# Comparison of Different Strategies For Arc Therapy Optimization

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**Abstract.** Radiotherapy (RT) has seen considerable changes in the last decades, offering an increased range of treatment modalities to cancer patients. Volumetric Modulated Arc Therapy (VMAT), one of the most efficient RT arc techniques, particularly with respect to dose delivery time, has recently considered noncoplanar arc trajectories while irradiating the patient, thanks to technological advances in the most recent generation of RT systems. In a preliminary work we have proposed a two-step optimization approach for noncoplanar arc trajectory optimization (ATO). In this paper, treatment plans considering 5-, 7-, 9-, 11- and 21-beam ensembles in the first step (optimal selection of noncoplanar irradiation directions) of the proposed approach are compared to assess the trade-offs between treatment quality and computational time required to obtain a noncoplanar VMAT plan. The different strategies were tested resorting to a head-and-neck tumor case already treated at the Portuguese Institute of Oncology of Coimbra (IPOC). Results obtained presented similar treatment quality for the different strategies. However, strategies that consider a reduced number of beams in the first step clearly outperform the other strategies in terms of computational times. Results show that for a similar tumor coverage, treatment plans with optimal beam irradiation directions obtained an enhanced organ sparing.

**Keywords:** noncoplanar optimization, imrt, arc therapy, treatment planning

# 1 Introduction

Cancer incidence will continue to rise worldwide, with an expected increase of 63.1% of cancer cases by 2040, compared with 2018 [1]. Radiotherapy (RT) is used in more than 50% of all cancer cases in high-income countries [2]. In Europe, currently, less than 75% of the patients who should be treated with RT actually are [3].

Radiation oncology relies on cutting edge technology to provide the best possible treatments. There are different treatment modalities available, taking advantage of technological advances that allow an increased control over the shape and intensity of the radiation. In external RT, radiation is generated by a linear accelerator mounted on a gantry that can rotate around the patient that lays immobilized in a treatment couch that can also rotate. RT systems use a multileaf collimator (MLC) to modulate the radiation beam into a discrete set of small beamlets whose individual intensities can be optimized – fluence map optimization (FMO) problem. In static intensity-modulated RT (IMRT), the nonuniform radiation fields obtained can be delivered while the gantry is halted at given beam irradiation directions that can be optimally selected – beam angle optimization (BAO). In rotational (or arc) IMRT, the patient is continuously irradiated with the treatment beam always on while the gantry is rotating around the patient.

Volumetric modulated arc therapy (VMAT) is a modern IMRT arc technique, particularly efficient with respect to dose delivery time [4]. Typically, VMAT uses beam trajectories that lay in the plane of rotation of the linear accelerator for a $0^\circ$ couch angle (coplanar trajectories). The most recent RT systems allow the simultaneous movement of the gantry and the couch while continuously irradiating the patient. The highly noncoplanar arc trajectories obtained combine the short treatment times of VMAT [4], with the improved organ sparing of noncoplanar IMRT treatment plans [5]. We have proposed an optimization approach composed of two steps for arc trajectory optimization (ATO) in a preliminary work [6]. A set of (seven) optimal noncoplanar beam irradiation directions is initially calculated in a first step. Then, anchored in these beam irradiation directions (anchor points), additional anchor points are iteratively calculated until 21 anchor points are obtained defining the noncoplanar arc trajectory. In this paper, the trade-offs between the computational time needed to find an optimal noncoplanar beam ensemble – fastest if a small number of beams are considered or slowest when more beams are included in the beam orientation optimization – and the overall quality of the treatment plans obtained are investigated. A head-and-neck tumor case treated previously at the Portuguese Institute of Oncology of Coimbra (IPOC) is used to compare VMAT treatment plans considering 5-, 7-, 9-, 11- and 21-beam ensembles in the first step of the proposed approach. The rest of the paper is organized as follows. The head-and-neck cancer case is described in section two. In the following section, the noncoplanar ATO strategy is briefly described. In section four, the computational results are presented. The last section is devoted to the conclusions.

## 2 Head-and-Neck Cancer Case

A complex head-and-neck tumor case already treated at IPOC is used in the computational tests. The planing target volume (PTV) is composed of both the tumor and the lymph nodes. Two different dose prescription levels were considered for each patient. A 70.0 Gy radiation dose was prescribed to the tumor ($PTV_{70}$) while a 59.4 Gy radiation dose was prescribed to the lymph nodes ($PTV_{59.4}$).

Treatment planning of head-and-neck cancer cases is difficult due to the large number of organs-at-risk (OARs) surrounding or even overlapping both the tumor and the lymph nodes. The list of OARs considered in our tests is composed of spinal cord, brainstem, oral cavity, and parotid glands. Spinal cord and brainstem are serial organs, i.e. organs that may see their functionality impaired even if only a small part is damaged while the oral cavity and the parotid glands are parallel organs, i.e. organs whose functionality is not impaired if only a small part is damaged. Thus, maximum-dose constraints are considered for serial organs while mean-dose constraints are considered for parallel organs. The remaining normal tissues, called Body, is also included in the optimization procedures to prevent dose accumulation elsewhere. Table 1 depicts the doses prescribed for the PTVs and the tolerance doses for the OARs included in the optimization.

**Table 1.** Doses prescribed for the PTVs and tolerance doses for the OARs included in the optimization.

| Structure | Tolerance Dose | | Prescribed |
|---|---|---|---|
| | Mean | Max | dose |
| $\mathbf{PTV_{70}}$ | – | – | 70.0 Gy |
| $\mathbf{PTV_{59.4}}$ | – | – | 59.4 Gy |
| **Right parotid** | 26 Gy | – | – |
| **Left parotid** | 26 Gy | – | – |
| **Oral cavity** | 45 Gy | – | – |
| **Spinal cord** | – | 45 Gy | – |
| **Brainstem** | – | 54 Gy | – |
| **Body** | – | 80 Gy | – |

## 3 Arc Trajectory Optimization

The ATO framework evolves in two steps. An optimal noncoplanar beam irradiation ensemble is calculated in the first step, using a previously developed BAO algorithm [7]. Then, anchored in these beam irradiation directions, additional beam directions are iteratively calculated in order to define the trajectory of the noncoplanar arc. The output of the FMO problem is the measure considered to guide both optimization procedures in each of the steps. Aiming at minimizing the possible discrepancies to fully deliverable VMAT plans, direct aperture optimization (DAO) is used in this work for fluence optimization rather than the conventional beamlet-based FMO. The DAO approach used in this work is presented next followed by the description of the two steps that compose the ATO approach.

### 3.1 Fluence Map Optimization – DAO

Direct aperture optimization produces a deliverable plan by calculating aperture shapes instead of beamlet intensities that need to be converted to aperture shapes. The use of DAO during treatment planning can thus decrease possible discrepancies to fully deliverable VMAT plans.

The head-and-neck case considered in this work was assessed in matRad [8], an open source RT treatment planning system written in Matlab. matRad provides an experimental DAO implementation that can be customized by selecting, from a set of options available, objectives, constraints or weights assigned to each structure. matRad uses a gradient-based DAO algorithm [9] that starts with a good initial solution obtained by conventional beamlet-based FMO including sequencing [10].

Considering the appropriate options in matRad, fluence optimization can be formulated as a convex voxel-based nonlinear model [11]:

$$\min_w \left[ \underline{\lambda}_i \left( T_i - \sum_{j=1}^{N} D_{ij} w_j \right)_+^2 + \overline{\lambda}_i \left( \sum_{j=1}^{N} D_{ij} w_j - T_i \right)_+^2 \right]$$

$$s.t. \quad 0 \leq w_j \leq w^{max}, \; j = 1, \ldots, N,$$

where $D_{ij}$ is the dose delivered by beamlet $j$ to voxel $i$, $w_j$ is the intensity of beamlet $j$, $T_i$ is the prescribed/tolerated dose for voxel $i$, $\overline{\lambda}_i$ and $\underline{\lambda}_i$ are overdose and underdose penalties, respectively, $w^{max}$ is the maximum weight (intensity) of a beamlet and $(\cdot)_+ = \max\{0, \cdot\}$. This FMO formulation implies that overdose or underdose may be clinically accepted at reduced levels, but are decreasingly acceptable for increased deviations from the prescribed/tolerated doses [11].

In matRad, FMO is addressed using IPOPT [12], an interior point optimizer solver developed by the COIN-OR initiative.

### 3.2 First Step – BAO

The BAO problem has been formulated by us considering all possible continuous beam irradiation directions rather than a discretized set of irradiation directions around the tumor. Thus, instead of a combinatorial optimization problem we tackle a continuous global optimization problem [7,13,14,15,16,17]. If $n$ is the number of irradiation directions previously defined, $\theta$ denotes a gantry angle and $\phi$ denotes a couch angle, the mathematical formulation of the continuous BAO problem is

$$\begin{aligned} &\min f\Big((\theta_1,\phi_1),\ldots,(\theta_n,\phi_n)\Big) \\ &s.t.\ \Big(\theta_1,\ldots,\theta_n,\phi_1,\ldots,\phi_n\Big) \in \mathbb{R}^{2n}, \end{aligned}$$

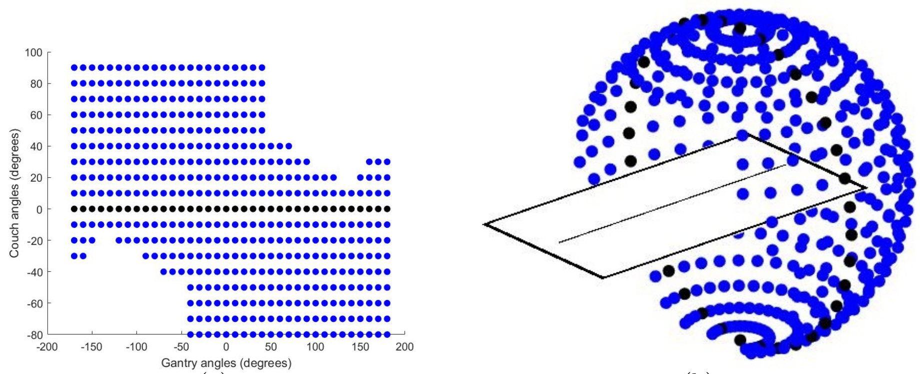
with $f$ an objective function where the best beam angle ensemble is attained at the function's minimum. The optimal FMO value is the measure used as the objective function, $f$, to provide guidance for the BAO procedure. Collision between the linear accelerator gantry and the patient may occur for some pairs of couch and gantry angles. In order to consider only feasible directions while maintaining an unbounded formulation, the following penalization is considered:

$$f\Big((\theta_1,\phi_1),\ldots,(\theta_n,\phi_n)\Big) = \begin{cases} +\infty & \text{if collisions occur} \\ \text{optimal FMO value} & \text{otherwise.} \end{cases}$$

The BAO continuous search space presents a property of symmetry due to the fact that irradiation order is irrelevant. If the beam directions are kept sorted, only a small portion of the entire BAO search space needs to be explored [7]. The continuous BAO formulation is suited for the use of derivative-free optimization algorithms. Pattern search methods (PSM) are a class of derivative-free optimization algorithms that need a reduced number of function evaluations to converge making them an excellent option to address the highly non-convex and time-consuming BAO problem [18,19]. For a thorough description of the complete BAO framework used in this first step see, e.g., Ref. [7].

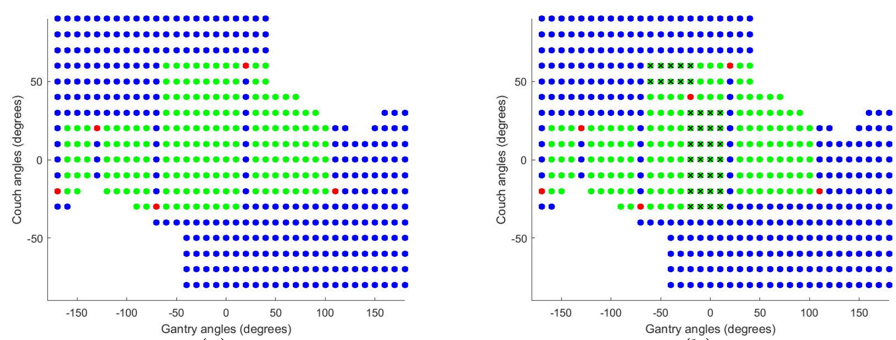
### 3.3 Second Step – ATO

For this second step, a discrete set of beams spaced 10° apart for both the couch and the gantry angles is considered. Fig. 1 displays, both in 2D (Fig. 1(a)) and in 3D (Fig. 1(b)), the resulting candidate beams. Note that pairs of gantry-couch directions that would cause collision between gantry and patient for head-and-neck cancer cases were removed. The ATO strategy proposed is anchored in beam directions (anchor points) calculated in the first step, adding novel anchor points based on optimal FMO values. For illustration purposes, Fig. 2 displays in red the 5-beam ensemble obtained in the first step. ATO iterative procedure starts with these 5 initial anchor points and halts when 21 beam directions are obtained, which corresponds to the number of anchor beams that is typically used to define the arc trajectory [20,21]. The ATO strategy is based on dosimetric considerations, similarly to the BAO approach, being guided by the optimal FMO values. In order to enhance the short delivery times characteristics of VMAT plans, the gantry/couch movements are constrained according to the following conditions:




(a) (b)

**Fig. 1.** Feasible equispaced candidate beams represented in 2D – 1(a) and in 3D – 1(b). Coplanar beams for a fixed 0° couch angle are displayed in black while noncoplanar candidate beams are displayed in blue.




(a) (b)

**Fig. 2.** The 5-beam ensemble, solution of the noncoplanar BAO problem is displayed in red while the possible beams considered in the iterative search of a novel anchor beam are displayed in green – 2(a). New anchor beam calculated for the largest set of candidate green beams is added while candidate green beams that fail the gantry/couch movement restrictions are removed – 2(b).

– The starting position of the couch and the gantry corresponds to the beam with lowest value of gantry angle of the noncoplanar BAO solution, i.e. is the leftmost beam in Fig. 2;
– The anchor beam to visit next has the lowest value of gantry angle among the beams that are yet to be visited;
– The last position of the couch and the gantry corresponds to the beam with highest value of gantry angle of the noncoplanar BAO solution, i.e. is the rightmost beam in Fig. 2;

– The movement of the gantry must always be towards the next beam while the couch can be halted of move towards the next beam.

Fig. 2(a) displays in green the candidates for novel anchor point to add when considering a 5-beam ensemble obtained in the first step as initial anchor points and following the definition of the possible gantry and coach movements. The most populated set is selected for searching the new anchor beam to add to the current arc trajectory for two reasons: to add anchor beams where more degrees of freedom exist and to reduce the computational time by reducing as much as possible the overall number of green points. Iteratively, each candidate beam of the most populated set is temporarily inserted in the trajectory and the optimal FMO value for the corresponding beam ensemble is calculated. The candidate beam that lead to the best beam ensemble in terms of optimal FMO value is selected, the candidate green points that become infeasible due to the gantry/couch movement restrictions are removed and a new iteration can proceed. Figure 2(b) illustrates one iteration of this arc trajectory optimization approach. This iterative procedure ends when the number of anchor beams is 21. The process of obtaining the optimized arc trajectory has been completely automated in order to get the required solution without additional human intervention. The pseudocode of the ATO algorithm is presented in Algorithm 1.

**Algorithm 1** Iterative arc trajectory algorithm

**Initialization:**

– Calculate the initial anchor beams resorting to the noncoplanar BAO algorithm;
– Identify the possible candidate green beams to iteratively calculate new anchor beams;

**Iteration:**

**While** less than 21 anchor beams exist **do**

1. Choose the set of candidate beams between two anchor beams with larger cardinality;
2. Add each beam of the previous set to the current set of anchor beams and calculate the optimal FMO value of the resulting beam ensemble;
3. Select as new anchor point the candidate green beam that corresponds to the beam ensemble with best optimal FMO value in the previous step;
4. The candidate beams that fail the gantry/couch movement restrictions are removed.

## 4 Computational Results

A personal computer with an Intel i7-6700 processor @ 2.60 GHz was used for the computational tests. All tests were performed in a sequential way to

**Table 2.** Optimal FMO results and computational time.

| #BAO | Plan | | | | | BAO | ATO | Total |
|---|---|---|---|---|---|---|---|---|
| beams | $\mathbf{IMRT_5}$ | $\mathbf{IMRT_7}$ | $\mathbf{IMRT_9}$ | $\mathbf{IMRT_{11}}$ | VMAT | time | time | time |
| **0 (Equi)** | 199.4 | 179.9 | 171.7 | 169.6 | 163.4 | – | – | – |
| **5** | **178.7** | 170.4 | 165.9 | 162.2 | 157.7 | 10.3 | 20.7 | 31.0 |
| **7** | – | **166.9** | 165.1 | 162.5 | 158.6 | 21.5 | 10.2 | 31.7 |
| **9** | – | – | **164.4** | 163.1 | 158.3 | 30.4 | 11.9 | 42.3 |
| **11** | – | – | – | **161.6** | 158.1 | 47.1 | 13.5 | 60.6 |
| **21** | – | – | – | – | **158.4** | 121.4 | – | 121.4 |

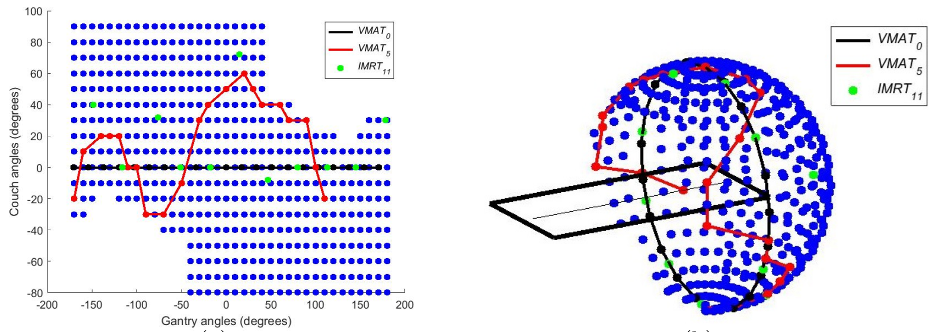
**Table 3.** Tumor coverage achieved by the different plans.

| | $VMAT_0$ | $VMAT_5$ | $IMRT_{11}$ |
|---|---|---|---|
| $PTV_{70}$ $\mathrm{D}_{95}$ | 66.66 | 66.74 | 66.63 |
| $PTV_{59.4}$ $\mathrm{D}_{95}$ | 58.13 | 58.12 | 57.98 |

allow a better comparison of computational times. Nevertheless, all algorithms are implemented in parallel and computational times are typically divided by 12 – the maximum number of cores that our Matlab license allows us to use.

VMAT treatment plans considering 5-, 7-, 9-, 11- and 21-beam ensembles in the first step of the proposed approach were obtained as well as IMRT plans with optimal 5-, 7-, 9- and 11-beam ensembles. Table 2 depicts the optimal FMO value obtained for each of these VMAT and IMRT plans as well as the computational times required for BAO and/or ATO. As expected, the optimal FMO value improves for treatment plans with more beam directions but at the cost of larger computational times required to obtain optimal irradiation directions using BAO (depicted in bold). For instance, computing an optimal 21-beam ensemble using BAO takes twice the time of computing an optimal 11-beam ensemble with only a small gain in terms of optimal FMO value. The optimal FMO value obtained for the VMAT plans following the different strategies – less beam directions in the first step up to all directions in the first step – present only small differences. However, in terms of computational times, the strategies that consider less beams in the first step clearly outperform the other strategies in terms of computational times.

The quality of the treatment plans is also acknowledged by a set of dose metrics. These dose metrics are compared for the noncoplanar VMAT plan with best optimal FMO value (and overall computational time) denoted $VMAT_5$, the coplanar VMAT plan denoted $VMAT_0$ and the best IMRT plan denoted $IMRT_{11}$, whose noncoplanar arc trajectory, coplanar arc trajectory and noncoplanar beams, respectively, are displayed in Fig. 3. Target coverage is one of the metrics typically used for the tumors, i.e. the amount of PTV that receives




(a) (b)

**Fig. 3.** Trajectories obtained by $VMAT_0$, the coplanar VMAT plan, and $VMAT_5$, the noncoplanar VMAT plan with best optimal FMO value (and overall computational time), and noncoplanar beams of $IMRT_{11}$, the best IMRT plan, in 2D – 3(a) and in 3D – 3(b).

**Table 4.** Organ's sparing achieved by the different plans.

| | Mean Dose (Gy) | | | Max Dose (Gy) | | |
|---|---|---|---|---|---|---|
| OAR | $VMAT_0$ | $VMAT_5$ | $IMRT_{11}$ | $VMAT_0$ | $VMAT_5$ | $IMRT_{11}$ |
| Brainstem (BS) | – | – | – | 45.94 | 44.99 | 46.12 |
| Spinal cord (SC) | – | – | – | 36.92 | 35.97 | 36.56 |
| Right parotid (RPrt) | 28.34 | 27.17 | 26.16 | – | – | – |
| Left parotid (LPrt) | 22.57 | 21.39 | 20.85 | – | – | – |
| Oral Cavity (OCav) | 29.42 | 27.72 | 27.65 | – | – | – |

95% of the dose prescribed ($D_{95}$). Typically, 95% to 98% of the PTV volume is required. Table 3 reports the tumor coverage metrics. We can observe that tumor coverage is very similar for the three plans with a small advantage for the *VMAT* plans. OARs sparing results are depicted in Table 4. Results obtained by the different plans fulfill most of the times the tolerance doses with no clear advantage of one approach for all structures. For instance, for serial organs, brainstem and spinal cord, $VMAT_5$ treatment plan obtained the best sparing while for parallel organs, oral cavity and parotids, $IMRT_{11}$ treatment plan obtained the best sparing results. Target coverage and organ sparing for the different structures are displayed in Fig. 4 allowing a more comprehensive view of the results.

## 5 Conclusions

Radiation oncology has seen considerable changes in the last decades, offering an increased range of treatment modalities to cancer patients. Efforts dedicated

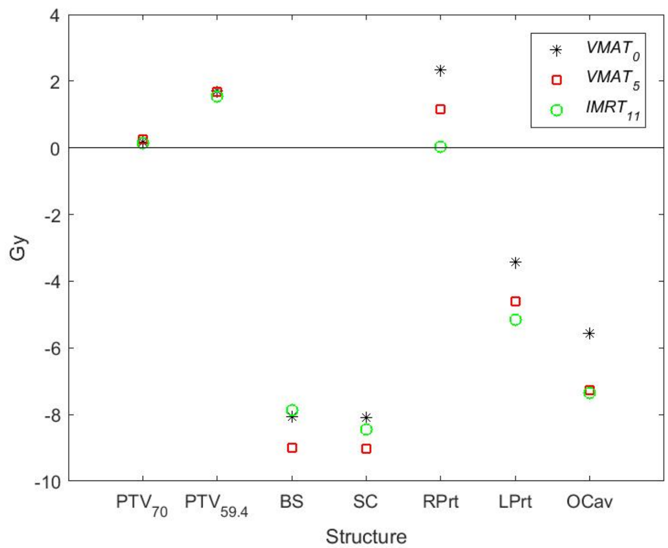



**Fig. 4.** Comparison of tumor coverage and organ sparing metrics obtained by $VMAT_0$, $VMAT_5$ and $IMRT_{11}$ treatment plans. The horizontal line displayed represents the difference between actual dose metric and the prescribed or tolerance (mean or maximum) dose metric for each structure. Target volume values should be above the line while organs values should be under the line.

to beam angle optimization in IMRT enable the conclusion that optimizing the beam directions will always result in plans that are at least as good as the ones planned by a human, but can be much better for some patients in terms of tumor target coverage and normal tissue sparing. Since performing beam angle optimization will not waste any human resources, it can be assessed as being a valuable tool in clinical routine [22]. More recently, VMAT treatment plan optimization for photon therapy has also been researched, namely deciding the best trajectory of arcs [6].

In this work, treatment plans considering 5-, 7-, 9-, 11- and 21-beam ensembles in the first step (BAO) of a two-step ATO approach are compared to assess the trade-offs between treatment quality and computational time required to obtain a noncoplanar VMAT plan. The different strategies tested obtained similar treatment quality as measured by the optimal FMO value. However, strategies that consider less beams in the first step clearly outperform the other strategies in terms of computational times. Generally, results show that for a similar tumor coverage, treatment plans with optimal beam directions obtained an enhanced organ sparing.

As future work, further test should be performed with more patients and considering different tumor sites in order to validate the conclusions withdrawn using a single head-and-neck cancer case.

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