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A MULTIPLE OBJECTIVE LINEAR PROGRAMMING MODEL FOR POWER GENERATION EXPANSION PLANNING

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SUMMARY

The planning of new units for electrical power generation is a problem which involves different and conflicting aspects. Besides cost, security issues and environmental concerns must be explicitly incorporated into the models. In this way mathematical models become more realistic, and they enhance the decision maker's comprehension of the complex and conflicting nature of the distinct aspects of the problem. A multiple objective linear programming model for power generation expansion planning is presented. The model considers three objective functions (net present cost of the expansion plans, reliability of the supply system, and environmental impacts) and three categories of constraints (load requirements, operational restrictions and budget). Three generating technologies are considered for power system expansion: oil, nuclear and coal.

KEY WORDS power generation planning; multiple objective linear programming; mathematical modelling; decision support systems

INTRODUCTION

The planning of new units for electrical power generation is a problem which involves different, conflicting, and incommensurate aspects, some of which are not directly quantifiable by an economic indicator. The models which express the qualitatively different aspects of these complex problems in currency units in order to encompass them in an economic objective function fail to capture explicitly the distinct aspects arising in the evaluation of power expansion policies. The problem is multiple objective in nature: economic, technological, environmental and reliability aspects, among others, must be explicitly taken into account. With the aid of multiple objective models, decision makers may grasp the conflicting nature of the objectives and the trade-offs to be made in order to obtain satisfactory compromise solutions from the set of nondominated solutions. In a multiple objective context the concept of optimal solution gives place to the one of nondominated solutions (feasible solutions for which no improvement in any objective function is possible without sacrificing on at least one of the other objective functions).

Although power generation expansion problems have been modelled as single objective programming problems (Caramanis *et al.*, 1982), by considering the total cost only, some attention has been paid recently to the development of multiple objective models (Clímaco and Almeida, 1981; Clímaco *et al.*, 1990[a]; Quaddus and Goh, 1985; Zionts and Deshpande, 1981). A multiple objective linear programming (MOLP) model was developed by some of the authors (Clímaco and Almeida, 1981), in which a method to characterize the whole nondominated surface by computing all the nondominated extreme points was used. Further experiments with more realistic formulations, thus augmenting the dimension of the MOLP model, showed the disadvantages of using methods to compute all nondominated extreme points.

The computational effort to determine all these solutions is not worthwhile generally. Even when it is possible to compute them, this effort is not necessary in many situations. In fact, presenting the decision

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maker (DM) with a large set of solutions, in many cases with just slight differences among the objective function values, may further complicate an already complex decision problem.

These experiments in power system planning, as well as a conceptual evaluation of the different multiple objective approaches, led to the development of a new interactive method named TRIMAP. In interactive methods decision phases involving the DM are alternated with computation phases. The DM intervenes in the solution search process by inputting information into the procedure, which in turn is used to guide the search process to compute a new solution which more closely corresponds to his/her preferences. Thus, interactive procedures reduce the computational burden and the number of irrelevant solutions generated. The interactive capabilities of TRIMAP seem to be very useful for exploiting power generation expansion problems, namely by enabling in an interactive way a progressive and selective identification of the set of nondominated solutions and using graphical displays.

The model considers three objective functions which quantify the total system cost, the environmental impact (both to be minimized), and the reliability of the supply system (to be maximized). Three categories of constraints were considered in order to satisfy 'instantaneous' demand, operational capacity and budget restrictions. The decision variables refer to the generating technologies: oil, nuclear and coal.

2. THE TRIMAP INTERACTIVE METHOD

The TRIMAP method (Clímaco and Antunes, 1987, 1989) is based on a progressive and selective learning of the set of nondominated solutions. The method combines three main procedures: weight space decomposition, introduction of constraints on the objective function space, and introduction of constraints on the weight space. Moreover, the constraints imposed on the objective function values are translated into the weight space. The dialogue with the DM is made mainly in terms of the objective function values, which is the type of information that does not place an excessive burden on the DM.

In each step the DM needs only to express some indications about the options to be considered in the further search for nondominated solutions. The interactive process continues until the DM has gathered 'sufficient knowledge' about the problem, thus improving his/her skills. There are no irrevocable decisions throughout the process, as it is always admissable to go 'backwards' at a later interaction. In this respect TRIMAP is different from other interactive methods in which a rigid sequence of computation and decision phases exist. It is also distinct regarding strategies for reducing the scope of the search and techniques to compute nondominated solutions. A comparative study is presented in Clímaco and Antunes (1990).

TRIMAP is not too demanding with respect to the information required from the DM in each interaction. The main concern is to put at the disposal of the DM a simple and flexible procedure, in order to aid him/her in absorbing the information (having in mind the limited capacity of human beings to process effectively large amounts of information). TRIMAP is designed for problems with three objective functions. Although this is a limitation, it allows for the use of graphical means which are particularly suited for the dialogue with the DM. This enlarges the DM's capacity to process the information, by simplifying the dialogue phase and capitalizing on his/her main strengths, namely visual inspection.

2.1 TRIMAP fundamentals

Let us consider the MOLP problem

$$\begin{aligned} &\text{`Max'} \quad && \mathbf{f}(\mathbf{x}) = C\mathbf{x} \\ &\text{subject to} \quad && \mathbf{x} \in X \equiv \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \geq 0,\ A\mathbf{x} = \mathbf{b}\} \end{aligned} \tag{1}$$

Given a vector-valued function $\mathbf{f}(\mathbf{x}) = C\mathbf{x}$: $\mathbb{R}^n \Rightarrow \mathbb{R}^p$ the set of efficient (or Pareto optimal) solutions is defined by

$$X_E = \{\mathbf{x} \in X \mid \nexists \mathbf{x}' \in X : \mathbf{f}(\mathbf{x}') \geq \mathbf{f}(\mathbf{x})\}$$

where $\mathbf{f}(\mathbf{x}') \geqq \mathbf{f}(\mathbf{x})$ means $f_k(\mathbf{x}') \geq f_k(\mathbf{x})$ for all k and $f_k(\mathbf{x}') > f_k(\mathbf{x})$ for at least one k ($k = 1,2,\ldots,p$). The objective vector $\mathbf{f}(\mathbf{x})$ is nondominated (noninferior) when $\mathbf{x} \in X_E$. Generally, the concept of nondominance refers to the objective space, whereas the concept of efficiency refers to the decision variable space. 'Max' denotes the operation of computing nondominated solutions.

The process of computing efficient solutions used in TRIMAP consists of solving weighted LP problems:

$$\begin{array}{ll} \max & \lambda_1 f_1(\mathbf{x}) + \lambda_2 f_2(\mathbf{x}) + \lambda_3 f_3(\mathbf{x}) \\ \text{s.t.} & \mathbf{x} \in X \end{array} \tag{2}$$

The admissible set of weights is defined by $\Lambda = \{\boldsymbol{\lambda} : \boldsymbol{\lambda} \in \mathbb{R}^3,\ \Sigma\lambda_k = 1,\ \lambda_k > 0,\ k = 1, 2, 3\}$.

The optimal solution to (2), if unique, yields an efficient solution to (1) which is an extreme point of the feasible region X. The admissible set of weights Λ can be graphically displayed as an equilateral triangle. The graphical representation of the $\boldsymbol{\lambda}$ set which gives rise to each nondominated extreme solution can be achieved through the decomposition of the triangle.

The region comprising the set of weights corresponding to a nondominated extreme solution is called indifference region. This is the region for which $\{\boldsymbol{\lambda}^{\mathrm{T}} W \geq 0,\ \boldsymbol{\lambda} \in \Lambda\}$ is consistent, where $W = C_B B^{-1} A - C$ is the reduced cost matrix, and C_B and B are the partitions of C and A, respectively, corresponding to the basic variables of the LP model. The DM can be indifferent to all the combinations of weights within it, because they lead to the same nondominated solution. A common boundary between two indifference regions means that the corresponding efficient extreme solutions are connected by an efficient edge. If a point $\boldsymbol{\lambda} \in \Lambda$ belongs to several indifference regions this means that these correspond to efficient solutions lying on the same face. The analysis of the weight space is thus a valuable tool in learning the shape of the nondominated surface, and it is used in TRIMAP as a means for collecting and presenting the information.

To make the most of the information supplied by TRIMAP it is indispensable, at least in an initial phase, that someone with technical knowledge of MOLP assists the DM in interpreting the comparative analysis of the graphs presented after each computation phase. For example, as in many real-world problems, in this energy planning problem the number of nondominated extreme points with very close objective function values is relatively high. The comparative analysis of the weight space and objective function space enables the detection of these situations and thence helps to avoid searching large numbers of close solutions that are of no interest to the DM.

2.2 An overview of TRIMAP

The search begins by computing the efficient solutions which optimize the three objective functions individually. This is intended to provide the DM as quickly as possible with information that allows him to have a global knowledge of the set of nondominated solutions, and consequently to introduce additional constraints on the objective function values, thus contributing to a rapid reduction in the scope of the search.

The selection of the set of weights to compute a new efficient solution can be indirect or direct. In the indirect form, the weights are computed after the selection by the DM of three nondominated extreme points. These solutions are used to construct a weighted function, the gradient of which is normal to a constant cost plane passing through the solutions in the objective space.

In the direct form, the selection of the weights is made by the DM's indication of the triangle zones not yet filled, for which he thinks it is important to continue the search. This can be done by introducing the values of the weights in a dialogue box or by positioning the mouse on the desired point of the triangle.

The introduction of additional limitations on the objective function values and their translation into the weight space is one of the most innovative features of TRIMAP. This enables the dialogue with the DM to be done in terms of the objective function values (the type of questions the DM easily understands), accumulating the resulting information in the weight space. Whenever the DM imposes the additional constraint $f_k(\mathbf{x}) \geq L_k$, $L_k \in \mathbb{R}$, the auxiliary problem

$$\begin{array}{ll} \max & f_k(\mathbf{x}) \\ \text{s.t.} & \mathbf{x} \in X_a \equiv \{\mathbf{x} \in X : f_k(\mathbf{x}) \leq L_k\} \end{array} \tag{3}$$

is solved. The union of the subregions of the weight space corresponding to alternative solutions to (3) determines the region of the weight space where the additional limitation on the objective function value is satisfied. If the DM is interested in the nondominated solutions which satisfy $f_k(\mathbf{x}) \geq L_k$ only, then it is sufficient to restrict the search to sets of weights within this region.

It is also possible to eliminate triangle regions by imposing absolute or relative restrictions directly on the weights.

If the objective function values do not have the same order of magnitude this will influence the decomposition of the weight space. In these cases it is advisable to normalize the functions.

2.3 TRIMAP user interface

In the development of the TRIMAP package, special emphasis has been placed on the creation of an intuitive, user-friendly, interactive environment that is easy to learn and use. The user keeps control of the solution search process by clicking the mouse on appropriate places of the screen (menu bar, buttons etc.). The fundamental structure of the computer interface is a menu bar at the top of the screen which lists the titles of the available pulldown menus, grouping the actions available to the user. Overlapping windows improve the availability of the information, and are used for displaying graphical and text information. Dialogue boxes are used to request further information from the user before a given command can be carried out or to convey some useful information to the user. In each interaction the DM has at his disposal different forms of graphical information. The triangle (weight space) displays the indifference regions corresponding to each nondominated solution already known. Eventual constraints on the variation of the weights are also presented, whether they are directly introduced into the weight space or result from additional constraints imposed on the objective function values. Another graph presents the nondominated solutions already computed projected on a plane of the objective function space. Complementary indicators of the nondominated solutions are also available: the objective function values, the values of the basic decision variables, the Chebyshev distance (L_∞) to the 'ideal solution' and the percentage of the triangle area occupied by the indifference region. The 'ideal solution' is the one that would optimize all the objective functions simultaneously (obtained by optimizing each objective function separately), and it is not feasible when the objectives are in conflict. The area of the indifference region is somehow a measure of the robustness of the solution regarding the variation of the weights. For further details about TRIMAP see Clímaco and Antunes (1987, 1989).

3. A MOLP MODEL FOR POWER GENERATION EXPANSION PLANNING

This model is concerned with the planning of new units for electricity generation regarding the dependence on load evolution only. The problem associated with the siting of new units and their effects on the transmission network expansion planning are not considered herein.

3.1 Power generation system model

A simple model to construct the reliability objective function is used, consisting of the classical two-state approach to the operating cycle of a repairable component (Billinton, 1982). This model considers that a generating unit is either operating in normal conditions or it is on outage.

Let m be the so-called mean-time-to-failure and r the mean-time-to-repair. The concept of forced outage rate (*FOR*) is used as the long-term parameter to characterize a unit's unavailability, which is given by $r/(r+m)$. By taking the mean values for a long period of time, the *FOR* definition may be

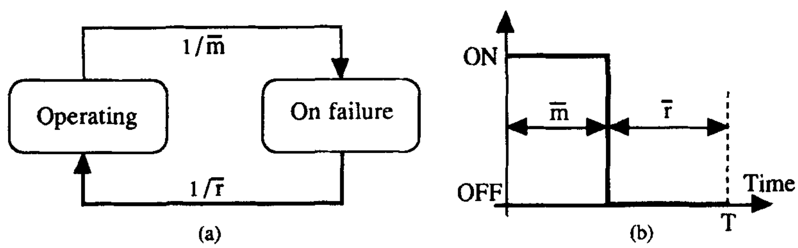


Figure 1. (a) Two-state model of a generating unit; (b) mean cycle

stated as:

$$FOR = \frac{\text{Amount of time on forced outage for a given long period (h)}}{\text{Total period duration (h)}}$$

The FOR of a generating group may be considered as the long-term probability of that group to be in the failure state. The generating system existing at a given point in time may be modelled by means of capacity outage probability tables (COPT). In these tables each line represents a system state. They may be organized, in a simplified form, with three values in each row: the first column contains possible values of capacity on outage; the second one contains the state individual probabilities; the third column is a set of cumulative values, each one representing the probability of an amount of power equal to or greater than the corresponding value on the first column to be on outage.

A more detailed model, considering intermediate derated states (or partial failure states) was not taken into account for two main reasons. Firstly, from the point of view of methodological presentation the number of states considered is irrelevant. Secondly, the probabilistic calculations that are performed for reliability evaluation are external to the MOLP model being used. In this manner more exact computations may be performed, including the partially derated states for generating units, as long as the communication formats with the MOLP model are respected.

3.2 Load model

The load model is based on a peak load duration curve (LDC). The LDC is adequate both for use with LP models and for reliability evaluation through the loss-of-load-probability (LOLP) index. The LDC is approximated to a histogram. As far as the reliability evaluation is concerned this histogram is organised in vertical slices of constant power. The LDC is viewed here in a different way, since the improved z – substitutes method is used in the optimization process. This method allows for a considerable reduction in the number of constraints usually needed in generation planning using LP models. In the improved z-substitutes method (Beglari and Laughton, 1974) the planning period is divided into several subperiods, each one corresponding to a LDC, which in turn is divided in a given number of intervals. If the order of merit of the generating groups does not change from one interval to the next, it may be stated that each group will never be operated in interval $s+1$ at a level of power higher than it was operated in interval s. Hence the description of the LDC may be achieved through the differences in height between contiguous slices instead of the absolute height of each and every slice. For S intervals, this reduces the number of constraints by a factor $1/S$ with respect to the total needed if the classical approach was used.

After the computation of each nondominated solution one of the outcomes (decision variables) consists of the contribution of each type of generating units to the load in each subperiod. The sum of all these contributions must be equal to the total LDC of the corresponding subperiod.

A general form of one of these contributions is displayed in Figure 2. Each $z_{is}^{r}(s=1,\ldots,6)$ is the reduction in power output level of the groups of type i, from interval $s-1$ to interval s within subperiod r. Groups of the same type are those with the same values of the characterizing parameters, using the

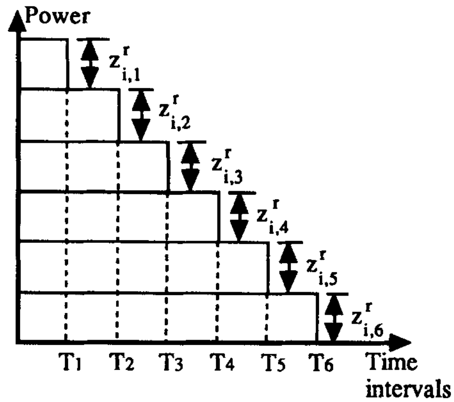


Figure 2. Contribution of the groups of type i in subperiod r to the global LDC of the same period

same fuel. The area under the LDC is computed by means of

$$\sum_{s=1}^{6} z_s\left(\sum_{k=1}^{s} T_k\right)$$

The LDC in each subperiod is computed according to reference load data and to growth rates, established by an external forecasting procedure.

3.3 Objective functions

Three objective functions have been considered in the model, which quantify the net present cost of the expansion plan (to be minimized), the reliability of the supply system (to be maximized) and a measure of the environmental impact (to be minimized).

3.3.1 Total expansion cost: This objective function has the form

$$f_1 = \sum_{j=1}^{J}\left[\sum_{r=1}^{j}\left(\frac{g_{j-(r-1)}}{100}\sum_{i=1}^{I} c_i x_i^r\right) + k_t\left(\sum_{s=1}^{S} s\sum_{i=1}^{I} b_i\, z_{is}^j + \sum_{s=1}^{S} s\sum_{a=1}^{A} b_a^*\, z_{as}^{j*}\right)\right] \tag{4}$$

The first sum inside the outermost parenthesis in (4) represents the investment costs. The term affected by constant k_t represents the operational cost of both the new units and of the ones already in service at the beginning of the planning period (PLPER). The meaning of the symbols in (4) are as follows:

j = index of subperiod inside the PLPER ($j = 1,\ldots,J$)
s = index of interval inside a subperiod ($s = 1,\ldots,S$)
i = index of unit type considered for additions ($i = 1,\ldots,I$)
a = index of unit type existing at the beginning of the PLPER ($a = 1,\ldots,A$)
k_t = constant denoting the number of hours within an interval
g_r = factor representing the capital charges in subperiod r as a percentage of initial investment
x_i^j = decision variable representing the total power output (MW) of units of type i to be installed in subperiod j
z_{is}^j = variable in the improved z-substitutes method (MW) corresponding to a group of type i in interval s of subperiod j
z_{as}^{j*} = the same as z_{is}^j with respect to units of type a
c_i = investment cost (\$/MW) associated with a group of type i
b_i = operational and maintenance charges (\$/MWh) of a group of type i
b_a^* = operational and maintenance charges (\$/MWh) of a group of type a

No capital charges are assumed for units already in service at the beginning of the PLPER, since they are constant to every plan.

The factor g_r is obtained by summing up the annualized capital charges (in percentage of initial investment) of subperiod r of the PLPER. The annualized capital charge at year n (considering an N year period) is given by

$$g_n = \left[\frac{(N+1)-n}{\sum_{k=1}^{N} [(N+1)-k]} + \left(1 - \frac{\sum_{k=1}^{n-1} [(N+1)-k]}{\sum_{k=1}^{N} [(N+1)-k]} \right) d \right] \times 100\%$$

where the first term corresponds to the fraction of the initial investment that must be amortized in each year n, and the second term represents the expenditure corresponding to the application of a discount rate d to the previous year debt. If subperiod r lasts from year p to year q of the PLPER of N years, then g_r in (4) becomes

$$g_r = \sum_{n=p}^{q} g_n,$$

with $p < q$ and $p,\, q = 1, \ldots, N$.

3.3.2 Reliability of the supply system: The probabilistic calculations involved in reliability assessment are intrinsically nonlinear. Rutz *et al.* (1983) proposed an approach where reliability was modelled as a constraint of an LP model for generation planning considering cost as the single objective function. A similar approach has been used in our model to formulate an objective function rather than a constraint. The fundamental parameter used to model the reliability of the generating units is the load-carrying capability (LCC) of new units. The LCC is the amount of power that the system load can increase with the addition of a new unit in order to maintain the reliability level as it was before the unit addition (Garver, 1967).

The reliability objective function to be maximized may be written as

$$f_2 = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{r=1}^{j} h_i^j \frac{x_i^r}{G_i} \tag{5}$$

h_i^j = LCC of a unit of type i in subperiod j of the PLPER
G_i = rated power of a unit of type i, considered for additions

In the context of TRIMAP the importance of the objective functions can be controlled by the DM, by means of the weights. Within the set of solutions the DM considers acceptable, some may not satisfy a reliability criterion if they are too biased by predominant weights on the other two objectives. This raises the need to have some means of evaluating a plan's performance against the risk of supply failure. This is accomplished by a reliability evaluation module external to TRIMAP, which also computes the LCC values to be used in (5).

The generation system and load models (COPTs and LDCs) are merged together in order to obtain a risk function for each subperiod j of the PLPER. This function relates the reliability index loss-of-load probability (LOLP) to outage magnitude (in MW). The LOLP concept may be illustrated with the help of the sample LDC of Figure 3, for a given subperiod j. The LOLP in subperiod j (L_j) is given by

$$L_j = \sum_{s=1}^{S} p_{ac}\,(C_G^j - P_s)\Delta T_s \tag{6}$$

where p_{ac} (ΔP) stands for the probability of occurrence of an outage of magnitude higher than ΔP (value taken from the COPT corresponding to the generating system of subperiod j), and C_G^j is the total generating capacity installed in the same subperiod.

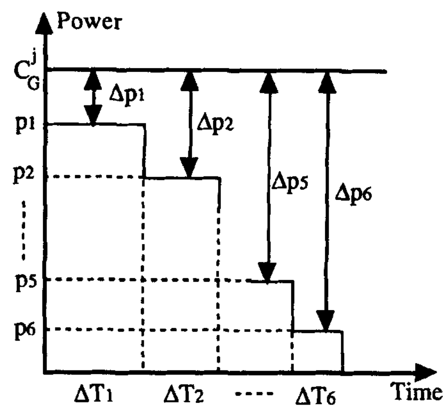


Figure 3. Samply LDC for the computation of LOLP

The risk function, a sample of which is qualitatively displayed in Figure 4, is approximated to an exponential function of the form:

$$L_w = ke^{-\delta_w/M_j} \tag{7}$$

where δ_w is the outage magnitude, k is a constant and M_j is a parameter that characterizes each generating system in a given subperiod j: (7) is transformed into (8)

$$\ln L_w = \ln k - \frac{\delta_w}{M_j} \tag{8}$$

and then curve-fitted to obtain M_j.

M_j (in MW) is then used for obtaining the LCC of each added generating unit, according to the expansion plan. Using the same notation for the LCC, as in (5)

$$h_i^j = G_i - M_j \ln\left[(1-u_i) + u_i\, e^{G_i/M_j}\right] \tag{9}$$

where u_i is the forced outage rate of a unit of type i.

Initially an expansion plan is estimated manually with no other concerns than using only units of the types available for additions and respecting the load constraints (11). This usually leads to a situation with poor reliability performance, which may be confirmed by running the reliability evaluation module. The LCCs obtained as output may be then used for defining f_2 and running TRIMAP for the first time.

An expansion plan selected in the first TRIMAP run is biased by LCC values corresponding probably to undesirable system operating conditions. This means that, to obtain a reasonable plan from the security of supply point of view, it may be necessary to accept a less good compromise in terms of cost

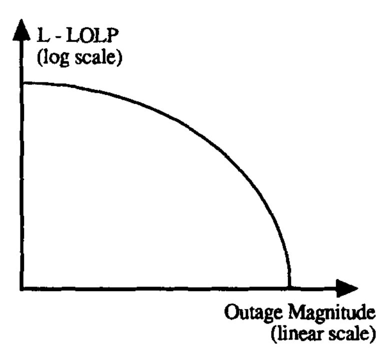


Figure 4. Sample risk function in a semi-log plot

and environmental impact than it would be possible with more realistic LCC values. The procedure used consists in evaluating the reliability performance of each acceptable plan selected by the DM. The simplest outcome is to choose only one plan with good risk performance in every subperiod of the PLPER. TRIMAP is then run again with the data revised after computing the new reliability parameters. The DM may then assess the risk of supply failure of any plan using the reliability evaluation module.

3.3.3 Environmental impact: Alternative generating technologies are very difficult to compare from an environmental perspective. House *et al.* (1981) refer to several different categories of deficiencies that may be found in studies of environmental impact of different energy technologies. Among them some could be pointed out, such as uncertainty, subjectivity, ignorance and incommensurability. They may be as well considered as characteristics of the data usually available. In order to deal with this objective function in the framework of an LP model, some assumptions had to be made in order to keep the problem manageable. A table by Holdren (1977, see House *et al.*, 1981) has been used, which establishes a tentative ranking of several generating technologies according to different criteria of impact evaluation. Four of these criteria have been considered (land use, large accident/sabotage, emissions/public health, and effect on eccosystems) to obtain a first ranking of three alternative generating technologies: oil, nuclear and coal. The first criterion has little to do with the energy output of the generating groups. The second one is associated with both energy output and the existence of the generating facilities. The remaining one may be related to energy output only. In this context two parameters were obtained to characterize each generating alternative: one penalizing installed capacity and the other penalizing energy output. A weighted sum of the indices in House *et al.* (1981) was made to compute each parameter. In both cases the weight assigned to the criterion large accident/sabotage has been intentionally set much higher than the others in order to penalize risk.

The environmental impact objective function is

$$f_3 = \sum_{i=1}^{I} v_i \sum_{j=1}^{J} x_i^j + \sum_{i=1}^{I} e_i \sum_{j=1}^{J} k_t \sum_{s=1}^{S} s\, z_{ij}^S + \sum_{a=1}^{A} e_a \sum_{j=1}^{J} k_t \sum_{s=1}^{S} s\, z_{aj}^{s*} \tag{10}$$

where e_i penalizes the energy output of groups of type i, considered for additions; e_a penalizes the energy output of existing groups of type a: v_i penalizes the installed capacity of groups of type i.

As the impact due to installed capacity of groups existing at the beginning of the PLPER is a constant, it has not been included in the formulation of the objective function.

3.4 Constraints

The first category of constraints requires that all the generating power available at each interval s (of a subperiod j of the PLPER) is at least equal to the load in the same interval. Both new and previously existent generation must be considered. The meaning of P_k is illustrated in Figure 3.

$$\sum_{i=1}^{I} \sum_{s=k}^{S} z_{is}^j + \sum_{a=1}^{A} \sum_{s=k}^{S} z_{as}^{j*} \geq P_k \quad (j=1,\dots,J), (k=1,\dots,S) \tag{11}$$

For each subperiod j there are S constraints of this type.

The second category includes operational constraints of the generating units imposing that the power output of a unit cannot exceed its rated power, previously affected by some availability factor. The right-hand terms of the constraints relative to the pre-existent units are constants.

$$\sum_{s=1}^{S} z_{is}^j \leq m_i \sum_{r=1}^{j} x_i^r \quad (i=1,\dots,I), (j=1,\dots,J) \tag{12}$$

$$\sum_{s=1}^{S} z_{as}^{j*} \leq m_a G_a^* \quad (a=1,\dots,A), (j=1,\dots,J) \tag{13}$$

where m_i is the availability factor of a unit of type i.

The total number of constraints of this type is $J \times (I+A)$.

A budgetary constraint establishing an upper bound on the cost objective function was also introduced.

4. AN ILLUSTRATIVE EXAMPLE

This section is aimed at illustrating how TRIMAP may be used to provide decision aid in a power generation expansion study using the MOLP model previously described. Although the data do not correspond to a real case study, the various types of parameters were quantified in a manner such as to be reasonably in agreement with real values. The budgetary constraint is 8×10^7 currency units, and the discount rate $d = 4\%$. Tables 1 to 4 contain the data used to construct the coefficients of the decision variables in the objective functions and constraints, grouping them according to their nature.

The estimated values of k_i^j (number of units of type i added in subperiod j) in Table 4 were used as input to the reliability evaluation module, that produced the h_i^j values in the same table. Then (5) has

Table 1. Generating system existing at the beginning of the PLPER

Type	Number	Rated power (MW)	Operational costs ($/KWh)	*FOR*	Environmental impact e_i	v_i	Availability factor
Coal	3	400	6·59	0·10	2·60	1·43	0·75
Coal	4	250	6·59	0·06	2·60	1·43	0·75
Oil	3	180	13·30	0·04	2·07	1·29	0·80

Table 2. Generating units considered for additions

Type	Rated power (MW)	Investment costs ($/MW)	Operational costs ($/KWh)	*FOR*	Environmental impact e_i	v_i	Availability factor
Oil	75	702	12·6	0·02	2·07	1·29	0·90
Nuclear	600	1695	5·0	0·09	3·80	4·57	0·60
Coal	350	1229	6·5	0·07	2·60	1·43	0·80

Table 3. Peak load duration curves (values in MW)

Subperiod ↓	1	2	3	4	5	6
0	2110	2019	1932	1849	1769	1693
1	2748	2629	2516	2408	2304	2205
2	3528	3375	3230	3091	2958	2831
3	4400	4209	4028	3855	3689	3531

Table 4. Reliability related data used in the first run of TRIMAP (h_i^j in MW)

	Subperiod 1			Subperiod 2			Subperiod 3		
	O	N	C	O	N	C	O	N	C
k_i^j	1	0	0	0	1	1	0	0	2
h_i^j	72·7	0	0	0	257·88	243·01	0	0	230·1

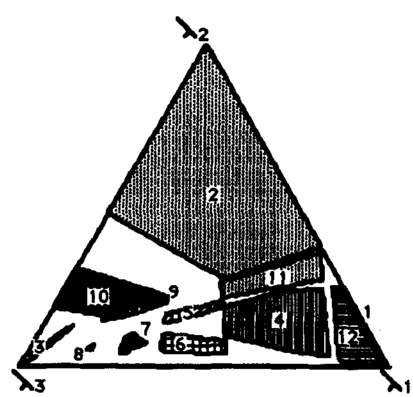
been parametrized for the first run of TRIMAP. Some solutions (expansion plans) at the output of the package were selected for reliability evaluation and one of them was chosen to construct the reliability objective function for the second run of TRIMAP.

4.1 Brief analysis of the results

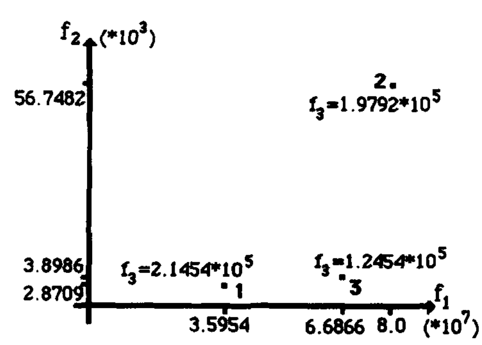
In its first step TRIMAP computes the nondominated extreme solutions corresponding to the optima of the objective functions (identified by 1, 2 and 3). Let us suppose that the DM, based on this information, decided to search for some more nondominated solutions by selecting in a direct manner sets of weights corresponding to not yet covered regions of the triangle. The DM may grasp in a progressive and interactive way the shape of the nondominated surface and begin to focus his/her attention in the parts of it where the solutions which better correspond to his/her preferences are located. Note that by analysing the indifference regions on the weight space and the corresponding values of the objective functions, the DM may conclude whether it is worthwhile to search for new solutions in certain zones of the triangle.

Let us suppose that the DM gained sufficient insights into the set of nondominated solutions, after performing the search displayed in Figure 5(a). For the sake of clarity only the solutions corresponding to the optima of the objective functions were represented on the projection of the objective functions space (Figure 5(b)).

The main conclusions from this search may be summarized as follows. Minimum cost is obtained with a mix of nuclear and coal units, whereas the minimum environmental impact corresponds to the addition



(a)



(b)

Figure 5(a). The weight space after a first search to gain insights into the problem. (b) Projection of the objective function space displaying the solutions which optimize the objective functions individually

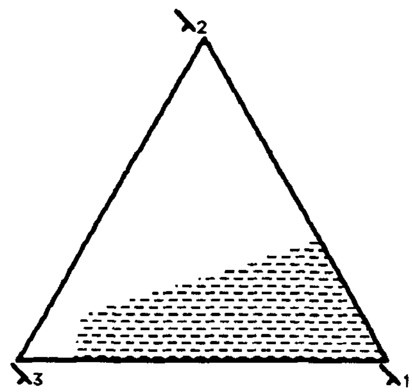


Figure 6. Region of the weight space where $f_1 \leq 4 \times 10^7$ is satisfied

of oil burning units only. The maximum of the reliability objective function is an unrealistic solution, where cost is completely sacrificed to reliability. Three distinct zones may be distinguished in the weight space. In an upper triangle zone a compromise between f_2 and f_3 keeps f_1 at the allowed maximum imposed by the budget constraint (as in solutions 2 and 10). In a lower right triangle zone, nuclear units appear in just a few solutions near the optimum of the cost objective (for instance, solution 12), and a smooth transition regarding cost occurs among solutions with and without nuclear technology (for instance, between solutions 12 and 4). The majority of the lower triangle zone is filled by indifference regions corresponding to expansion plans with additions of coal units only (from solution 4 to solutions 8 and 9). In a lower left zone near the optimum of the environmental impact objective function, a sharp transition occurs from all-oil plans to all-coal plans (for instance, from solution 3 to solution 8). A sharp variation of cost occurs among these three zones.

Figure 6 illustrates the TRIMAP feature that allows the DM to introduce additional constraints on the objective function values to reduce the scope of the search as a result of the knowledge about the problem gathered throughout the interactive process. A upperbound for $f_1 (f_1 \leq 4 \times 10^7)$ was introduced to identify the zone of the triangle where this additional constraint is satisfied in order to perform a more exhaustive search of solution plans within it (Figure 7).

The objective function values corresponding to some solutions are presented in Table 5. The amounts of power generation additions in a subset of the latter are presented in Table 6.

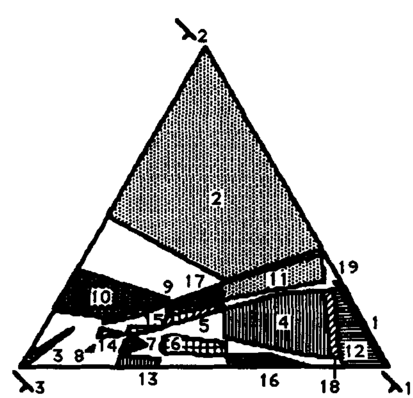


Figure 7. Making a more exhaustive search in the reduced weight space

* Figures 5 to 7 are copies of the screen displays with some minor editing to improve clarity regarding the identification of the solutions.

Table 5. Objective function values of solutions on Figures 5 and 7

Solution	$f_1(10^7)$	$f_2(10^3)$	$f_3(10^5)$
1	3·5954	2·8709	2·1454
2	8·0000	56·7482	1·9792
3	6·6866	3·8986	1·2454
4	3·7308	4·6535	1·5408
5	3·7702	5·3687	1·5406
6	3·7330	4·4636	1·5355
7	3·7373	4·4006	1·5334
8	3·7375	4·4006	1·5333
9	3·7906	5·7719	1·5450
10	8·0000	56·7451	1·9790
11	3·7902	5·7698	1·5452
12	3·5968	2·8429	2·0554
13	3·7374	4·2399	1·5326
14	3·7627	4·8627	1·5334
15	3·7619	4·9908	1·5355
16	3·7328	4·4625	1·5357
17	3·7904	5·7698	1·5451
18	3·6452	3·5957	1·8414
19	3·6045	3·0702	2·1498

Table 6. Generation additions (MW) for some solutions (the index refers to the subperiod)

Solution	Oil 1	Nuclear 1	Coal 1	Coal 2	Coal 3
1		3673·3			441·3
3	3994·4				
4			1370·0	975·0	851·3
5			1370·0	1826·3	
6			1370·0	975·0	625·0
7			1371·3	975·0	547·5
8			1370·0	976·3	547·5
9			1371·3	2065·0	
11			1370·0	2065·0	
12		3126·7			851·3
13			1371·3	783·3	738·3
14			1371·3	1523·4	
15			1371·3	1600·0	
16			1370·0	973·8	626·3
17			1370·0	2065·0	
18		1826·7		975·0	851·3
19		3673·3			678·3

5. CONCLUSIONS

The planning of the expansion of electrical power systems is a problem which involves different aspects, some of them being not directly quantifiable by an economic indicator. A MOLP model has been presented aimed at taking into account explicitly the main aspects which need to be weighted by DMs when evaluating alternative policies for power generation expansion. Flexible and interactive methodologies and computer-oriented models must be at the core of decision aid tools in order to provide effective decision support to DMs in complex real-world problems, namely by enabling different scenarios to be evaluated in a user-friendly fashion. TRIMAP differs from other interactive procedures because it does not possess a rigid pre-specified sequence of computation and decision phases, but it permits actions to be performed as requested by the DM.

The conflicting objectives considered in this model are the minimization of the net present cost of the expansion plans, the maximization of the reliability of the supply system and the minimization of the environmental impacts. These objectives are optimized subject to budget, load and operational constraints. The technologies considered for power generation expansion are oil, nuclear and coal based units. The MOLP model presented in this paper confirms the main advantages of using multiple objective models also reported elsewhere (Quaddus and Goh, 1985; Zionts and Deshpande, 1981) rather than encompassing all the aspects which need to be weighted by DMs in a single objective function.

Some extensions to this work are currently under way. A two-stage procedure for multiple objective evaluation of power expansion planning is presented in Clímaco *et al.* (1990). In the first stage a procedure similar to the one described in this paper is used to select a set of alternative expansion plans. These are considered as basic solutions which are then expanded by considering new criteria of evaluation for which the data has intrinsic uncertainty. This new discrete alternative multiple criteria problem is then studied using any method devoted to tackle this type of problem. Some of the authors are also working on the integration of demand-side management issues in the MOLP model described in this paper, including the reformulation of the objective functions and the consideration of new categories of constraints. Other extensions include the use of different interactive methods to study these problems in order to make the most of the potentialities of each method in distinct phases of the interactive decision process. The information generated by each method is then combined in such a way which can be useful for the DM in subsequent phases of this process.

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