risk measure.

New developments of the Basel Committee (Basel III – 2013 revised version), following the financial crisis that started in 2008, culminated in the acceptance and recommendation of the ES as the new reference risk measure for all the Banking and Insurance industries. In a recent article, (Kellner and Rösch, 2016) portrait this evolution – a replacement of VaR ($\alpha = 0.99$) by ES ($\alpha = 0.975$) – and explore their robustness, how the two risk measures react to different sources and behaviours of risk modelling (probability density functions). (Lim et al., 2011) point out the ES measure is not reliable in a mean-ES or global ES minimization problems because of ES estimation errors, and that these errors are amplified in the very optimization process. They conclude that the ES is fragile in portfolio optimization due to these estimation errors. (Koch-Medina and Munari, 2016) also cast some doubts about the merit of the ES, stating that the recommendation to use one of both the VaR and ES may be misleading in assessing the real risk agents are exposed to in the financial industry. It is also noted how the mere recommendation to use one of these measures might not be sufficient to provide enough assurance for the avoidance of excessive risk taking by the banks and insurance companies. In the authors' view, ES should not be considered a magic solution to risk exposure but rather an important contribution to improve risk assessment. Other cautious policies regarding risk avoidance should also be considered to strengthen the financial industry against new eventual crises.

(Kellner and Rösch, 2016) go even farther in analysing the latest suggestions of the Basel Committee on Banking Supervision and the consequences of a complete dismissal of the VaR measure in favour of the ES. The outcome of the research shows that the ES with $(1-\alpha)=0.975$ versus VaR with $(1-\alpha)=0.99$ is more sensitive towards regulatory arbitrage and parameter misspecification, that is to say, the ES might induce a higher potential for regulatory arbitrage (search of regulatory loopholes to avoid unfavourable legislation) and favours a higher risk exposure due to parameter misspecification. The authors suggest this higher risk exposure might be consequence of « (…) a trade-off between a model's ability to better capture the heavy tailed behaviour of risks and a higher vulnerability to model risk».

Still, and despite the shortcomings of using ES as a risk measure, it represents a progress. A significant number of comparative analyses between VaR and ES have been conducted and the ES is consistently considered a better risk assessment measure. (Balbás et al., 2009), (Ho and Cadle, 2008) and (Chen et al., 2014) are a few examples that present this parallel following the sub-prime crisis that began late 2007 / early 2008.

Attempts to improve the accuracy and robustness of ES estimators or methods of estimation have been conducted in particular throughout the last decade with some success. This is the case of (Quaranta and Zaffaroni, 2008) which use techniques of robust optimization to deal with uncertainty applied to the Italian Stock Market. (Inui and Kijima, 2005) propose an efficient extrapolation method to estimate the ES, while (Wong, 2008) and (Righi and Ceretta, 2015) base their risk avoidance techniques on Monte Carlo simulations with backtest. In another recent article, (Gerlach and Chen, 2017) combine Monte Carlo simulation with Markov chains to propose an asymmetric Gaussian density distribution for returns' behaviour and subsequently apply this assumption on the estimation process. A similar process is presented by (Lönnbark, 2016), applied to an Auto-Regressive (AR) estimation model (GARCH – Generalized Auto-Regressive Conditional Heteroscedasticity) also with recourse to Monte Carlo simulations and assuming a skewed t-distribution for returns. (Degiannakis and Potamia, 2017) go a step further and develop a methodology to calculate ES (and VaR) with both *intra-day high frequency* and *inter-day* data, within AR(1)-GARCH(1,1)-skT and AR(1)-HAR-RV-skT frameworks, for stock indices, commodities and exchange rates, where HAR-RV stands for Heterogeneous Auto-Regressive model of

Realized Volatility and skT for *skewed t-distribution* of returns. The authors found the GARCH specification based on the inter-day information set is the superior model for more accurately forecasting the multiple-days-ahead VaR and ES measurements, at a 95% confidence level. (So and Wong, 2012) present alternative statistical methods to estimate the ES under the assumption of GARCH models for returns' behaviour and suggest an alternative risk measure: the *Median Shortfall*.

For a more comprehensive analysis of the ES (and VaR), it is recommended to read (Nadarajah et al., 2014), where a review of the major developments in this area which have occurred since the 1990's and the most relevant estimation methods, and also (Nadarajah et al., 2016), where the authors tabulate a series of expressions for both these measures (over one hundred parametric distributions) with illustrations using empirical data.

2.3.1.6 Mean - Mean Absolute Deviation

The *Mean Absolute Deviation* (MAD) emerged as an alternative to variance as a risk measure, one that is simpler to compute. (Michalowski and Ogryczak, 1998) emphasises the computational attractiveness of the MAD-based portfolio optimization model, due to its simplicity by resulting in a linear programming problem. Considering a portfolio P with expected return $E(R_P)$ and return R_{Pt} at moment t, the mathematical expression defining the MAD is:

$$MAD(R_P) = \frac{1}{T}\sum_{t=1}^{T}|R_{Pt} - E(R_P)|$$

where

$|\cdot|$represents the modulus or absolute value.

Comparing the MAD with the variance, we may see the former possesses an advantage in its computation (and execution time) over the latter: while the first has a linear formulation, the second is a quadratic function. This may not constitute such a great advantage when dealing with a small number of assets, but it makes a significant difference when hundreds or even thousands of stocks are taken into account in the selection process.

This very situation is addressed by (Konno and Yamazaki, 1991) who solve a large scale portfolio optimization problem considering over 1.000 stocks, and by (Mansini et al., 2014), where the advantages of the linearity in the MAD method are presented and this method is compared with other important risk measures such as variance or CVaR in a realistic environment with a large number of assets.

The MAD gained considerable interest among academics and financial practitioners since (Sharpe, 1971). In this article, Sharpe draws attention to the advantages of using a risk measure such as the MAD, namely the aforementioned linear approach to the portfolio optimization problem rather than the traditional quadratic from Markowitz, and also the useful information produced in the process as by-product.

The MAD seems to be a robust measure of risk regarding different markets. (Zenios and Kang, 1993) shows how it suits well the assumptions of consumer behaviour for mortgage-backed securities and derivative markets (MAD is consistent with asymmetric distribution of returns of mortgage securities and derivative products, and stylized facts associated to these markets: the propensity of homeowners to anticipate their mortgages' payments and the option adjusted premia associated with mortgage backed securities). The use of MAD generated better results than the traditional portfolio optimization in the mortgage related Insurance industry. Also in a risk minimization perspective, (Liu and Zhang, 2009) presents several advantages of the MAD-based model for hedging portfolios. Empirical experiments using data from NYBOT show that the hedging strategies based on the MAD model have better hedging effectiveness than traditional ones. But the MAD model shows robustness not only regarding the inner market characteristics: in (Li et al., 2016) the MAD is used with a slight variation that increases robustness in relation to market direction, whatever the market. Computational experiments show the method can discern high return assets within the pool of selection.

(Konno and Shirakawa, 1994) show how MAD possesses the same properties inherent to the standard deviation of returns under similar assumptions. This same assertion is deduced in (Konno, 2005), where all CAPM relations for the M-V model also hold for the MAD model. So MAD can be considered and may be eligible as a better risk proxy in the portfolio optimization context. (Silva et al., 2017), for example, combine the MAD with CVaR into a new risk assessing method, and although it does not produce the portfolios with the best returns, it generates portfolios with lesser risk than other methods, a situation that might be of particular interest in highly volatile markets.

More recently, the MAD portfolio optimization model has been gaining ground (Konno, 2005) and establishing itself as mainstream, namely because of its adoption in international large portfolio optimization models, such as the ALM – long term asset liability management model and the mortgage backed security portfolio optimization model.

Another interesting evolution of the MAD model has been the tendency to use a mean absolute *semi-deviation* (or *negative deviation*), where only adverse return measurements (below the mean) are taken in consideration on the computation of the risk measure. This is a very similar procedure to the calculation of the semivariance compared to the variance. The results of works such (Michalowski and Ogryczak, 1998), (Kamil et al., 2010), (Moon and Yao, 2011) or (Liu and Qin, 2012) have shown semi-deviation as a feasible robust alternative to the MAD or any other risk measure.

2.3.2 Performance Assessment

There is a substantial number of financial performance assessment indicators. For the purpose of this thesis, and since the list of available indicators is so extensive, we will discuss only the two different indicators related to our empirical work, the Sharpe Ratio and the CALMAR ratio.

2.3.2.1 Sharpe Ratio and Information Ratio

The *Sharpe Ratio* (SR) was one of the first landmark return-to-risk assessment measures developed early in the 1960’s (Sharpe, 1966). It establishes the relation between the deviation of the expected asset’s return with reference to a predetermined benchmark, usually a risk-free asset, and the standard deviation of the returns:

$$SR(R_A) = \frac{E(R_A - R_F)}{\sigma(R_A)}$$

where

$SR(R_A)$ stands for the Sharpe Ratio of Asset A;

R_A represents the returns of Asset A;

R_F represents the returns of a Risk-Free Asset, the benchmark.

Later on, (Sharpe, 1994) acknowledged the inconsistency between the basis for the computation of the expected return and the basis for the computation of the standard deviation, adapting the formula in both numerator and denominator to the differential of returns (asset's minus the benchmark's). Mathematically, we may calculate the SR with the following expression, after Sharpe's reformulation, also designated as *Information Ratio* (IR), which is considered a generalization of the original SR:

$$SR^*(R_A) = \frac{E(D)}{\sigma(D)} = IR(R_A)$$

where

$SR^*(R_A)$ stands for the reformulated Sharpe Ratio of asset A,

$IR(R_A)$ represents the Information Ratio of asset A,

$D = R_A - R_B$,

and R_B represents the benchmark's returns.

The SR is a commonly used tool to assess financial performance. In (Coates and Page, 2009), the authors use the SR to evaluate the performance of traders of Downtown London (also known as "The City") and to infer how distant is the City market from the tenets proclaimed by the EMH, which basically says they should not outperform the market in a broader sense. The findings of this article go against the EMH, since experienced traders present higher SRs than the overall market. (Goldberg, 2015) uses the SR in a chronological perspective, assessing the consistency and preservation of specific SR values throughout time, considering the 1987 financial crash, the 2000's internet bubble and the 2008 subprime crisis. This study concludes that SRs are time variant and very distinct according to the markets they are applied to.

Some authors have theorized variations to the original SR devised by Sharpe. (Ardia and Boudt, 2015), for instance, suggests a modified SR to evaluate financial performance when in presence of non-normality conditions of returns. The authors use the modified SR definition as the quotient of the differential between the excess return of the asset and its modified VaR. Empirical data suggests there is a complementarity between the SR and modified SR for financial performance assessment. Another article, (Chow and Lai, 2015), suggests the use of conditional SRs, as a means to perceive downside risk. Empirical results show that these conditional SRs are able to detect and discriminate downside performance, something the original SR cannot afford to achieve. These variations therefore seem to add value to the conventional SR.

2.3.2.2 CALMAR Ratio

The *Calmar Ratio* (CR) – "CALMAR" short *CAL*ifornia *M*anaged *A*ccount *R*eports) – is a risk measure developed by Terry W. Young in 1991 that uses cumulative returns of an asset relatively to its maximum drawdown. The generic formula of the Calmar ratio may be presented as:

$$CR(R_A) = \frac{E(R_A)}{MDD_T(R_A)} = \frac{1}{T} \cdot \frac{\sum_{t=1}^{T} R_{At}}{MDD_T(R_A)}$$

where

R_{At} is the daily rate of return of asset A;

and

$$MDD_T(R_{At}) = \max_{1 \le K \le T} \left\{ \max_{K < L \le T} \left\{ \sum_{t=1}^{K} R_{At} - \sum_{t=1}^{L} R_{At} \, ; 0 \right\} ; 0 \right\}$$

formula that is equivalent to:

$$MDD_T(R_{At}) = \max_{1 \le K < L \le T} \left\{ \sum_{t=1}^{K} R_{At} - \sum_{t=1}^{L} R_{At} \, ; 0 \right\} \qquad (3)$$

where

$MDD_T(R_{At})$ represents the *Maximum DrawDown* of asset A throughout period T.

The original formula depicted by (Young, 1991) considered a specific period of 3 years (36 months), computed with monthly rates of return. The original formula may be depicted as:

$$CR^{original}(R_A) = \frac{1}{36} \cdot \frac{\left(\sum_{t=1}^{36} R_{At}^m\right)}{MDD_{36}(R_A^m)}$$

where

R_{At}^m is the monthly rate of return of Asset A;

and

$$MDD_{36}(R_{At}^m) = \max_{1 \le K < L \le 36} \left\{ \sum_{t=1}^{K} R_{At}^m - \sum_{t=1}^{L} R_{At}^m \, ; 0 \right\}$$

$MDD_{36}(R_{At}^m)$ represents the *Maximum DrawDown* of asset *A* throughout the 36 months, with monthly rates of return. The MDD is the largest drop from peak to trough in a time span, the worst fall in the asset's cumulative rate of return observed during the trading activity, i.e., of all the falls (designated by the expression $\max_{K<L\le 36}\{\sum_{t=1}^{K} R_{At}^m - \sum_{t=1}^{L} R_{At}^m \, ; 0\}$), MDD represents the largest. (Magdon-Ismail, 2004), for instance, regards the MDD measure as one of the most important among all available in the financial spectrum, particularly when applied with a normalized (1 year) Calmar.

The CR is commonly used by the Financial Industry as an auxiliary measure in a profitability-risk context, mostly because of its simple formula, ease to compute and also widespread acceptance among industry practitioners. Therefore it is relatively frequent to see comparing studies of the CR alongside with other ratios such as the *Sharpe Ratio*.

In two sequential studies, (Eling and Schuhmacher, 2007) and (Eling, 2008), the authors make a comparative analysis of several return-risk methods, including the CR, within the Hedge/Mutual Fund Industries, concluding for the superiority of the Sharpe Ratio, particularly as far as robustness is concerned. Nevertheless, the authors also admit that the difference is not really that relevant in the context of these industries (in his own words «the choice of performance measure is not critical to fund evaluation») and therefore other return-risk measures could be adopted without great loss. In a later article (Schuhmacher and Eling, 2011), the authors go even further, classifying, from a decision theoretical perspective, drawdown measures as equally good as the SR. These results are confirmed years later for US Fixed-Income, Equity and Asset Allocation Mutual Funds by (Ornelas et al., 2012).

There are several other articles – some in fact very recent – that employ the CR as a method of risk-return analysis in several financial-based industries, such as (Auer and Schuhmacher, 2013), (Dunis and Miao, 2006), (Wilinski et al., 2013), (Olszweski, 2014) or (Wilinski et al., 2014), making this measure as one of the most relevant available methods of return and risk assessment for financial markets.

3 Evolutionary Techniques applied to Finance

This chapter presents a review of the major evolutionary techniques that are currently applied to problems of optimization in the financial area, both with single and multiple objective functions. For over four decades, several operations research (OR) techniques have been used in the analysis of financial data. Among the most common soft OR techniques applied to financial markets are fuzzy logic, neural networks and evolutionary algorithms. Evolutionary Algorithms (EA) are a kind of metaheuristics that emulate the processes of biological mechanics of genetics, namely *reproduction* and *mutation* observed in living beings. This type of heuristic usually provides good results both in time efficiency and in solution quality; nevertheless, it does not guarantee absolute optima. These techniques may be used solely, on an isolated form, or combined, like for instance in (Leigh et al., 2002) or (Armano et al., 2002). (Leigh et al., 2002), for instance, combine neural networks for pattern recognition with Genetic Algorithms (GAs) to improve the correlation between the actual price increase of the NYSE composite index and the price increase estimated by the neural network model. (Armano et al., 2002) also use GAs along with Artificial Neural Networks (ANN) in a different perspective – GAs for quasi-stationary regimes' (trends') identification, and ANN for market price movement prediction. It is difficult to know which technique is the best. Usually, the better use we make out of them, the better the outcome. Nevertheless, (Chen et al., 2007) provide some insight on the reasons for the preference for evolutionary algorithms – they seem to grant better results in optimization problems, due partly to their evolutionary nature (retaining the best, discarding the worst), partly to their mechanics, performing global search.

3.1 Generic Common Aspects to Population-Based Evolutionary Techniques

3.1.1 Population

Populations are nuclear to understand these algorithms' mechanisms, for in their core we find the solutions we want to optimize. A *population* of an Evolutionary Algorithm is constituted by a predetermined number of *solutions*, *individuals* or *chromosomes*. Each chromosome is composed by a set of parameters – designated by *genes* – that represent a solution for the problem being considered. In a sense, genes stand for the variable values inherent to the problem, and likewise, chromosomes represent a specific combination of these genes (variables) as a possible solution for the problem.

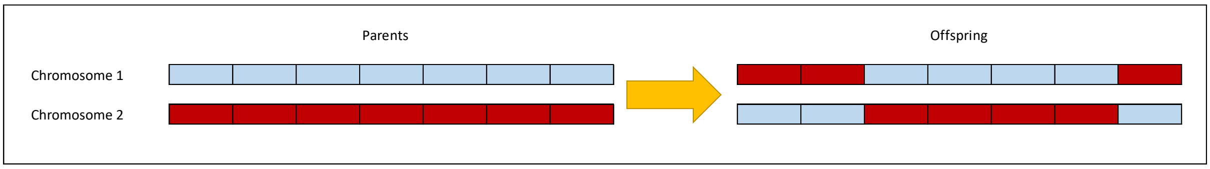
The optimization process is based on the generation of an *initial population* and its evolution through an iterative procedure where a systematic selection process is performed in each iteration, according to a pre-established *evaluation function* that measures the fitness of each chromosome. The initial population is usually generated with some random process. At the end of the optimization process we ought to have a good or very good solution. Often these metaheuristics do not attain the global optimum; rather they efficiently – with limited computational effort and execution time – achieve what may be considered an appropriate solution; for many such problems, without the use of the metaheuristic, achieving the optimal solution via hard heuristics or traditional quantitative methods would be virtually impossible. This is the major advantage of metaheuristics: efficiency.

3.1.2 Evolution and Reproduction

3.1.2.1 Crossover

Crossover is the operation of interaction of two (or more) chromosomes, designated as *parents*, to result into one, two (typically) or more different chromosomes called *offspring*. To each chromosome or individual of the population of any given iteration is defined a probability of being chosen for crossover – a *crossover probability* for individual $i = p_c^i$. It is also predetermined how many crossover points will be required in the optimization process, let us say n_c. The location of these crossover points within the chromosomes extent may be deterministic or random. It is not unusual to use a single crossover point with random location. Applying n_c crossover points means each chosen chromosome will be divided into $n_c + 1$ segments of genes. Then the algorithm selects two (or more) chromosomes with probability p_c^i, divides each one into $n_c + 1$ segments and crosses the segments alternately from the two parents, forming two different offspring chromosomes. An example with 2 parents and 2 crossover points can be seen in Figure 1.

Figure 1. Example of Crossover operation between two chromosomes with two different crossover points.



Source: Author.

In this process it is vital that offspring chromosomes inherit genetic material from all selected parents. This will allow the diversification of the resulting individuals and promote diversification of the elements constituting the population for the next generation, and thus avoiding premature convergence. Equally important is the validity of the resulting offspring, in the sense that the generated chromosomes must be feasible solutions in the context of the problem, and this very condition should be considered in the conceptualization of the crossover operator as an important criterion.

3.1.2.2 Mutation

Mutation, usually in very small amounts, represents a complement of diversification or the possibility of fine-tuning of a single chromosome (usually one gene within). For an individual i, it occurs with a predetermined probability p_m^i, such that every gene in each chromosome has this probability of changing its value. This change may be within a predetermined fixed or variable amplitude. The probability p_m^i is considerably small (around 1%) and the gene value variations themselves are also relatively small. The purpose of this mechanism is to locally search for better solutions and perfect them as well as to escape from local optima. The mutation operator should be able to search all possible solutions (search space) and be able to produce feasible solutions when considering problems with constraints.

When a feasible solution is not met, a way to overcome this difficulty is to repeat the procedure until one is attained. Other possible way may be to control the mutation operator in such terms that generated solutions are always feasible. When this is not possible, infeasible solutions may be penalized in the fitness function.

3.1.2.3 Replacement

Replacement refers to the process of substitution of a chromosome by another in any given iteration of the optimization process in order to form the population for the next iteration. It involves the parent and the offspring populations. Extreme strategies are (Talbi, 2009): *steady-state replacement* – where a unique chromosome with better evaluation replaces the worst solution of the population; *generation replacement* – where all offspring solutions replace all the population of the previous generation. Between these, there are many different strategies, stochastic or deterministic, which replace some of individuals of the population. Special kinds of replacement may also be considered, which include not only the parent and offspring populations, but also new individuals, e.g., the *immigrant* used in (Mendes et al., 2012), where a new random solution is included into a generation's population to introduce new genetic material and prevent premature convergence in the optimization process. Either to constitute the population for the next iteration (where all or some parents may be replaced by offspring) or to choose the parents in the reproduction step, a *selection* method must be employed.

3.1.3 Most Common Selection Methods

3.1.3.1 Roulette Wheel Selection

This method of chromosome selection, also designated as *fitness proportionate selection*, is based on the relative fitness of chromosomes: every solution i of the population has the probability p_i equal to f_i/AF to be selected, where f_i is the fitness of chromosome i and AF represents the aggregate fitness of all the chromosomes of the population at that specific moment. Figuratively, we may consider a pie with an outer roulette where each individual has a slice proportional to their probability (that is, each slice represents p_i% of the whole pie). In this manner, solutions with higher fitness have higher probability of being chosen. The selection of x chromosomes is then obtained by spinning the roulette x independent times.

3.1.3.2 Tournament Selection

In a tournament selection procedure, y chromosomes are randomly selected in a first phase (each individual with the same probability of being chosen). These y qualify for the second phase of the procedure, where the solution with the highest fitness is selected. To obtain x selected chromosomes, we perform independently the tournament procedure (the 2 phases) x times. We may determine specific cases (such as the binary tournament, where $y = 2$) and/or devise variations of this general mechanism – for instance, select more chromosomes in each run, create a tournament in phase 2, or any other.

3.1.3.3 Rank-Based Selection

In a rank-based selection mechanism, a rank/order is attributed to each chromosome according to their fitness. To the chromosome with the highest ranking is given the highest probability of being chosen, to the second in rank the 2nd highest probability and so forth until all chromosomes have been given a probability and the sum of all the probabilities is equal to one. The attribution of probabilities may be produced by several means with various reasonings but always respecting the condition that the highest the rank the highest the probability and also the probabilities of the population sum to one. For instance, (Talbi, 2009) suggests the following formula to compute each chromosome's probability:

$$p_i = \frac{2 - S}{N} \cdot \frac{2r_i(S - 1)}{N(N - 1)}$$

where

p_i is the probability of chromosome i being chosen;

r_i is the rank of chromosome i;

S is the selection pressure, $1 < S \leq 2$;

N is the size of the population.

The selection pressure in this formula induces a bias towards better solutions, i.e., the greater S the higher will be the probability given to solutions that present better fitness.

3.1.3.4 Random Selection

Simple random selection may be considered in two different situations: 1) when every chromosome within a population has the very same probability of being chosen; 2) when for each chromosome at any given iteration there is the same probability of being chosen and not chosen (50%) – in this case the number of selected chromosomes is variable.

3.1.4 Typical Optimization Procedure

Evolutionary Algorithms possess a general *structure* that does not differ substantially from each other. The common initializer is usually a random generation of a base population. This generation may be more or less constrained, according with the purpose at hand or the conditions imposed by the problem itself. After this first step, the evolutionary algorithm proceeds with the generation of offspring, applying first the crossover operator, followed by mutation, and finally, replacement. In the next iteration, the generation of new offspring is repeated for the new population and the iterative process is repeated until a stopping condition is met. The solution with the best fitness is elected as the *optimized solution*. We may see this in the scheme presented below, which translates into pseudo-code the general structure of the algorithm.

Initialize Algorithm – generate initial population P_0

Do Until Stop criteria is met (from iterations $i = 1$ to $i = L$)

Evaluate P_{i-1} for ***Selection*** purposes

$P_i(c) \leftarrow$ Apply ***Crossover*** in ***Selection*** of P_{i-1}

$P_i(m) \leftarrow$ Apply ***Mutation*** in ***Selection*** of $P_i(c)$

$P_i(r) \leftarrow$ Apply ***Replacement*** in ***Selection*** of $P_i(m)$ and P_{i-1}

$P_i = P_i(r)$

$i = i + 1$

End Do

Optimal Solution $S^* \leftarrow$ Solution with the best fitness from P_L

where

P_i is the population at beginning of iteration i;

$P_i(c)$ represents the offspring population at iteration i after the application of the crossover operator;

$P_i(m)$ stands for the offspring population at iteration i after the application of the mutation operator;

$P_i(r)$ is the population at iteration i after the application of the replacement operator;

P_L is the population at the last iteration (L).

3.2 Single Objective Evolutionary Techniques Applied to Financial Markets

3.2.1 Genetic Algorithms

Genetic Algorithms (GA) are some of the simplest and most intuitive metaheuristics designed in computational sciences. First conceived by John H. Holland and further developed by Kenneth Alan De Jong during the 1970's in his Doctoral Thesis at the University of Michigan (DeJong, 1975), GAs – at the time designated as "adaptive" algorithm by De Jong – grew their influence as an efficient tool to solve non-conventional and complex problems to consolidate their status as basilar metaheuristics just on the turn of the century. Having its origins in a pure computer and communications sciences environment and financed by a grant from the National Aeronautics and Space Administration (NASA), it was natural that the first GAs adopted a configuration based in computational language (binary code). Nowadays, however, there is a multitude of other approaches to GA configuration, and it is rather common to see genes using float (real number) variables instead of binary variables.

GAs, with more or less complexity, end up using the same basic structure – the same depicted in the pseudo-code of subsection 3.1.4. But the optimization process with a GA is often a difficult task. In order to get the utmost of the algorithm, the GA operators must be adapted to the peculiarities of the problem at hand. Although the framework or structure is well established, a great amount of work must be put in the fine-tuning of the population size and selection procedure, and the particular kind of crossover and mutation mechanisms – (Balas and Niehaus, 1998) tried several variations and developed a GA that outperforms the most common ones in the resolution of a set of theoretical problems. (Ghosh, 2012) focuses on the importance of initial population generation and its diversity and developed a method to avoid premature convergence and improve the chances of achieving the global optimum. (Hong et al., 2000) pay more attention to the mutation operator. In their view, mutation is the mechanism that guaranties diversity of the population in later phases of the optimization process, and therefore, should be influential enough to be able to diversify the population at any given iteration. The authors even propose the use of different mutation operators and their trial and error fine-tuning. The article culminates with the idea that the best way to exploit mutation is by setting a *dynamic mutation*-based algorithm – simultaneous use different mutation operators, applied according to a mutation *ratio*. The mutation operators' ratios vary according to the fitness of the respective offspring each operator produces.

(Fu et al., 2013) study how a traditional GA may be improved in a context of portfolio management. The authors use several distinct GAs (among which a hierarchical GA) to optimize the TA indicators' parameters and portfolio weights. The article succeeds in showing the versatility and adjustment capabilities of GAs in complex problem solving.

In (Kim and Shin, 2007) the authors merge two types of artificial neural networks (ATNN - adaptive time delay neural networks, and TDNN - time delay neural networks) with a GA to form a combinatorial metaheuristic for stock market price prediction by detecting time patterns in historical data. In this specific metaheuristic the purpose of the GA is to optimize the number of time delays and network architectural factors, serving as a supporting mechanism for optimization. The use of the GA combined with the artificial neural networks (ANNs) improves the results obtained by the sole ANNs, increasing their accuracy in identifying price patterns. (Evans et al., 2013) used an ANNs- and GAs-based model applied to Foreign Exchange Market (FOREX), in particular to three of the most traded currency pairs: GBP/USD, EUR/GBP, and EUR/USD. The authors conclude, with a 95% confidence level, for the non-random nature of these markets and present an overall 23.3% annualized net rate of return. In a recent article, (Chang and Lee, 2017) combines also a Markov decision process with a GA build a new analytical framework and create a decision support system for the development of trading strategies.

It is also common to find studies where a GA performance is compared with other metaheuristics. (Mokhatab Rafiei et al., 2011), for example, draws a parallel between three different models: one based in a GA, a second in an ANN and a third based in multiple discriminant analysis (MDA) with the purpose to distinguish bankrupt-prone from non-bankrupt-prone corporations. In this research, the ANN-based model revealed itself as the most reliable reaching 98.6% and 96.3% accuracy rates for in-sample (IS) and out-of-sample (OOS) data. The GA model attained 92.5% and 91.5% accuracy rates for IS and OOS respectively and the worst performance came from MDA with 80.6% and 79.9%.

The survey by (Aguilar-Rivera et al., 2015) presents a thorough inquiry on the advantages and limitations of multiple Darwinian metaheuristics applied to the financial area, namely genetic algorithms, genetic programming, multi-objective evolutionary algorithms, learning classifier systems, co-evolutionary approaches, and estimation of distribution algorithms. The authors also conclude there has been a change of interest in each algorithm over time; GAs however remained the most popular category. In another survey, (Cavalcante et al., 2016) present a review of the most popular metaheuristics (ANNs, GAs, ML, …) applied to financial markets and published in scientific articles from 2009 to 2015. Besides the literature review, this work establishes a systematic approach for the creation of an intelligent trading system and challenges the scientific community of this area to different possible developments for the near future.

(Štěpánek et al., 2012) use a GA to assess the possibility of abnormal returns in stock markets within a behavioural approach. The GA was mainly used to detect and predict behaviours, functions and associated parameters of market agents and in this way deduce price movements. The advantage of the used tool is the dynamic interaction of parameters in each iteration of the algorithm, portraying the GA simulation (or optimization process) to be a real dynamic development of agents' behaviours and decision processes through a certain time span.

The advantages of GAs are well known and have warranted so far their popularity in the computation-related academic community. Nonetheless, the GA is not a perfect tool and in some cases may fall short in performance and effectiveness when solving specific problems. In this case, the combination of GAs with other metaheuristics may become handy; other times, we might need to take an altogether different path and use completely distinct techniques. Genetic programming might be a feasible alternative.

3.2.2 Genetic Programming

Genetic Programming (GP) is a single-objective evolutionary technique, developed by (Koza, 1992), that presents a substantial and relevant differences regarding all other evolutionary approaches – the algorithm unfolds in an inverted tree-like shape where the branches represent possible paths the algorithm may choose. Each branch itself leads to a different function or complete program and not a static value for a parameter of the chromosome. When applied to finance, GP is commonly used as a trading rule discovery and evolvement approach. The structure may be more or less flexible, according to research purposes, in the way the tree branches are exchanged within the structure itself (crossover), the applied level of layers, and the mutation of the program in each branch. Too much flexibility and lack of constraints might lead to excessive complexity of the tree structure and inefficiency of the algorithm.

GP is a relatively established technique in the finance academic field. (Kaboudan, 2000) proposed, for instance, a single day-trading strategy based on the optimization provided by GP, where regressions applied in each branch were used to predict prices for the following day, with profitable returns for a pool of six stocks. (Potvin et al., 2004), on the other hand, presented a GP-based framework in order to generate profitable short term trading rules. The out-of-sample (OOS) results in this paper came short of what was expected and proved to be more effective when the observed markets were not on a rising trend.

A similar approach was conducted by (How et al., 2009), but instead of dealing with individual stocks, the authors focused on three different indexes: Russell 1000 (large cap = top 1000 corporations of the Russell 3000 in terms of capitalisation), Russell 2000 (small cap = 2000 corporations with smaller cap from Russell 3000) and Russell 3000 (broad market). The purpose was to devise TA trading strategies based on the use of a GP and test how the average dimension of the studied corporations of each index might induce different OOS outcomes in terms of financial return of the defined rules. OOS results showed the market/index with more potential for predictability was the small cap (Russell 2000). But even for that market, after incorporating trading costs, profitable trading opportunities disappeared.

Focusing on risk, in (Esfahanipour and Mousavi, 2011) a GP is used to generate risk-adjusted TA trading rules, by adopting a conditional Sharpe ratio for that purpose (a CVaR-based risk measure). The method granted better results in comparison to a simple Buy and Hold (B&H) strategy. (Lensberg and Schenk-Hoppé, 2013) presents a GP algorithm used for optimization of hedging strategies (minimization of risk), under nonlinear transaction costs. The algorithm showed a good ability to adapt to new pieces of information of the hedging strategy and also robustness regarding the parameter values.

There are also variations to the original GP algorithm. For instance, (Li et al., 2006) proposes a Financial GP as a forecasting tool based on the original GP and the (discrete) *Wavelet Transform*, an image or signal processing method inherited from the engineering area, with interesting results in terms of prediction capabilities. (Luna et al., 2012) uses a grammar-guided GP for mining trading rules. The algorithm allows a great deal of flexibility and adaptability to specific problems, although keeping an accentuated elitist nature (selects in each iteration the best individuals with evaluation above a certain threshold). (Manahov et al., 2014) presents another variation of GP, a version of the strongly typed genetic programming learning algorithm to devise trading rules. With recourse to the three Russell indexes, arriving at similar conclusions of (How et al., 2009) – superiority of models developed for small-cap stocks.

On another tone, (Bouaziz et al., 2016) focused on combining extended genetic programming with a hybrid artificial bee colony algorithm (itself a composite of artificial bee colony procedures with opposite-based particle swarm optimization. A similar approach is conducted in (Hsu, 2011), where a hybridization of a self-organizing map, a neural network and genetic programming is developed in order to predict stock prices. Another composite metaheuristic involving GP is used in (Yang et al., 2014): a GP algorithm based on the least squares method (GP-LSM), cumulating the flexibility of GP and the ability to improve the convergence speed concomitantly to the prediction accuracy (because of its nature and capacity to optimize the fitness of nonlinear models). By comparing the GP-LSM with other models, namely the seasonal auto regression integrated moving average (SARIMA) and back propagation artificial neural networks (BP-ANN), the authors of the article have shown how their model performs better than the other considered models.

GP presents several good characteristics and is flexible enough to produce joint models with other metaheuristics/functions. An overview of the GP evolution since its inception until recent days is documented in (Kouchakpour et al., 2009). GP is highly regarded in the scientific community but it has a major drawback: the *computational effort* it requires. In the next subsections we will move on to two distinct single objective alternatives to GP, Evolution Strategies and Evolutionary Programming.

3.2.3 Evolution Strategies

Evolution Strategies (EST) are associated with the work of (Schwefel, 1984), although first introduced by (Rechenberg, 1964), in Berlin – Technical University. This optimization method was developed independently from other evolutionary techniques and presents several peculiar characteristics. While GAs were initially designed for solution with discrete configuration (namely binary gene configuration), ESTs were developed in the context of continuous optimization within a real-value vector framework. The mutation factor usually assumes a random nature with Gaussian distribution (zero mean); the selection procedure uses a uniform random distribution, and replacement, which may assume two different generic forms: one, $(\mu + \lambda)$, in this case, the population λ of offsprings will compete with the initial parents' population μ for comparison and selection of the best population with size μ among the pool $(\mu + \lambda)$; or a second form, (μ, λ), where the initial parent population μ is completely substituted by the new offspring population λ for the next iteration and eventually the best individuals of μ until the size of the initial population is achieved (which happens whenever $\lambda < \mu$). ESTs also allow the possibility of self-adaptation of the mutation step size.

(Beyer and Schwefel, 2002) analyse this branch of computational evolution, from its inception in the 1960's in Germany to the 2000's, concluding with a speculation of how subsequent developments in ESTs will come to be. (Schwefel, 1981) and (Schwefel, 1984) give a systematic presentation of this kind of algorithm. The EST has the ability to improve and accelerate the learning process of self-adaptation, particularly when $\lambda > \mu$. (Grill and Hartmann, 2000) focus on the efficiency of this kind of algorithms, where the coexistence of independent individuals (strategies) and the versatility of the algorithm proportionate the reduction of processing time and an enhanced applicability to particularly difficult and large scale problems. On another perspective, (Arnold and Beyer, 2003) evaluate the EST performance with the presence of Gaussian *noise*, and the EST reveals greater degree of robustness than other algorithms (direct pattern search, multi-directional search and implicit filtering) when dealing with high levels of noise.

Several authors tried to introduce into EST some sort of variation in order to improve the algorithm. For instance, (Mezura-Montes and Coello, 2008) develop a variation of a traditional EST approach and use other four different ESTs: the variation of a $(\mu + 1)$ – EST designated as "$V(\mu + 1) - EST$"; $(\mu + \lambda)$ – EST without correlated mutation; $(\mu + \lambda)$ – EST with correlated mutation; (μ, λ) – EST without correlated mutation; and (μ, λ) – EST with correlated mutation. They compare these four ESTs with each other and with other state-of-the-art metaheuristics. The variation shows an interestingly competitive performance either regarding the other ESTs as well as the state-of-the-art benchmark metaheuristics. (Diouane et al., 2015) also propose a variation within the globally convergent ESTs with empirical results showing a competitive, efficient and robust performance when compared with state-of-the-art optimization solvers. Another variation – the GES (Grouped Evolution Strategies – presented in (Kashan et al., 2015) shows its merits in particular as a flexible and robust optimization methodology for complex problems.

3.2.4 Evolutionary Programming

Evolutionary Programming (EP) emerged as an evolutionary technique in the 1960's with the work of (Fogel et al., 1966). It was developed mainly to improve computational performance through evolution of finite state machines. There is a major difference that diverges this optimization method from EST, which lies in the absence of any form of *recombination/crossover*. The evolution of the population is basically induced from *mutation*, which presents a Gaussian behaviour with zero mean. The selection of the next iteration's population is stochastic using a *fitness proportionate selection*, choosing a population of size μ of the fittest individuals from the pool $(\mu + \mu')$, where μ is the beginning population and μ' is the mutated population of each iteration.

The same author that developed EP presented an article where a parallel between GA and EP performance was drawn – (Fogel, 1995). In this work, empirical results show the EP consistently outperformed the GA in a statistically significant way, under nonparametric hypothesis testing. Another comparison amongst GA and EP was conducted by (Abu-Zitar and Nuseirat, 2002), namely to understand their robustness when dealing with machine learning related problems. In this case, the performance of both techniques was similar and effective in achieving good results. In both techniques, fine-tuning was an essential part of performance improvement.

The mutation operator is of paramount importance for EP. It has been subject of study, in particular in how it may be fine-tuned to avoid premature convergence to local optima or lack of diversity in the population at early stages of the optimization process. (Alam et al., 2012) address this specific question and introduce a new variation of EP, the DGEP - *Diversity Guided EP* as an optimization approach where the mutation step-size is influenced by the diversity information of the population at any given iteration. Empirical tests have shown interesting results, in particular in comparison to other forms of EP such as the CEP - *Classical EP*, ALEP - *Adaptive Lévy EP*, and IFEP - *Improved Fast EP*; ALEP and IFEP use respectively a *Lévy* and a *Cauchy* probability functions for the mutation operator as alternatives to the *Gaussian* probability function.

(Das et al., 2013) introduce another variation: instead of selecting deterministically the i^{th} population member to apply a mutation of step-size conditioned by the Gaussian probability function, the authors propose a mixture of this procedure with the possibility of choosing randomly another population member to use upon the mutation operator, with a 50% chance of choosing each procedure. Within the second possibility, the parent selection would be according to their respective fitness. This new bivariate parent selection scheme, *p-best Adaptive Fast EP (AFEP)*, shows statistical superiority regarding final accuracy, speed and robustness, when compared with the previously mentioned CEP, ALEP, and also with the *Fast EP* (FEP), *Modified EP* (MEP), *Adaptive Fast EP* (AFEP) and finally *EP based on Reinforcement Learning* (RLEP).

3.3 Multi-Objective Evolutionary Algorithms and Portfolio Optimization

Multi-Objective Evolutionary Algorithms (MOEAs) represent a class of metaheuristics particularly adequate for portfolio optimization, since they can address the duality *risk-return* so characteristically present in portfolio optimization. There are many MOEAs, but some of them have emerged as more consistent in their effectiveness and efficiency. We will devote special attention to influential MOEAs that have consistently produced good performances over the years and were also used in the empirical studies of this work, namely the *Nondominated Sorting Genetic Algorithm II* (NSGA-II), introduced by (Deb et al., 2002), and the *Strength Pareto Evolutionary Algorithm 2* (SPEA-2), developed by (Zitzler et al., 2001).

3.3.1 Nondominated Sorting Genetic Algorithm II

The NSGA-II was developed as an improvement to the original NSGA. It was first presented in (Deb et al., 2002). It was a reaction to several criticisms to the original algorithm, specifically, the *high computational complexity of nondominated sorting, lack of elitism* and the *need of specifying the sharing parameter* (σ_{share}). The improved algorithm follows these steps (Deb et al., 2002), pp. 184-186:

P_t – Population at iteration T
T – Maximum number of iterations

Generate a Population P_0 of size N
For $t = 0 \; to \; T$

- *Evaluate Population* P_t
- *Recombination* of Population P_t;
- $Q_t \leftarrow$ *Mutation* of Population P_t after *Recombination*;
- *Generate* $R_t = P_t \cup Q_t$ of size 2N;
- *Evaluate Population* R_t – *Rank* (fitness) is given to each solution according to:
  - *Non-domination sorting:* (i_{rank}) – 1st phase of ordering by applying the *fast-non-dominated-sort* procedure; new sets of solutions are delimited according to their nondominance rank: F_1, F_2, F_3 ...; order the elements from the lowest to the highest rank – a solution i is preferable to a solution j $(i \prec_n j)$ if $(i_{rank} < j_{rank})$ (the lower the rank the better the fitness);
  - Apply the *Crowding Distance procedure:* $(i_{distance})$, 2nd phase of ordering for selection: a solution i is preferable to a solution j if $[(i_{rank} = j_{rank})$ and $(i_{distance} > j_{distance})]$,

- *Select* P_{t+1} from the sorted R_t – the first N solutions are chosen for the Population of the next iteration;

Next t

Final Solutions = P_T

The drawbacks pointed out to the original NSGA were therefore overcome or at least attenuated by this new algorithm: the computational complexity was reduced from $O(MN^3)$ to $O(MN^2)$, improving the optimization procedure in terms of required computational effort; the elitism was improved by the new *fast-non-dominated sort* and *crowding distance* selection procedures; and finally the specification of the sharing parameter is no longer needed, since this procedure is altogether abandoned and substituted by a crowded-comparison approach.

Some studies have emphasized the interesting performance of this algorithm when compared with other MOEAs. NSGA-II has been considered particularly effective when adopting a few modifications, namely the *predator-prey approach* (Deb et al., 2006). New improvements to this algorithm have been introduced in (D'Souza et al., 2010), leading to the reduction of its running time and complexity, and therefore making it more attractive for solving practical problems.

This algorithm has attained quite a success in the resolution of the asset portfolio optimization problem. In this respect, two articles can be highlighted: (Anagnostopoulos and Mamanis, 2011) and (Metaxiotis and Liagkouras, 2012). In the first, an exercise of portfolio optimization is conducted with recourse to six MOEAs, among which stands also the NSGA-II. In all studied markets the NSGA-II performed at a high level, reaching the true Pareto front of the problems. (Metaxiotis and Liagkouras, 2012) on the other hand, make an extensive review of the most common MOEAs applied so far to portfolio management. The NSGA-II is present in the vast majority of reviewed articles.

Other works such as (Lwin et al., 2014) show the importance of this algorithm for solving portfolio optimization problems. In this last paper, and for most of the criteria of performance evaluation, the NSGA-II shows strong robustness, qualifying as one of the best algorithms in consideration by the author. Other articles reinforce this idea – (Duran et al., 2009), (Mishra et al., 2009), (Mishra et al., 2010), (Mishra et al., 2016), (Lwin et al., 2017)

– and it is not difficult to state how the NSGA-II remains as one core MOEA, a reference for performance evaluation in this kind of problem.

3.3.2 Strength Pareto Evolutionary Algorithm 2

The SPEA-2 (Zitzler et al., 2001) results from an improvement of the original SPEA, an algorithm first developed by Zitzler and presented in his PhD thesis (Zitzler, 1999). In this new algorithm the fitness assignment strategy is fine-tuned, considering for each individual solution how many solutions it dominates and is dominated by. SPEA-2 incorporates a nearest neighbour density estimation technique that enhances the optimization process. Its process can be described as a sequence of the following steps (Zitzler et al., 2001):

Input: N – Population Size
$\bar{N}$ – Archive Size
T – Maximum number of iterations
Output: **A** – Non-dominated set

Initialization: Generate an initial population P_0; set $t = 0$ and create an empty archive $\bar{P}_0 = \emptyset$

For $t = 0$ *to* T

- *Fitness assignment*: Calculate fitness values of individuals in P_t and $\overline{P_t}$
- *Environmental selection*: Copy all nondominated individuals in P_t and $\overline{P_t}$ to $\bar{P}_{t+1}$. If the size of $\bar{P}_{t+1}$ exceeds $\bar{N}$ then reduce $\bar{P}_{t+1}$ by means of the *truncation operator* (mechanism establishing the condition that guarantees the size of the archive does not exceed the predefined threshold); otherwise, if the size of $\bar{P}_{t+1}$ is less than $\bar{N}$, then fill $\bar{P}_{t+1}$ with dominated individuals in P_t and $\overline{P_t}$

- *Termination*: If $t > T$ or another stopping criterion is satisfied then set **A** to the set of decision vectors represented by the nondominated individuals in $\bar{P}_{t+1}$. ***Stop***.
- *Mating selection*: Perform binary tournament selection with replacement on $\bar{P}_{t+1}$ in order to fill the mating pool and set $\bar{P}_{t+1}$ to the resulting population.
- *Variation*: Apply recombination and mutation operators to the mating pool and set P_{t+1} to the resulting population.

Next t

Final Solutions = **A**

The main differences of the new improved version of the algorithm lie in following facts: 1) the use of a fine-grained fitness assignment strategy which incorporates density information – (Zitzler, 1999), pp. 34-36, versus (Zitzler et al., 2001), pp. 6-7; 2) the archive size is fixed (contrary to the original SPEA, which could change its archive size through the iterations); 3) the original SPEA's clustering technique used to limit the population size to N is replaced by a truncation method; and finally 4) only the solutions present in the archive are used as possible parents for mating – comparison between (Zitzler, 1999), pp. 32-40, and (Zitzler et al., 2001).

The SPEA-2 represents another reference MOEA. In fact, NSGA-II and SPEA-2 have been commonly used as benchmarks for the evaluation of new multiobjective algorithms. SPEA-2 has consistently proved to be a reliable alternative as far as MOEAs are concerned. Nevertheless, some studies report a less-than-efficient performance, particularly when compared to the NSGA-II, and when applied to portfolio optimization. For instance, in (Anagnostopoulos and Mamanis, 2011), SPEA-2's results stay consistently just below NSGA-II's, although not very far away. Other situations have arisen in portfolio optimization with SPEA-2: in (Mishra et al., 2009), (Diosan, 2005) and (Lwin et al., 2013), its optimized fronts present lesser extension than those of the competitor algorithms, something that may create some difficulties in the evaluation of the optimized solutions' quality. Still, SPEA-2 remains as one of the most consistent and important MOEAs up to date. In Chapter 5 we will compare the performance of these two MOEAs (NSGA-II and SPEA-2) applied to portfolio optimization within four distinct Stock Markets.

3.3.3 Other MOEAs

(Metaxiotis and Liagkouras, 2012) present a survey of the most important MOEAs applied to portfolio management. Among them there are the already mentioned NSGA-II and SPEA-2, and a list of other reputed MOEAs. In this set we may consider:

- *Vector Evaluation Genetic Algorithm* (VEGA)
- *Niched Pareto Genetic Algorithm II* (NPGA-II)
- *Multi-Objective Genetic Algorithm* (MOGA)
- *Pareto Envelope-based Selection Algorithm II* (PESA-II)

(Anagnostopoulos and Mamanis, 2011) identify in addition to some of the aforementioned algorithms, the

- *e-Multiobjective Evolutionary Algorithm* (e-MOEA),

a new algorithm that presents some interesting and promising results for the portfolio optimization problem. (Lwin et al., 2014) uses also the

- *Pareto Archived Evolution Strategy* (PAES)

as an alternative algorithm, by comparing its performance with the NSGA-II, the SPEA-2, the PESA-II, and a newly developed algorithm that shows outstanding results, the

- *Learning-Guided Multi-Objective Evolutionary Algorithm With External Archive* (MODEwAwL),

an algorithm that beats all the contesters by far in a set of different performance indicators (*inverted generational distance*, *generational distance*, *diversity metric* and *hypervolume*). (Saborido et al., 2016) suggests the use of

- *Multi-objective Evolutionary Algorithm Based on Decomposition* (MOEA/D)
- *Global Weighting Achievement Scalarizing Function Genetic Algorithm* (GWASF-GA)

as alternatives to the NSGA-II.

As we may see, there is a list of important MOEAs perfectly adequate for portfolio optimization, and new alternative algorithms used in this area of Finance are emerging continuously, something that suggests a bright future for this branch of Applied OR.

4 Optimizing TA Trading Strategies with a Genetic Algorithm[4]

In this chapter an empirical study is conducted in order to draw some conclusions about the Forex market behaviour. As remarked earlier, we will use data relative to the three major Forex crosses. To attain our goal we will develop a GA in order to search the feasible space of solutions and achieve a set of optimized solutions for each scenario [TA category; Market; Period]. These solutions (translatable into trading strategies) will later on be applied to out-of-sample (OOS) data, so we may know how effective they are.

The chapter starts with a presentation of the Trading Model assumptions, where all the rationale employed in the model is described, including the adopted TA indicators and rules for trading. It will follow a presentation of our GA, its structure, pseudo code and mechanics. In a third part, we will present and discuss the results obtained both in-sample (IS) and OOS and draw conclusions about them.

4.1 The Trading Model

The model tries to replicate the trading environment in the Forex market with typical TA indicators and trading strategies commonly used by the industry. The purpose is to obtain and compare *optimized solutions* in three different TA categories – *momentum*, *trend* and *breakout* – and assess which ones produce better average outcomes. A *solution* consists of one indicator (belonging to one of these categories) with a given setting for its parameters.

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[4] Chapter 4 presents the empirical work of this Thesis published in the article Macedo, L.L., Godinho, P. and Alves, M.J. (2016), ‘A Comparative Study of Technical Trading Strategies Using a Genetic Algorithm’, *Computational Economics*, available at: https://doi.org/10.1007/s10614-016-9641-9.

Due to the complexity of the optimization task, a metaheuristic technique was chosen. A genetic algorithm is a suitable technique to achieve the proposed goal. It can easily embody the multi-dimensionality of the problem, that is, it can optimize simultaneously the *solution structure* (selection of the TA indicator) and the *solution parameters* (parameters of the indicator). In addition, the randomness factor in population generation and the variation of the parameters can help to avoid the trap of local optima and premature convergence in the optimization process, at the same time as the population evolves to yield solutions with a better fit. Our methodology seems better suited to the purpose at hand than other metaheuristics applied to Forex, such as artificial neural networks (ANNs), e.g. (Andreou et al., 2002); genetic programming (GP), e.g. (Neely and Weller, 2003), (Wilson and Banzhaf, 2010); grammatical evolution (GE), e.g. (Brabazon and O'Neill, 2004). This is because GAs allow optimization of parameters of isolated TA indicators. GP is better for optimizing TA rules, combining them into more complex structures and even designing whole trading systems. ANNs are better suited for pattern recognition, establishing evolving relations through complex input-output models. GP or GE are more useful for finding new optimized rules, which is not the objective in mind – we deliberately assume the trading rules as predefined and stable, since it is those rules used by the trading industry that we want to verify. Moreover, with GP or GE, new trading rules could emerge that are completely unrelated to the TA philosophy.

To achieve the proposed objective, the GA will optimize the *evaluation function* (*EF*) represented as:

$$EF = {Profit(A_T)}\big/{MDD(A_T)}$$

where A_T is the account at the end of time T (end of the period) and $MDD(A_T)$ is the *maximum drawdown* of the account at the same time, defined in (3), Section 2.3.2.2. The *MDD* is the largest drop from peak to trough in a time span, the worst fall in account value observed for the trading activity.

The intention is to maximize the ratio *EF* (in-sample), where the numerator has the measure of total profit, and the denominator has a risk measure – the maximum drawdown (*MDD*). Once the maximization process has been completed the optimized trading strategies (indicator and parameters) are applied to out-of-sample data. The analysis will be conducted with regard to a measure of return, *Return on Account* (*RoA*), computed as:

$$RoA = \frac{Profit\ (A_t)}{Initial\ Capital} * 100\%$$

4.1.1 Assumptions

The study concentrates on three major currency markets: EUR/USD, GBP/USD and USD/JPY. We use daily quotes. Throughout the article, currency crosses are expressed under the ISO 4217 code format ("BBB/CCC", where "BBB" = Base currency and "CCC" = Counter currency). Traded quantities are designated in base currency. A standard lot trades 100 000 base currency units, with a leverage of 1:100, meaning that for every traded lot we allocate the equivalent to 1 000 base currency units to margin, expressed in counter currency. The interest rate differential, for rollover computation, is defined as the arithmetic difference between base currency and counter currency daily interest rates. Rollovers are calculated at the end of the day (time t) and multiplied by the respective closing price to become expressed in counter currency terms. For simplification, absolute profits, percentage in point (*pip*), margins, standard deviations and trading costs are designated in counter currency. At any given time *t*, the trading signal devised by an indicator assigns a trading position Y_t (*long*, *short* or *out-of-the-market*, indicated by signals "1", "–1" and "0", respectively) at the beginning of time $t + 1$. The difference between Y_t and Y_{t-1} defines the action to take at time $t + 1$. For instance, if Y_{t-1} = –1 and Y_t = –1, the short position remains unchanged and no action is taken; if Y_{t-1} = –1 and Y_t = +1 the position is reversed from a short position to a long position and 2 lots of 100 000 currency units are bought ($Y_t - Y_{t-1}$ = +2); if Y_{t-1} = –1 and Y_t = 0, this implies a change from a short position of 1 lot to an out-of-the-market situation by buying 1 lot ($Y_t - Y_{t-1}$ = +1) in $t + 1$, and so on. Execution prices in $t + 1$ are, by assumption, considered at the opening price. To compute *MDD* and the *RoA* in percentage terms we consider a starting trading account size (equity) of 50 000.00 currency units for the EUR/USD and GBP/USD markets, 5 000 000.00 units for USD/JPY. For every period, the account size restarts with the mentioned equity figures. Whenever an account hits zero value, an order is triggered to clear all open positions and exit the market.

4.1.2 Example of indicator application and RoA Calculation

We present an example of the computation of a solution *RoA* (Table 1). For simplification purposes, we have restricted the period of potential transactions to a 15-day span and ignored trading costs. SMA(*n*) represents a simple moving average with a moving window of *n* observations.

Table 1. Computation of a solution's RoA.

(1) Day	(2) OPEN	(3) MAX	(4) MIN	(5) CLOSE	(6) SMA(3)	(7) SMA(5)	(8) SMA(3) - SMA(5)	(9) Position (Y_t)	(10) Profit (DP_t)	(11) Account Value
1	1.2765	1.2831	1.2716	1.2823				0	0.00	50 000.00
2	1.2824	1.2836	1.2774	1.2793				0	0.00	50 000.00
3	1.2793	1.2835	1.2739	1.2799	1.2805			0	0.00	50 000.00
4	1.2799	1.2911	1.2773	1.2870	1.2821			0	0.00	50 000.00
5	1.2871	1.2894	1.2803	1.2819	1.2829	1.2821	0.0008533	1	0.00	50 000.00
6	1.2820	1.2895	1.2762	1.2789	1.2826	1.2814	0.0012000	1	-310.00	49 690.00
7	1.2789	1.2903	1.2767	1.2870	1.2826	1.2829	-0.0003400	-1	810.00	50 500.00
8	1.2867	1.2912	1.2744	1.2791	1.2817	1.2828	-0.0011133	-1	730.00	51 230.00
9	1.2791	1.2797	1.2705	1.2725	1.2795	1.2799	-0.0003467	-1	660.00	51 890.00
10	1.2725	1.2765	1.2704	1.2719	1.2745	1.2779	-0.0033800	-1	60.00	51 950.00
11	1.2718	1.2805	1.2692	1.2785	1.2743	1.2778	-0.0035000	-1	-660.00	51 290.00
12	1.2786	1.2866	1.2766	1.2853	1.2786	1.2775	0.0011067	1	-680.00	50 610.00
13	1.2850	1.2888	1.2810	1.2832	1.2823	1.2783	0.0040533	1	-150.00	50 460.00
14	1.2832	1.2847	1.2777	1.2824	1.2836	1.2803	0.0033733	1	-80.00	50 380.00
15	1.2824	1.2940	1.2818	1.2863	1.2840	1.2831	0.0008267	1	390.00	50 770.00
									770.00	

Source: Author.

Assume the algorithm has generated (in the training phase) a solution within the *trend* category where the selected indicator is an SMA crossover, taking long positions when SMA(3) > SMA(5) and short positions otherwise. This strategy is applied to the out-of-the sample data in Table 1. The moving averages are applied to the close prices and are presented in columns (6) and (7) of Table 1. The difference between the two averages is given in column (8) and the respective positions to assume on each day are in column (9), where "1" represents long and "-1" short positions. In this case we can only have a position signal after the 5th day, as the longest SMA needs 5 days to compute its first value. On day 5 we can see the system signals a long position, which will be taken on the next day at the opening price. Therefore, at the end of day 6 we have 1 trading lot of 100 000 base currency units bought at 1.2820 and valued at 1.2789. This represents a loss of -310.00 counter currency units [(1.2789 – 1.2820)*100 000= -310.00]. On day 7 we keep a long position of one lot (signal at day 6) and so the profit will be equal to the difference of the closing price on day 7 and the closing price on day 6, i.e. [(1.2870 – 1.2789)*100 000= +810.00]. When the position changes from long to short on day 8 (signal on day 7) we sell 2 lots at 1.2867 – one to offset the previous trading position and the other to take a short position. At the end of day 8 we have produced 730.00 in counter currency profits: (1.2867 - 1.2870)*100 000 + (1.2867 - 1.2791)*100 000 = -30.00 (loss for offsetting previous position) + 760.00 (profit from the short position). Following this reasoning, at the end of the period we would have a net cumulative profit of 770.00, which represents a *RoA* of 770 / 50 000 = 1.54%.

Below are presented the formulae used to calculate the daily profit (DP_t), where are taken into consideration all possible scenarios for the position held at each time period.

Continuous long position, $Y_{t-2} = 1$ and $Y_{t-1} = 1$:

$$DP_t = \left(P_t^{close} - P_{t-1}^{close}\right)$$

Reversing from a short to a long position, $Y_{t-2} = -1$ and $Y_{t-1} = 1$:

$$DP_t = \left(P_t^{close} - P_t^{open}\right) + \left(P_{t-1}^{close} - P_t^{open}\right)$$

Continuous short position, $Y_{t-2} = -1$ and $Y_{t-1} = -1$:

$$DP_t = -\left(P_t^{close} - P_{t-1}^{close}\right) = \left(P_{t-1}^{close} - P_t^{close}\right)$$

Reversing from a long to a short position, $Y_{t-2} = 1$ and $Y_{t-1} = -1$:

$$DP_t = -\left[\left(P_t^{close} - P_t^{open}\right) + \left(P_{t-1}^{close} - P_t^{open}\right)\right] \\ = \left(P_t^{open} - P_t^{close}\right) + \left(P_t^{open} - P_{t-1}^{close}\right)$$

From a long position to out of the market, $Y_{t-2} = 1$ and $Y_{t-1} = 0$:

$$DP_t = \left(P_t^{open} - P_{t-1}^{close}\right)$$

From a short position to out of the market, $Y_{t-2} = -1$ and $Y_{t-1} = 0$:

$$DP_t = -\left(P_t^{open} - P_{t-1}^{close}\right) = \left(P_{t-1}^{close} - P_t^{open}\right)$$

From out of the market to a long position $Y_{t-2} = 0$ and $Y_{t-1} = 1$:

$$DP_t = \left(P_t^{close} - P_t^{open}\right)$$

From out of the market to a short position, $Y_{t-2} = 0$ and $Y_{t-1} = -1$:

$$DP_t = -\left(P_t^{close} - P_t^{open}\right) = \left(P_t^{open} - P_t^{close}\right)$$

P_t^{open} and P_t^{close} stand for open and close prices at a given day t.

The account profit is computed as:

$$Profit\ (A_t) = \sum_{t=1}^{T} DP_t$$

4.1.3 Technical Indicators

Technical indicators have been presented in Subsection 2.2.2.2. In this chapter, we consider TA indicators of three categories: *trend*, *momentum* and *breakout*. Regarding the *trend* category, the indicators were mainly based on crossing moving averages: *double simple moving average crossover*, *double exponential moving average crossover*, *triple simple moving average crossover*, *directional movement index* and *moving average convergence divergence*. Within the *momentum* category the following indicators were considered: *relative strength index*, *Williams' %R* and *stochastic oscillator*. The *breakout* indicators used in this work are *average true range* (ATR), *Bollinger Bands*, *close price, exponential moving average +/- k standard deviations* and *double exponential moving average breakout*.

Table 2. TA indicators used, respective parameters to be optimized and constraints associated.

Indicator	Category	Parameters	Parameter Constraints
Relative Strengh Index (RSI)	Momentum	ub_{RSI} ; lb_{RSI} ; n_{RSI}	$0<ub_i\leq1$; $0\leq lb_i<1$; $lb_i<ub_i$; $1\leq n_i\leq30$
Williams' %R	Momentum	$ub_{\%R}$; $lb_{\%R}$; $n_{\%R}$	$-1<ub_i\leq0$; $-1\leq lb_i<0$; $lb_i<ub_i$; $1\leq n_i\leq30$
Stochastic Oscillator (SO)	Momentum	$\alpha_{F\%D}$; $\alpha_{S\%D}$; $n_{F\%K}$; $n_{F\%D}$; $n_{S\%D}$	$0<\alpha_i\leq1$; $1\leq n_i\leq30$
Double SMA Crossover	Trend	n_{SMA1} ; n_{SMA2}	$1\leq n_i\leq30$
Double EMA Crossover	Trend	α_{EMA1} ; α_{EMA2} ; n_{EMA1} ; n_{EMA2}	$0<\alpha_i\leq1$; $1\leq n_i\leq30$
Triple SMA Crossover	Trend	α_{TSMA1} ; α_{TSMA2} ; α_{TSMA3} ; n_{TSMA1} ; n_{TSMA2} ; n_{TSMA3}	$0<\alpha_i\leq1$; $1\leq n_i\leq90$
Moving Average Convergence Divergence (MACD)	Trend	$\alpha_{FastEMA}$; $\alpha_{SlowEMA}$; α_{Signal} ; $n_{FastEMA}$; $n_{SlowEMA}$; n_{Signal}	$0<\alpha_i\leq1$; $1\leq n_i\leq30$
Directional Movement Index (DMI)	Trend	α_{DI_ATR} ; α_{EMA+DM} ; α_{EMA-DM} ; n_{DMI}	$0<\alpha_i\leq1$; $1\leq n_i\leq30$
Average True Range (ATR)	Breakout	$\alpha_{CP\_EMA\_ATR}$; α_{ATR} ; $n_{CP\_EMA\_ATR}$; n_{ATR}	$0<\alpha_i\leq1$; $1\leq n_i\leq30$
Bollinger Bands (BB)	Breakout	n_{SMA_BB} ; k_{BB}	$2\leq n_i\leq30$; $0\leq k_i\leq5$
Close Price EMA ± kσ	Breakout	$\alpha_{CP\_EMA\_StDv}$; $n_{CP\_EMA\_StDv}$; $k_{Long\_CP\_EMA\_StDv}$; $k_{Short\_CP\_EMA\_StDv}$	$0<\alpha_i\leq1$; $2\leq n_i\leq30$; $0\leq k_i\leq5$
Double EMA Breakout	Breakout	α_{DbEMA1} ; α_{DbEMA2} ; n_{DbEMA1} ; n_{DbEMA2}	$0<\alpha_i\leq1$; $1\leq n_i\leq30$

Source: Author.

There are five distinct kinds of parameters on which the indicators depend:

- $\alpha_i = {}^{w_{t-1}}/_{w_t}$: ratio between weights of observations in moment $t-1$ and moment t for the computation of an Exponential Moving Average;
- n_i: number of observations of the moving window for the calculation of indicator i;
- ub_i: upper bound of indicator i;
- lb_i: lower bound of indicator i;
- k_i: number of standard deviations used for the computation of indicator i.

The indicators, parameters and constraints are summarized in Table 2. The trading rules based on these indicators were presented in Subsection 2.2.2.2 and are also summarized in Table 3.

Table 3. Adopted TA trading rules, associated to the respective indicators.

Indicator	Trading Rules (moment *t* , to take effect in *t+1*)
Relative Strengh Index (RSI)	$Y_t=1$ if $RSI_t \leq lb_{RSI}$ and $RSI_{t-1} \geq lb_{RSI}$; $Y_t=-1$ if $RSI_t \geq ub_{RSI}$ and $RSI_{t-1} \leq ub_{RSI}$; $Y_t=Y_{t-1}$ otherwise
Williams' %R	$Y_t=1$ if $\%R_t \leq lb_{\%R}$ and $\%R_{t-1} \geq lb_{\%R}$; $Y_t=-1$ if $\%R_t \geq ub_{\%R}$ and $\%R_{t-1} \leq ub_{\%R}$; $Y_t=Y_{t-1}$ otherwise
Stochastic Oscillator (SO)	$Y_t=1$ if $Fast\%D_t > Slow\%D_t$; $Y_t=-1$ if $Fast\%D_t < Slow\%D_t$; $Y_t=0$ otherwise
Double SMA Crossover	$Y_t=1$ if $SMA1_t > SMA2_t$; $Y_t=-1$ if $SMA1_t < SMA2_t$; $Y_t=0$ otherwise (SMA1 = Fast MA ; SMA2 = Slow MA)
Double EMA Crossover	$Y_t=1$ if $EMA1_t > EMA2_t$; $Y_t=-1$ if $EMA1_t < EMA2_t$; $Y_t=0$ otherwise (EMA1 = Fast MA ; EMA2 = Slow MA)
Triple SMA Crossover	$Y_t=1$ if $SMA1_t > SMA2_t$ and $SMA1_t > SMA3_t$; $Y_t=-1$ if $SMA1_t < SMA2_t$ and $SMA1_t < SMA3_t$; $Y_t=0$ otherwise (SMA1 = Fast MA ; SMA2 = Intermediate MA ; SMA3 = Slow MA)
Moving Average Convergence Divergence (MACD)	$Y_t=1$ if $MACD_t > Signal_t$; $Y_t=-1$ if $MACD_t < Signal_t$; $Y_t=0$ otherwise
Directional Movement Index (DMI)	$Y_t=1$ if $+DI_t > -DI_t$; $Y_t=-1$ if $+DI_t < -DI_t$; $Y_t=0$ otherwise
Average True Range (ATR)	$Y_t=1$ if $Close\ Price_t > EMA+ATR_t$; $Y_t=-1$ if $Close\ Price_t < EMA-ATR_t$; $Y_t=0$ otherwise
Bollinger Bands (BB)	$Y_t=1$ if $Close\ Price_t > SMA\_BB+k\_BB_t$; $Y_t=-1$ if $Close\ Price_t < SMA\_BB-k\_BB_t$; $Y_t=0$ otherwise
Close Price EMA ± kσ	$Y_t=1$ if $Close\ Price_t > EMA+k\_Long.\ \sigma_t$; $Y_t=-1$ if $Close\ Price_t < EMA-k\_Short.\ \sigma_t$; $Y_t=0$ otherwise
Double EMA Breakout	$Y_t=1$ if $Close\ Price_t > EMA1_t$ and $Close\ Price_t > EMA2_t$; $Y_t=-1$ if $Close\ Price_t < EMA1_t$ and $Close\ Price_t < EMA2_t$; $Y_t=0$ otherwise (EMA1 = Fast MA ; EMA2 = Slow MA)

$Y_t=1$: long position
$Y_t=0$: out of the market
$Y_t=-1$: short position

Source: Author.

4.1.4 Chromosome Configuration

A solution (chromosome) is composed of a set of parameters, confined to a category of TA indicators. For every solution, the GA activates only one indicator (represented by boolean variables); the rest remain latent as *shadow* indicators, with their respective parameters. Indicators of the same category are thus competing with each other. In the generic example shown in Figure 2, the hypothetical category in question consists of three different indicators: Boolean variables d_1, d_2, d_3 identify whether the respective indicators are active or not (only one can be active); parameters p_{ij} refer to the parameters of each current indicator (p_{ij} is the parameter j of the indicator i). In this example, indicator 2 is active.

Figure 2. Representation (encoding) of a single generic solution of a hypothetical category, where indicator 2 is active and indicators 1 and 3 are inactive (latent indicators).

d_1	p_{11}	p_{12}	p_{13}	d_2	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	d_3	p_{31}	p_{32}
$d_1=0$				$d_2=1$						$d_3=0$		

Source: Author.

This configuration allows more flexibility than the use of a single indicator because it provides an opportunity for indicator interchange and hence more diversity in the optimization process. We chose to use the optimization of individual indicators and not a combination of two or more so that the validity of each single TA indicator could be discerned.

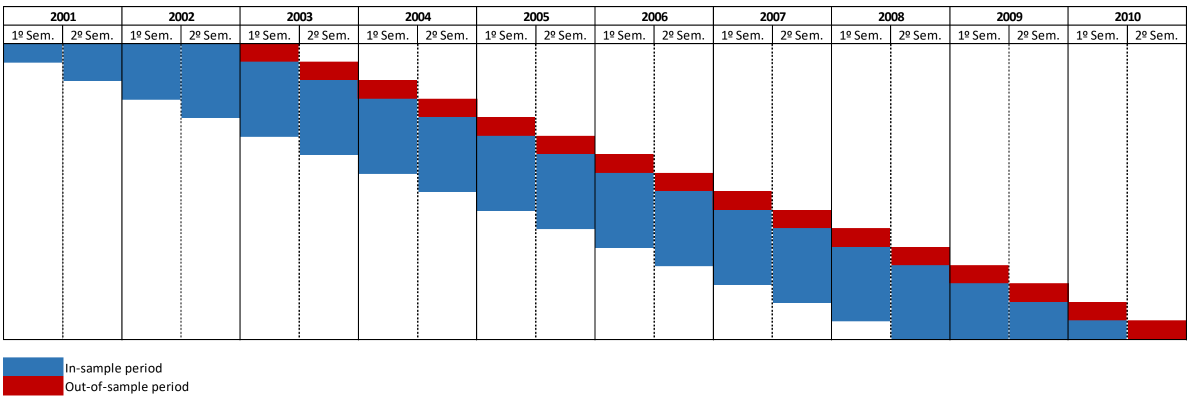
4.1.5 Algorithm Structure and Optimization Rationale

In the training phase, the objective is to identify the combination of parameters that maximizes *EF*, defined as a risk-adjusted return. The procedure includes a training phase, where the category/indicator selection and the parameter optimization are fulfilled by the

GA, and a testing phase. In this last phase, the previously optimized solutions are applied to the out-of-sample data. Different experiments for each category of indicators within each single market are performed. Every experiment consists of an in-sample training period of two years and an out-of-sample testing timespan of one semester, according to the schedule in Figure 3.

For each market, indicator category and timespan, the genetic algorithm will produce a set of N optimized solutions, each resulting from an independent run, n=1,…,N. We have considered N=50 for each of the 16 timespans. The purpose is to assess the average return (RoA) of each set to draw conclusions. A solution is composed of an active indicator and its parameters.

Figure 3. Rolling window timespans.



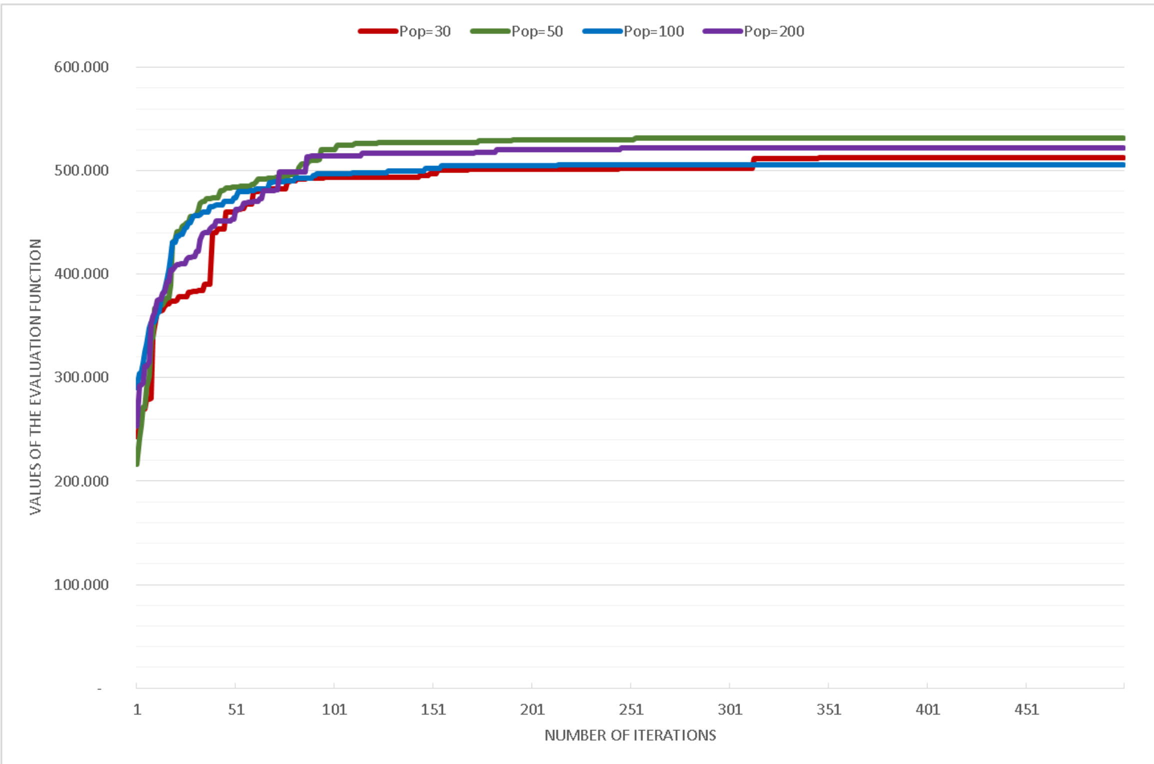
Source: Author.

4.1.5.1 Algorithm

It is a difficult task to define the most appropriate size of a population and a suitable number of iterations in a GA. While small populations and/or small numbers of iterations run the risk of under-covering the solution space (being trapped in local optima), large populations and/or large numbers of iterations may require an excessive computational effort. Schaffer et al. (1989) concluded that for a representative series of multiple genetic algorithms, an appropriate population size should be set between 20 and 30. A similar result was reported by (Haupt and Haupt, 2000), who argue that small population sizes combined with larger mutation rates perform better, not only by achieving better results but also by doing so in a shorter execution time. (Papadamou and Stephanides, 2007) also suggested a population of 30 as a suitable population size for GAs applied to TA indicator analysis, taking into consideration performance and computational effort.

We did a preliminary test using the same markets, taking a population size of 30 and a very large number of iterations in order to set an appropriate number of iterations to use. The results showed that the algorithm produced large improvements up to the 100th iteration for most of the trials, but, after that iteration, the improvements were generally not significant. Further experiments were conducted in our algorithm with different population sizes: 50, 100 and 200 chromosomes, all with 500 iterations.

Figure 4. Comparison of the GA performance with different population sizes (30, 50, 100 and 200). Each line represents the average performance of a sample of 10 independent runs for each population scenario in the EUR/USD market.



Source: Author.

The average performances are presented in Figure 4 and show evidence that after the 100th iteration there are only small improvements. We may see that around the 80th iteration average performances of the GA become very similar. In this context, we have decided to use 100 iterations and a population size of 30 in the optimization process.

The following scheme describes the optimization process. The algorithm starts with the random generation of a population of J trading strategies that will be subject to crossover and mutation through I iterations. At the end of the I^{th} iteration, the algorithm will produce an in-sample optimized solution. As stated above, J and I will be 30 and 100, respectively.

i= Iteration number, $i = 1, \ldots, I$

j= Solution number within the Population

P(i) = Population per market for each TA indicators' category at the end of iteration i

S_{ij}= j^{th} solution (trading strategy) at iteration i; $i = 1, \ldots, I$; $j = 1, \ldots, J$;

P(i) =$\{S_{ij}; j = 1,2, \ldots, J\}$

Generate P(0), the original population, formed by J random trading strategies (solutions)

For $i = 1$ *to* I

    For $j = 1$ *to* J

        $S_{ij} \leftarrow$ *Crossover* $S_{(i-1)l}$ *with* $S_{(i-1)k}$*; l and k randomly selected*

        $S^*_{ij} \leftarrow$ *Best of* ($S_{(i-1)j}$; S_{ij}) *according to the evaluation function EF*

        $S'_{ij} \leftarrow$ *Mutation of* S^*_{ij}

        $S^{**}_{ij} \leftarrow$ *Best of* (S^*_{ij}; S'_{ij}) *according to the evaluation function EF*

    Next *j*

    P(i) =$\{S^{**}_{ij}; j = 1,2, \ldots, J\}$

Next *i*

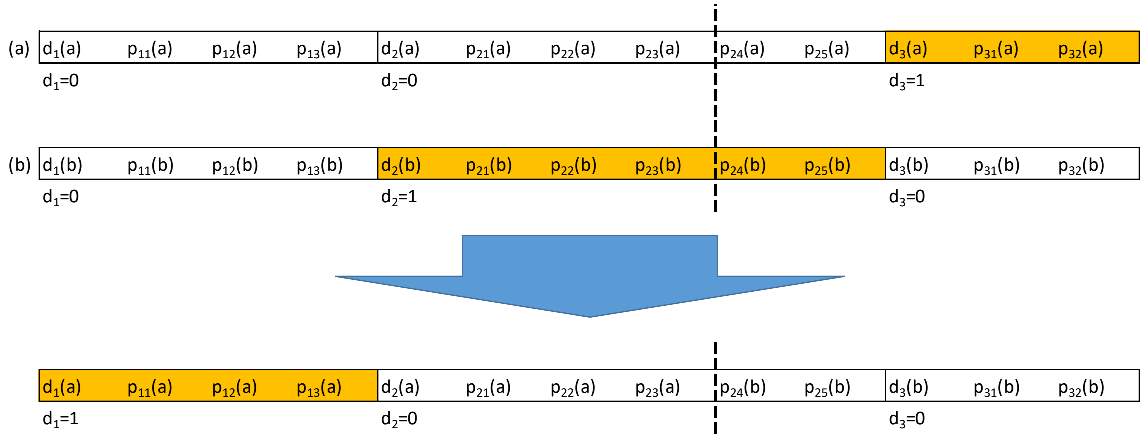
Output $S^{***} \leftarrow$ *Best Strategy of P(i)*

In the end, considering all markets, categories and periods, we have a set of 7 200 optimized trading strategies S^{***} covering all scenarios (*N· #markets · #categories · #periods* = 50*3*3*16 = 7 200), to be tested with out-of-sample data.

4.1.5.2 Crossover and Mutation

The adopted crossover operator is a binary crossover with a single randomly selected cutting point. For every population of *J* valid solutions, the algorithm picks the first solution (our *reference element*), which will be compared with an offspring generated by the crossover operator. Next, the algorithm randomly selects two different solutions from the population and applies the crossover operator, originating a single offspring. The offspring receives the genetic material from the first parent up to the cutting point and from the second parent thereafter. The offspring must have only one indicator active. Whenever this condition does not hold the algorithm goes on to select one active indicator (randomly, to prevent excessive elitism) – see an example in Figure 5.

Figure 5. Crossover operator: a solution obtained by crossover where indicator 1 becomes active (randomly selected). Genetic material from indicators 2 and 3 remains latent.



Source: Author.

The algorithm compares the *EF* value of the offspring with the *reference element*'s value and the best of them is selected to integrate the population of the next iteration. Then, the algorithm applies mutation with a certain probability to this solution. The solution obtained after mutation replaces the original solution in the population of the next iteration only if it has a better *EF* value. The algorithm picks the next solution (2nd) as a *reference* *element* and repeats the procedure until all *J* solutions of the current population have been evaluated against *J* alternative offspring.

Regarding the mutation operator, the probability of mutation of each gene (parameter of the indicator) is 1% except for the Boolean variables d_i. There is also the possibility of genetic material recovery provided by a random re-selection of the active indicator. This consists of a recalculation of the d_i values that is always performed at every iteration. In this process each indicator has the same probability of being chosen. Only one d_i can take the value "1". The type of mutation depends on the specificities of each parameter, according to the following conditions:

- $\alpha_i, \Delta ub_i, \Delta lb_i \in]-0.05000; +0.05000[$, following a uniform density probability function;
- $\Delta n_i \in \{-1; +1\}$, with equal probability of choosing each element;
- $k_i \in]-0.50000; +0.50000[$, following a uniform density probability function.

Variations in parameters α_i, ub_i, lb_i and k_i occur in figures up to five decimal places, as shown above in the intervals. The major contribution of this mutation operator and re-selection procedure lies in the possibility of recovering latent or *shadow* genetic material, which may prevent a premature convergence of the algorithm. Our algorithm presents a linear time complexity *O*(*n*), where *n* stands for data input (daily open-high-low-close price quotes). The time required to execute our algorithm increases in proportion to the used amount of input data, making it a less time consuming algorithm.

4.2 Empirical Results and Discussion

In this section, we will apply the optimized solutions obtained by the GA in the in-sample data to the out-of-sample data. The results in terms of the average RoAs will be compared with those obtained from the *preeminent indicator* (the most frequently selected indicator in each set of 50 optimized solutions), considering the usual industry values for the parameters. We can therefore evaluate the GA's ability to optimize the parameter values. The commonly used parameter values, according to the industry – (Aronson, 2007), (Colby, 2003), (Kirkpatrick and Dahlquist, 2011), (Murphy, 1999), (Schwager, 1996) – are:

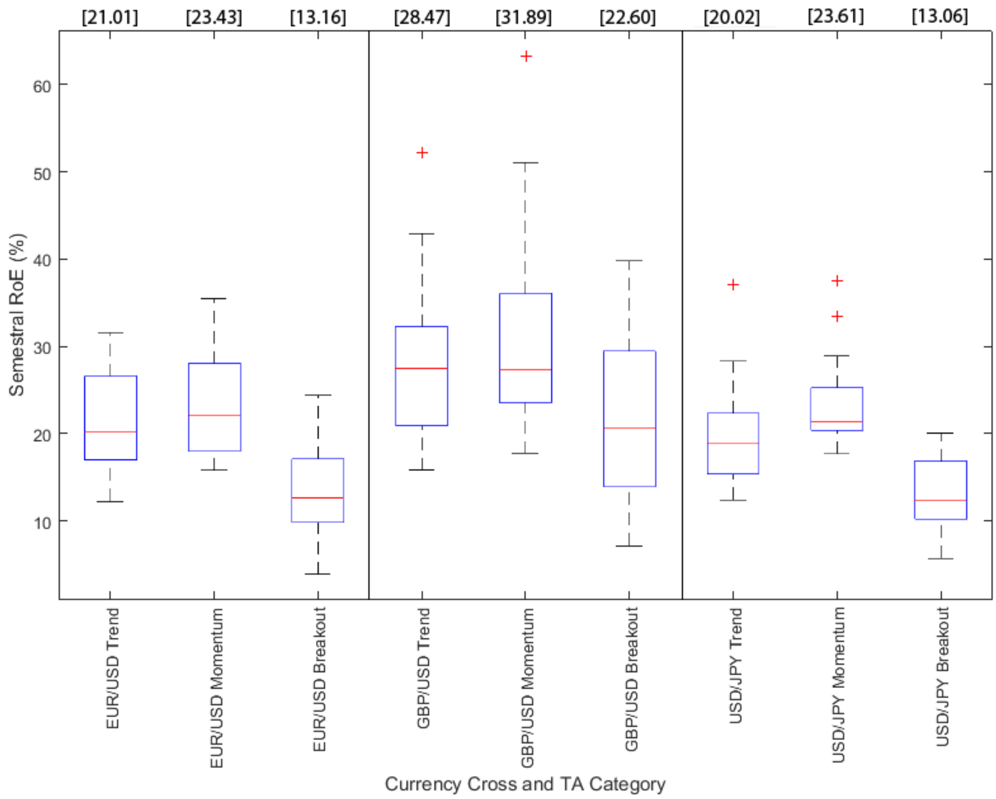
RSI: $n = 14$; upper threshold = 70%; lower threshold = 30%;
Williams'%R: $n = 10$; upper threshold = -20%; lower threshold = -80%;
Stochastics: $n_{Fast\%K} = 5$; $\alpha_{Fast\%D} = 1$; $n_{Fast\%D} = 3$; $\alpha_{Slow\%D} = 1$; $n_{Slow\%D} = 3$;
Moving Average Crossovers: $n_1 = 5$ (short term – a trading week); $n_2 = 20$ (medium term – one trading month); and if necessary $n_3 = 60$ (long term – three trading months);
For all EMAs, consider $\alpha = 0.8$;
MACD: $n_{FastEMA} = 12$; $n_{SlowEMA} = 26$; $n_{Signal} = 9$; all $\alpha = 0.8$;
DMI: $n_{DMI} = 14$; all $\alpha = 0.8$;
ATR: $n_{CP\_EMA\_ATR} = 5$; $n_{ATR} = 14$; all $\alpha = 0.8$;
BB: $n = 10$; all $k = 2$;
$CP \pm k\sigma$: $n = 5$; all $k = 1$; $\alpha = 0.8$.

The training process was executed in the simplest possible way, trading one standard lot without trading costs and stops. In Forex markets we deal with two distinct kinds of costs: 1) *spreads* and 2) *rollover*, usually a fraction of the spread measured in pips (around 0.2 pips in EUR/USD or GBP/USD markets, for instance) that accrues to interest rate differentials, increasing unfavorable or attenuating favorable differentials. In our case, the rollover (effective cost) was excluded from the training phase but not the interest rate differential itself, which may be a cost or a profit to the trader. In-sample RoAs are in 6 month adjusted rates (in proportion), so they may be related and compared to out-of-sample RoAs.

4.2.1 Trading Strategy Return without Trading Costs

The analysis of in-sample results allows us to conclude that all kinds of TA techniques present positive returns; *breakout* systems have consistently the worst in-sample *RoA* performances within each market and *momentum* and *trend* optimized strategies show very similar figures. Figure 6 presents a boxplot with the returns of all combinations of currency crosses and TA categories. In each boxplot, the bottom, middle and top of the boxes represent the 25th, 50th and 75th percentiles, respectively; the top and bottom whiskers stand for the maximum and minimum, respectively, excluding outliers; outliers are represented by '+'. The related results are presented in the Appendix.

Figure 6. Return (%) of the optimized solutions, in-sample without costs, from the 1st semester of 2001 to the 2nd semester of 2008. On the top, in brackets, are presented the average semestral returns (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 1.

In this figure it is possible to see the median value in the middle of each box. The bottom of the box represents the lower quartile (25th percentile of RoAs), meaning 25% of all RoAs stand below that value, while the top of the box stands for the upper quartile (75th percentile *RoA* value), meaning 75% of all RoA observations are lower than that mark. The lower and the upper whiskers stand for the minimum and maximum values, excluding outliers. Outliers are represented with crosses (plus sign) and represent extreme values, less than 1.5 times of lower quartile or higher than 1.5 times of upper quartile. On the top of each figure, in brackets, are shown the average RoA values for each currency cross / TA category combination. We may see how the optimization procedure produces very different outcomes by market, enabling us to conclude that each market has its own singularities and inherent characteristics promoting or preventing the exploitation of TA profitable trading opportunities, in tune with Lo's AMH.

Table 4 presents the profitabilities for out-of-sample data, per semester. The table is organized by market, and within each market we have three columns with the *mean* profitabilities by TA category followed by the respective return *median* values. Below the median stands the respective *p-value* to the non-parametric Wilcoxon test, where "H_0: The median is statistically equal to zero" and "H_1: The median is not statistically equal to zero", to assess the statistical significance of the measures of central tendency. A very small *p-value* is indicative against the null hypothesis. On a fourth column of each market it is presented the correspondent semester price variation of the first currency in relation to the second one.

Regarding the mean profitabilities, it is noticeable how all TA categories produce out-of-sample aggregate positive outcomes. These positive semestral returns show consistency, in particular with reference to the GBP/USD market. This market produces the best outcomes, with trend-based profitabilities staying well ahead. When comparing TA strategy-based results with price variation, we may see in the EUR/USD market that average results are consistently better when the EUR appreciates against the USD. In four out of the 11 semesters in which the EUR rises against the USD, all TA categories present good average RoAs, while the same only happens once in the five semesters in which the dollar appreciates against the euro. The other semesters present mixed results, depending on the TA category. The same is true regarding the GBP/USD when the GBP appreciates against the USD, although the *trend* category behaves positively in some cases when the opposite happens (see 2005-2nd and 2008-2nd semesters). In the USD/JPY market there is not a defined tendency – when the JPY appreciates strongly against the USD (2003-2, 2004-2, 2008-1, 2008-2 or 2010-2) the trading returns vary from heavy losses to strong profits. Mixed results are attained when the USD appreciates against the JPY.

Table 4. Return (%) of the optimized solutions, out-of-sample without costs.

RoA (%)	EUR/USD						
	Trend		Momentum		Breakout		Price
Semester	Mean	Median	Mean	Median	Mean	Median	Variation
2003-1	16,6	*15,4*	18,8	*21,0*	16,8	*18,8*	9,8
		<0.0001		*<0.0001*		*<0.0001*	
2003-2	6,3	*3,4*	16,2	*15,1*	25,7	*28,0*	10,3
		<0.0001		*<0.0001*		*<0.0001*	
2004-1	24,2	*34,0*	0,1	*-4,4*	-31,5	*-32,0*	-3,1
		<0.0001		*0,8205*		*<0.0001*	
2004-2	1,7	*2,7*	1,0	*-5,1*	28,4	*28,7*	11,1
		0,1876		*0,6745*		*<0.0001*	
2005-1	-14,1	*-14,1*	-14,9	*-13,4*	-17,3	*-18,4*	-10,8
		<0.0001		*<0.0001*		*<0.0001*	
2005-2	5,4	*8,6*	6,7	*3,4*	0,2	*1,5*	-2,0
		0,0042		*0,0001*		*0,0831*	
2006-1	-3,1	*-2,1*	-2,1	*-1,3*	-3,9	*-5,5*	8,0
		0,0042		*0,2487*		*0,0001*	
2006-2	-1,9	*-1,5*	3,4	*5,0*	-0,7	*-0,3*	3,2
		0,0781		*0,0081*		*0,2220*	
2007-1	1,8	*1,5*	1,1	*0,9*	-2,7	*-2,8*	2,6
		0,0081		*0,0136*		*0,0002*	
2007-2	-13,3	*-14,8*	-0,7	*1,7*	14,0	*14,6*	7,7
		<0.0001		*0,6535*		*<0.0001*	
2008-1	16,1	*14,7*	5,2	*-2,9*	8,1	*10,2*	8,1
		<0.0001		*0,2861*		*<0.0001*	
2008-2	-1,9	*-10,7*	-11,3	*-14,8*	9,9	*6,8*	-11,3
		0,7758		*0,0349*		*0,0001*	
2009-1	-4,6	*-4,0*	9,0	*9,7*	-5,7	*-2,1*	0,5
		0,0733		*0,0001*		*0,0052*	
2009-2	-10,3	*-9,8*	-12,3	*-12,1*	-5,7	*-4,4*	2,1
		<0.0001		*<0.0001*		*<0.0001*	
2010-1	5,6	*14,6*	-3,8	*-8,0*	-2,0	*-4,5*	-14,7
		0,0207		*0,1018*		*0,0014*	
2010-2	16,0	*18,9*	-4,4	*-5,6*	8,6	*10,0*	9,4
		<0.0001		*0,0017*		*0,0012*	
Aggregate	44,49		12,00		42,32		30,90
Semestral Average	*2,78*		*0,75*		*2,64*		*1,93*

Table 4. (cont.) Return (%) of the optimized solutions, out-of-sample without costs.

RoA (%)	GBP/USD						
	Trend		Momentum		Breakout		Price
Semester	Mean	Median	Mean	Median	Mean	Median	Variation
2003-1	0,4	*1,2*	8,2	*6,6*	12,9	*13,4*	2,9
		0,7981		*<0.0001*		*<0.0001*	
2003-2	24,1	*24,2*	10,8	*14,2*	48,8	*49,6*	8,2
		<0.0001		*<0.0001*		*<0.0001*	
2004-1	-30,5	*-35,2*	2,1	*-1,4*	-17,8	*-15,9*	2,0
		<0.0001		*0,5988*		*<0.0001*	
2004-2	5,8	*5,8*	5,3	*7,9*	14,3	*16,5*	5,3
		0,0002		*0,1660*		*<0.0001*	
2005-1	9,8	*11,0*	-0,2	*-1,0*	-5,6	*-10,4*	-6,7
		<0.0001		*0,7391*		*0,0010*	
2005-2	23,1	*25,8*	8,6	*9,1*	3,7	*3,7*	-3,9
		<0.0001		*0,0001*		*0,0001*	
2006-1	-3,6	*-5,0*	7,3	*11,9*	-0,6	*-0,8*	7,4
		0,0421		*0,0019*		*0,2332*	
2006-2	16,3	*20,7*	5,5	*5,0*	13,4	*13,0*	6,0
		<0.0001		*0,0003*		*<0.0001*	
2007-1	5,6	*8,2*	-7,9	*-7,2*	0,2	*2,3*	2,5
		0,0001		*<0.0001*		*0,0849*	
2007-2	13,5	*12,6*	9,7	*7,8*	-8,4	*-7,8*	-1,1
		<0.0001		*<0.0001*		*<0.0001*	
2008-1	-21,9	*-25,5*	4,6	*-0,5*	-24,5	*-20,5*	0,3
		<0.0001		*0,3039*		*<0.0001*	
2008-2	49,6	*85,2*	-16,2	*-10,5*	22,5	*-2,8*	-26,5
		<0.0001		*0,0013*		*0,0618*	
2009-1	28,6	*32,0*	36,7	*38,9*	-2,6	*0,7*	12,5
		<0.0001		*<0.0001*		*0,5921*	
2009-2	-17,9	*-19,8*	-18,5	*-18,2*	3,2	*0,7*	-1,9
		<0.0001		*<0.0001*		*0,2447*	
2010-1	1,0	*2,0*	2,6	*2,5*	0,0	*-0,6*	-7,5
		0,1490		*0,0484*		*0,5463*	
2010-2	-5,1	*-5,7*	-5,9	*-6,4*	0,0	*1,1*	4,4
		<0.0001		*0,0057*		*0,7030*	
Aggregate	98,84		52,65		59,55		3,85
Semestral Average	*6,18*		*3,29*		*3,72*		*0,24*

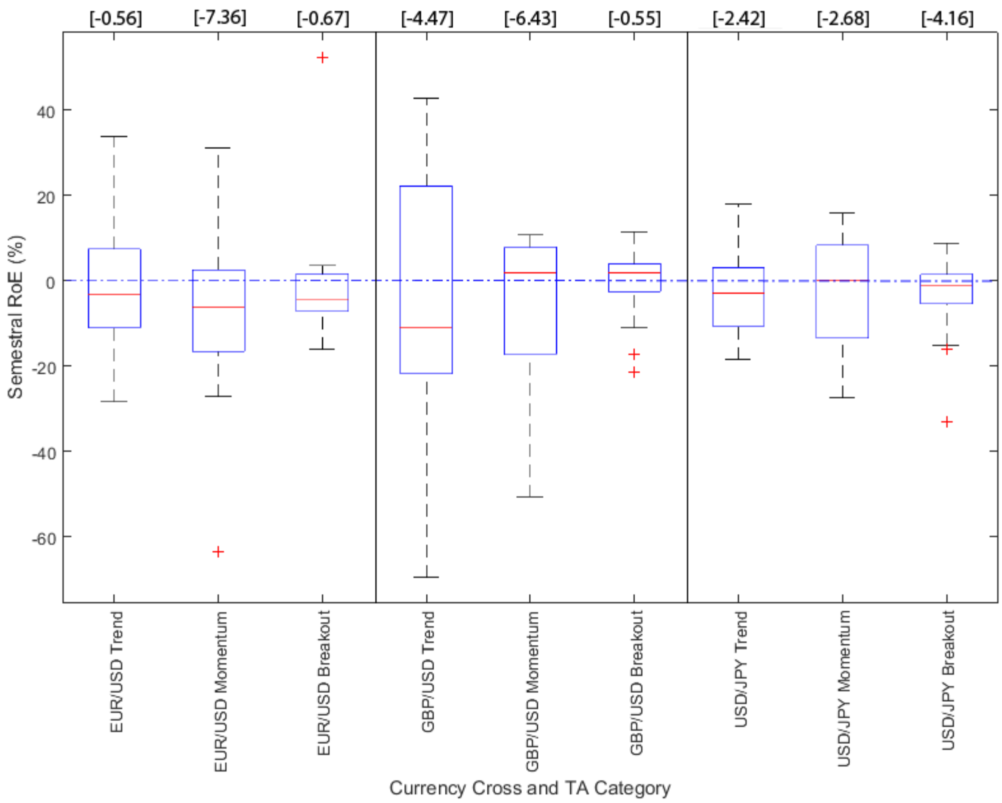
Table 4. (cont.) Return (%) of the optimized solutions, out-of-sample without costs.

RoA (%)	USD/JPY						
	Trend		Momentum		Breakout		Price
Semester	Mean	Median	Mean	Median	Mean	Median	Variation
2003-1	2,0	*0,6*	9,2	*8,6*	3,4	*3,7*	0,8
		0,1942		*<0.0001*		*<0.0001*	
2003-2	5,5	*5,0*	3,7	*3,4*	1,5	*2,6*	-10,5
		<0.0001		*<0.0001*		*0,0618*	
2004-1	11,4	*16,2*	5,9	*5,2*	11,5	*8,6*	0,9
		0,0001		*<0.0001*		*<0.0001*	
2004-2	-21,1	*-25,7*	-19,2	*-20,6*	-22,4	*-27,8*	-5,1
		<0.0001		*<0.0001*		*<0.0001*	
2005-1	1,4	*2,3*	-5,1	*-4,9*	-1,9	*-0,1*	7,9
		0,0703		*0,0003*		*0,0884*	
2005-2	8,3	*9,5*	-10,2	*-13,0*	-3,3	*-1,3*	6,3
		0,0002		*<0.0001*		*<0.0001*	
2006-1	-1,4	*-1,9*	5,7	*4,6*	4,6	*4,0*	-2,8
		0,4602		*<0.0001*		*0,0001*	
2006-2	-4,0	*-4,5*	0,4	*-2,2*	8,4	*10,5*	4,1
		0,0004		*0,5657*		*<0.0001*	
2007-1	0,9	*-0,5*	15,6	*14,5*	9,4	*9,0*	3,4
		0,7391		*<0.0001*		*<0.0001*	
2007-2	-9,4	*-9,7*	5,6	*4,3*	-12,4	*-12,9*	-9,3
		<0.0001		*0,0001*		*<0.0001*	
2008-1	4,0	*13,6*	4,5	*7,4*	-5,3	*-5,2*	-5,0
		0,2369		*0,0067*		*<0.0001*	
2008-2	31,2	*25,4*	4,2	*4,3*	25,8	*31,3*	-14,5
		<0.0001		*0,7906*		*<0.0001*	
2009-1	-19,1	*-18,3*	-12,8	*-14,1*	-10,5	*-9,5*	6,1
		<0.0001		*<0.0001*		*<0.0001*	
2009-2	1,7	*-0,6*	-1,6	*-0,4*	1,6	*0,9*	-3,3
		0,9269		*0,2408*		*0,0050*	
2010-1	-3,8	*-4,2*	5,5	*6,0*	2,4	*1,4*	-4,8
		0,0001		*<0.0001*		*0,2009*	
2010-2	-3,3	*-0,8*	-8,3	*-9,2*	-0,2	*-0,1*	-8,3
		0,0659		*<0.0001*		*0,9193*	
Aggregate	4,23		3,05		12,64		-34,37
Semestral Average	*0,26*		*0,19*		*0,79*		*-2,15*

Source: Author.

Figure 7 presents a box-plot of the TA *preeminent indicator* profitabilities for each period, market and category. Figure 8 shows another box-plot with the difference between the returns obtained by the GA optimization process and those of the preeminent indicators, i.e., the excess returns from the optimized solutions.

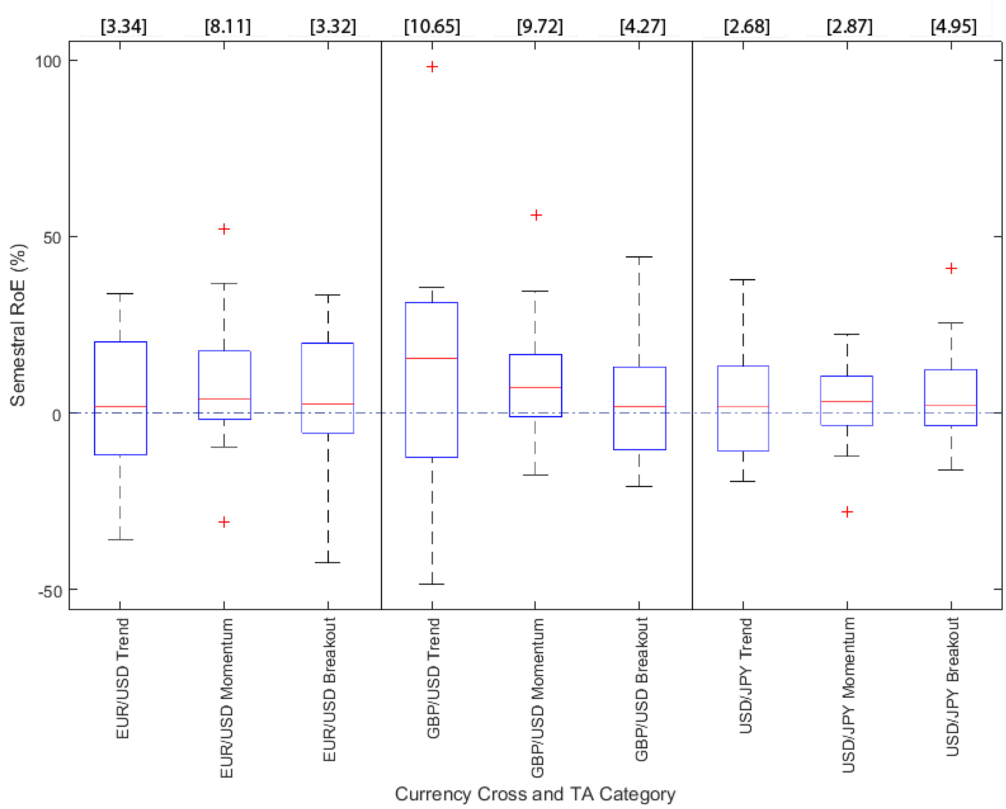
Figure 7. Return (%) of the preeminent TA indicator, out-of-sample without costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average RoAs (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 2.

It may also be seen in Figure 8 how excess returns are positive in the majority of cases. This shows our GA was able to fine-tune the parameters in a way that allowed it, in most cases, to beat the correspondent TA indicators using the parameter values commonly accepted by the trading industry. The consistency is also corroborated by the cumulative positive excess returns. All cumulative returns are above 40% and, in some cases (in the GBP/USD market for instance), they are much higher.

Figure 8. Excess Returns (%) of the optimized solutions with respect to the preeminent TA indicator of each period, out-of-sample without costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average excess returns (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 3.

A comparison of out-of-sample average returns with the average returns of a large set (10 000) of random solutions was also performed. These solutions were generated by randomly creating trading position signals out of the 3 possible ("+1", "-1" or "0" to designate *long*, *short* or *out-of-the-market* positions), each with equal probability of being selected. All the averages of random solutions are very close to zero and our out-of-sample results present significantly better cumulative figures. The results are shown in Table A 4.

Regarding the predictive power of TA indicators *per se*, the out-of-sample overall results without costs (Table 4), present somewhat attractive figures for the timespan of eight years, with the majority of periods showing positive outcomes in all markets. Return varies within markets and with the kind of applied strategy. The USD/JPY presents lower marks in almost all categories, suggesting fewer opportunities for sustained trading profits. This fact may also suggest a greater level of market development in terms of efficiency – a notion that is consistent with the AMH theorized by Andrew Lo. Among all three sorts of trading strategies, *trend* category seems to produce the best outcome.

Table 5. Out-of-sample return correlations between TA categories, by market.

		Trend	Momentum	Breakout	Price Var.
EUR/USD	Trend	1			
	Momentum	0.472	1		
	Breakout	-0.002	0.420	1	
	Price Var.	0.209	0.548	0.562	1
GBP/USD	Trend	1			
	Momentum	0.199	1		
	Breakout	0.604	-0.044	1	
	Price Var.	-0.332	0.612	-0.032	1
USD/JPY	Trend	1			
	Momentum	0.468	1		
	Breakout	0.834	0.612	1	
	Price Var.	-0.280	-0.168	-0.142	1

Source: Author.

Correlations between return of categories within the studied markets (Table 5) show a positive tendency – the only two negative correlations are very close to zero (EUR/USD *breakout* vs. *trend*: -0.002; GBP/USD *breakout* vs. *momentum*: -0.044); the others are mildly (e.g., GBP/USD *momentum* vs. *trend*: +0.199) to highly positive (see for instance USD/JPY *breakout* vs. *trend* or *breakout* vs. *momentum*: +0.834 and +0.612, respectively). This suggests optimized solutions for a given period in each market tend to generate outcomes with similar signal and overall proportion, with particular focus to the USD/JPY, no matter what TA category of indicators is used.

Also in Table 5, price variation vs TA category return correlations show in the EUR/USD and USD/JPY how returns are negatively correlated with USD price variation, i.e., RoAs in all categories tend to be positive when USD price decreases against the other currency. In the GBP/USD there is a strong positive correlation of GBP price variation and TA momentum category (equivalent to a strong *negative* correlation of *USD* price variation and momentum category), but this tendency does not hold regarding the other TA categories: the correlation varies from -0.332 with *trend* to +0.612 with *momentum* and almost zero correlation with *breakout* strategy categories.

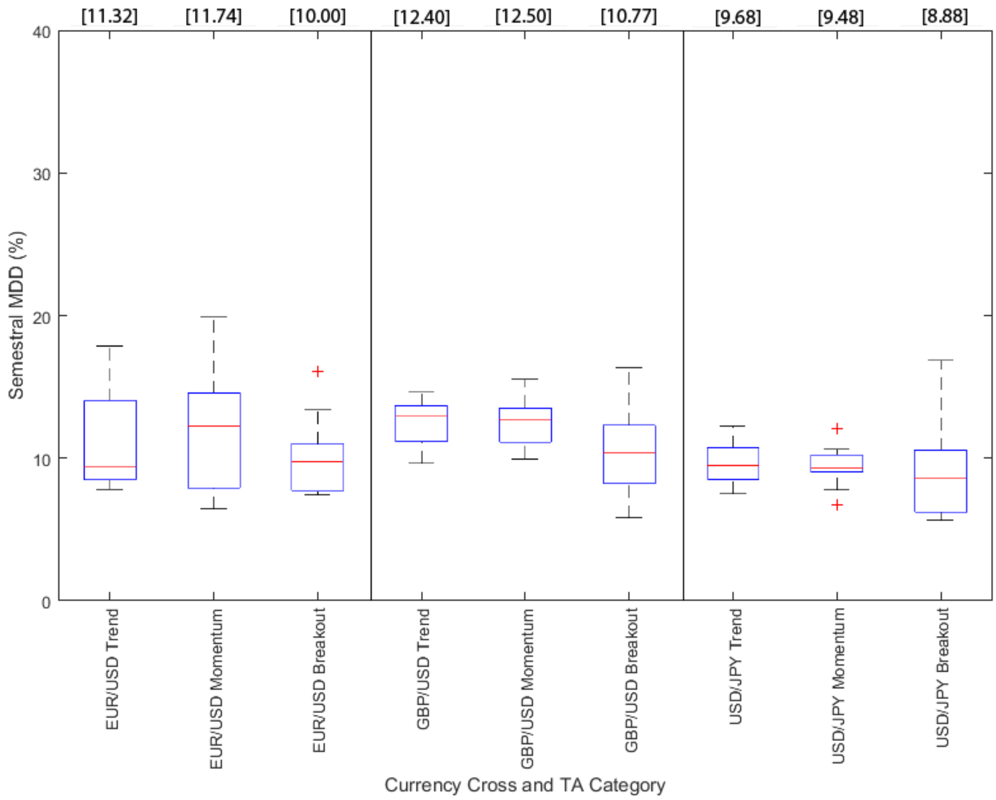
Table 6. Out-of-sample return correlations between markets, split by TA category.

		EUR/USD	GBP/USD	USD/JPY
	EUR/USD	1		
Trend	GBP/USD	-0.447	1	
	USD/JPY	0.230	0.151	1
	EUR/USD	1		
Momentum	GBP/USD	0.637	1	
	USD/JPY	0.093	-0.305	1
	EUR/USD	1		
Breakout	GBP/USD	0.593	1	
	USD/JPY	-0.349	0.181	1
	EUR/USD	1		
Price Var.	GBP/USD	0.689	1	
	USD/JPY	-0.182	0.346	1

Source: Author.

The correlation between markets intends to assess the consistency of TA trading strategy's categories throughout all studied markets, i.e., whenever a TA category works (does not work) in a period in a specific market, it should (should not) work in the other markets. We can see in Table 6 there is not consistency in profitabilities of TA strategies throughout the studied markets for each TA category, which in turn may imply that markets seem to possess, at any given period, distinct inherent characteristics preventing them from reacting evenly to similar trading strategies. This might be a consequence of structural market divergences or circumstancial differences following a process of change (different stages of market efficiency).

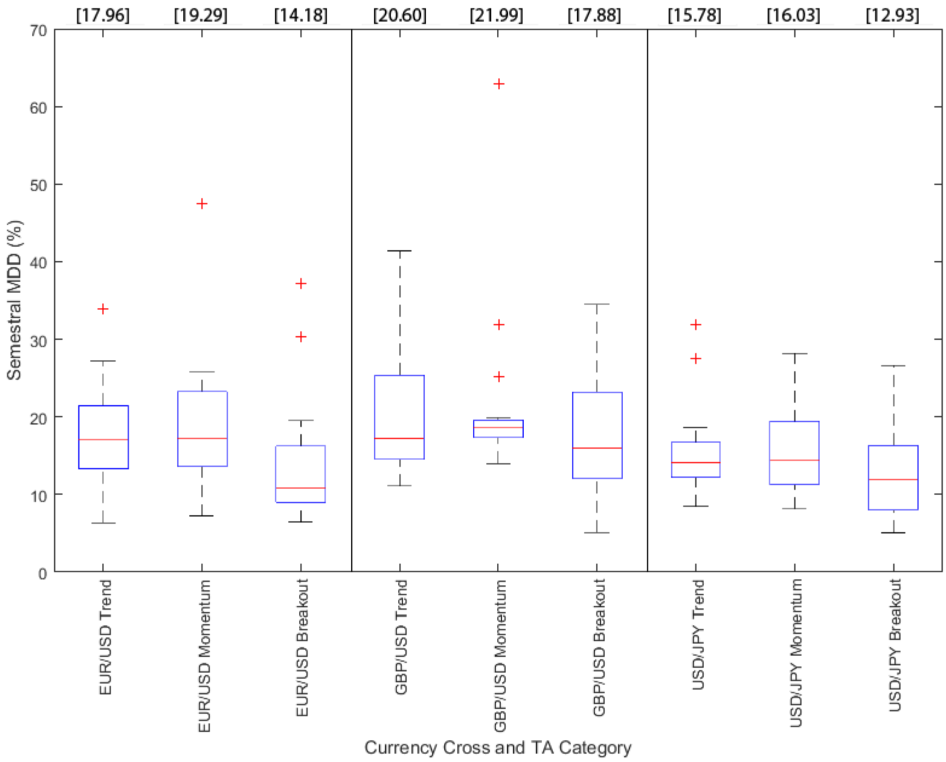
Figure 9. MDD (%) for the optimized solutions, in-sample without costs, from 1st semester 2001 to the 2nd semester 2008. In brackets are presented the average MDDs (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 5.

Regarding risk, measured by MDD (Figure 9 and Figure 10), we may see how in-sample results seem to be much more concentrated around the median. Out-of-sample MDD values are more widespread and present a considerable greater number of outliers than in-sample results. The divergence between in-sample and out-of-sample risk levels shown by *momentum* and *trend* systems versus *breakout* systems increases in out-of-sample *MDD*. This might suggest *breakout* strategies (which present less risk) possess inherent risk-mitigating characteristics that may rely, for instance, in their signal generator ability to promptly react to price change, and with that avoid more effectively unfavorable market moves. When put into a market's perspective, we may acknowledge the USD/JPY shows better *MDD* percentages, qualifying as the less risky market. The GBP/USD shows signs of being the riskiest of all studied markets, but it should not be forgotten this is the market that allows greatest return, so we may detect a direct relation between risk and return applied to the use of TA.

Figure 10. MDD (%) for the optimized solutions, out-of-sample without costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average MDDs (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 6.

4.2.2 Trading Strategy Return Considering Spreads and Rollover Costs

Table 7. Return (%) of the optimized solutions, out-of-sample, with costs.

RoA (%)	EUR/USD					
	Trend		Momentum		Breakout	
Semester	Mean	Median	Mean	Median	Mean	Median
2003-1	14,2	*13,6*	15,4	*17,5*	15,1	*17,7*
		<0.0001		*<0.0001*		*<0.0001*
2003-2	0,2	*-3,5*	12,1	*10,8*	24,0	*26,2*
		0,5463		*<0.0001*		*<0.0001*
2004-1	19,0	*28,6*	-2,7	*-5,9*	-33,9	*-34,0*
		<0.0001		*0,6191*		*<0.0001*
2004-2	-3,3	*-2,6*	-2,7	*-8,7*	27,0	*27,5*
		0,0038		*0,2078*		*<0.0001*
2005-1	-18,7	*-18,5*	-18,3	*-17,2*	-18,9	*-19,7*
		<0.0001		*<0.0001*		*<0.0001*
2005-2	1,0	*4,0*	3,1	*-0,0*	-1,4	*0,1*
		0,2184		*0,0422*		*0,8431*
2006-1	-8,3	*-8,1*	-4,4	*-2,9*	-5,2	*-6,7*
		<0.0001		*0,0160*		*<0.0001*
2006-2	-7,3	*-5,4*	0,9	*3,0*	-2,4	*-2,0*
		<0.0001		*0,6191*		*0,0003*
2007-1	-1,2	*-2,1*	-1,8	*-1,6*	-4,1	*-4,5*
		0,0248		*0,0074*		*<0.0001*
2007-2	-15,3	*-16,6*	-2,7	*-0,8*	12,2	*12,8*
		<0.0001		*0,0959*		*<0.0001*
2008-1	10,0	*8,1*	2,9	*-5,2*	6,0	*7,4*
		<0.0001		*0,7030*		*0,0002*
2008-2	-6,1	*-17,5*	-13,3	*-16,3*	8,0	*5,1*
		0,0998		*0,0116*		*0,0010*
2009-1	-7,4	*-6,4*	5,9	*6,6*	-7,5	*-3,3*
		0,0038		*0,0027*		*0,0003*
2009-2	-13,3	*-12,7*	-16,3	*-16,9*	-7,0	*-5,5*
		<0.0001		*<0.0001*		*<0.0001*
2010-1	3,5	*12,6*	-7,0	*-10,7*	-3,3	*-5,9*
		0,0812		*0,0093*		*0,0002*
2010-2	13,8	*17,2*	-7,2	*-9,2*	7,1	*8,3*
		<0.0001		*<0.0001*		*0,0037*
Aggregate	-19,19		-36,06		15,74	
Semestral Average	*-1,20*		*-2,25*		*0,98*	

Table 7. (cont.) Return (%) of the optimized solutions, out-of-sample, with costs.

RoA (%)	GBP/USD					
	Trend		Momentum		Breakout	
Semester	Mean	Median	Mean	Median	Mean	Median
2003-1	-2,3	*-1,3*	5,2	*4,2*	10,2	*9,6*
		0,0358		*0,0004*		*<0.0001*
2003-2	21,5	*22,0*	7,7	*11,2*	46,9	*48,0*
		<0.0001		*0,0012*		*<0.0001*
2004-1	-32,8	*-37,4*	-0,5	*-3,6*	-19,8	*-17,8*
		<0.0001		*0,7832*		*<0.0001*
2004-2	3,6	*3,8*	2,4	*5,4*	12,8	*15,0*
		0,0242		*0,3823*		*<0.0001*
2005-1	7,5	*8,8*	-2,4	*-3,1*	-7,2	*-11,5*
		0,0004		*0,1236*		*0,0001*
2005-2	20,9	*24,3*	6,5	*7,0*	2,2	*2,3*
		<0.0001		*0,0028*		*0,0009*
2006-1	-6,5	*-6,9*	5,4	*10,0*	-1,9	*-2,1*
		0,0013		*0,0133*		*0,0107*
2006-2	14,3	*18,6*	3,9	*3,5*	11,8	*12,3*
		<0.0001		*0,0088*		*<0.0001*
2007-1	3,0	*5,6*	-10,2	*-9,5*	-1,7	*0,6*
		0,0004		*<0.0001*		*0,9654*
2007-2	10,8	*9,1*	7,4	*5,8*	-10,2	*-9,6*
		<0.0001		*0,0002*		*<0.0001*
2008-1	-25,1	*-28,8*	2,2	*-3,3*	-27,2	*-22,6*
		<0.0001		*0,7246*		*<0.0001*
2008-2	46,7	*82,4*	-18,5	*-12,4*	20,6	*-3,6*
		<0.0001		*0,0006*		*0,0798*
2009-1	25,4	*28,9*	33,6	*36,2*	-5,1	*-1,6*
		<0.0001		*<0.0001*		*0,2113*
2009-2	-21,4	*-23,5*	-22,3	*-22,2*	0,7	*-1,3*
		<0.0001		*<0.0001*		*0,7832*
2010-1	-2,7	*-2,0*	-0,8	*-1,4*	-1,5	*-1,7*
		0,1689		*0,5921*		*0,0254*
2010-2	-10,7	*-11,3*	-10,7	*-10,5*	-3,9	*-3,1*
		<0.0001		*<0.0001*		*0,0050*
Aggregate	52,16		8,88		26,71	
Semestral Average	*3,26*		*0,56*		*1,67*	

Table 7. (cont.) Return (%) of the optimized solutions, out-of-sample, with costs.

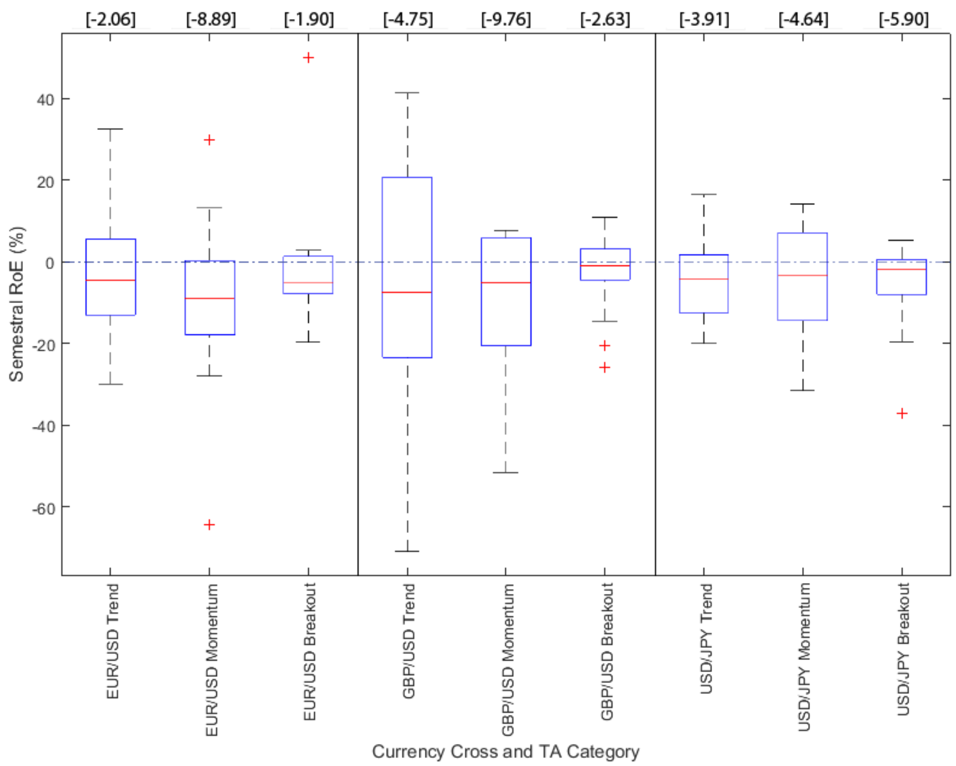
RoA (%)	USD/JPY					
	Trend		Momentum		Breakout	
Semester	Mean	Median	Mean	Median	Mean	Median
2003-1	-0,9	*-4,2*	6,9	*6,1*	-0,0	*1,1*
		0,5789		*<0.0001*		*0,2732*
2003-2	2,6	*2,4*	0,3	*0,2*	-1,0	*0,6*
		0,0133		*0,7758*		*0,2690*
2004-1	8,9	*14,2*	2,8	*2,1*	9,5	*6,8*
		0,0005		*0,0384*		*<0.0001*
2004-2	-24,3	*-28,8*	-22,5	*-23,8*	-25,4	*-31,5*
		<0.0001		*<0.0001*		*<0.0001*
2005-1	-0,8	*0,7*	-7,4	*-6,6*	-3,4	*-1,9*
		0,9269		*<0.0001*		*<0.0001*
2005-2	5,4	*6,9*	-13,5	*-17,0*	-4,9	*-2,8*
		0,0047		*<0.0001*		*<0.0001*
2006-1	-4,6	*-5,2*	2,8	*2,0*	3,2	*3,1*
		0,0104		*0,0814*		*0,0052*
2006-2	-6,9	*-6,8*	-2,0	*-4,8*	6,9	*8,9*
		<0.0001		*0,0349*		*<0.0001*
2007-1	-3,8	*-5,8*	13,0	*11,3*	7,7	*7,3*
		0,0007		*<0.0001*		*<0.0001*
2007-2	-13,6	*-14,1*	3,1	*1,4*	-14,6	*-15,2*
		<0.0001		*0,0393*		*<0.0001*
2008-1	-1,2	*8,7*	1,3	*5,4*	-7,1	*-7,0*
		0,5722		*0,4312*		*<0.0001*
2008-2	25,7	*23,5*	1,4	*2,7*	24,6	*30,2*
		<0.0001		*0,8431*		*<0.0001*
2009-1	-24,2	*-23,7*	-17,7	*-19,5*	-12,5	*-11,2*
		<0.0001		*<0.0001*		*<0.0001*
2009-2	-2,2	*-4,8*	-3,6	*-3,0*	-0,1	*-0,7*
		0,0462		*0,0025*		*0,6744*
2010-1	-6,7	*-7,8*	3,5	*4,0*	0,7	*-0,2*
		<0.0001		*0,0002*		*0,9654*
2010-2	-7,8	*-7,9*	-11,1	*-11,3*	-2,8	*-2,7*
		<0.0001		*<0.0001*		*0,0765*
Aggregate	-54,29		-42,64		-19,13	
Semestral Average	*-3,39*		*-2,67*		*-1,20*	

Source: Author.

In the simulation with costs we have considered spreads of 2 pips and rollover costs of 0.2 pips for the EUR/USD and GBP/USD markets; 200 pips and 20 pips respectively for the USD/JPY market. These spreads are to be taken on a *per turn* basis, i.e., they are in reference to a single market's action of buying or selling. Rollover costs are added (subtracted) to (from) unfavorable (favorable) interest rate differentials.

With the inclusion of reasonable trading costs (Table 7), we notice how apparently attractive out-of-sample profits simply disappear. The medians of the observed results remain, in general, statistically different from zero, but the aggregate average returns suffer deeply. An exception seems to be the GBP/USD market, with the use of *trend* strategies, which still presents some interesting results. The outcomes suggest these markets might be, considering more realistic assumptions, relatively efficient.

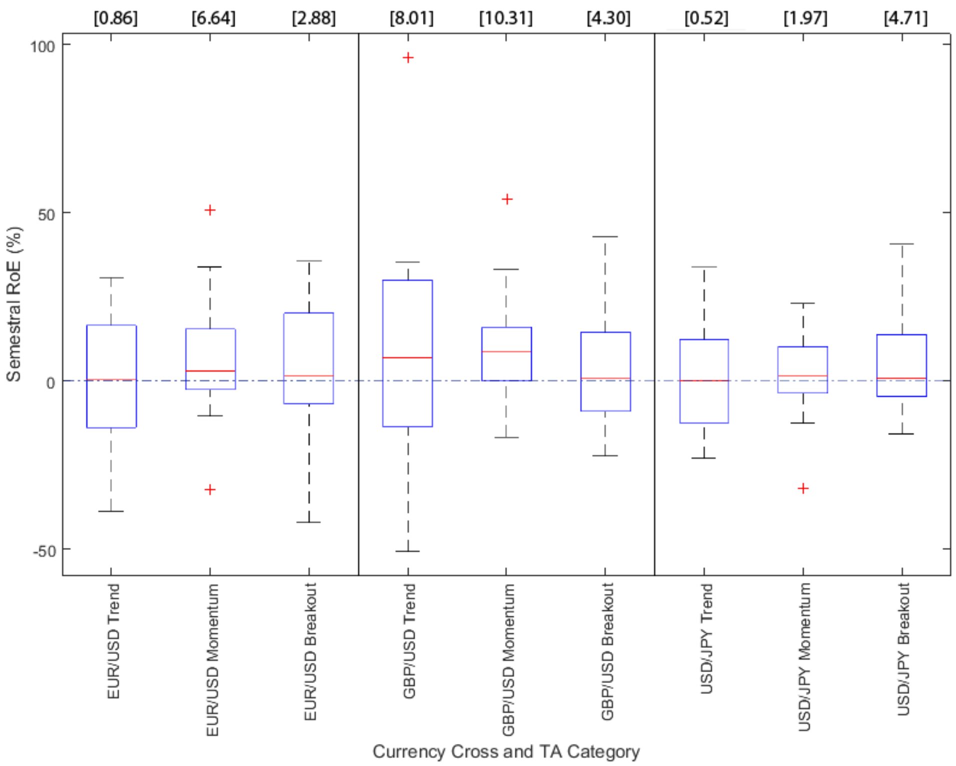
Figure 11. Return (%) of the preeminent TA indicator in each period, out-of-sample, with costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average RoAs (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 7.

We can see in Figure 11 and Figure 12 how the returns of optimized solutions remain superior compared to those of the correspondent *preeminent indicators* applying the industry parameters. In spite of that, the inclusion of trading costs in our analysis suggests a decline of the excess returns of the strategies provided by the GA compared to figures without costs, with the exception of *momentum* and *breakout* techniques in the GBP/USD market, where there is a small improvement in RoAs (compare Figure 8 with Figure 12).

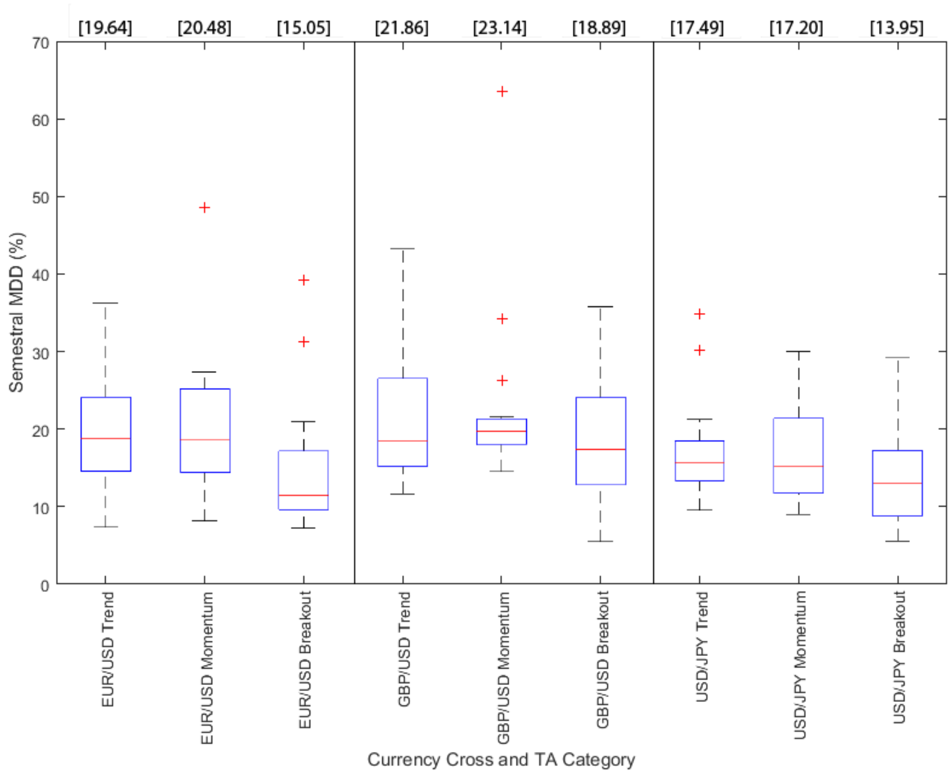
Figure 12. Excess Returns of the optimized solutions compared to the preeminent TA indicator of each period, out-of-sample, with costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average excess returns (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 8.

Another conclusion we may draw refers to the inclusion of trading costs – *breakout* strategies demonstrate more resiliency (their outcomes suffer less). This assessment is reinforced by the lower observed risk levels stressed by the *MDD* values, comparing Figure 10 with Figure 13.

Figure 13. MDD (%) for the optimized solutions, out-of-sample, with costs, from 1st semester 2003 to the 2nd semester 2010. In brackets are presented the average MDDs (%) for each combination currency cross / TA category.



Source: Author. The related results are presented in Table A 9.

Momentum and *trend* strategies' risk levels remain close to each other. *Breakout* systems seem to produce consistently smaller MDD values. Nevertheless, the inclusion of trading costs only increases the MDD by about 1.5 to 2 percentage points, a change that does not acutely affect average risk levels.

4.3 Conclusion

The original GA proposed in this chapter presents a good advantage in comparison to the most commonly used GAs: a wide set of solutions in the search process with the possibility of genetic material recovery from shadow indicators allows greater diversity of inherited genetic material and prevents a precocious convergence in the optimization process. The use of a large number of in-sample/out-of-sample timespans with reference to the overall trading period minimizes the likelihood of obtaining results misled by data mining. For the aggregate period of 2003-2010, the out-of-sample results obtained from the optimized solutions outperform substantially the corresponding most frequently used TA indicators with traditional industry parameters. Results for the Forex crosses vary widely within the considered markets and the TA trading strategy categories – which suggests not all kinds of trading strategies present the same predictive power; and not all markets perform equally or show the same inner characteristics. This may be a symptom of the existence of different stages of efficiency development, an idea compatible with Andrew Lo's AMH. The inclusion of trading costs in Forex trading changes significantly the landscape in terms of average return – the majority of results turns negative, and the existence of profitable trading opportunities seems elusive when considering more realistic assumptions. This suggests markets may be more efficient than return without costs implied, an observation showing strong evidence in favour of the EMH for the three major Forex markets. There is also a negative correlation between USD price variation and TA categories RoAs, with a few exceptions in the GBP/USD market.

The somewhat interesting return figures and the statistical significance of attained results do not provide the conditions or sustenance to assert the validity of TA as an effective isolated tool in trading activities within the three major Forex markets, particularly when considering more realistic terms. This may be seen as an argument in favour of market efficiency. (Shmilovici et al., 2009) also tested efficiency in several Forex markets concluding in favour of market efficiency, particularly when dealing with intraday data. Also (Ozturk et al., 2016) tested the EUR/USD and GBP/USD with crossover, Bollinger Bands and divergence TA indicators, reaching similar results to the presented in this chapter – limited positive results and profits in 60% of the number of trades, but without trading costs. Our findings are consistent with (Kuang et al., 2014) and (Fang et al., 2014), that conclude there are no strong evidences of TA indicator's predictive power. We may also see in (Yu et al., 2013) how the inclusion of trading costs affects return, turning profits into losses, just as shown in our empirical work.

5 Portfolio Optimization with Multi-Objective Evolutionary Algorithms[5]

Portfolio optimization has been one of the most hectic areas in the field of Finance. All major Financial Institutions deal with this kind of problem, usually focused in two distinct objectives: how to maximize returns and simultaneously minimize associated risk. Therefore it is no surprise to see how much the academic literature centres its analysis on this topic, and how so many methods of optimization spring from Universities, Conferences and the global Financial Industry.

One of the most successful techniques applied to portfolio optimization has been the use of multi-objective evolutionary algorithms. These metaheuristics are particularly adequate to this kind of optimization problem, since they are able to produce Pareto fronts representing the existing trade-offs between return and risk. (Metaxiotis and Liagkouras, 2012) present an interesting review of the most important MOEAs applied specifically to portfolio optimization.

In this chapter we propose a portfolio model, identifying the adopted return and risk measures, the MOEAs selected for performance comparison, the TA indicators used in the simulation and the data and methodology applied in the developed work. Presentation of empirical results and their discussion will ensue.

5.1 Mean-Semivariance Portfolio Optimization

As explained is Chapter 2, traditionally portfolio optimization problems use variance (or standard deviation) as a measure of risk (Subsection 2.3.1.1). Although commonly

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[5] Chapter 5 presents the empirical work of this Thesis published in the article: Macedo, L.L., Godinho, P. and Alves, M.J. (2017), 'Mean-semivariance portfolio optimization with multiobjective evolutionary algorithms and technical analysis rules', *Expert Systems with Applications*, Elsevier Ltd, Vol. 79, pp. 33–43.

accepted, this measure is not the most appropriate for assessing risk, since it considers equally adverse deviations (below average) as well as favourable ones (above average). An adequate alternative, since investors are concerned with adverse variations, is the mean-semivariance framework for portfolio optimization, explained in detail in Subsection 2.3.1.2.

The difficulty with the computation of semivariance resides on the endogenous nature of the portfolio semicovariance matrix, which depends on the weights given to each asset, i.e., a change in weights affects the periods in which the portfolio underperforms the benchmark. When using MOEAs, the difficulty of computing the semivariance is overcome by nature, since the computation is made by iterations and, in each iteration, the weights of the portfolio(s) are known *a priori* because they are generated by the algorithm. Knowing the weights, we may compute the portfolio average return (used as the benchmark) and consequently determine in which periods the portfolio underperforms the benchmark. Therefore, in this respect, MOEAs seem to be adequate tools for portfolio optimization under a mean-semivariance framework.

The multiobjective portfolio optimization problem that we consider can be formulated as follows:

$$\max\ E(R_P) = \sum_{i=1}^{n} w_i E(R_i) = \sum_{t=1}^{T}\sum_{i=1}^{n} w_i \frac{R_{it}}{T}$$

$$\min\ S(R_P) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j S_{PC}$$

Subject to:

$$\sum_{i=1}^{n} w_i = 1$$

$$S_{PC} = E(\min(0, R_P - C)^2) = \frac{1}{T}\sum_{t=1}^{T}[\min(0, R_{Pt} - C)]^2$$

where P stands for portfolio, w_i is the weight of asset i in the portfolio, R_P is the portfolio overall return, R_{it} is the return of asset i in period t, n is the number of assets, S is the semivariance (as defined by Markowitz) and T is the number of periods considered in the analysis. In this work, the benchmark is $C = E(R_P)$ and the w_is represent *constant weights* without taking into consideration rebalancing costs. These constant weights are adopted in all trading strategies. For the purpose of simplicity, when we mention Buy-and-Hold (B&H) hereafter it is meant as holding the position with constant weights, i.e., after each day there is a rebalancing process such that all changes in the weights – due to daily profits and losses of the portfolio assets – are neutralized and the weights remain as initially (unaltered or *constant*).

5.2 Multiobjective Evolutionary Algorithms Applied to Portfolio Optimization

Both NSGA II and SPEA 2 have been considered as two of the best currently available MOEAs and they are often used in comparative studies within multiobjective problems, e.g. (Mishra et al., 2010), (Anagnostopoulos and Mamanis, 2011), (Lwin et al., 2014). One particular area in which these algorithms have been successfully applied is stock portfolio optimization. Considering two objective functions, *risk* and *return*, the problem of portfolio optimization possesses the very characteristics MOEAs are tailored for, especially when taking into consideration some more realistic assumptions, like cardinality (maximum number of admissible stocks in the portfolio), round-lot constraints (stocks/assets often trade in standard units, called the lots, composed by a predetermined number of shares, usually but not necessarily 100 – (Skolpadungket et al., 2007), or the incorporation of trading costs. In this kind of problems, both SPEA 2 and NSGA II have been references in the literature, and they usually present consistently better results than alternative techniques – (Duran et al., 2009), (Mishra et al., 2009), (Metaxiotis and Liagkouras, 2012). We will use these algorithms implemented in *Matlab Release 2016a*.

5.3 Technical Analysis Indicators and Trading Strategies

This empirical work uses 3 different TA indicators (MACD, RSI and BB), which give origin to four distinct associated trading strategies applied to stock market portfolio optimization:

- *Moving Average Convergence/Divergence* (MACD),
- *Relative Strength Index* (RSI),
- *Conventional Bollinger Bands* (BB) and
- *Contrarian Bollinger Bands* (CBB)

all to be compared with the B&H[6] scenario, as a reference for normal profits.

The use of TA in trading has been viewed with scepticism from the Academic world regarding the attainment of above normal returns. Several studies present strong evidence in favour of TA, either based on indicator analysis, such as (Brock et al., 1992) and (Pinto et al., 2015), or based on chart analysis, like (Lo et al., 2000). However, other studies, such as (Allen and Karjalainen, 1999) and (Neely, 2003), acknowledge little value in TA-based strategies; this is particularly so when some more realistic assumptions - like the existence of transaction costs – are considered (Macedo et al., 2016). In this context, it is important to gather further empirical evidence for or against the validity of TA as an effective tool to exploit market inefficiencies, namely in stock markets. As mentioned, we decided to test the optimization algorithms under four common TA strategies and their respective trading rules:

- MACD (Subsection 2.2.2.2.5) – The common technical strategy associated with this indicator states that we should have a long[7] position when the *MACD* value rises above the *Signal line* and be short[8] in the market when the opposite situation occurs. In our case, since we are considering stocks preventing the usage of short positions,

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[6] As mentioned in the previous subsection, in Chapter 5 "B&H" does not represent a pure B&H strategy but rather a B&H with rebalancing such that the daily profits and losses are redistributed in order to maintain constant weights.

[7] A *long* position is a net buying position in the market.

[8] A *short* position is a net selling position.

we will be long if *MACD>Signal* and be out of the market if *MACD≤Signal*. We will not assume short positions.

- RSI (Subsection 2.2.2.2.6) – In our specific case, and since we are dealing with stock portfolios, the following rule of trading will be adopted: we will be long in the market if RSI>30% and the price is on an upward move (price in the present day is larger than price in the previous day), and we will be out of the market otherwise.
- BB (Subsection 2.2.2.2.9) – In the conventional case, a long position will be taken in day t if price in $t-1$ is simultaneously above $SMA_{t-1}^{n_{SMABB}} - k_{BB} \cdot \sigma$ and below $SMA_{t-1}^{n_{SMABB}}$ to avoid false triggering. If these conditions are not met, we will be out of the market.
- CBB (Subsection 2.2.2.2.9) – In the *CBB* case, we will assume a long position in moment t if price in $t-1$ is below $SMA_{t-1}^{n_{SMABB}} + k_{BB} \cdot \sigma$.

These are commonly adopted trading strategies in the industry, but other valid strategies could be used. For a more detailed study of these indicators, see for instance (Kirkpatrick and Dahlquist, 2011) or (Kaufman, 2013).

5.4 Data and Methodology

This study uses data from several Stock Exchanges provided by *DataStream* (now *Eikon*), a part of Thompson-Reuters corporation. We have aggregated stocks of countries with economic similarities (dimension, perceived efficiency and liquidity) to form four different markets: *market Tier 1* – stocks of countries in development (Argentina, Brazil and South Africa), *market Tier 2* – stocks of peripheral developed countries (Greece, Portugal and Belgium), *market Tier 3* – stocks of fully developed countries (UK, Australia and The Netherlands), and finally, stocks of the *US market*. This study uses daily adjusted closing price data (in EUR) of stocks from the following countries (with the number of corporations within brackets): Argentina [15], Brazil [17], South Africa [13], Greece [15], Portugal [15], Belgium [15], UK [15], Australia [15], The Netherlands [15] and the US [49]. Prices span a period of almost 16 years (15 ¾ years, from 2000-01-03 to 2015-10-01) for all stocks. Corporations were selected by order of appearance in *DataStream* queries and according to the required condition of data availability (with data from year 2000 on). Lists of all corporations/assets are presented in the Appendix (Tables A9 to A12).

Data is organized in matrices of dimension $T * n$, being T the total time span duration and n the number of assets of each market to optimize in a portfolio. In our specific case, $T = 4109$ and $n = 45$ for *markets Tiers* 1 to 3 and $n = 49$ for the *US market*. The rates of return will be considered as continuous in time and compound, and therefore they will follow the model of continuous compound rates as formulated below:

$$P_{it} = P_{i(t-1)} \cdot exp^{R_{it}}$$

where

P_{it} is the price of asset i at day t;

R_{it} is the rate of return of asset i at day t.

The rate of return at day t, for any asset i, is determined by:

$$R_{it} = \ln(P_{it}) - \ln\left(P_{i(t-1)}\right)$$

This expression will be applied to all price matrices in order to obtain rates of return of all assets in all considered markets, generating four new matrices of $(T - 1) * n$, starting at day 2000-01-04. A new column (4108*1) of daily returns of 0.0000766551 is added to each matrix. This column represents the returns of a risk-free asset (*Cash*), which corresponds to a yearly rate of return of 0.02 (or 2%). The final matrices will have dimensions 4108*50 for the *US market* and 4108*46 for *markets Tiers* 1 to 3. These raw matrices represent the rates of return in a pure B&H scenario for each considered market.

Applying the rules mentioned in 5.3 to a market matrix we obtain its correspondent matrix of binary elements (with "1" for long positions and "0" for out-of-the-market) of each trading strategy and market. To these matrices is added a new column of "1"s for the Cash asset, which does not vary according to TA trading rules. By executing an element-wise multiplication of these binary matrices with the previous B&H matrix of returns we generate a new matrix for each trading strategy for the market at question, i.e., matrices *MACD*, *RSI*, *BB* and *CBB* for each market. At the end of this procedure we have 20 matrices of returns (4 markets times 5 trading strategies, including the B&H scenario).

These 20 return matrices all include trading costs – daily returns are deducted of their respective trading costs. Currently, with the emergence of Internet-based trading apps and brokerage firms' fierce competition, trading costs may assume infinitesimal percentages. For the purpose of this work, we consider a percentage trading cost over trading amounts of 0.05% per round, or 0.10% per round-turn. A round-turn is a complete cycle of buying and selling an asset. Each of these return matrices is then split into 2 different matrices, one for IS data (first 10 years, 2000-01-04 to 2009-12-31) and another for OOS data (last 5 ¾ years, from 2010-01-02 to 2015-10-01).

In the optimization process using NSGA-II (see Subsection 3.3.1) and SPEA-2 (details in Subsection 3.3.2), the genes that form a single chromosome (*portfolio solution*) represent the asset weights in the portfolio. A chromosome is therefore a vector $(w_1, w_2, \ldots, w_n)$, with $w_i \in \mathbb{R}$, $w_i \geq 0$, $i = 1, 2, \ldots, n$, and $\sum_{i=1}^{n} w_i = 1$; n = 46 in *Markets Tier 1* to *3* and n = 50 in the *US market*. The sum of all weights $w_i, i = 1, \ldots, n$ is normalized to 1 after crossover and mutation and before fitness evaluation, i.e. $w_i = w_i'/W$, $= 1, \ldots, n$, $W = \sum_{i=1}^{n} w_i'$, where w_i' represents the weight of the i^{th} asset after the algorithm operations and w_i is the normalized weight. It is important to reinforce that these are *constant weights* and rebalancing costs are not taken into account.

The adopted parameters for the algorithms NSGA-II and SPEA-2 are the following:

- Initial population: $n_{pop} = 100$ chromosomes;
- Maximum number of elements in the archive of *non-dominated* solutions: $n_{arch} = 100$ chromosomes; this archive is the outcome of the algorithm (the *optimized solutions*)
- Maximum number of iterations: $n_{iter} = 300$
- Crossover probability: $p_c = 0.9$
- Mutation probability: $p_m = 0.02$

The experiment setting follows the steps below:

1. For all markets (*Tiers 1, 2, 3* and *US*) and strategies (B&H, CBB, BB, MACD and RSI), do:

a. Optimize populations of 100 randomly generated solutions by applying the multiobjective algorithm (NSGA-II / SPEA-2) to IS data; perform 25 independent runs, each run producing a set (front) of at most 100 *non-dominated* solutions (portfolios);
b. Compute the *hypervolume* (HV) measure (Zitzler and Thiele, 1999) for the set of non-dominated solutions and select the set with *median* HV value among the 25 independent runs;
c. Use the weights of the solutions in the selected population and apply them to OOS data in order to generate the respective OOS population of solutions;

2. Compare the optimized populations (one per market) for the 5 strategies, for both the IS and the OOS data.

The outcome of the optimization process using each MOEA in the IS data is a set of 25 fronts of 100 *optimized solutions* per each combination *market-strategy*. Comparing all fronts would be unpractical Therefore, a criterion was used to select a representative front for each combination: we computed the *hypervolume* (HV) value of each front (Zitzler and Thiele, 1999) and the front with the median HV value was selected as the representative of the MOEA optimization outcome for each combination. HV measures the volume of the multidimensional region that is dominated by the set of non-dominated solutions that is being assessed. This quality indicator can assess both convergence and diversity of the non-dominated solutions, and larger values of HV indicate better approximation sets. We applied the code of (Fonseca et al., 2006) to compute the HV values.

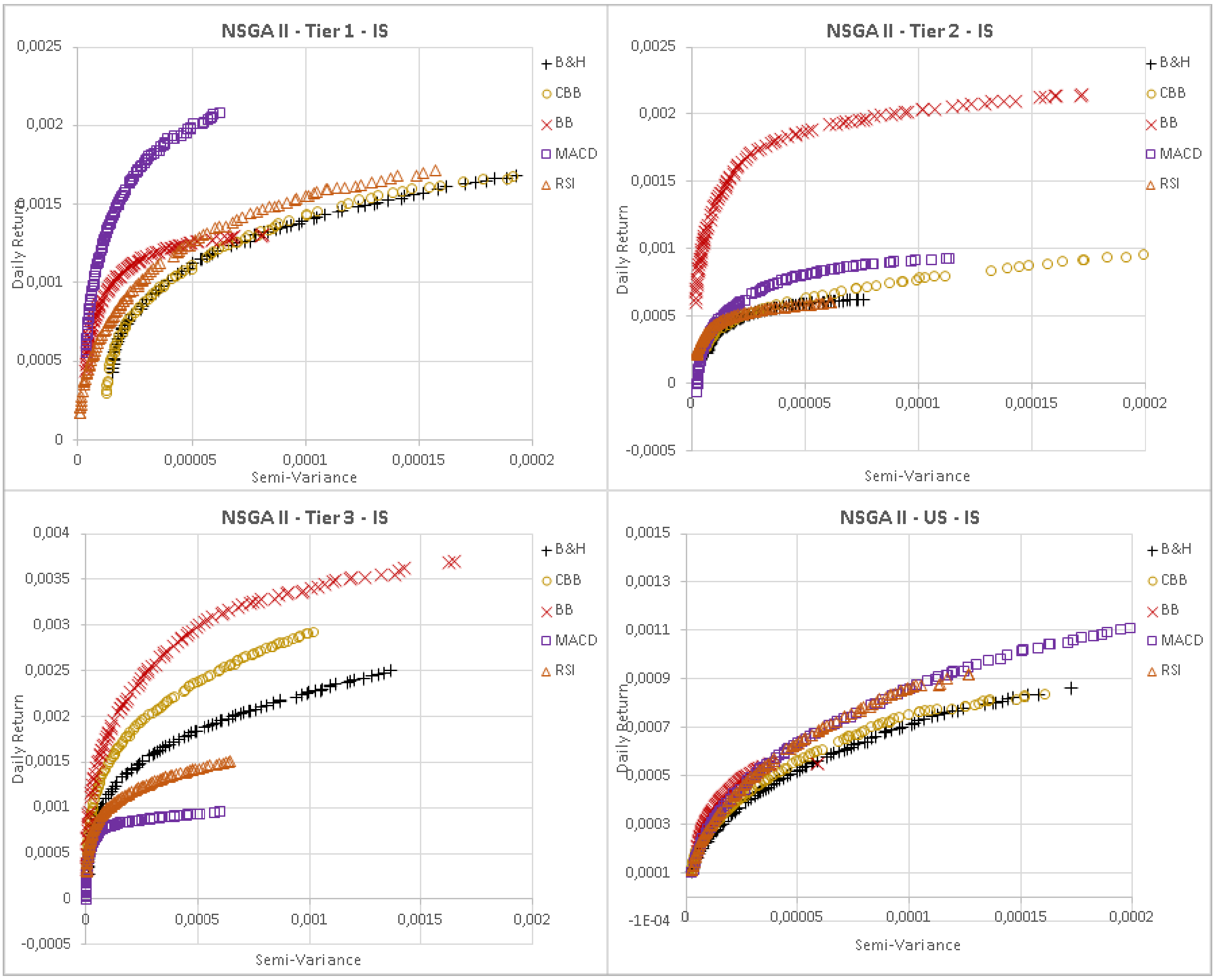
5.5 Empirical Results and Discussion

The main goals of this empirical work are to assess:

1) which of the algorithms presents a better performance both *in* and *out-of-sample* (IS vs. OOS);
2) what is the impact of the chosen TA strategies on the non-dominated frontiers;

Figure 14 and Figure 18 present the IS and OOS outcomes (*risk-return* frontiers of non-dominated portfolios with median HV value), respectively, for all five strategies and for each market, resulting from the application of the NSGA II algorithm. Figure 15 and Figure 19 show the IS and OOS outcomes resulting from the SPEA 2 algorithm. In-sample frontiers result from the optimization process itself; out-of-sample results depict in-sample solutions applied to out-of-sample data.

Figure 14. NSGA II frontiers of non-dominated solutions, in-sample, under 5 strategies (B&H, CBB, BB, MACD and RSI), with costs, regarding all 4 aggregate markets (Tiers 1, 2, 3 and US).



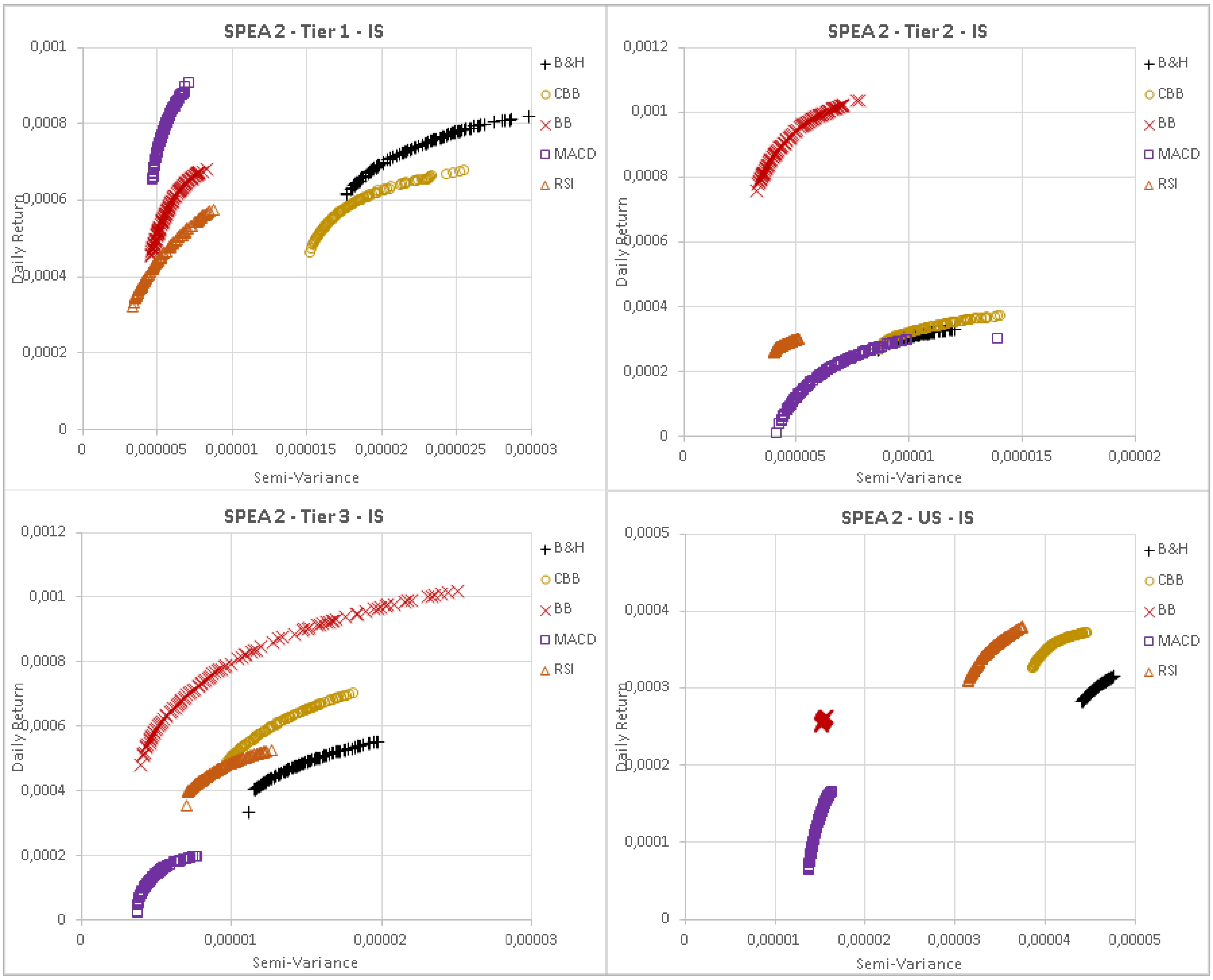
Source: Author.

Regarding the in-sample results obtained with NSGA II (Figure 14), the US market is the one generating the most similar non-dominated frontiers among all strategies, and they are very close to the B&H median frontier, suggesting a great level of efficiency in this market. The strategy using BB trading rules presents the best IS median non-dominated frontiers for the markets Tier 2 and 3. Conversely, CBB is the strategy that produces the median frontiers closest to the B&H scenario, suggesting little advantage in using its associated trading strategy.

The market presenting better BB IS results is the Tier 3, which in theory would be the 2nd most efficient market (following the US stock market). This is interesting since it would be expected that the less efficient the market, the more likely it would be to present exploitable trading opportunities and, therefore, the better the non-dominated frontiers that would result from the optimization process with IS data. This might suggest trading opportunities may not arise in less efficient markets as it was supposed to, and/or market liquidity plays an important role in materialising profit opportunities. Tier 3 also shows how the use of some trading strategies (MACD and RSI) may hurt the overall output, presenting results that are even worse than the B&H scenario. However, we must stress that IS results may be the result of some overfitting, so we must be careful in not giving too much importance to the conclusions regarding the IS performance of the strategies.

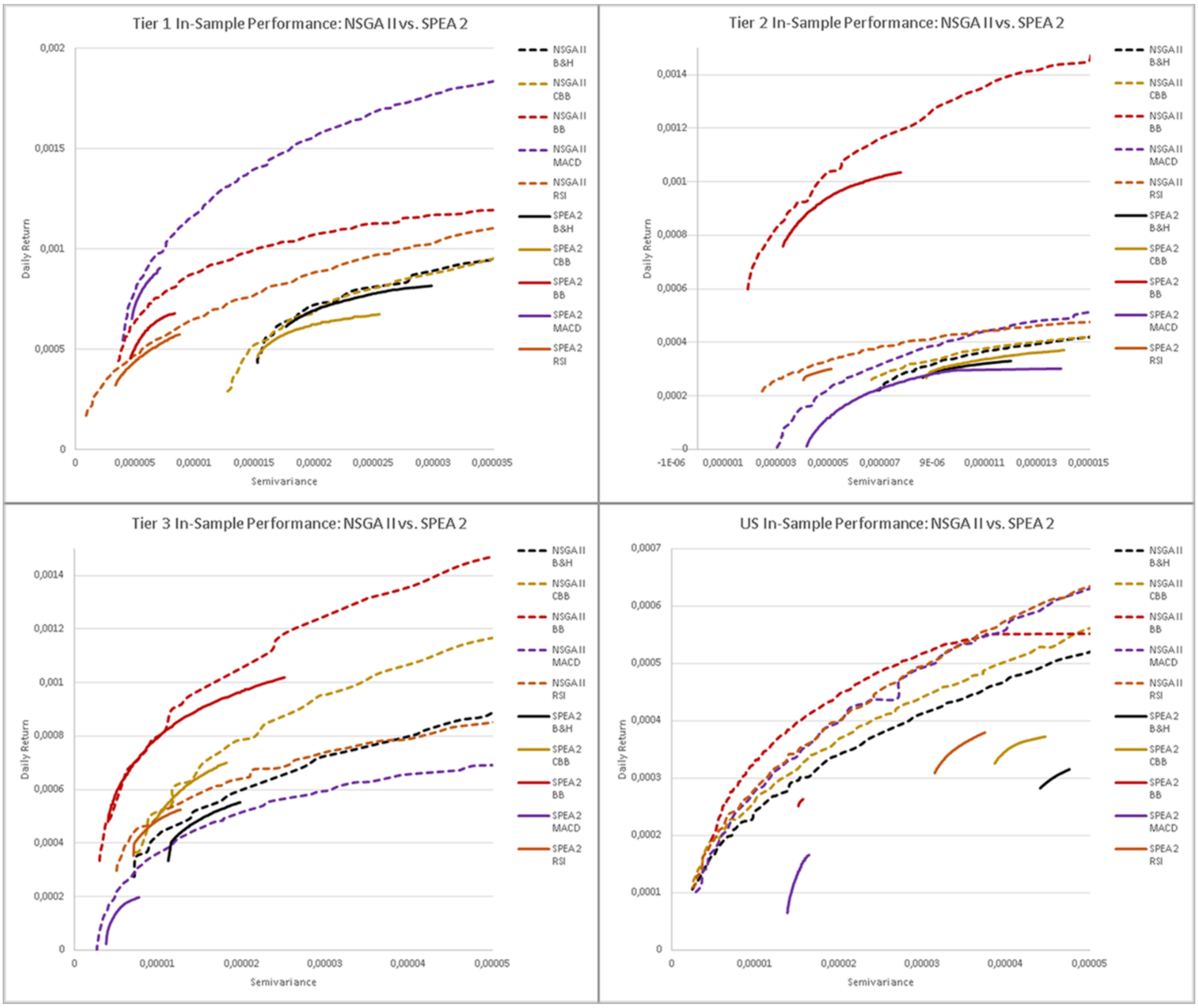
The IS results of SPEA 2 (Figure 15) show a different spectre: the solutions are much more concentrated within a small semi-variance interval, which prevents the possibility of deducing sustained conclusions. Still, we may see BB presents the best median front in markets Tier 2 and 3. The limited extension of the non-dominated sets does not allow much further comparison within each market.

Figure 15. SPEA 2 frontiers of non-dominated solutions, in-sample, under 5 strategies (B&H, CBB, BB, MACD and RSI), with costs, regarding all 4 aggregate markets (Tiers 1, 2, 3 and US).



Source: Author.

Figure 16. Performance comparison between NSGA II and SPEA 2 algorithms for the five TA strategies (B&H, CBB, BB, MACD and RSI) in each market, within the four stock markets (Tiers 1, 2, 3 and US).



Source: Author.

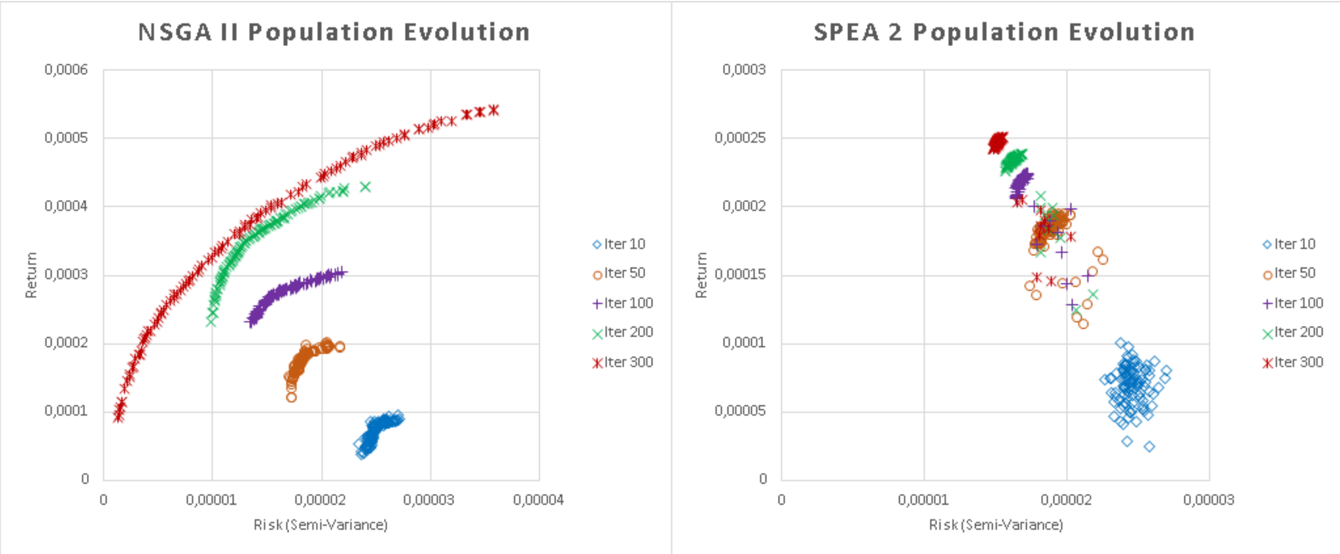
The very limited extension of the IS fronts obtained by SPEA 2 is reinforced in Figure 16, which presents a comparison of NSGA II and SPEA 2 performances in IS data. Notice that the scales of semi-variance have been reduced with respect to Figure 14 in order to allow legibility of SPEA 2 fronts; otherwise, it would be almost impossible to see their shapes. The dashed lines represent the NSGA II frontiers for each strategy; the continuous lines stand for the SPEA 2 frontiers. We can observe that in markets Tier 1 to 3 the non-dominated frontiers obtained by both algorithms are very close to each other, but NSGA II consistently outperforms SPEA 2, for all strategies.

The main conclusions obtained with this comparison (NSGA II outperforming SPEA 2 and SPEA 2 frontiers presenting a lesser extent than those of the NSGA II) are similar to the conclusions reached by (Mishra et al., 2009), (Diosan, 2005) or (Lwin et al., 2013) in portfolio optimization problems. In order to have a better perception of this fact, we have further analysed the evolution of the population throughout generations using NSGA II and SPEA 2 in a run for the conventional Bollinger Bands (BB) strategy in the US market (the case that has shown the greatest divergence in terms of front length). Populations in iterations 10, 50, 100, 200 and 300 of the optimization process are presented in Figure 17, so a notion of the population evolution may be inferred. It can be observed that SPEA 2 population starts with a relatively dispersed population but, as it approaches the later iterations, it tends to maintain the front extension or to narrow it slightly, concentrating the population in the middle of the Pareto front. On the other hand, NSGA II stimulates diversity in the population at the same time it approaches the Pareto front.

A possible reason for this discrepancy between the two algorithms may be the way diversity is promoted in the selection procedure of each algorithm. NSGA II uses a crowding distance, which measures the distance of each individual to its nearest neighbours on the objective function space. A non-dominated solution with a smaller value of the crowding distance is more crowded by other solutions and it will be less preferred to integrate next population than another non-dominated solution with higher crowding distance. SPEA 2 uses a density estimation technique. Density information is incorporated in the fitness function to discriminate between individuals having identical raw fitness. The way the density information is calculated and considered in the selection procedure is different from the crowding distance technique of NSGA II. The latter algorithm emphasizes the boundary solutions of the non-dominated set, finding solutions closer to the outlying edges of the Pareto front. This fact may lead NSGA II to provide a broader range of solutions.

This difference in behaviour is not fully replicated in all combinations market / trading strategy. It seems that some markets and/or strategies are more prone to induce this divergence in the performance of the algorithms, leading us to conclude that data also plays an active relevant role in this matter.

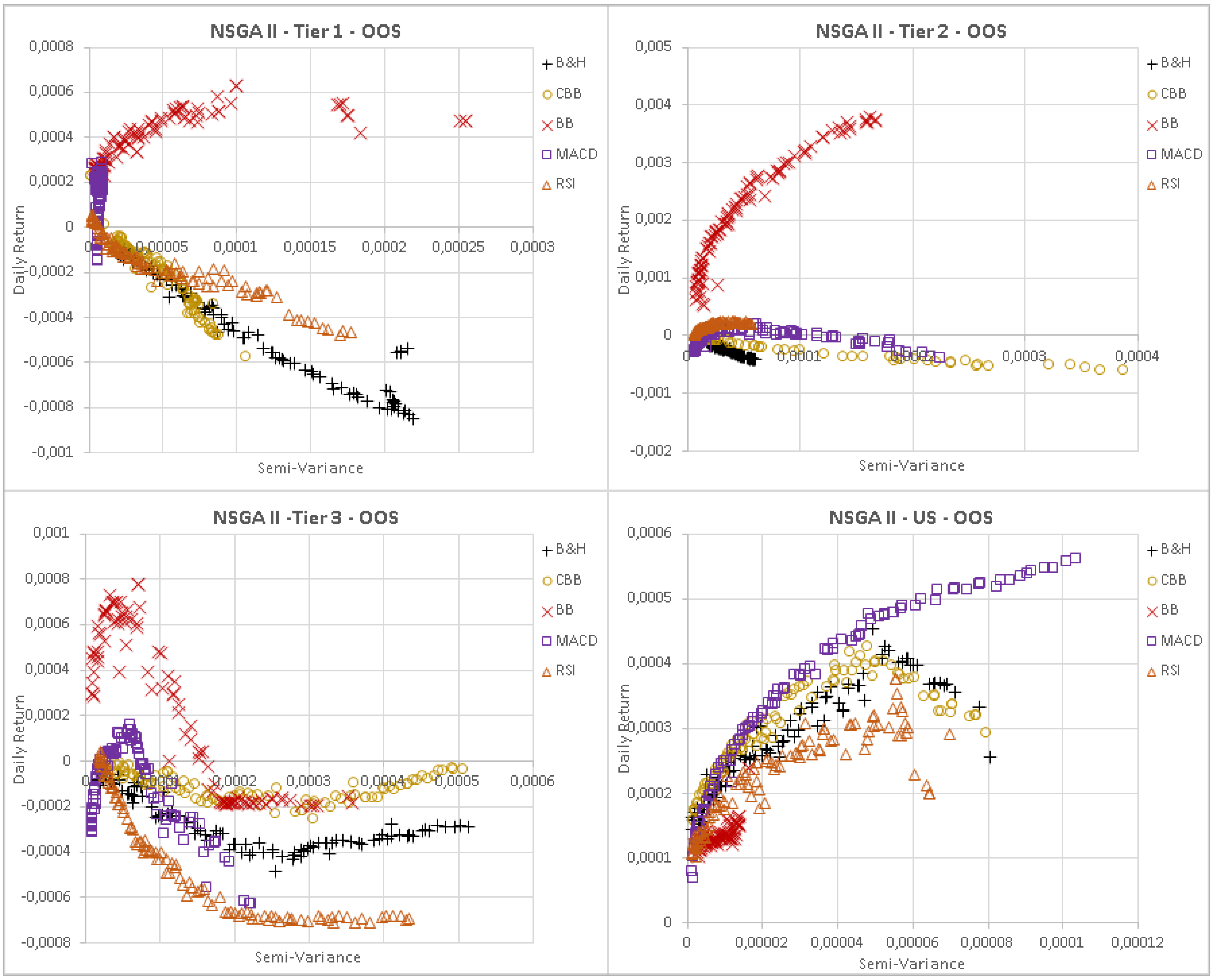
Figure 17. Comparison of the evolution of NSGA II and SPEA 2 populations throughout iterations for the BB strategy in US market.



Source: Author.

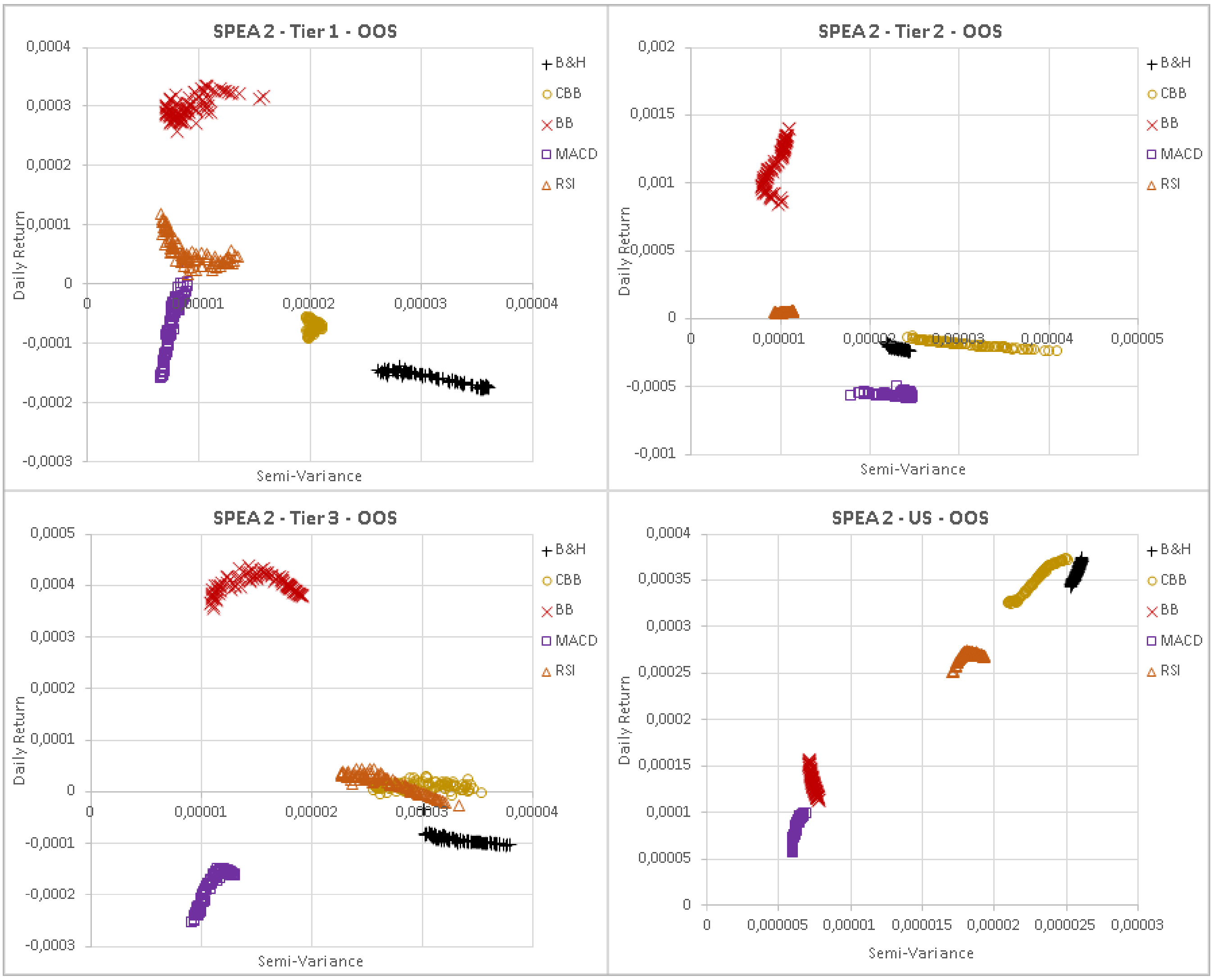
Regarding OOS (Figure 18 and Figure 19), overall results obtained by the BB strategy are very interesting. Considering the application of NSGA II a, the BB strategy emerges as the most consistent and giving better OOS results among all the TA strategies that were considered. This conclusion confirms the IS results, which in turn suggests the good IS results were not exclusively due to potential overfitting. MACD also reveals some interesting results in OOS data, performing better than B&H in most markets. The RSI indicator shows better performances in less developed markets. The CBB strategy shows mixed results, with better performances in the US market and Tier 3, and staying close to B&H in less developed markets (Tiers 1 and 2).

Figure 18. NSGA II out-of-sample frontiers corresponding to the in-sample non-dominated solutions, under 5 strategies (B&H, CBB, BB, MACD and RSI), with costs, regarding all 4 aggregate markets (Tiers 1, 2, 3 and US).



Source: Author.

Figure 19. SPEA 2 out-of-sample frontiers corresponding to the in-sample non-dominated solutions, under 5 strategies (B&H, CBB, BB, MACD and RSI), with costs, regarding all 4 aggregate markets (Tiers 1, 2, 3 and US).



Source: Author.

Similarly to the IS behaviour, OOS results are very limited for solutions generated by SPEA 2. The analysis of SPEA 2 OOS results (Figure 19) shows that the BB trading strategy produces the best outcome among all strategies, supporting this trading strategy as the best one within the studied TA techniques. In addition, the results of SPEA 2 suggest that the RSI strategy performs OOS better than B&H in all markets. The MACD may be considered the strategy with the less interesting results of all, particularly in markets Tier 2 and 3.

5.6 Conclusion

Regarding the empirical work presented in this chapter, overall results show that there is quite a difference between the outcome obtained with NSGA II and with SPEA 2. The former algorithm produces much wider IS non-dominated frontiers, which have similar shapes. On the other hand, the latter algorithm presents IS non-dominated frontiers with different shapes and positions and the solutions are more concentrated presenting low diversity. NSGA II systematically outperforms SPEA 2, in an in-sample comparison. Results also show how the use of TA indicators with associated trading strategies may influence the frontiers of non-dominated portfolios. In most markets, *conventional Bollinger Bands* (BB) presents better out-of-sample results, deeming it as the most profitable TA trading tool and the strategy of election among all studied.

6 Conclusion

The study portrayed in this Thesis addresses the relevance of TA in the context of trading. It tries to validate a link between TA indicators and above normal results. To achieve that intent, we have used several evolutionary techniques applied to the optimization of the parameters of these indicators.

Two distinct empirical approaches were conducted:

1. a first one, where an original GA was developed to optimize parameters of three different categories of TA indicators (*trend*, *momentum* and *breakout*) within the Forex market (three different crosses: EUR/USD, GBP/USD and USD/JPY) and trading with a single trading account;
2. a second one, where, with the use of two established MOEAs (the NSGA-II and the SPEA-2), the weights of portfolios were optimized for 4 different TA trading strategies (MACD, RSI, BB and CBB) and compared with the B&H, all strategies with constant weights, in the context of portfolio optimization of stock markets of distinct levels of development and liquidity.

The results obtained were diverse and it would be interesting to address once more the objectives identified earlier in Chapter 1 (Introduction) and draw a conclusion about each and every mentioned goal. For convenience, all the objectives will be replicated here, each followed by a comment about the related achieved empirical results:

1. *To compare the performance of diverse markets, to know how efficient markets are;*

In this study, we assessed efficiency by analysing the potential for achieving excess returns using strategies based on technical analysis. Our experimental simulation generated a broad spectrum of divergent results in this matter; if by one token, several Forex markets seemed very efficient particularly when considering trading costs it is also true that some other appeared less efficient allowing exploitation of good profit opportunities, such as the GBP/USD. The same happens in portfolio management with Stock markets: The US market seemed much more efficient than markets Tier 1, 2 or 3, with little possibility to attain substantial profits. The main conclusion is that it was possible to detect different degrees of efficiency in different markets.

2. *To find out if TA is effectively useful for trading, both for portfolio management and direct account trading activities;*

We could find trading opportunities in several markets when optimizing portfolios attaining interesting financial results for some trading strategies, namely BB, even considering established trading costs figures. Also in forex it was possible to achieve good results for the GBP/USD market. Therefore, the simulation allowed to gather some evidence showing TA may induce trading opportunities in some markets at some periods of time. Nevertheless it is also important to notice the overall temporal lack of consistency in the gains obtained with recourse to TA.

3. *To determine which of the studied TA indicators shows more potential to be used as a predicting tool;*

In Forex markets, the category of indicators that has shown consistently better IS results was *momentum*; curiously, with OOS data, the category showing more resiliency and robustness was *trend* for the EUR/USD and GBP/USD markets and *breakout* for the USD/JPY market. The inclusion of trading costs changes the verdict for the EUR/USD market from *trend* to *breakout* category. Regarding portfolio optimization, the overall best trading strategy may be considered the BB, which attains better IS and OOS results in most of the markets.

4. *To assess if evolutionary techniques help in any way to improve financial performance (via parameters fine-tuning), with reference to traditional parameters used in TA;*

This assessment can only be made with reference to the application of the GA used in Chapter 4, since it was in this simulation we optimized the parameters of each indicator. The conclusion we arrived at is that the GA we developed allows to produce better performances when compared with the use of TA indicators with traditionally accepted parameters.

5. *To compare performances of up-to-date MOEAs in portfolio optimization problems, which involve risk-return trade-offs and where a Pareto front can be obtained;*

The results obtained by the two MOEAs revealed a superiority of the NSGA-II performance. This superiority is reflected in both the extension of the non-dominated front and its proximity to the Pareto front.

6. *To verify if existing costs affect significantly the attained results in all studied markets;*

The admission of trading costs in the Forex markets has shown an important effect in the final results. In this case the introduction of reasonable costs into the trading activity does affect the results. On the other hand, in portfolio optimization, the inclusion of regular trading costs did not affect substantially overall final results. So we cannot state a definitive conclusion about this factor in trading profitability.

As a final word, it is important to state how this work, in spite of producing several important conclusions, stimulates further research in this area, since some of the results were not completely conclusive. It would be interesting to explore other types of markets, such as Bonds, Futures, Commodities, and different Stock Markets from diverse countries: for instance, make some comparative analysis by market segment (including different corporations' stocks of the same segments). Other alternative path of research could be the development and optimization of different trading models, both in a single account and portfolio optimization perspectives, employ other metaheuristics or combinations of metaheuristics with other techniques such as artificial neural networks. But the scope could go even further, and make for example some combination of Fundamental and Technical Analysis indicators. There is an endless array of possibilities of research in this area and the path to follow will largely depend on the relevance of the subject of inquiry.

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Appendix

TABLE A 1. Profitability (%) of the optimized solutions, in-sample without costs.

Profitability (%)	EUR/USD			GBP/USD			USD/JPY		
Period	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2001-1 to 2002-2	13,9	18,0	10,0	15,9	18,3	17,7	22,8	23,3	18,0
2001-2 to 2003-1	20,0	23,7	14,9	17,5	20,7	21,0	21,9	26,3	17,1
2002-1 to 2003-2	20,6	23,0	24,5	29,1	25,0	34,3	19,1	24,1	14,9
2002-2 to 2004-1	30,9	26,5	17,6	25,7	30,4	27,9	20,5	24,2	16,6
2003-1 to 2004-2	31,6	26,7	20,0	27,9	34,2	31,0	15,2	20,5	10,7
2003-2 to 2005-1	21,0	20,3	10,2	28,4	28,8	18,5	15,6	21,0	5,9
2004-1 to 2005-2	16,8	15,9	6,5	27,3	25,8	12,8	19,5	17,9	5,7
2004-2 to 2006-1	17,4	17,2	7,1	27,7	25,2	14,1	14,6	17,7	11,8
2005-1 to 2006-2	17,2	18,4	3,9	26,4	24,0	13,6	12,4	20,4	18,5
2005-2 to 2007-1	13,1	17,9	9,6	19,1	17,7	20,4	14,0	20,6	12,3
2006-1 to 2007-2	12,2	16,5	10,4	22,7	23,3	12,6	17,9	20,7	7,7
2006-2 to 2008-1	17,2	21,3	16,6	17,0	34,0	7,2	18,5	21,7	12,3
2007-1 to 2008-2	27,8	31,2	19,2	42,9	51,1	39,8	37,0	37,5	20,1
2007-2 to 2009-1	28,1	35,5	14,5	52,2	63,3	37,2	28,3	33,4	15,3
2008-1 to 2009-2	25,3	29,5	13,5	40,1	50,7	26,2	25,7	28,9	9,8
2008-2 to 2010-1	22,9	33,3	11,8	35,5	37,9	27,2	17,3	19,7	12,3
Cumulative	336,21	374,84	210,49	455,53	510,18	361,57	320,36	377,80	208,96
Annual Average	*42,03*	*46,85*	*26,31*	*56,94*	*63,77*	*45,20*	*40,05*	*47,22*	*26,12*

Source: Author.

TABLE A 2. Profitability (%) of the preeminent TA indicator in each period, out-of-sample without costs.

Profitability (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	-3,6	-12,4	2,1	-21,8	6,8	-21,5	0,9	0,6	8,8
2003-2	22,4	-20,7	-7,8	26,1	-45,3	4,6	-10,4	15,9	3,2
2004-1	0,5	31,1	-5,4	-23,5	-13,6	2,9	12,4	-16,5	-3,4
2004-2	-18,4	-27,0	-0,8	-21,8	6,8	3,2	-10,8	-27,4	-33,0
2005-1	-2,7	-21,5	3,5	4,8	7,6	11,5	-11,6	-0,4	-0,1
2005-2	-28,4	13,5	-3,7	-8,0	-0,6	0,9	2,7	-13,1	2,1
2006-1	-4,0	-4,6	-5,0	42,7	7,8	-1,6	3,4	2,6	-1,7
2006-2	-12,7	-1,3	1,2	-19,4	-5,0	-1,4	-17,4	8,9	-5,3
2007-1	-15,5	-4,2	-4,2	-25,3	9,5	1,5	-18,4	12,3	-16,1
2007-2	7,0	-7,8	-9,6	-21,9	4,3	2,4	1,4	7,7	3,7
2008-1	-5,0	14,7	-16,0	-4,0	9,9	-11,0	18,0	-8,3	-4,1
2008-2	33,8	-63,4	52,3	18,1	-50,7	11,1	-6,6	-13,9	-15,2
2009-1	7,8	6,4	3,5	-69,5	10,7	7,4	0,2	15,0	0,3
2009-2	-9,4	-6,7	-7,1	30,6	-21,2	-17,4	12,7	-12,5	-0,8
2010-1	6,2	-5,9	-6,3	35,2	-6,4	-3,4	-6,0	4,7	0,7
2010-2	13,0	-8,1	-7,3	-14,1	-23,3	2,2	-9,1	-18,5	-5,6
Cumulative	-9,01	-117,70	-10,78	-71,58	-102,83	-8,78	-38,70	-42,85	-66,55
Annual Average	*-1,13*	*-14,71*	*-1,35*	*-8,95*	*-12,85*	*-1,10*	*-4,84*	*-5,36*	*-8,32*

Source: Author.

TABLE A 3. Excess Returns of the optimized solutions compared to the preeminent TA indicator of each period, out-of-sample without costs.

Excess Returns (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	**20,2**	**31,2**	**14,7**	**22,1**	**1,4**	**34,5**	**1,1**	**8,6**	**-5,4**
2003-2	**-16,2**	**36,8**	**33,5**	**-2,0**	**56,0**	**44,2**	**15,8**	**-12,3**	**-1,7**
2004-1	**23,7**	**-31,0**	**-26,1**	**-7,0**	**15,7**	**-20,7**	**-1,1**	**22,4**	**14,9**
2004-2	**20,1**	**28,0**	**29,2**	**27,6**	**-1,5**	**11,1**	**-10,3**	**8,3**	**10,6**
2005-1	**-11,4**	**6,6**	**-20,7**	**5,0**	**-7,8**	**-17,1**	**13,0**	**-4,7**	**-1,8**
2005-2	**33,8**	**-6,8**	**3,9**	**31,0**	**9,3**	**2,8**	**5,6**	**2,8**	**-5,4**
2006-1	**1,0**	**2,5**	**1,1**	**-46,3**	**-0,5**	**1,1**	**-4,8**	**3,1**	**6,3**
2006-2	**10,8**	**4,6**	**-1,9**	**35,7**	**10,6**	**14,8**	**13,4**	**-8,5**	**13,7**
2007-1	**17,3**	**5,3**	**1,4**	**30,9**	**-17,4**	**-1,4**	**19,3**	**3,3**	**25,5**
2007-2	**-20,3**	**7,1**	**23,7**	**35,4**	**5,4**	**-10,8**	**-10,9**	**-2,1**	**-16,0**
2008-1	**21,1**	**-9,5**	**24,1**	**-17,9**	**-5,3**	**-13,5**	**-14,0**	**12,8**	**-1,2**
2008-2	**-35,7**	**52,1**	**-42,4**	**31,5**	**34,5**	**11,5**	**37,8**	**18,2**	**41,0**
2009-1	**-12,4**	**2,6**	**-9,2**	**98,1**	**26,0**	**-10,0**	**-19,3**	**-27,9**	**-10,8**
2009-2	**-0,8**	**-5,7**	**1,4**	**-48,5**	**2,6**	**20,5**	**-11,0**	**10,9**	**2,4**
2010-1	**-0,6**	**2,1**	**4,4**	**-34,2**	**9,0**	**3,5**	**2,3**	**0,8**	**1,7**
2010-2	**3,0**	**3,6**	**15,9**	**9,0**	**17,5**	**-2,1**	**5,8**	**10,2**	**5,4**
Cumulative	**53,50**	**129,69**	**53,10**	**170,42**	**155,47**	**68,33**	**42,93**	**45,90**	**79,19**
Annual Average	*6,69*	*16,21*	*6,64*	*21,30*	*19,43*	*8,54*	*5,37*	*5,74*	*9,90*

Source: Author.

TABLE A 4. Profitability (%) of randomly generated solutions, out of sample, one standard lot, without costs, uncompounded semestral rates.

Profitability (%)	EUR/USD	GBP/USD	USD/JPY
Semester			
2003-1	**0,2**	**0,0**	**-0,1**
2003-2	**-0,0**	**-0,0**	**0,0**
2004-1	**-0,0**	**0,1**	**-0,1**
2004-2	**0,1**	**0,2**	**-0,1**
2005-1	**-0,1**	**-0,1**	**0,1**
2005-2	**-0,0**	**0,1**	**-0,2**
2006-1	**-0,1**	**-0,1**	**0,2**
2006-2	**-0,1**	**-0,3**	**0,0**
2007-1	**0,1**	**0,1**	**0,0**
2007-2	**0,2**	**0,1**	**0,0**
2008-1	**0,0**	**-0,1**	**-0,0**
2008-2	**0,3**	**0,2**	**0,1**
2009-1	**-0,2**	**-0,4**	**0,0**
2009-2	**-0,1**	**-0,1**	**0,1**
2010-1	**-0,2**	**0,1**	**0,0**
2010-2	**-0,1**	**0,1**	**0,0**
Cumulative	**-0,19**	**-0,14**	**0,15**
Annual Simple Average	*-0,02*	*-0,02*	*0,02*
Annual Compound Average	***-0,02***	***-0,02***	***0,02***

Source: Author.

TABLE A 5. MDD (%) for the optimized solutions, in-sample without costs.

Maximum Drawdown (%)	EUR/USD			GBP/USD			USD/JPY		
Period	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2001-1 to 2002-2	8,4	7,6	10,6	13,6	9,9	8,5	8,8	9,0	10,6
2001-2 to 2003-1	7,9	6,5	9,5	9,7	10,7	7,8	8,5	9,1	9,0
2002-1 to 2003-2	7,8	9,2	10,0	9,6	10,8	7,9	7,5	6,7	5,6
2002-2 to 2004-1	9,5	12,4	11,5	14,6	13,4	11,9	8,5	7,8	7,4
2003-1 to 2004-2	8,7	15,5	12,5	13,5	14,8	11,7	10,3	9,2	5,7
2003-2 to 2005-1	11,6	14,0	16,0	13,0	11,7	11,6	9,4	8,7	5,9
2004-1 to 2005-2	12,5	13,3	7,5	11,8	11,4	5,9	8,3	9,0	6,1
2004-2 to 2006-1	12,0	12,1	10,6	10,6	11,8	7,9	11,1	10,1	8,2
2005-1 to 2006-2	8,5	9,4	8,5	11,8	13,4	8,6	10,4	10,3	6,3
2005-2 to 2007-1	9,4	8,1	9,2	12,1	13,3	10,3	8,1	10,5	6,7
2006-1 to 2007-2	7,8	7,7	7,4	9,6	10,5	10,4	10,3	9,1	9,6
2006-2 to 2008-1	9,1	7,7	7,8	13,2	13,8	16,4	11,8	9,7	10,0
2007-1 to 2008-2	15,5	13,9	10,3	14,1	15,5	15,0	8,9	9,5	10,6
2007-2 to 2009-1	17,6	15,6	7,6	13,7	12,3	15,3	11,0	12,1	12,8
2008-1 to 2009-2	16,9	15,1	7,7	14,6	13,6	10,4	9,6	10,2	16,8
2008-2 to 2010-1	17,9	19,9	13,4	12,8	13,1	12,7	12,3	10,6	10,6
Semester Average	*11,3*	*11,7*	*10,0*	*12,4*	*12,5*	*10,8*	*9,7*	*9,5*	*8,9*

Source: Author.

TABLE A 6. MDD (%) for the optimized solutions, out-of-sample without costs.

Maximum Drawdown (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	12,4	10,0	8,9	19,1	18,2	13,5	11,1	10,5	13,9
2003-2	13,0	15,0	10,4	11,1	17,4	9,4	8,5	9,3	8,3
2004-1	13,7	21,4	37,2	41,4	25,2	34,5	12,9	12,3	8,4
2004-2	18,2	16,4	8,9	17,7	18,9	20,2	27,6	25,5	26,6
2005-1	21,1	24,7	19,5	17,5	19,1	18,2	11,7	14,6	7,7
2005-2	15,3	13,8	15,2	14,4	17,1	13,3	14,7	20,0	7,4
2006-1	22,4	16,2	9,2	19,1	18,7	10,8	16,1	14,2	6,1
2006-2	11,8	13,2	6,5	12,8	14,0	5,1	11,2	8,1	5,1
2007-1	6,4	7,2	7,0	13,2	18,0	8,4	13,4	11,9	11,9
2007-2	19,4	13,4	12,5	14,6	17,2	23,8	18,7	14,6	21,7
2008-1	15,2	18,2	13,9	35,4	19,9	30,3	17,3	18,0	17,4
2008-2	33,9	47,5	30,3	31,7	62,9	22,6	16,1	27,5	13,1
2009-1	27,2	19,4	17,2	16,5	16,1	26,8	31,9	28,2	20,8
2009-2	21,8	22,9	9,1	31,8	31,8	21,8	12,7	12,1	15,3
2010-1	19,7	25,7	9,7	16,9	18,6	13,5	13,5	10,7	11,1
2010-2	15,8	23,6	11,3	16,3	18,6	13,8	15,0	18,9	12,1
Semester Average	*18,0*	*19,3*	*14,2*	*20,6*	*22,0*	*17,9*	*15,8*	*16,0*	*12,9*

Source: Author.

TABLE A 7. Profitability (%) of the preeminent TA indicator in each period, out-of-sample, with costs.

Profitability (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	-4,9	-13,7	1,8	-23,5	5,3	-25,8	-0,6	-3,1	5,3
2003-2	21,0	-21,8	-11,5	24,8	-46,3	4,1	-11,8	12,8	2,8
2004-1	-1,0	29,8	-5,7	-25,1	-16,6	2,5	11,0	-20,2	-6,5
2004-2	-19,8	-27,9	-1,3	-5,7	-11,2	-2,3	-13,4	-31,4	-37,1
2005-1	-4,1	-22,5	2,8	3,6	6,3	10,9	-12,9	-3,7	-0,5
2005-2	-29,8	12,1	-4,1	-9,2	-1,8	0,4	1,6	-13,9	1,8
2006-1	-5,5	-5,7	-5,8	41,3	4,9	-2,1	1,9	1,4	-2,1
2006-2	-14,0	-2,5	0,9	-21,0	-8,2	-2,0	-19,4	7,6	-8,9
2007-1	-17,2	-5,2	-4,7	-26,7	6,5	1,1	-19,8	11,0	-19,5
2007-2	5,8	-8,4	-13,2	-23,2	3,2	-1,1	0,2	6,8	1,2
2008-1	-6,5	13,2	-19,7	-5,4	6,7	-14,4	16,5	-9,5	-6,9
2008-2	32,6	-64,2	50,0	16,7	-51,6	10,5	-8,1	-14,9	-16,0
2009-1	5,5	3,3	2,8	-70,8	7,8	3,9	-1,3	14,2	-0,3
2009-2	-11,8	-9,9	-7,7	29,3	-24,4	-20,6	11,6	-13,5	-1,3
2010-1	5,1	-9,3	-7,1	34,2	-9,7	-6,4	-7,3	3,9	0,1
2010-2	11,7	-9,4	-7,8	-15,4	-26,7	-0,7	-10,9	-21,7	-6,3
Cumulative	-32,93	-142,27	-30,38	-76,06	-156,10	-42,01	-62,59	-74,16	-94,43
Annual Average	*-4,12*	*-17,78*	*-3,80*	*-9,51*	*-19,51*	*-5,25*	*-7,82*	*-9,27*	*-11,80*

Source: Author.

TABLE A 8. Excess Returns of the optimized solutions compared to the preeminent TA indicator of each period, out-of-sample, with costs.

Excess Returns (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	19,1	29,2	13,3	21,2	-0,1	36,0	-0,3	9,9	-5,3
2003-2	-20,8	33,9	35,6	-3,3	54,0	42,9	14,4	-12,5	-3,8
2004-1	20,0	-32,4	-28,2	-7,7	16,1	-22,3	-2,1	23,1	16,1
2004-2	16,5	25,2	28,4	9,3	13,6	15,1	-10,8	8,9	11,8
2005-1	-14,6	4,2	-21,7	3,8	-8,7	-18,1	12,1	-3,7	-2,9
2005-2	30,8	-9,1	2,7	30,1	8,3	1,8	3,8	0,3	-6,7
2006-1	-2,7	1,3	0,6	-47,9	0,6	0,2	-6,5	1,3	5,3
2006-2	6,7	3,4	-3,2	35,3	12,2	13,9	12,5	-9,6	15,8
2007-1	16,0	3,4	0,6	29,7	-16,7	-2,8	16,0	2,0	27,2
2007-2	-21,1	5,7	25,5	33,9	4,1	-9,2	-13,8	-3,6	-15,7
2008-1	16,6	-10,2	25,7	-19,6	-4,5	-12,8	-17,7	10,8	-0,1
2008-2	-38,7	51,0	-42,1	30,0	33,1	10,1	33,8	16,3	40,6
2009-1	-12,9	2,5	-10,4	96,2	25,8	-8,9	-23,0	-31,8	-12,2
2009-2	-1,4	-6,5	0,7	-50,7	2,0	21,2	-13,8	9,9	1,2
2010-1	-1,7	2,4	3,8	-36,9	9,0	4,9	0,6	-0,3	0,6
2010-2	2,1	2,2	14,9	4,7	16,0	-3,2	3,1	10,6	3,5
Cumulative	13,74	106,21	46,12	128,21	164,98	68,72	8,29	31,52	75,30
Annual Average	*1,72*	*13,28*	*5,77*	*16,03*	*20,62*	*8,59*	*1,04*	*3,94*	*9,41*

Source: Author.

TABLE A 9. MDD (%) for the optimized solutions, out-of-sample, with costs.

Maximum Drawdown (%)	EUR/USD			GBP/USD			USD/JPY		
Semester	Trend	Momentum	Breakout	Trend	Momentum	Breakout	Trend	Momentum	Breakout
2003-1	13,0	10,4	9,3	20,3	19,3	14,3	12,3	11,2	15,4
2003-2	15,0	16,2	11,1	11,7	18,3	9,9	9,5	10,7	9,4
2004-1	14,2	22,4	39,2	43,2	26,3	35,8	13,7	13,6	9,1
2004-2	20,5	18,1	9,2	18,5	19,9	21,1	30,2	28,2	29,2
2005-1	24,5	26,6	21,0	18,5	19,8	18,9	12,4	15,7	8,5
2005-2	16,8	14,7	16,1	14,9	17,7	14,2	15,8	22,3	8,4
2006-1	25,0	17,3	9,9	20,5	19,7	11,4	17,4	15,1	6,6
2006-2	14,2	14,1	7,3	13,4	14,5	5,5	12,9	9,0	5,6
2007-1	7,3	8,2	7,9	14,0	19,3	9,1	15,7	12,2	12,6
2007-2	20,5	14,0	13,0	15,5	17,9	24,7	21,3	15,2	23,4
2008-1	17,2	19,1	14,7	37,8	21,1	32,6	19,4	18,9	18,4
2008-2	36,2	48,5	31,3	32,5	63,6	23,4	17,2	28,4	13,3
2009-1	28,8	20,5	18,2	17,2	16,9	28,0	34,9	30,0	21,9
2009-2	23,8	25,8	10,1	34,0	34,2	23,0	14,6	12,9	16,1
2010-1	20,7	27,4	10,5	18,6	20,1	14,3	15,0	11,2	12,0
2010-2	16,5	24,6	11,9	19,0	21,6	15,9	17,6	20,4	13,3
Semester Average	*19,6*	*20,5*	*15,1*	*21,9*	*23,1*	*18,9*	*17,5*	*17,2*	*13,9*

Source: Author.

TABLE A 10. List of Market Tier 1 stocks used in the portfolio optimization problem.

Market Tier 1		
Argentina	***Brazil***	***South Africa***
AGROMETAL	AMAZONIA ON	ADVTECH
BANCO SANTANDER RIO "B"	AMPLA ENERGIA E SERVICOS ON	AFRICAN OXYGEN
CAPEX	ATOM PARTICIPACOES	ANGLOGOLD ASHANTI
CAPUTO	B MERC BRASIL PN	ARCELORMITTAL SA.
CARLOS CASADO "B"	BANCO DO NORD ON	CAXTON & CTP PB&PRT.
CELULOSA	BANCO ESTADO ESP. SANTO BANEST ON	DATATEC
COLORIN	BIC MONARK ON	FIRSTRAND
ENDESA COSTANERA	BNCO ALFA INVEST PN	GOLD FIELDS
GOFFRE	BOMBRIL PN	HARMONY GOLD MNG.
IRSA	CEMEPE INVEST PN	MMI HOLDINGS
MORIXE HERMANOS	CIA TECIDOS SANTANENSE PN	NEDBANK GROUP
NUEVO BANCO SUQUIA "B"	CNCO.ALFA DE ADMO. SR.F PN	RMB
SNIAFA COMR.FINCA. INMB. "A"	CONST AD LIN PN	TSOGO SUN
SOCIEDAD COMERCIAL DEL PLATA	CORREA RIBEIRO PN	
TRANSPORTADORA DE GAS DEL SUR	DHB INDUSTRIA E COMERCIO ON	
	OI PN	
	REDE ENERGIA ON	

Source: Author.

TABLE A 11. List of Market Tier 2 stocks used in the portfolio optimization problem.

Market Tier 2		
Greece	***Portugal***	***Belgium***
AEGEK	CIMENTOS DE PORTL.SGPS	BEFIMMO
AEOLIAN INVESTMENT FUND	CIPAN LIMITED DATA	COFINIMMO
ALTEC HOLDINGS	COMPTA	COLRUYT
ATHENA	COPAM LIMITED DATA	DEXIA
ATTICA HOLDINGS	EDP ENERGIAS DE PORTUGAL	FLUXYS BELGIUM "D"
ATTI-KAT	ESTORIL SOL "B"	IMMOBEL
AXON HOLDINGS	FENALU LIMITED DATA	PICANOL
BANK OF GREECE	IMMOBL.CON.GRAO-PARA	RETAIL ESTATES
EDRASIS PSALLIDAS	LISGRAFICA	SABCA
EKTER	LITHO FORMAS PORTUGUESA LIMITED DAT	SIPEF
ELTRAK PROPERTY	OREY ANTUNES	SOLVAC
ELVIEMEK LAND DEVELOPMENT LOGIST.PK	SOCIETY AGUAS DA CURIA LIMITED DATA	SPADEL
EMPORIKOS DESMOS	SONAE SGPS	TESSENDERLO
FLEXOPACK	SONAGI LIMITED DATA	UMICORE
FLR MLS C SARANTOPOULOS	TOYOTA CAETANO	VAN DE VELDE

Source: Author.

TABLE A 12. List of Market Tier 3 stocks used in the portfolio optimization problem.

Market Tier 3		
UK	***Australia***	***Netherlands***
AVIVA	AGL ENERGY	AFC AJAX
BRITISH AMERICAN TOBACCO	ALUMINA	AND INTL.PUBLISHERS
BRITISH LAND	AUST.FNDTN.INV.COMPANY	ARCADIS
BT GROUP	COCA-COLA AMATIL	ASML HOLDING
IMPERIAL BRANDS	DEXUS PROPERTY GROUP	BATENBURG TECHNIEK
KINGFISHER	FORTESCUE METALS GP.	BINCKBANK
LLOYDS BANKING GROUP	GPT GROUP	BRILL
PEARSON	JAMES HARDIE INDS.CDI.	EUROCOMMERCIAL
PRUDENTIAL	NEWCREST MINING	HEINEKEN
RECKITT BENCKISER GROUP	OIL SEARCH	HEINEKEN HLDG.
ROLLS-ROYCE HOLDINGS	QBE INSURANCE GROUP	RELX
SSE	SANTOS	ROYAL DUTCH SHELL A
TESCO	STOCKLAND	STERN GROEP
VODAFONE GROUP	TRANSURBAN GROUP	USG PEOPLE
WPP	WESTFIELD	WOLTERS KLUWER

Source: Author.

TABLE A 13. List of U.S. Market stocks used in the portfolio optimization problem.

US Market		
ADOBE SYSTEMS	CONOCOPHILLIPS	MERCK & COMPANY
AMAZON.COM	COSTCO WHOLESALE	MICROSOFT
AMERICAN EXPRESS	DOW CHEMICAL	MORGAN STANLEY
AMGEN	DUKE ENERGY	ORACLE
APPLE	EXXON MOBIL	PEPSICO
AT&T	FEDEX	PFIZER
BANK OF AMERICA	FORD MOTOR	PROCTER & GAMBLE
BANK OF NEW YORK MELLON	GENERAL ELECTRIC	REGENERON PHARMS.
BIOGEN	GOLDMAN SACHS GP.	STARBUCKS
BLACKROCK	HEWLETT-PACKARD	TARGET
BOEING	HOME DEPOT	TEXAS INSTRUMENTS
CATERPILLAR	INTEL	TIME WARNER
CHEVRON	INTERNATIONAL BUS.MCHS.	UNION PACIFIC
CISCO SYSTEMS	JOHNSON & JOHNSON	WAL MART STORES
CITIGROUP	JP MORGAN CHASE & CO.	WALT DISNEY
COCA COLA	LOCKHEED MARTIN	
COLGATE-PALM.	MCDONALDS	

Source: Author.