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On the Gains of Using High Frequency Data in Portfolio Selection

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Abstract

This paper analyzes empirically the performance gains of using high frequency data in portfolio selection. Assuming Constant Relative Risk Aversion (CRRA) preferences, with different relative risk aversion levels, we compare low and high frequency portfolios within mean-variance, mean-variance-skewness and mean-variance-skewness-kurtosis frameworks. Using data on fourteen stocks of the Euronext Paris, from January 1999 to December 2005, we conclude that the high frequency portfolios outperform the low frequency portfolios for every out-of-sample measure, irrespectively to the relative risk aversion coefficient considered. The empirical results also suggest that for moderate relative risk aversion the best performance is always achieved through the jointly use of the realized variance, skewness and kurtosis. This claim is reinforced when trading costs are taken into account.

Keywords: Portfolio selection; utility maximization criteria; higher moments; high frequency data.

JEL classification: C55; C61; G11.

1. INTRODUCTION

The reliability of the classical mean-variance portfolio selection model is drawn upon the assumptions of a normal returns distribution or a quadratic utility function (Markowitz, 1952), however these conditions are seldom verified in practice. At least since Mandelbrot (1963), one of the stylized facts of financial time series is that returns distributions exhibit fat tails. Consequently, it seems that investors take into account the non-normal features of the returns distribution, showing preference for positive skewness (see, e.g., the seminal work of Arditti, 1967) and disliking high kurtosis (see, e.g., the empirical work of Maringer and Parpas, 2009). On the other hand, several empirical studies suggest that there are performance

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gains when higher moments (namely skewness and kurtosis) are considered in portfolio selection (see, e.g., Amaya *et al.*, 2015; de Athayde and Flores, 2004; Brito *et al.*, 2017a, 2017b; Harvey *et al.*, 2010; Maringer and Parpas, 2009).

For many years, GARCH (see Bollerslev, 1986; Engle, 1982; Nelson, 1991) and stochastic volatility models (see Taylor, 1986) have been widely used in the financial services industry. More recently, motivated by the increasing availability of high frequency data, Andersen *et al.* (2001) and Barndorff-Nielsen and Shephard (2002) paved the way for the use of realized estimators. Since then, researchers and quants began to dedicate special attention to the estimation of the realized variance, i.e., began to use intraday data to estimate the variance as the sum of squared returns. The realized variance offers considerable estimation power since it is a model-free measure and converges theoretically to the quadratic variation. It was early observed by Merton (1980) that the accuracy of the variance estimation increases with the sample frequency, due to the continuity of its sample path. In fact, many empirical papers, such as Fleming *et al.* (2003) and Liu (2009), have supported this claim.

A similar approach to the one used for the realized variance can be designed for the estimation of higher moments. The realized skewness can be defined as the sum of the 3rd power (see, e.g., Neuberger, 2012) and the realized kurtosis can be defined as the sum of the 4th power (see, e.g., Amaya *et al.*, 2015) of intraday returns. But an important question remains open to discussion: Are there performance gains in portfolio selection when using jointly the three realized moments (variance, skewness and kurtosis)? This paper contributes empirically to answer this question. Motivated by the works of Brandt *et al.* (2009) and Brito *et al.* (2017a, 2017b), the portfolio selection problem is defined in a Constant Relative Risk Aversion (CRRA) world, where the returns distribution is characterized not only by the first two moments but also by the skewness and kurtosis. Therefore, three different frameworks are considered: MV (mean-variance), MVS (mean-variance-skewness) and MVSK (mean-variance-skewness-kurtosis). The methodological design is the following: For each framework and a given relative risk aversion level two utility-maximizing portfolios are built, one based on daily data (which we designate by low frequency portfolio) and another based on intraday data (the high frequency portfolio); then, the performances of the low and high frequency portfolios are compared using eight out-of-sample measures: utility, mean return, standard deviation, skewness, kurtosis, Sharpe ratio, turnover and net Sharpe ratio.

The contribution of this paper is twofold. First, within the three frameworks and for different relative risk aversion levels, we try to find evidence on the advantage of using high frequency data to the portfolios' performance. Second, bearing in mind that previous studies (see, e.g., Fleming *et al.*, 2003; Liu, 2009) suggest that using the realized variance helps to improve the portfolio performance, we investigate if introducing the realized skewness and kurtosis also has a significant positive effect in the portfolios' performance.

The analysis is conducted on the same dataset used in Brito *et al.* (2017a) formed by fourteen stocks, belonging (at current date, August 2018) to the CAC 40 Index, for a seven-year period (January 1999 to December 2005). The empirical evidence is quite clear: For the three frameworks (MV, MVS and MVSK), the high frequency portfolios outperform the low frequency portfolios for every out-of-sample measure, irrespectively to the relative risk aversion coefficient considered. The empirical results also suggest that for moderate relative risk aversion levels, the best performance is always achieved through the jointly use of the realized variance, skewness and kurtosis.

The remainder of the paper proceeds as follows. Section 2 formulates the basic investor's portfolio selection problem and presents three different approximations for the investor's expected utility. Section 3 develops the MV, MVS and MVSK frameworks under CRRA preferences. Section 4 explains the procedures for estimating higher moments using high frequency data. Section 5 presents an empirical application on fourteen stocks of the CAC 40 Stock Market Index. Finally, Section 6 concludes the paper.

2. UTILITY MAXIMIZATION AND THE INVESTOR'S PROBLEM

Following the same notation as in Brito *et al.* (2017a, 2017b), suppose that the investor has a certain wealth to invest in a set of N stocks. In this setting, the portfolio at time t is defined by a $N \times 1$ vector, w_t, of weights representing the proportions of the total wealth invested into the N stocks. Let $E_t(r_{i,t+1})$, $i = 1, \dots, N$, denote the expected return of stock i at time $t+1$. The portfolio is assumed to be linear in $w_{1,t}, \dots, w_{N,t}$, and thus its expected return, at time $t+1$, is given by $E_t(r_{p,t+1}) = \sum_{i=1}^{N} w_{i,t} E_t(r_{i,t+1})$.

According to the utility maximization criterion, and denoting the investor's utility by $u(\cdot)$, the investor's problem can be formulated as

$$\begin{gathered}\max_{w_t \in \mathcal{R}^N} E_t[u(r_{p,t+1})] = E_t\left[u\left(\sum_{i=1}^{N} w_{i,t} r_{i,t+1}\right)\right] \\ such\ that \sum_{i=1}^{N} w_{i,t} = 1\ , w_{i,t} \geq 0, \qquad i = 1, \dots, N\end{gathered} \tag{1}$$

Short selling is not allowed in Problem (1), since in real markets there are some practical and regulatory constraints on short trading positions (especially within the European Union). Furthermore, the use of these non-negative constraints usually results in more robust portfolios (see, e.g., DeMiguel *et al.*, 2009a; 2009b). However, we must point out that allowing for short selling would not change the rationale of the model.

In Problem (1), the investor's expected utility, $E_t[u(r_{p,t+1})]$, needs to be estimated. In this paper, and following Brito *et al.* (2017b), we consider the approximations for the investor's expected utility based on the second, third and fourth order Taylor expansions around the expected return of the portfolio, $E_t(r_{p,t+1})$. The expansions are truncated at the fourth order because there are no theoretical grounds, in terms of the investor's preferences, to include higher polynomial terms (for further details see Dittmar, 2002; Kimball, 1993; Martellini and Ziemann, 2010).

Considering the second order Taylor's expansion of the expected utility, $E_t[u(r_{p,t+1})]$, around the expected return of the portfolio, $E_t(r_{p,t+1})$, we have

$$E_t[u(r_{p,t+1})] \approx \theta_1[E_t(r_{p,t+1})] - \theta_2[E_t(r_{p,t+1})]\upsilon_t(r_{p,t+1}) \tag{2}$$

where $\theta_1[E_t(r_{p,t+1})] = u[E_t(r_{p,t+1})]$, $\theta_2[E_t(r_{p,t+1})] = -u''[E_t(r_{p,t+1})]/2$, and $v_t(r_{p,t+1}) = E_t[r_{p,t+1} - E_t(r_{p,t+1})]^2$ is the portfolio variance. Since:

$$v_t(r_{p,t+1}) = E_t[r_{p,t+1} - E_t(r_{p,t+1})]^2 = E_t\left[\sum_{i=1}^{N} w_{i,t} r_{i,t+1} - E_t\left(\sum_{i=1}^{N} w_{i,t} r_{i,t+1}\right)\right]^2 \tag{3}$$

then

$$v_t(r_{p,t+1}) = \sum_{i=1}^{N}\sum_{j=1}^{N} E_t[(r_{i,t+1} - \mu_{i,t})(r_{j,t+1} - \mu_{j,t})]\, w_{i,j} w_{j,t} \tag{4}$$

or, in a more condensed form

$$v_t(r_{p,t+1}) = w_t^T \Sigma_t w_t \tag{5}$$

where Σ_t is the covariance matrix.

Considering the third order Taylor's expansion, the investor's expected utility is approximated by

$$E_t[u(r_{p,t+1})] \approx \theta_1[E_t(r_{p,t+1})] - \theta_2[E_t(r_{p,t+1})]v_t(r_{p,t+1}) + \theta_3[E_t(r_{p,t+1})]s_t(r_{p,t+1}) \tag{6}$$

where $\theta_3[E_t(r_{p,t+1})] = u'''[E_t(r_{p,t+1})]/6$, and $s_t(r_{p,t+1}) = E_t[r_{p,t+1} - E_t(r_{p,t+1})]^3$ denotes the portfolio skewness, which can be computed as a three dimensional tensor. Following de Athayde and Flores (2004) it is possible to transform this tensor into a $N \times N^2$ matrix:

$$s_t(r_{p,t+1}) = E_t[r_{p,t+1} - E_t(r_{p,t+1})]^3 = w_t^T \Phi_t (w_t \otimes w_t) \tag{7}$$

where Φ_t is the coskewness matrix and $\otimes$ denotes the Kronecker product. The coskewness matrix of dimension $N \times N^2$ can be represented by N matrices, $A_{i,t}$, of dimensions $N \times N$ such that

$$\Phi_t = [A_{1,t} | A_{2,t} | \cdots | A_{N,t}] \tag{8}$$

where

$$A_{i,t} = \begin{bmatrix} a_{i11,t} & a_{i12,t} & \cdots & a_{i1N,t} \\ a_{i21,t} & a_{i22,t} & \cdots & a_{i2N,t} \\ \vdots & \vdots & \ddots & \vdots \\ a_{iN1,t} & a_{iN2,t} & \cdots & a_{iNN,t} \end{bmatrix} \tag{9}$$

and each element, $a_{ijk.t}$, is given by:

$$a_{ijk,t} = \frac{1}{t}\sum_{\tau=1}^{t}\left[r_{i,\tau} - E_t\left(r_{i,\tau}\right)\right]\left[r_{j,\tau} - E_t\left(r_{j,\tau}\right)\right]\left[r_{k,\tau} - E_t\left(r_{k,\tau}\right)\right] \tag{10}$$

with $i, j, k = 1, \dots, N$.

Finally, considering the fourth order Taylor expansion of the expected utility, $E_t\left[u\left(r_{p,t+1}\right)\right]$, around the expected return of the portfolio, $E_t\left(r_{p,t+1}\right)$, it follows that

$$\begin{aligned} E_t\left[u\left(r_{p,t+1}\right)\right] \approx \theta_1\left[E_t\left(r_{p,t+1}\right)\right] - \theta_2\left[E_t\left(r_{p,t+1}\right)\right]v_t\left(r_{p,t+1}\right) + \theta_3\left[E_t\left(r_{p,t+1}\right)\right]s_t\left(r_{p,t+1}\right) \\ - \theta_4\left[E_t\left(r_{p,t+1}\right)\right]k_t\left(r_{p,t+1}\right) \end{aligned} \tag{11}$$

where $\theta_4\left[E_t\left(r_{p,t+1}\right)\right] = -u''''\left[E_t\left(r_{p,t+1}\right)\right]/24$, and $k\left(r_{p,t+1}\right) = E_t\left[r_{p,t+1} - E_t\left(r_{p,t+1}\right)\right]^4$ is the portfolio kurtosis. Analogously to the portfolio skewness, $k\left(r_{p,t+1}\right)$ can be computed as

$$k_t\left(r_{p,t+1}\right) = E_t\left[r_{p,t+1} - E_t\left(r_{p,t+1}\right)\right]^4 = w_t^T \Psi_t\left(w_t \otimes w_t \otimes w_t\right) \tag{12}$$

where Ψ_t is the cokurtosis matrix, corresponding to N^2matrices, $B_{ij,t}$, of dimensions $N \times N$ such that

$$\Psi_t = \left[B_{11,t}|B_{12,t}|\cdots|B_{1N,t}|B_{21,t}|B_{22,t}|\cdots|B_{2N,t}|\cdots|B_{N1,t}|B_{N2,t}|\cdots|B_{NN,t}\right] \tag{13}$$

with

$$B_{ij,t} = \begin{bmatrix} b_{ij11,t} & b_{ij12,t} & \cdots & b_{ij1N,t} \\ b_{i21,t} & b_{ij22,t} & \cdots & b_{ij2N,t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{ijN1,t} & b_{ijN2,t} & \cdots & b_{ijNN,t} \end{bmatrix} \tag{14}$$

and where each element, $b_{ijkl,t}$, is given by

$$b_{ijkl,t} = \frac{1}{t}\sum_{\tau=1}^{t}\left[r_{i,\tau} - E_t\left(r_{i,\tau}\right)\right]\left[r_{j,\tau} - E_t\left(r_{j,\tau}\right)\right]\left[r_{k,\tau} - E_t\left(r_{k,\tau}\right)\right]\left[r_{l,\tau} - E_t\left(r_{l,\tau}\right)\right] \tag{15}$$

with $i, j, k, l = 1, \dots, N$.

3. CRRA PREFERENCES AND HIGHER MOMENTS

Supposing that the investor has CRRA preferences, her utility is given by

$$u(r_{p,t+1}) = \begin{cases} \dfrac{(1+r_{p,t+1})^{1-\gamma}-1}{1-\gamma} & if \quad \gamma > 1 \\ \log(1+r_{p,t+1}) & if \quad \gamma = 1 \end{cases} \tag{16}$$

where γ represents the relative risk aversion coefficient (a higher value of γ implies more risk aversion). A CRRA-utility allows the incorporation of preferences toward higher moments in a parsimonious manner (Brandt *et al.*, 2009; Brito *et al.*, 2017a, 2017b). According to the previous section, three different frameworks are considered, denoted by MV, MVS and MVSK, where the approximated expected utility is given by Equation (2), Equation (6) and Equation (11), respectively. Thereby, within the MV framework the investor's problem is

$$\begin{aligned} &\max_{w_t \in \mathcal{R}^N} \theta_1[E_t(r_{p,t+1})] - \theta_2[E_t(r_{p,t+1})]\upsilon_t(r_{p,t+1}) \\ &such\ that \sum_{i=1}^{N} w_{i,t} = 1\ , w_{i,t} \geq 0, \qquad i = 1, \dots, N \end{aligned} \tag{17}$$

where

$$\begin{aligned} \theta_1[E_t(r_{p,t+1})] &= \begin{cases} \dfrac{[1+E_t(r_{p,t+1})]^{1-\gamma}-1}{1-\gamma} & if \quad \gamma > 1 \\ \log(1+r_{p,t+1}) & if \quad \gamma = 1 \end{cases} \\ \theta_2[E_t(r_{p,t+1})] &= \begin{cases} \dfrac{\gamma[1+E_t(r_{p,t+1})]^{-(\gamma+1)}}{2} & if \quad \gamma > 1 \\ \dfrac{1}{2(1+r_{p,t+1})^2} & if \quad \gamma = 1 \end{cases} \end{aligned} \tag{18}$$

In turn, within the MVS framework the investor's problem is given by

$$\begin{aligned} &\max_{w_t \in \mathcal{R}^N} \theta_1[E_t(r_{p,t+1})] - \theta_2[E_t(r_{p,t+1})]\upsilon_t(r_{p,t+1}) \\ &\qquad +\theta_3[E_t(r_{p,t+1})]s_t(r_{p,t+1}) \\ &such\ that \sum_{i=1}^{N} w_{i,t} = 1\ ,\ w_{i,t} \geq 0, \qquad i = 1, \dots, N \end{aligned} \tag{19}$$

where

$$\theta_3\left[E_t\left(r_{p,t+1}\right)\right] = \begin{cases} \dfrac{\gamma(\gamma+1)\left[1+E_t\left(r_{p,t+1}\right)\right]^{-(\gamma+2)}}{6} & if \quad \gamma > 1 \\ \dfrac{1}{3\left(1+r_{p,t+1}\right)^3} & if \quad \gamma = 1 \end{cases} \tag{20}$$

Finally, within the MVSK framework the investor's problem is formulated as

$$\begin{gathered} \max_{w_t \in \mathcal{R}^N} \theta_1\left[E_t\left(r_{p,t+1}\right)\right] - \theta_2\left[E_t\left(r_{p,t+1}\right)\right]\upsilon_t\left(r_{p,t+1}\right) \\ +\theta_3\left[E_t\left(r_{p,t+1}\right)\right]s_t\left(r_{p,t+1}\right) - \theta_4\left[E_t\left(r_{p,t+1}\right)\right]k_t\left(r_{p,t+1}\right) \\ such\ that \sum_{i=1}^{N} w_{i,t} = 1\ , \quad w_{i,t} \geq 0, \quad i = 1, \dots, N \end{gathered} \tag{21}$$

where

$$\theta_4\left[E_t\left(r_{p,t+1}\right)\right] = \begin{cases} \dfrac{\gamma(\gamma+1)(\gamma+2)\left[1+E_t\left(r_{p,t+1}\right)\right]^{-(\gamma+3)}}{24} & if \quad \gamma > 1 \\ \dfrac{1}{4\left(1+r_{p,t+1}\right)^4} & if \quad \gamma = 1 \end{cases} \tag{22}$$

The solutions of Problem (17), Problem (19) and Problem (21), when the moments and co-moments are estimated using daily returns, are called hereafter low frequency portfolios and are denoted by $w^{(low)}$.

4. PARAMETERS ESTIMATION WITH HIGH FREQUENCY DATA

Arguably the use of high frequency data reduces the estimation error of the parameters in the portfolio selection problem. Therefore, inspired by the works of , Andersen *et al.* (2001), Brito *et al.* (2017a), and Neuberger (2012), we use the realized variance, the realized skewness and the realized kurtosis of the portfolio as inputs in Problem (17), Problem (19) and Problem (21).

Supposing that at day $t+1$ there are Q intraday sampling periods, the realized variance of stock i (with $i = 1, \dots, N$) is defined as

$$rv_{i,t+1}^{Q} = \sum_{q=1}^{Q} r_{i,t+(q/Q)}^{2} \tag{23}$$

where $r_{i,t+(q/Q)}$ represents the return of stock i in the intraday period $t+(q/Q)$.

Analogously, the realized skewness, at day $t+1$, of stock i is defined as

$$rs_{i,t+1}^{Q} = \sum_{q=1}^{Q} r_{i,t+(q/Q)}^{3} \tag{24}$$

and the corresponding realized kurtosis is defined as

$$rk_{i,t+1}^{Q} = \sum_{q=1}^{Q} r_{i,t+(q/Q)}^{4} \tag{25}$$

According to the previous definitions, the daily portfolio realized variance is computed as

$$rv_{t+1} = w_{t+1}^{T} \mathrm{R}\Sigma_{t+1} w_{t+1} \tag{26}$$

where $\mathrm{R}\Sigma_{t+1}$ represents the realized covariance matrix, with each entry, $rc_{ij,t+1}$, given by

$$rc_{ij,t+1} = \frac{1}{t} \sum_{\tau=1}^{t} \sum_{q=1}^{Q} r_{i,\tau+(q/Q)} r_{j,\tau+(q/Q)} \tag{27}$$

Proceeding in the same way as described in Section 2, for the computation of the portfolio skewness, the daily portfolio realized skewness is computed as

$$rs_{t+1} = w_{t+1}^{T} \mathrm{R}\Phi_{t+1} (w_{t+1} \otimes w_{t+1}) \tag{28}$$

where $\mathrm{R}\Phi_{t+1}$ is the realized coskewness matrix. The realized coskewness matrix corresponds to N matrices $RA_{i,t+1}$ of dimension $N \times N$ such that

$$\mathrm{R}\Phi_{t+1} = \left[RA_{1,t+1} | RA_{2,t+1} | \cdots | RA_{N,t+1} \right] \tag{29}$$

where

$$RA_{i,t+1} = \begin{bmatrix} ra_{i11,t+1} & ra_{i12,t+1} & \cdots & ra_{i1N,t+1} \\ ra_{i21,t+1} & ra_{i22,t+1} & \cdots & ra_{i2N,t+1} \\ \vdots & \vdots & \ddots & \vdots \\ ra_{iN1,t+1} & ra_{iN2,t+1} & \cdots & ra_{iNN,t+1} \end{bmatrix} \tag{30}$$

with each element, $ra_{ijk,t+1}$ given by

$$ra_{ijk,t+1} = \frac{1}{t} \sum_{\tau=1}^{t} \sum_{q=1}^{Q} r_{i,\tau+(q/Q)} r_{j,\tau+(q/Q)} r_{k,\tau+(q/Q)} \tag{31}$$

for $i, j, k = 1, \ldots, N$.

The daily portfolio realized kurtosis can be obtained by computing the following products

$$rk_{t+1} = w_{t+1}^T \mathrm{R}\Psi_{t+1}(w_{t+1} \otimes w_{t+1} \otimes w_{t+1}) \tag{32}$$

where $\mathrm{R}\Psi_{t+1}$ represents the realized cokurtosis matrix, which corresponds to N^2 matrices $RB_{ij,t+1}$ of dimension $N \times N$ such that

$$\begin{gathered} R\Psi_{t+1} = \left[RB_{11,t+1} \middle| RB_{12,t+1} \middle| \dots \middle| RB_{1N,t+1} \middle| RB_{21,t+1} \middle| RB_{22,t+1} \middle| \dots \right. \\ \left. \middle| RB_{2N,t+1} \middle| \dots \middle| RB_{N1,t+1} \middle| RB_{N2,t+1} \middle| \dots \middle| RB_{NN,t+1} \right] \end{gathered} \tag{33}$$

with

$$RB_{ij,t+1} = \begin{bmatrix} rb_{ij11,t+1} & rb_{ij12,t+1} & \cdots & rb_{ij1N,t+1} \\ rb_{ij21,t+1} & rb_{ij22,t+1} & \cdots & rb_{ij2N,t+1} \\ \vdots & \vdots & \ddots & \vdots \\ rb_{ijN1,t+1} & rb_{ijN2,t+1} & \cdots & rb_{ijNN,t+1} \end{bmatrix} \tag{34}$$

and where each element, $rb_{ijkl,t+1}$, is given by

$$rb_{ijkl,t+1} = \frac{1}{t} \sum_{\tau=1}^{t} \sum_{q=1}^{Q} r_{i,\tau+(q/Q)} r_{j,\tau+(q/Q)} r_{k,\tau+(q/Q)} r_{l,\tau+(q/Q)} \tag{35}$$

with $i, j, k, l = 1, \dots, N$.

The investor's problems with realized moments within the MV, MVS and MVSK frameworks, are given, respectively, by

$$\begin{gathered} \max_{w_t \in \mathcal{R}^N} \theta_1 [E_t(r_{p,t+1})] - \theta_2 [E_t(r_{p,t+1})] rv_t(r_{p,t+1}) \\ such\ that \sum_{i=1}^{N} w_{i,t} = 1 \ , w_{i,t} \geq 0, \qquad i = 1, \dots, N \end{gathered} \tag{36}$$

$$\begin{gathered} \max_{w_t \in \mathcal{R}^N} \theta_1 [E_t(r_{p,t+1})] - \theta_2 [E_t(r_{p,t+1})] rv_t(r_{p,t+1}) \\ + \theta_3 [E_t(r_{p,t+1})] rs_t(r_{p,t+1}) \\ such\ that \sum_{i=1}^{N} w_{i,t} = 1 \ , w_{i,t} \geq 0, \qquad i = 1, \dots, N \end{gathered} \tag{37}$$

$$\begin{gathered}\max_{w_t \in \mathcal{R}^N} \theta_1\left[E_t\left(r_{p,t+1}\right)\right]-\theta_2\left[E_t\left(r_{p,t+1}\right)\right] r v_t\left(r_{p,t+1}\right) \\ +\theta_3\left[E_t\left(r_{p,t+1}\right)\right] r s_t\left(r_{p,t+1}\right)-\theta_4\left[E_t\left(r_{p,t+1}\right)\right] r k_t\left(r_{p,t+1}\right) \\ \text{such that} \sum_{i=1}^N w_{i,t}=1 \text{ , } w_{i,t} \geq 0, \qquad i=1, \ldots, N\end{gathered} \tag{38}$$

Notice that for the estimation of the daily mean return using intraday data, only the first and last price observations (open and closing daily transaction prices) will matter.

The solutions of Problem (36), Problem (37) and Problem (38), are referred as high frequency portfolios and are denoted hereafter as $w^{(high)}$.

All the objective functions of Problem (17), Problem (19), Problem (21), Problem (36), Problem (37) and Problem (38) are continuous nonlinear but smooth functions, thereby all the presented problems can be solved using a nonlinear constrained optimization algorithm. This paper uses a sequential quadratic programming (SQP) algorithm implemented in MATLAB®.

5. EMPIRICAL ANALYSIS

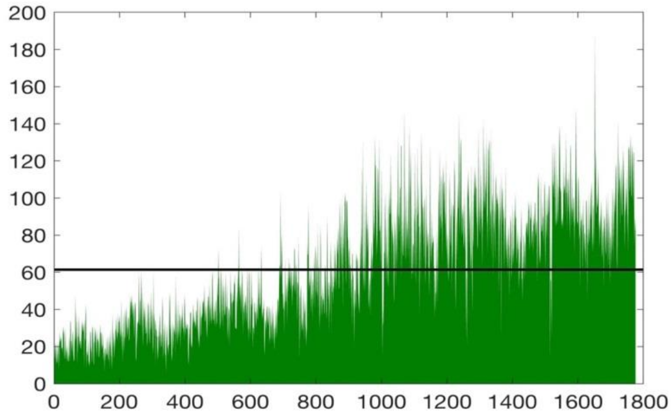
5.1 Data description

In each framework presented in the previous sections, the performances of the low and high frequency portfolios are compared using a dataset from the CAC 40 Index of the Euronext Paris (formerly, before 2000, called the Paris Bourse). The dataset was provided by the European Financial Institute (EUROFIDAI) and corresponds to intraday price observations of fourteen stocks (see Table no. 1). These stocks were always traded in the Euronext Paris during the sample period, but they did not always necessarily belong to the CAC 40 Index. The intraday data were gathered during each trading session (09:00 a.m. - 17:30 p.m., local time), from January 1999 to December 2005 (1777 trading days). In the raw dataset, the intraday price observations were unsynchronized, which can lead to serious biases in the estimation of the moments and co-moments of the stocks returns (see Campbell *et al.*, 1997, pp. 84-98, for further details). The data was synchronized using a well-known algorithm called the all refresh-time method (described in Barndorff-Nielsen *et al.*, 2011). After the synchronization procedure, there were on average about 61 prices changes per day (see Figure no. 1), which correspond to an average frequency of one observation per 8 minutes.

Table no. 1 – The fourteen stocks from the France Stock Market Index (CAC 40)

Stock Designation	
AIR LIQUIDE	LVMH
AXA	MICHELIN
CARREFOUR	PERNOD RICARD
DANONE	SAINT-GOBAIN
ESSILOR INTL	SANOFI-AVENTIS
FRANCE TELECOM	TOTAL
L'OREAL	UNIBAL

Notes: This table lists the stocks used in the empirical analysis. The intraday data on these stocks, from January 1999 to December 2005, were provided by the European Financial Institute (EUROFIDAI).



Notes: This figure reports the average number of intraday price changes (on the fourteen stocks) per day. The horizontal axis corresponds to the number of trading days. The solid horizontal line represents the overall average number of price changes per day.

Figure no. 1 – Average number of intraday price changes per day

5.2 Out-of-sample performance

The comparison between the performances of the low frequency portfolios $\left(w^{(low)}\right)$ and the high frequency portfolios $\left(w^{(high)}\right)$ was conducted using a rolling-window approach (see, e.g., DeMiguel *et al.*, 2009a), for a total of 771 evaluation periods (days). Firstly, for each relative risk aversion level (with $\gamma = 1, 4, 5, 10$), the low frequency portfolios (solutions of Problem (17), Problem (19) and Problem (21)) and the high frequency portfolios (solutions of Problem (36), Problem (37) and Problem (38)) were computed, for the in-sample window, from the first trading day of January 1999 to the last trading day of December 2002. Each portfolio was held fixed and its return was observed over the next trading day (first trading day of January 2003). Then the first trading day of January 1999 was discarded and included the first trading day of January 2003 into the sample. This process was repeated until exhausting the 771 trading days from January 2003 to December 2005. With this procedure, the time series of daily returns for each $w^{(low)}$ portfolio and for the corresponding $w^{(high)}$ portfolio were recorded, resulting in a total of 24 time series of out-of-sample portfolio returns, one for each combination of risk aversion level, moments framework and sampling frequency.

This paper considers four relative risk aversion coefficients. The $\gamma = 1$ case corresponds to the important case of the optimal growth portfolio, which has been often used in the financial literature, at least since the ground breaking paper of Kelly (1956); $\gamma = 4$ and $\gamma = 5$ are reasonable values as estimated in Bliss and Panigirzoglou (2004), and $\gamma = 10$ corresponds to an extreme case, also commonly used in the literature (see, e.g., Brandt *et al.*, 2009).

The recorded time series of out-of-sample daily returns for each portfolio ($w^{(low)}$ and $w^{(high)}$) are used to compute the out-of-sample utility, $\hat{u}$, given by

$$\hat{u} = \begin{cases} \dfrac{(1+\widehat{m})^{1-\gamma}-1}{1-\gamma} & if \quad \gamma > 1 \\ \log(1+\widehat{m}) & if \quad \gamma = 1 \end{cases} \tag{39}$$

where $\widehat{m}$ represents the out-of-sample mean return. The results are reported in Table no. 2.

Table no. 2 – Utility ($\hat{u}$)

Framework	**MV**		**MVS**		**MVSK**	
Relative risk aversion level	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$
$\gamma = 1$	9.52	10.26	9.44	10.26	9.42	10.26
$\gamma = 4$	6.90	9.64	6.77	9.64	6.74	9.64
$\gamma = 5$	7.09	10.17	7.05	10.17	7.07	10.17
$\gamma = 10$	9.63	12.28	9.68	12.27	9.75	12.27

Notes: This table reports, for each framework (mean-variance (MV), mean-variance-skewness (MVS) and mean-variance-skewness-kurtosis (MVSK)) and relative risk aversion level (γ), the annualized out-of-sample utility ($\hat{u}$) of the low frequency ($w^{(low)}$) and high frequency ($w^{(high)}$) portfolios. The values are in percentage.

For all relative risk aversion levels and frameworks considered, the high frequency portfolios always outperform the corresponding low frequency portfolios in terms of out-of-sample utility. Table no. 2 shows that when $\gamma = 1$, the differences between the $w^{(low)}$ and the $w^{(high)}$ portfolios are 74bp (basis points), 82bp and 84bp for the MV, MVS and MVSK frameworks, respectively; when $\gamma = 4$, the differences between the $w^{(low)}$ and the $w^{(high)}$ portfolios are 273bp, 287bp and 290bp, for the MV, MVS and MVSK frameworks, respectively; when $\gamma = 5$ these differences are 307bp, 311bp and 310bp; and when $\gamma = 10$ these differences are 265bp, 259bp and 253bp. Hence the minimum difference occurs in the MV framework for $\gamma = 1$, while the maximum difference occurs in the MVS framework for $\gamma = 5$. For $\gamma = 1, 4$ and 5, the main effect of including higher moments (skewness or kurtosis) is the deterioration of the $w^{(low)}$ portfolios out-of-sample utility, producing an increase in the performance superiority of the $w^{(high)}$ portfolios. For $\gamma = 10$ the reverse effect is observable.

The investor wants to achieve the portfolio with the highest mean return and skewness and the lowest volatility and kurtosis, therefore the superiority of the high frequency portfolios may be the result of its dominance in any of these dimensions. Strikingly, regardless of the relative risk aversion coefficient and moments framework, the high frequency portfolio is able to outperform out-of-sample the corresponding low frequency portfolio in terms of mean return, standard deviation, skewness and kurtosis (see Table no. 3).

Table no. 3 – Out-of-sample moments

Framework	**MV**		**MVS**		**MVSK**	
Relative risk aversion level	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$
Mean ($\widehat{m}$)						
$\gamma = 1$	9.52	10.26	9.44	10.26	9.42	10.26
$\gamma = 4$	6.91	9.64	6.77	9.64	6.75	9.65
$\gamma = 5$	7.10	10.18	7.06	10.18	7.07	10.18
$\gamma = 10$	9.65	12.31	9.70	12.30	9.77	12.30
Standard deviation ($\widehat{std}$)						
$\gamma = 1$	19.33	18.14	19.30	18.14	19.29	18.14
$\gamma = 4$	17.94	16.55	17.92	16.55	17.93	16.55
$\gamma = 5$	17.87	16.23	17.84	16.23	17.83	16.23
$\gamma = 10$	17.40	15.21	17.43	15.21	17.41	15.21
Skewness ($\hat{s}$)						
$\gamma = 1$	-0.88	-0.36	-0.89	-0.36	-0.89	-0.36
$\gamma = 4$	-1.66	-0.96	-1.70	-0.96	-1.74	-0.95
$\gamma = 5$	-2.24	-1.10	-2.26	-1.10	-2.29	-1.10
$\gamma = 10$	-3.14	-1.37	-3.11	-1.38	-3.14	-1.38
Kurtosis ($\hat{k}$)						
$\gamma = 1$	14.40	8.11	14.52	8.12	14.50	8.12
$\gamma = 4$	18.68	10.25	19.11	10.25	19.60	10.21
$\gamma = 5$	27.08	11.74	27.16	11.76	27.62	11.74
$\gamma = 10$	41.96	14.87	41.14	14.97	41.66	14.97

Notes: This table reports, for each framework (mean-variance (MV), mean-variance-skewness (MVS) and mean-variance-skewness-kurtosis (MVSK)) and relative risk aversion level (γ), the annualized out-of-sample mean return ($\widehat{m}$), the annualized out-of-sample standard deviation ($\widehat{std}$), the daily out-of-sample skewness ($\hat{s}$) and the daily out-of-sample kurtosis ($\hat{k}$) of the low frequency ($w^{(low)}$) and high frequency ($w^{(high)}$) portfolios. The values of the mean and standard deviation are in percentage.

In order to assess the statistical significance of the results, the bootstrap p-values for the difference between all the above statistics of each pair of portfolios ($w^{(low)}$ and $w^{(high)}$), within each framework, were also computed. None of the differences were statistically significant for $\hat{u}$, $\widehat{m}$, $\widehat{std}$, $\hat{s}$ and $\hat{k}$. However, these results present quite strong evidence favoring the use of high frequency data, in the sense that for any possible out-of-sample performance measure, involving any of the four moments (mean, variance, skewness and kurtosis), the high frequency portfolio will always exhibit a better performance than the corresponding low frequency portfolio. A commonly used performance metric is the out-of-sample Sharpe ratio, which, assuming that the risk-free rate is zero, is computed as:

$$\widehat{sr} = \frac{\widehat{m}}{\widehat{std}} \tag{40}$$

The results for the out-of-sample Sharpe ratios presented in Table no. 4 show that for reasonable relative risk aversion levels of $\gamma = 4$ and $\gamma = 5$ (Bliss and Panigirzoglou, 2004) the differences between the Sharpe ratios are statistically significant (at the 5% level) in every framework.

Table no. 4 – Sharpe ratio ($\widehat{sr}$)

Framework	MV		MVS		MVSK	
Relative risk aversion level	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$
$\gamma = 1$	29.36	33.60	29.16	33.62	29.11	33.60
	(0.5097)		(0.4781)		(0.4757)	
$\gamma = 4$	23.23	34.70	22.80	34.70	22.72	34.73
	(0.0480)		(0.0378)		(0.0368)	
$\gamma = 5$	23.93	37.25	23.86	37.25	23.90	37.27
	(0.0394)		(0.0392)		(0.0392)	
$\gamma = 10$	33.03	47.63	33.14	47.58	33.39	47.59
	(0.0986)		(0.0970)		(0.1008)	

Notes: This table reports for each framework (mean-variance (MV), mean-variance-skewness (MVS) and mean-variance-skewness-kurtosis (MVSK)) and relative risk aversion level (γ), the daily out-of-sample Sharpe ratios ($\widehat{sr}$) of the low frequency ($w^{(low)}$) and high frequency ($w^{(high)}$) portfolios. All Sharpe ratios values are multiplied by a factor of 10^3. In parenthesis are the bootstrap p-values of the difference between the Sharpe ratio of the low frequency portfolio ($w^{(low)}$) and the corresponding high frequency portfolio ($w^{(high)}$). These bootstrap p-values were computed according to the Ledoit and Wolf (2008) robust methodology.

In most cases the out-of-sample Sharpe ratios, both for the low and high frequency portfolios, increase with the relative risk aversion level (the exceptions are the changes of the Sharpe ratio for the $w^{(low)}$ portfolios, when γ increases from 1 to 4). This result support the findings of Martellini and Ziemann (2010).

The portfolio turnover can be defined as the average, over all time periods, of the absolute changes in weights across the N available stocks:

$$tr = \frac{1}{\#periods} \sum_{t=1}^{\#periods} \sum_{i=1}^{N} \left(|w_{i,t+1} - w_{i,t}^{h}| \right) \tag{41}$$

where $w_{i,t}^{h}$ and $w_{i,t+1}$ are the portfolio weights before and after rebalancing at time $t + 1$, respectively. The quantities $w_{i,t}^{h}$ are computed as

$$w_{i,t}^{h} = w_{i,t-1} \frac{1 + r_{i,t}}{1 + r_{p,t}} \tag{42}$$

The low and high frequency portfolios turnover results, tr, are reported in Table no. 5.

Table no. 5 – Turnover ($\boldsymbol{tr}$)

Framework	MV		MVS		MVSK	
Relative risk aversion level	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$
$\gamma = 1$	80.23	77.32	80.88	77.30	81.02	77.29
$\gamma = 4$	57.38	51.18	57.26	51.20	56.98	51.17
$\gamma = 5$	52.68	46.33	52.40	46.31	52.12	46.30
$\gamma = 10$	36.30	29.66	35.74	29.68	35.24	29.66

Notes: This table reports, for each framework (mean-variance (MV), mean-variance-skewness (MVS) and mean-variance-skewness-kurtosis (MVSK)) and relative risk aversion level (γ), the daily turnover (tr) of the low frequency ($w^{(low)}$) and high frequency ($w^{(high)}$) portfolios. All the turnover values are multiplied by a factor of 10^3.

Similar patterns to the ones presented before for other out-of-sample measures are also found here, i.e. the turnover decreases with γ in each framework and sampling frequency. For each sampling frequency and γ, the turnover is remarkably stable across the three frameworks, and, most importantly, the high frequency portfolios outperform the corresponding low frequency portfolios.

Therefore, in the presence of proportional transaction costs, the high frequency portfolios provide a saving in trading costs, arguably implying that the superiority of these portfolios increase after considering these costs. In order to directly observe this effect, the proportional trading cost is set equal to 50bp per trade, as assumed in DeMiguel *et al.* (2009a). Accordingly, the cost of the portfolio rebalancing at each time t+1 is computed as

$$cost_{t+1} = 0.5\% \sum_{t=1}^{N} \left(|w_{i,t+1} - w_{i,t}^{h}| \right) \tag{43}$$

Following DeMiguel *et al.* (2009b), in the presence of proportional transaction costs, the one-period investor's wealth change is given by

$$W_{t+1} = W_t \left(1 + r_{p,t+1}\right)\left(1 - cost_{t+1}\right) \tag{44}$$

Hence $\frac{W_{t+1}}{W_t} - 1$ corresponds to the return net of transaction costs. Accordingly, the net Sharpe ratio can be defined as

$$\widehat{sr}_{net}^{ref} = \frac{\widehat{m}_{net}}{\widehat{std}_{net}} \tag{45}$$

where $\widehat{m}_{net}$ and $\widehat{std}_{net}$ represent the mean and standard deviation of the out-of-sample returns after transaction costs, respectively. When the numerator is negative, the Sharpe ratio should be refined in order to achieve the correct ranking of the portfolios. This can be accomplished using the methodology proposed by Israelsen (2005). Thus, the refined net Sharpe ratio is given by

$$\widehat{sr}_{net}^{ref} = \frac{\widehat{m}_{net}}{\widehat{std}_{net}^{\,\widehat{m}_{ref}/\mathrm{abs}(\widehat{m}_{ref})}} \tag{46}$$

where abs(·) is the absolute value function. Note that when $\widehat{m}_{net}$ is positive, Equation (45) and Equation (46) are equivalent.

Table no. 6 reports the results for the refined net Sharpe ratios. The results are similar to the results observed for the Sharpe ratio (see Table no. 4), but the observed performance differences between low and high frequency portfolios are now larger.

Table no. 6 – Refined net Sharpe ratio $\left(\widehat{sr}_{net}^{ref}\right)$

Framework	MV		MVS		MVSK	
Relative risk aversion level	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$	$w^{(low)}$	$w^{(high)}$
$\gamma = 1$	-0.00057	-0.000077	-0.00064	-0.000073	-0.00066	-0.000075
	(0.6075)		(0.5361)		(0.5195)	
$\gamma = 4$	-0.00030	9.91	-0.00035	9.90	-0.00035	9.93
	(0.0178)		(0.0152)		(0.0184)	
$\gamma = 5$	0.31	14.36	0.31	14.37	0.47	14.40
	(0.0178)		(0.0152)		(0.0184)	
$\gamma = 10$	16.30	31.99	16.69	31.93	17.16	31.95
	(0.0404)		(0.0502)		(0.0560)	

Notes: This table reports for each framework (mean-variance (MV), mean-variance-skewness (MVS) and mean-variance-skewness-kurtosis (MVSK)) and relative risk aversion level (γ), the daily out-of-sample refined net Sharpe ratios ($\widehat{sr}_{net}^{ref}$) of the low frequency ($w^{(low)}$) and high frequency ($w^{(high)}$) portfolios. All the daily refined net Sharpe ratios values are multiplied by a factor of 10^3. In parenthesis are the bootstrap p-values of the difference between the net Sharpe ratio of the low frequency portfolio ($w^{(low)}$) and the corresponding high frequency portfolio ($w^{(high)}$). These bootstrap p-values were computed according to the Ledoit and Wolf (2008) robust methodology.

All the results obtained for the different performance evaluation measures ($\hat{u}$, $\hat{m}$, $\widehat{std}$, $\hat{s}$, $\hat{k}$, $\widehat{sr}$, tr and $\widehat{sr}_{net}^{ref}$), suggest that the use of high frequency data improves the out-of-sample performance of the portfolios, most especially when trading costs are considered. This claim seems quite robust, as it does not depend on the framework, i.e. the moments used, nor does it depend on the relative risk aversion level.

Table no. 7 shows, for each out-of-sample measure and relative risk aversion level, the best framework when using high frequency data.

Table no. 7 – High frequency portfolio $\left(w^{(high)}\right)$ performance

Performance measure	$\gamma = 1$	$\gamma = 4$	$\gamma = 5$	$\gamma = 10$
Utility ($\hat{u}$)	MVS	MVSK	MVSK	MV
Mean ($\hat{m}$)	MVS	MVSK	MVSK	MV
Standard deviation $\left(\widehat{std}\right)$	MV	MVSK	MVSK	MV
Skewness ($\hat{s}$)	MV	MVSK	MVSK	MV
Kurtosis $\left(\hat{k}\right)$	MV	MVSK	MVSK	MV
Sharpe ratio ($\widehat{sr}$)	MVS	MVSK	MVSK	MV
Turnover (tr)	MVSK	MVSK	MVSK	MVSK
Net Sharpe ratio $\left(\widehat{sr}_{net}^{ref}\right)$	MVS	MVSK	MVSK	MV

Notes: This table reports, for each out-of-sample performance evaluation measure ($\hat{u}$, $\hat{m}$, $\widehat{std}$, $\hat{s}$, $\hat{k}$, $\widehat{sr}$, tr and $\widehat{sr}_{net}^{ref}$), and relative risk aversion level (γ), the framework, mean-variance (MV), mean-variance-skewness (MVS) or mean-variance-skewness-kurtosis (MVSK), where the high frequency portfolio ($w^{(high)}$) achieves the best performance.

The summary information in Table no. 7 highlights a very interesting pattern: for moderate relative risk aversion levels, $\gamma = 4$ and $\gamma = 5$, the best performance is always achieved when all the moments, the realized variance, skewness and kurtosis, are jointly used.

Summarizing, the results presented in this paper show that, for the typical investor, with moderate risk aversion, the consideration of higher moments improves the performance of the portfolios (see, for instance Harvey *et al.*, 2010, for low frequency portfolios), and, that those higher moments, such as skewness and kurtosis, can be better estimated using high frequency data (see, e.g. Neuberger, 2012; Amaya *et al.*, 2015).

6. CONCLUSIONS

Nowadays the use of big data seems to offer a competitive advantage in many fields. Particularly in Finance, the increasing availability of huge high frequency datasets encourages the emergence of new investment strategies built on all the trading information.

This paper contributes to the existing literature by analyzing the practical benefits of using intraday information in portfolio selection. The analysis is conducted in a CRRA utility maximization world, where investors can have different risk aversion levels. The comparison between low frequency portfolios, where the inputs of the optimization problem are obtained from daily data, and high frequency portfolios, where these inputs are obtained from intraday data, is accomplished considering three frameworks according to different information sets: mean-variance, mean-variance-skewness and mean-variance-skewness-kurtosis.

The empirical results, based on fourteen blue-chip stocks traded in the Euronext Paris, show a superior daily out-of-sample performance of the high frequency portfolios, irrespectively of the framework and level of risk aversion considered. This result is transversal to all the out-of-sample performance measures (utility, mean return, standard deviation, skewness, kurtosis, Sharpe ratio, turnover and net Sharpe ratio). For moderate relative risk aversion levels, the differences between the high frequency and low frequency portfolios, in terms of the Sharpe ratio and most particularly in terms of the net Sharpe ratio, are always statistically significant (at the 5% level). The superior performance of high frequency portfolios, measured by the net Sharpe ratio, highlights that the advantage of using high frequency data in portfolio selection problems is even more pronounced when real market conditions, such as transaction costs, are brought into the analysis.

Assuming moderate risk aversion, which arguably defines most investors, the best high frequency portfolios are always the ones that consider not only the mean and variance but also the skewness and kurtosis, irrespectively of the performance metric used. Hence, for the typical investor, the portfolio selection problem is only adequately defined if all these moments are included into the information set.

The literature is unanimous in pointing out that returns are non-normal and therefore are better described by skewed fat-tailed distributions. Although, there is no empirical evidence suggesting that high frequency data has additional information that can be used to estimate expected returns at lower frequencies, there is a prolific literature on the use of high frequency data for estimating the volatility. This paper contributes to the existing literature, by clearly supporting the claim that high frequency data can also be used to produce better estimates of the skewness and kurtosis, which in turn can be used to create better portfolios. In conclusion, this paper provides compelling empirical evidence on the existence of real gains when using high frequency data and higher moments in portfolio selection.

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References

Amaya, D., Christoffersen, P., Jacobs, K., and Vasquez, A., 2015. Does Realized Skewness Predict the Cross-Section of Equity Returns? *Journal of Financial Economics, 118*(1), 135-167. http://dx.doi.org/10.1016/j.jfineco.2015.02.009

Andersen, T. G., Bollerslev, T., Diebold, F. X., and Ebens, H., 2001. The Distribution of Realized Stock Return Volatility. *Journal of Financial Economics, 61*(1), 43-76. http://dx.doi.org/10.1016/s0304-405x(01)00055-1

Arditti, F. D., 1967. Risk and the Required Return On Equity. *The Journal of Finance, 22*(1), 19-36. http://dx.doi.org/10.1111/j.1540-6261.1967.tb01651.x

Barndorff-Nielsen, O. E., Hansen, P. R., Lunde, A., and Shephard, N., 2011. Multivariate Realized Kernels: Consistent Positive Semi-Definite Estimators of the Covariation of Equity Prices with Noise and Non-Synchronous Trading. *Journal of Econometrics, 162*(2), 149-169. http://dx.doi.org/10.1016/j.jeconom.2010.07.009

Barndorff-Nielsen, O. E., and Shephard, N., 2002. Econometric Analysis of Realized Volatility and Its Use in Estimating Stochastic Volatility Models. *Journal of the Royal Statistical Society: Series B (Statistical Methodology), 64*(2), 253-280. http://dx.doi.org/10.1111/1467-9868.00336

Bliss, R., and Panigirzoglou, N., 2004. Option-Implied Risk Aversion Estimates. *The Journal of Finance, 59*(1), 407-446. http://dx.doi.org/10.1111/j.1540-6261.2004.00637.x

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics, 31*(3), 307-327. http://dx.doi.org/10.1016/0304-4076(86)90063-1

Brandt, M. W., Santa-Clara, P., and Valkanov, R., 2009. Parametric Portfolio Policies: Exploiting Characteristics in the Cross-Section of Equity Returns. *Review of Financial Studies, 22*(9), 3411-3447. http://dx.doi.org/10.1093/rfs/hhp003

Brito, R. P., Sebastião, H., and Godinho, P., 2017a. Portfolio Choice with High Frequency Data: CRRA Preferences and the Liquidity Effect. *Portuguese Economic Journal, 16*(2), 65-86. http://dx.doi.org/10.1007/s10258-017-0131-3

Brito, R. P., Sebastião, H., and Godinho, P., 2017b. Portfolio Management with Higher Moments: The Cardinality Impact. *International Transactions in Operational Research*. http://dx.doi.org/10.1111/itor.12404

Campbell, J. Y., Lo, A. W., and MacKinlay, A. G., 1997. *The Econometrics of Financial Markets*. Chichester: Princeton University Press.

de Athayde, G., and Flores, R., 2004. Finding a Maximum Skewness Portfolio: A General Solution to Three-Moments Portfolio Choice. *Journal of Economic Dynamics & Control, 28*(7), 1335-1352. http://dx.doi.org/10.1016/s0165-1889(02)00084-2

DeMiguel, V., Garlappi, L., Nogales, F. J., and Uppal, R., 2009a. A Generalized Approach to Portfolio Optimization: Improving Performance by Constraining Portfolio Norms. *Management Science, 55*(5), 798-812. http://dx.doi.org/10.1287/mnsc.1080.0986

DeMiguel, V., Garlappi, L., and Uppal, R., 2009b. Optimal Versus Naive Diversification: How Inefficient Is The 1/N Portfolio Strategy? *Review of Financial Studies, 22*(5), 1915-1953. http://dx.doi.org/10.1093/rfs/hhm075

Dittmar, R., 2002. Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns. *The Journal of Finance, 57*(1), 369-403. http://dx.doi.org/10.1111/1540-6261.00425

Engle, R. F., 1982. Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica, 50*(4), 987-1007. http://dx.doi.org/10.2307/1912773

Fleming, J., Kirby, C., and Ostdiek, B., 2003. The Economic Value of Volatility Timing Using "Realized" Volatility. *Journal of Financial Economics, 67*(3), 473-509. http://dx.doi.org/10.1016/s0304-405x(02)00259-3

Harvey, C. R., Liechty, J., Liechty, M. W., and Müller, P., 2010. Portfolio Selection with Higher Moments. *Quantitative Finance, 10*(5), 469-485. http://dx.doi.org/10.1080/14697681003756877

Israelsen, C. L., 2005. A Refinement to the Sharpe Ratio and Information Ratio. *Journal of Asset Management, 5*(6), 423-427. http://dx.doi.org/10.1057/palgrave.jam.2240158
Kelly, J., 1956. A New Interpretation of Information Rate. *The Bell System Technical Journal, 35*(4), 917-926. http://dx.doi.org/10.1002/j.1538-7305.1956.tb03809.x
Kimball, M., 1993. Standard Risk Aversion. *Econometrica, 61*(3), 589-611. http://dx.doi.org/10.2307/2951719
Ledoit, O., and Wolf, M., 2008. Robust Performance Hypothesis Testing with the Sharpe Ratio. *Journal of Empirical Finance, 15*(5), 850-859. http://dx.doi.org/10.1016/j.jempfin.2008.03.002
Liu, Q., 2009. On Portfolio Optimization: How and When Do We Benefit From High-Frequency Data? *Journal of Applied Econometrics, 24*(4), 560-582. http://dx.doi.org/10.1002/jae.1062
Mandelbrot, B., 1963. The Variation of Certain Speculative Prices. *The Journal of Business, 36*(4), 394-419. http://dx.doi.org/10.1086/294632
Maringer, D., and Parpas, P., 2009. Global Optimization of Higher Order Moments in Portfolio Selection. *Journal of Global Optimization, 43*(2/3), 219-230. http://dx.doi.org/10.1007/s10898-007-9224-3
Markowitz, H. M., 1952. Portfolio Selection. *The Journal of Finance, 7*(1), 77-91. http://dx.doi.org/10.1111/j.1540-6261.1952.tb01525.x
Martellini, L., and Ziemann, V., 2010. Improved Estimates of Higher-Order Comoments and Implications for Portfolio Selection. *Review of Financial Studies, 23*(4), 1467-1502. http://dx.doi.org/10.1093/rfs/hhp099
Merton, R. C., 1980. On Estimating the Expected Return On the Market: An Explanatory Investigation. *Journal of Financial Economics, 8*(4), 323-361. http://dx.doi.org/10.1016/0304-405X(80)90007-0
Nelson, D. B., 1991. Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica, 59*(2), 347-370. http://dx.doi.org/10.2307/2938260
Neuberger, A., 2012. Realized Skewness. *Review of Financial Studies, 25*(11), 3423-3455. http://dx.doi.org/10.1093/rfs/hhs101
Taylor, S. J., 1986. *Modelling Financial Time Series*. Chichester, UK: Wiley.

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